# Corrections and updates for third printing of Osborne and Rubinstein's "A Course in Game Theory" (MIT Press, 1994)

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### Corrections

Page,	Line	Correction
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$J^{(j)}$		
xiii, 4	Solutions to the exercises are now available at https://www.economi	
	utoronto.ca/osborne/cgt/.	
xiii, $-4$	Replace "that" with "than".	
XV	Martin J. Osborne's email address is now martin.osborne@utoronto.	
	ca and his mailing address is Department of Economics, University of	
	Toronto, 150 St. George Street, Toronto, Canada, M5S 3G7.	
XV	Ariel Rubinstein's email address is now rariel@tauex.tau.ac.il, his	
	website is https://arielrubinstein.tau.ac.il, and his second ad-	
	dress is Department of Economics, New York University, New York, NY	
	10003, USA.	
XV	The authors now maintain a web site for the book: https://www.	
	economics.utoronto.ca/osborne/cgt/.	
6	Zermelo did not define the algorithm discussed in the second paragraph;	
	the references to him should be deleted.	
7, -13	Insert "nonempty" before "disjoint".	
13	In the caption of Figure 13.1, replace "strategies" by "actions".	
23, 14	Replace the sentence "Note that" with "Note that by part (c), the	
	players' Nash equilibrium strategies may be found by solving the prob-	
	lems $\max_x \min_y u_1(x, y)$ and $\max_y \min_x u_2(x, y)$ ."	
30	Add, at the start of line 5, "The result in Exercise 20.4 is due to	
	Nash (1951)."	
32	Replace the first two sentences of the paragraph above (32.2) with: "Note	
	that each function $U_i$ is multilinear. That is, for any mixed strategy	
	profile $\alpha$ , any mixed strategies $\beta_i$ and $\gamma_i$ of player <i>i</i> , and any number $\lambda \in$	
	[0,1], we have $U_i(\alpha_{-i}, \lambda\beta_i + (1-\lambda)\gamma_i) = \lambda U_i(\alpha_{-i}, \beta_i) + (1-\lambda)U_i(\alpha_{-i}, \gamma_i)$ ."	

- 40, 6 Replace  $((\frac{1}{3}, \frac{2}{3}), (\frac{2}{3}, \frac{1}{3}))$  with  $((\frac{2}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3}))$ .
- 42 In line 3 of Exercise 42.1 replace "distribution" with "distributed".
- 45, 7 Replace "he" with "she".
- $50,\,-2 \qquad \text{Replace ``}0 \leq \gamma \leq 1 " \text{ with ``}0 < \gamma \leq 1 ".$
- 55, -10 Replace " $\mu_3^2(A)$ " with " $\mu_3^2(B)$ ".
- 56, -9 Replace "the following exercises" with "Exercises 56.4 and 56.5".
- 60, 22 Replace "player 1" with "player i".
- 60, 23 Replace " $U_i(a_{-i}, a_i^*)$ " with " $u_i(a_{-i}, a_i^*)$ ".
- 62, 2 Delete second "that".
- 68, -2 Add "infinite" before "decimal". (A number has a unique infinite decimal expansion.)
- 90, -20 Replace "If the length of every history is finite" with "If the longest history is finite".

98 In the display in Lemma 98.2 replace  $\succeq_i$  with  $\succeq_i|_h$ .

- 99, 3 Replace "the longest" with "a longest".
- 99, 4 Replace  $\Gamma(h^*)$  with  $\Gamma(h', h^*)$  on this line and on lines 6, 8, and 10.
- 99, 5 Replace  $s^*|_{h'}$  with  $s^*_i|_{h'}$ .
- 99, 9 Replace  $s_i^*|_{h^*}$  with  $s_i^*|_{(h',h^*)}$ .
- 100, 7–8 Delete "(a result first proved by Zermelo (1913))".
- 100, 9 The official version of chess is not finite, because a player has the *option* of declaring a draw once a position is repeated three times. Our argument applies to the version of chess in which a draw is automatic in this case.
- 104, -9, -8 Replace "she" with "he".
  - 107 Replace the first clause of the sentence starting on line 8 with "Player P(C(t)) prefers S(t+3) to S(t+1) to S(t+2) for  $t \le T-3$ ".
  - 108 On the fifth line of Exercise 108.1 insert "strategic form of the" before "modification".
  - 108, -2 Delete "only".
  - 114, -2 Replace "Kuhn (1953)" with "Kuhn (1950, 1953)".
  - 115, 2 Replace "Kuhn (1953)" with "Kuhn (1950, 1953)".
  - 122 Add to A3 the requirement that the Pareto frontier of X be connected.
  - 123 In the first display  $(M_i(G_i))$  replace "a SPE" with "an SPE".
  - 123 Replace the second sentence of the proof of Step 1 with "By A3 and the continuity of the preference relations, the domain of  $\phi$  is an interval and  $\phi$  is continuous, one-to-one, and decreasing."
  - 131, 3 Omit "the".
  - 131, 4 Change the parenthetical clause to read "(in which he makes the same proposal whenever he is the proposer, uses the same rule to accept proposals whenever he is the first responder, and uses the same rule to accept proposals whenever he is the second responder)".

