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MICRO COMPREHENSIVE EXAMINATION

Economics 202, 203, and 204

You must turn in this examination no later than 3:00 p.m. today. The examination consists of two parts: select 3 out of the 4 questions of Part A and 2 out of the 4 questions of Part B. All questions receive equal weight. Please use different Blue Books for different questions.

Be as concise and lucid as possible.

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Part A: Answer 3 out of the following 4 questions.

Question One

Consider an oligopoly consisting of a fixed number (n) of identical firms producing a perfectly homogeneous product, and each seeking to maximize profits. The common cost function is given by:

$$C(y_i) = 2y_i \text{ (where } y_i \text{ is the output of firm } i\text{),}$$

while the industry demand function is defined by

$$p = 10 - y,$$

where y is the total amount purchased and p is the product price.

- (a) Find the market outcome if the industry is perfectly competitive.
- (b) Find the outcome if the industry is run as a perfectly collusive monopoly.
- (c) Suppose now that each firm makes its production decisions taking the output levels of others as given. Find the symmetric Nash equilibrium in this situation. How does it vary as the number of firms varies?

What happens as n gets large?

- (d) Finally, consider the case in which each firm makes its production decisions taking the price of other firms as given. Would you expect the outcome to be the same as in part (c)? In particular, is it possible that there might be no Nash equilibrium in this case? Explain your answer.

Question Two

Suppose you have a utility function

$$U(Y) = -\alpha Y^{-\beta} \quad \alpha > 0, \beta > 0$$

and at a given moment of time you have $Y = \$1000$. Being risk averse (are you?) you are cautious and in considering investing this \$1000 you are offered an opportunity to earn the following:

- 10% with probability $1 - P$

+ 10% with probability P

- (a) What is approximately the minimal value of P which will induce you to invest your \$1000?
- (b) Is there any relation between your answer to question (a) and the degree of your risk aversion?
- (c) Suppose α is replaced by (2α) so that your utility function is changed to

$$U(Y) = -(2\alpha)Y^{-\beta}$$

How would you change your answers to questions (a) and (b) above?

Question Three

A monopolist faces the following demand function

$$\text{price} = a_0 - a_1 Q \quad (a_0, a_1 > 0)$$

and operates with the following cost function

$$\text{total cost} = b_0 + b_1 Q + b_2 Q^2 + t Q \quad (b_0, b_1, b_2 > 0)$$

where t is the excise tax rate levied by the government and Q is the amount of the commodity produced and sold.

- (a) Suppose the government, knowing the cost and revenue curves of the monopolist, maximizes the tax revenue from this source. What tax rate should it choose, given the assumptions that the monopolist takes t as exogenous and is a profit maximizer? (Make sure that the tax rate you find maximizes rather than minimizes tax revenue.)

- (b) Now suppose that one part of the (total) cost of collecting these taxes rises proportionately with the monopolist's output and, in addition, that the monopolist and the government's senior civil servants collude to maximize the sum of the monopolist's profits and of net tax receipts. How does this collusion affect the monopolist's output?
- (c) Propose one other model purporting to describe the behavior of the monopolist and the government officials that you would investigate if you sought an accurate prediction of the effect of the excise tax.

Question Four

- (a) Suppose the relationship between a firm's inputs (L_1 and L_2) and its output (X) is described by the following functional form:

$$X = A L_1^{\alpha_1} L_2^{\alpha_2} (L_1 L_2)^{\alpha_3}$$

where A , α_1 , α_2 , and α_3 are fixed parameters. Under what conditions is output a homogeneous function of L_1 and L_2 ? Is the production function
|| separable? What is the value of the elasticity of substitution between L_1 and L_2 ? Provide the reasoning for your answers.

- (b) "Economists should never be concerned with fixed costs because they are irrelevant in the short run and non-existent in the long run."

Explain and evaluate this proposition.

- (c) In Freeman Wills Crofts' Crime at Guildford, the celebrated Inspector French of Scotland Yard learns something of the wage policy of a firm making safes:

"It appeared that the firm were fully alive to the possibility of fraud or theft on the part of their men. For this reason only old hands who had been with them for many years, and of whose honesty they were completely satisfied, were entrusted with the fitting of the keys. These men, moreover, were paid a high rate of wages, so as to reduce temptation as far as possible." (Penguin edition, pp. 56-7.)

Comment briefly on this account. Given the informational problem of ascertaining the honesty of employees, would this relatively high wage rate obtain in a competitive market equilibrium?

Part B: Answer 2 out of the following 4 questions.

