Given name: ____________________ **Family name:** ____________________

Student number: ____________________ **Signature:** __________________________

UNIVERSITY OF TORONTO
Faculty of Arts and Science

ECO 326H1S Section L0101 (Advanced Economic Theory—Micro)
Instructor: Martin J. Osborne

MIDTERM EXAMINATION
February 2006

Duration: 1 hour 50 minutes

No aids allowed

This examination paper consists of **6** pages and **5** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS.

For graders' use:

	Score
1 (18)	
2 (20)	
3 (20)	
4 (20)	
5 (22)	
Total (100)	

1. Determine whether each of the following statements is true or false and provide a reason for your answer. (Reasons are required for credit!)

(a) [6] For a two player strategic game in which each player has two actions, neither player's action in any (pure strategy) Nash equilibrium is strictly dominated.

Circle one: True False

Reason:

Solution: True: If the action a of player i strictly dominates her action a^*, then in particular a is not a best response to *any* of the other player's actions, and hence is not an equilibrium action.

(b) [6] For a two player strategic game in which each player has two actions, neither player's action in any (pure strategy) Nash equilibrium is weakly dominated.

Circle one: True False

Reason:

Solution: False: consider, for example, the game in Figure 1, in which B weakly dominates A for each player, but (A, A) is a Nash equilibrium.

	A	B
A	$0, 0$	$0, 0$
B	$0, 0$	$1, 1$

Figure 1.

(c) [6] Consider a strategic game G that has two players, two actions for each player, and a unique (pure strategy) Nash equilibrium. Suppose that the strategic game G' differs from G only in the players' payoffs; all payoffs in G' are larger than the corresponding payoffs in G. Claim: it is possible that G' has a unique (pure strategy) Nash equilibrium in which both players are *worse off* than they are in the equilibrium of G.

Circle one: True False

Reason:

Solution: The question has two interpretations. Under one interpretation, the preferences of each player in G and G' are the same, so that the Nash equilibrium of G' is the same as the Nash equilibrium of G (and thus it is not possible for both players to be worse off in the equilibrium of G' than they are in the equilibrium of G).

Under the second interpretation, the players' preferences are not restricted to be the same in G and G'. In this case it is possible that both players are worse off in the equilibrium of G' than they are in the equilibrium of G: the action pair (T, L) is the unique Nash equilibrium of the game in the left panel of Figure 2; the action pair (B, R) is the unique Nash equilibrium of the game in the right panel, in which each payoff is greater than the corresponding payoff in the game in the left panel.

	L	R
T	$5,5$	$2,0$
B	$0,2$	$1,1$

	L	R
T	$6,6$	$3,7$
B	$7,2$	$4,4$

Figure 2.

2. [20] Two players can contribute to a public good. If the pair of contributions is (c_1, c_2) then player i's payoff, for $i = 1, 2$, is $C - C^2 - (c_i)^2$, where $C = c_1 + c_2$. Each player's contribution can be any nonnegative number. Find the Nash equilibrium (equilibria?) of the strategic game that models this situation.

   Solution: Player 1 chooses c_1 to maximize

   $$c_1 + c_2 - (c_1 + c_2)^2 - (c_1)^2$$

   given c_2. That is, player 1 chooses c_1 to maximize

   $$c_1 + c_2 - 2c_1c_2 - (c_2)^2 - 2(c_1)^2.$$

   The solution satisfies

   $$1 - 2c_2 - 4c_1 = 0,$$

   so that player 1's best response function is given by

   $$b_1(c_2) = \tfrac{1}{4}(1 - 2c_2).$$

   Symmetrically, player 2's best response function is given by

   $$b_2(c_1) = \tfrac{1}{4}(1 - 2c_1).$$

   An equilibrium (c_1^*, c_2^*) satisfies

   $$\begin{aligned} c_1^* &= b_1(c_2^*) \\ c_2^* &= b_2(c_1^*). \end{aligned}$$

   Solving this system we find that

   $$c_1^* = c_1^* = \tfrac{1}{6}.$$

   Thus the game has a unique Nash equilibrium, $(c_1^*, c_2^*) = (\frac{1}{6}, \frac{1}{6})$.

3. Suppose that two firms that produce the same perfectly divisible good compete in a market. The total demand for the good when the price is p is $13 - p$ (so that the price when the total amount sold is Q is $13 - Q$). Each firm i can produce any amount $q_i \leq 5$ at the cost q_i; neither firm can produce more than 5. (That is, the technology of each firm differs from the one considered in the book because each firm has limited capacity; each firm's unit cost is 1 up to its capacity.)