- 137 Replace the first point of the itemization in Definition 137.1 by
  - $H = \{\emptyset\} \cup (\bigcup_{t=1}^{\infty} A^t) \cup A^{\infty}$  (where  $\emptyset$  is the initial history and  $A^{\infty}$  is the set of infinite sequences  $(a^t)_{t=1}^{\infty}$  of action profiles in G)

and in the third point replace "the set  $A^{\infty}$  of infinite sequences  $(a^t)_{t=0}^{\infty}$  of action profiles in G" by " $A^{\infty}$ ". Remove the period at the end of the display in the third point.

- 137, -3 Change the summation to  $\sum_{t=1}^{\infty} \delta^{t-1} v_i^t$ .
- 138, 1 Remove  $\lim_{T\to\infty}$ .
- 138, 1 (except printings 1–3) Change upper limit of sum from T to  $\infty$ .
- 138, 9 Replace t with T (twice).
- 143, -6 Replace "A payoff profile w in G" with "A feasible payoff profile w of G".
- 143, -6 In printings 4 and later replace "A payoff profile w" with "A feasible payoff profile w of G". (Note that, according to our definitions, a feasible payoff profile may not be a payoff profile.)
- 144 In Proposition 144.1, replace "an enforceable payoff profile of G" with "an enforceable convex combination of payoff profiles of G". [The coefficients in the convex combination are not necessarily rational.]
- 145, 6 Insert "other than i" before "deviated".
- 145 The statement of Proposition 145.2 is correct, but is improved by changing the second sentence to read: "For all  $\varepsilon > 0$  there exists  $\underline{\delta} < 1$  such that for all  $\delta > \underline{\delta}$  there exists a payoff profile w' of G for which  $|w'-w| < \varepsilon$ and w' is a Nash equilibrium payoff profile of the  $\delta$ -discounted infinitely repeated game of G."
- 152, 11 Change "it" to "if".
- 156, 3 Replace the sentence starting "Consider" and the following sentence with "Consider a strategy  $\hat{s}_i$  of player *i* that differs from  $s_i$  only in that after the history  $(a^1, \ldots, a^{t-1})$  it chooses  $a_i$ , and after any longer history *h* it chooses an action that, given the profile  $s_{-i}(h)$  of actions planned by the other players after the history *h*, yields at least *i*'s minmax payoff. The outcome of  $(s_{-i}, \hat{s}_i)$  is a terminal history  $\hat{a}$  that is identical to *a* through period t - 1;  $u_i(\hat{a}^t) > u_i(a^t)$  and  $u_i(\hat{a}^r) \ge u_i(a^r)$  for  $r \ge t + 1$ ."
- 159–160 The sketch of the proof of Proposition 160.1 is flawed. It has been replaced. (The text has been rewritten, moving the result to page 159, where it appears as Proposition 159.1.)
- 171, -3 Change  $\ell \leq t^*$  to  $\ell < t^*$ .
- 185, 5 After the colon modify the text to read: "for example, if  $\gamma = 2$ , |N| = 2, and  $\theta_i = 1$  for both players then the associated game has also, in addition to (1, 1), inefficient equilibria (e.g. (0, 0))".
- 200, -17 Delete "by a player".

- 200, -11 Replace "after the history h" with "after the history h if  $P(h) \in N$  and chance if P(h) = c."
  - 201 Replace the paragraph starting on line -2 with: "Note that Definition 200.1 extends our definition of an extensive game with perfect information and chance moves (see Section 6.3.1) in the sense that the extensive game with perfect information and chance moves  $\langle N, H, P, f_c, (\succeq_i)_{i \in N} \rangle$  may naturally be identified with the extensive game  $\langle N, H, P, f_c, (\mathcal{I}_i)_{i \in N}, (\succeq_i)_{i \in N} \rangle$  in which every member of the information partition of every player is a singleton."
- 215, 5 Replace "the sets of actions" with "the sequences of actions".
- 216, -6 Replace the mathematical expression with  $p^2 \cdot 0 + p \cdot (1-p) \cdot 1 + (1-p) \cdot 0 = p(1-p)$ .
- 217, -2 Replace "Kuhn (1953)" with "Kuhn (1950, 1953)".
- 218, 3 Replace "Kuhn (1953)" with "Kuhn (1950, 1953)".
- 221, -2 L at the end of the line should be R.
- 226, 1 Replace from the start of the line to the end of the sentence with "equilibrium since the associated assessment violates sequential rationality at player 2's (singleton) information set."
- 226, 5 Replace  $\beta_2^{\varepsilon}(C)(c) = 1 2\varepsilon$  with  $\beta_2^{\varepsilon}(C)(d) = 2\varepsilon/(1-\varepsilon)$ .
- 229 In the fifth line of Exercise 229.1, replace "two" with "three".