Question Five

Last year the Council of Economic Advisors proposed three methods "by which prices may be established for use of air and water: (1) subsidies for pollution abatement [reduction]; (2) charges for emission of pollutants, and (3) sales of transferable environmental usage rights." Under the last system, the government would set up a specific limit on the total amount of pollutants that could be emitted. It would then issue the equivalent amount of certificates, each of which would give the holder the right to emit a unit of pollutants. Such certificates could be sold by the government agency at an auction to polluting industries.

- (a) Compare these three methods in terms of their outcomes from distributional and allocational points of view, and of their informational requirements of management.
- (b) Suppose the general public, including those especially concerned about the environment (such as conservation groups) can participate in the last scheme's market for certificates by buying certificates and throwing them away. What effect would this have on the efficiency and information requirements of the third scheme?

Question Six

Consider two economies I and II that have identical technologies represented by an aggregate production function $F(K,L)$ (without technological change) that satisfy all the conditions to ensure the existence of a balanced growth path.

Both economies are in a state of balanced growth but they do differ in two respects: the growth rates of their population and their savings ratio.

In economy I: The growth rate of population is 2%.

In economy II: The growth rate of population is 3%.

In economy I: The observed savings ratio is .1

In economy II: The observed savings ratio is .12

Equipped with this information, can you evaluate the following comparisons of the two economies in states of balanced growth:

- (a) Which economy has a higher growth rate of output and capital? Why?
- (b) Which economy has a higher wage rate? Why?
- (c) In which economy is the interest rate higher? Why?

Q/A

- (d) If you knew that in economy II total savings were equal to total capital return, what can you say about the interest rate in that economy?
- (e) Supposing you were considering the same conditions as above except that in addition, a process of Harrod Neutral technological change took place, thus the production function was $F(K, A(t)L)$. Which of the two economies has a higher growth rate and which has a higher interest rate?

Question Seven

Consider a consumer with a utility function $u(y)$ defined over his income y . Suppose the individual is considering a bet in which he risks his entire income y against the possibility of gaining a small amount h . The maximal probability p of ruin which he will be willing to accept would have to be very small in order for him to be indifferent between such a bet and simply retaining his income y and not taking the bet. Moreover, the more unwilling he is to risk ruin, the smaller p will be. For this reason we think of p as an inverse measure of the individual aversion to risking ruin and thus $\left(\frac{1}{p}\right)$ can be thought of as the Fear of Ruin.

- (a) Using a first-order approximation, show that for small h the ratio

$$\frac{p}{h} \text{ is approximately } \frac{u'(y)}{u(y)}$$

$$[\text{more precisely } \lim_{h \rightarrow 0} \frac{p}{h} = \frac{u'(y)}{u(y)}]$$

- (b) Suppose $u(y) = Ay^{.8}$. Assume that the individual has income $y = \$200$. What is the maximal probability of losing this entire \$200 which he will accept in return for gaining \$1? What if his income is \$400? \$800?
- (c) Most people have an increasing fear of ruin as their income increases simply due to the fact that they have more to lose. However, can an individual simultaneously
- (i) have an increasing fear of ruin (increasing with income),
 - (ii) not be risk averse?

Can you give an example and/or prove or explain your answer?

- (d) Similarly, can an individual simultaneously
- (i) have a decreasing fear of ruin,
 - (ii) be risk averse?

Can you give an example and/or prove or explain your answer?

Question Eight

Suppose a competitive firm transforms a single input (z) into two outputs (q_1 and q_2) according to a well-behaved, fully differentiable, production function.

- (a) What are the first-order and second-order conditions determining the profit maximizing levels of q_1 and q_2 ?
- (b) A tax is now introduced on each unit of q_1 sold. How will this tax change (1) the firm's demand for the input z , (2) the supply of the taxed commodity q_1 , and (3) the supply of the untaxed commodity q_2 ?
- (c) Now suppose that all competitive firms jointly producing q_1 and q_2 are subject to this tax on q_1 . On the assumption that q_1 and q_2 are independent in demand, derive an expression for the effect of this tax on the equilibrium price of q_1 . What is the sign of this expression?
(By "independent in demand," we mean that the demand for q_1 depends upon p_1 but not upon p_2 and the demand for q_2 depends upon p_2 but not upon p_1 .)
- (d) Let each and every firm's production function be $z = Aq_1^\alpha q_2^\beta$ where $A > 0$, $\alpha > 0$, $\beta > 0$, and $\alpha + \beta < 1$ are fixed parameters. With this information, evaluate the expression derived in part (c) of this question.