**Given name:**____________________ **Family name:**____________________

**Student number:**____________________ **Signature:**____________________________

**UNIVERSITY OF TORONTO**
**Faculty of Arts and Science**

**ECO 326H1S Section L0101 (Advanced Economic Theory—Micro)**
Instructor: Martin J. Osborne

**MIDTERM EXAMINATION**
**October 2008**

**Duration: 1 hour 50 minutes**

**No aids allowed**

This examination paper consists of **16** pages and **5** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

**TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS.**

For graders' use:

| | Score |
|---|---|
| 1 (20) | |
| 2 (20) | |
| 3 (20) | |
| 4 (20) | |
| 5 (20) | |
| Total (100) | |

1. Consider the following strategic game with vNM preferences. Throughout the question, consider mixed strategies as well as pure strategies.

| | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| $T$ | $1, 2$ | $4, 0$ | $2, 1$ |
| $M$ | $4, 0$ | $1, 2$ | $2, 1$ |
| $B$ | $2, 4$ | $2, 4$ | $1, 1$ |

(a) [4] Find the actions of each player, if any, that are strictly dominated.

(b) [4] Find the actions of each player, if any, that are weakly dominated but not strictly dominated.

*Question continues on next page*

(c) [12] Find all the Nash equilibria of the game.

2. [20] Two people compete for a prize. Each person $i$ chooses her effort level $e_i$, a nonnegative number. Person 1 is more skilled than person 2. Precisely, her probability of winning when the pair of efforts is $(e_1, e_2)$ is

$$\frac{\theta e_1}{\theta e_1 + e_2}$$

and person 2's probability of winning is

$$\frac{e_2}{\theta e_1 + e_2},$$

where $\theta > 1$. Each person values the prize at $V$. Person 1's payoff is

$$\frac{\theta e_1}{\theta e_1 + e_2} V - e_1$$

and person 2's payoff is

$$\frac{e_2}{\theta e_1 + e_2} V - e_2.$$

Does the strategic game that models this situation have a Nash equilibrium in which $e_1 = e_2$? If so, find all such equilibria; if not, argue why no such equilibrium exists.

*Space for answer continues on next page*

3. [20] Consider an example of Cournot's model of oligopoly in which there are two firms. As for the main example in the book, the inverse demand function is linear where it is positive: $P(Q) = \alpha - Q$ for $Q \leq \alpha$, and $P(Q) = 0$ for $Q > \alpha$. The cost function of each firm is linear, but unlike the example in the book, the unit costs of the firms differ: firm 1's cost function is $C_1(q_1) = c_1 q_1$ and firm 2's cost function is $C_2(q_2) = c_2 q_2$, where $c_1 < \alpha$ and $c_2 < \alpha$.

   Find the Nash equilibrium (equilibria?) of Cournot's game for this example *under the assumption that* $c_2 < 2c_1 - \alpha$. (You do **not** need to consider any other case.)

*Space for answer continues on next page*

4. In an *average-price* sealed-bid auction, the player whose bid is highest wins and pays the *average* of the highest two bids. As in the models in the book, if there is a tie for the highest bid, the winner is the one with the lowest index among those submitting the highest bid. The remaining assumptions are also the same as for the models in the book; in particular, the valuations are ordered $v_1 > v_2 > \cdots > v_n$.

   (a) [5] Does such an auction have a Nash equilibrium in which the price is $v_2$?

   (b) [5] Does such an auction have a Nash equilibrium in which the price is $v_1$?

*Question continues on next page*

(c) [5] Does such an auction have a Nash equilibrium in which some player bids more than $v_1$?

*Question continues on next page*

(d) [5] Is a bid of more than $v_i$ by player $i$ weakly dominated in such an auction?

5. Two players play the following game. Each chooses a number from the set $\{0, 1, \ldots, K\}$. These choices are independent. If the sum of the two numbers is odd, then player 2 pays player 1 that sum in dollars. If the sum is even, then player 1 pays player 2 the sum in dollars. Thus, for example, if player 1 chooses the number 3 and player 2 chooses the number 2, then player 2 pays 5 dollars to player 1. Assume that each player's von Neumann–Morgenstern payoff is equal to her own change in wealth and model the situation as a strategic game.

   (a) [3] Suppose that $K = 1$. Does the game have a pure strategy Nash equilibrium?

*Question continues on next page*

(b) [7] Find all mixed strategy Nash equilibria when $K = 1$.

*Question continues on next page*

(c) [10] Now suppose that $K = 2$. Does this game have a mixed strategy Nash equilibrium in which both players mix only between actions 0 and 1?

*Space for answer continues on next page*

**You may use the next two pages for rough work.**

**For rough work (will not be graded)**

**End of examination**
**Total pages: 16**
**Total marks: 100**