229 In the sixth line of Exercise 229.1, replace "S" with "s".

- 236, 7 Replace the sentence starting "The following example ...." with: "The following example shows, however, that there are games in which no perfect Bayesian equilibrium satisfies the requirement: in all perfect Bayesian equilibria of the game we describe, a player who at some point assigns probability zero to some history later assigns positive probability to this history."
- 237 After "0." on line 9 of Exercise 237.1, add "Take the set of possible offers to be finite, including 2 and 5."
- 241, -8 Delete "if".
  - 252 At the end of Example 252.1 the perturbed strategy of player 3 should be  $\sigma_3^{\varepsilon}(R) = \sigma_3(R)$  if  $\sigma_3(R) < 1$ , and  $\sigma_3^{\varepsilon}(R) = 1 - \varepsilon$  if  $\sigma_3(R) = 1$ .
  - 260 After "for all coalitions S and T" on the last line add ", where  $v(\emptyset) = 0$ ".
  - 261 In Exercise 261.1b, insert "nonnegative" before "feasible payoff profiles".
  - 263 On line 12, insert "for all  $S \in C$ " before "by the left-hand inequality", and replace the last sentence of this paragraph with "Thus x(N) = v(N), so that the payoff profile x is in the core."

- 274 Add, between lines 7 and 8, the following paragraph. "Now, for every agent *i* we have  $x_i \omega_i + \varepsilon \in Q$  for every  $\varepsilon > 0$ , so that  $p(x_i \omega_i + \varepsilon) \ge 0$ . Taking  $\varepsilon$  small, we conclude that  $px_i \ge p\omega_i$  for all *i*. But *x* is an allocation, so  $px_i = p\omega_i$  for all *i*."
- 274, -4 Replace "Shubik (1969)" with "Shubik (1969a)".
- 275, 13 Replace "from Moulin (1986, p. 237)" with "from Shapley and Shubik (1969b)".
- 281, -2 Replace "member" with "members".
- 287, 7 Replace  $\geq$  at end of line with  $\leq$ .
- 289 Replace the last sentence with the following two sentences. "Consider a zerosum homogeneous weighted majority game  $\langle N, v \rangle$  in which  $w_i = 0$  for every player *i* who does not belong to any minimal winning coalition. Show that the nucleolus of  $\langle N, v \rangle$  consists of the imputation *x* defined by  $x_i = w_i/w(N)$  for all  $i \in N$ ."
- 307, 7–8 Replace  $\succeq'_1, \succeq_2$  with  $\succeq'_i, \succeq_j$  (twice).
- 307, 15 Replace  $p \cdot x \succeq_j x^*$  with  $p \cdot x \succ_j x^*$ .
- 327 Replace "Recherce" with "Recherche" on line -10.
- 335, 3 Replace "[85]" with "[84]".
- 335 Add A. Tversky to the list of editors of the book in which Rubinstein (1995) appears.
- 336, 3 Replace "[115]" with "[114]".
- 337, 16 Replace "Shapley, L. S., and M. Shubik (1969)" with "Shapley, L. S., and M. Shubik (1969a)".
  - 339 Delete reference to Zermelo (1913).
  - 344 Entry for "Dominant action" should be Exercise 18.3.

#### Additional References

The modifications on pages 114, 115, 217, and 218 require the following additional reference.

Kuhn, H. W. (1950), "Extensive Games", Proceedings of the National Academy of Sciences of the United States of America 36, 570–576. [114, 115, 217, 218]

The modification on page 275 requires the following additional reference.

Shapley, L. S., and M. Shubik (1969b), "On the Core of an Economic System with Externalities", American Economic Review 59, 678– 684. [275]

### Updates

Battigalli (1993) is now

Battigalli, P. (1996), "Strategic Independence and Perfect Bayesian Equilibria", Journal of Economic Theory, 70, 201-234.

Glazer and Perry (1992) is now

Glazer, J. and M. Perry (1996), "Virtual Implementation in Backwards Induction", *Games and Economic Behavior* **15**, 27–32.

Hart and Mas-Colell (1992) is now

Hart, S. and A. Mas-Colell (1996), "Bargaining and Value", *Econometrica* 64, 357–380.

Hendon, Jacobsen, and Sloth (1993) is now

Hendon, E., H. J. Jacobsen, and B. Sloth (1996), "The One-Shot-Deviation Principle for Sequential Rationality" Games and Economic Behavior 12, 274–282.

Kohlberg and Reny (1993) is now

Kohlberg, E., and P. J. Reny (1997), "Independence on Relative Probability Spaces and Consistent Assessments in Game Trees", *Journal* of Economic Theory 75, 280–313.

Shaked (1987) is now

Shaked, A. (1994), "Opting Out: Bazaars versus 'Hi Tech' Markets", Investigaciones Económicas 18, 421–432.

Thompson (1952) is reprinted in *Classics in Game Theory* (H. W. Kuhn, ed.), Princeton University Press, 1997, pp. 36–45.