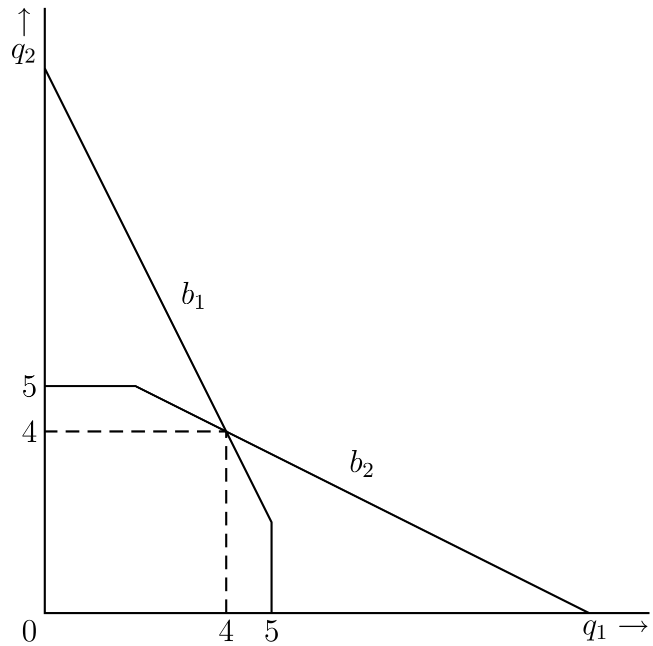
(a) Consider the strategic game in which the firms simultaneously choose outputs, and the price is $13 - Q$ when the firms' total output is Q.

i. [9] Find the firms' best response functions and illustrate them in a diagram, ***using the axes on the next page***.

Solution: The payoff of firm 1 is

$$q_1(13 - q_1 - q_2) - q_1$$

and the firm is restricted to choose $q_1 \leq 5$. This payoff is maximized at $q_1 = 6 - \frac{1}{2}q_2$ if this output is at most 5; otherwise the payoff-maximizing output is 5. (Draw a diagram of the firm's payoff.)



ii. [2] Find the Nash equilibrium (equilibria?) of this game.

Solution: In the absence of the capacity constraint, each firm produces 4 units in the unique Nash equilibrium. The capacity constraints cause the best response functions to be truncated at the output 5, which does not affect their point of intersection. Thus the unique Nash equilibrium of the game is $(4, 4)$, as when no capacity constraints exist.

(b) [9] Consider the strategic game in which the firms simultaneously choose prices. Assume that if the prices are the same, the total demand is split equally between the firms. Assume also that if (a) the prices differ, (b) the total demand at the lower price exceeds 5, and (c) the higher price is less than 13, then the firm with the higher price faces positive demand. Is the pair of prices $(1, 1)$ a Nash equilibrium?

Circle one: Yes No

Reason (required for credit!):

Solution: No: for this pair of prices each firm's profit is 0, whereas if one of the firms raises its price a little, it obtains a positive profit.

4. [20] Consider a variant of Hotelling's model of electoral competition in which there are three candidates and *no candidate has the option of withdrawing from the competition.* (That is, each candidate's only choice is which position to take.) As in Hotelling's model, a candidate prefers to win than to tie for first place than to lose.

   Assume that the set of possible positions is the interval from 0 to 1 and that the distribution of the citizens' favorite positions is uniform. (That is, for any x with $0 \leq x \leq 1$, the fraction of the citizens' favorite positions less than x is x.)

   Either find a (pure strategy) Nash equilibrium of the strategic game that models this situation *or* show that the game has no Nash equilibria.

   Solution: The game has Nash equilibria. An example is the action profile in which one candidate chooses $\frac{1}{3}$, one chooses $\frac{2}{3}$, and one chooses $\frac{3}{4}$. The candidate at $\frac{1}{3}$ wins outright and hence has no deviation that increases her payoff. If either of the other candidates changes her position, either the outcome does not change or the third candidate becomes the outright winner.

5. Each of two players attaches the value v to an object. The players simultaneously bid for the object. Each player has only *two* possible bids, h and l, with $h > l$. If the bids differ, the object is assigned to the highest bidder; if the bids are the same, the object is assigned to each bidder with probability $\frac{1}{2}$. *Both* players, regardless of whether they win or lose the auction, pay an amount of money equal to the *smaller* bid. In addition, each bidder pays a fixed participation cost c. The payoff of a player who obtains the object with probability π and pays the price p is $\pi v - p - c$.

   (a) [4] Model this situation as a strategic game. (A figure is sufficient.)

   Solution:

	h	l
h	$\frac{1}{2}v - h - c, \frac{1}{2}v - h - c$	$v - l - c, -l - c$
l	$-l - c, v - l - c$	$\frac{1}{2}v - l - c, \frac{1}{2}v - l - c$

   (b) [10] Assume $\frac{1}{2}v < h - l$. Find all Nash equilibria in pure and mixed strategies of the game.

   Solution: The strategy profiles (h, l) and (l, h) are pure strategy Nash equilibria. Player i prefers l to h when player j chooses k and prefers h to l when j chooses l, so in any mixed strategy equilibrium both players choose each action with positive probability. Let π be the probability that j assigns to h. Then the expected payoff to player i from h is given by

$$\pi(\tfrac{1}{2}v - h - c) + (1 - \pi)(v - l - c)$$

and the expected payoff to player i from l is given by

$$\pi(-l-c)+(1-\pi)(\tfrac{1}{2}v-l-c).$$

Equating these payoffs and solving for π yields $p=\frac{1}{2}v/(h-l)$. The strategy pair in which both players choose h with this probability is the only mixed strategy Nash equilibrium.

(c) [8] Now assume that each player has the option of not participating in the auction. When a player chooses to not participate her payoff is 0 regardless of the action taken by the other player. For each equilibrium you found in part (b), determine the values of c for which the equilibrium remains an equilibrium when the bidders can choose not to participate.

Solution: The pure strategy Nash equilibria in (b) are no longer equilibria because the player who loses the auction receives a negative payoff $(-l-c)$ and prefers not to participate and receive a payoff of 0.

The mixed strategy equilibrium in (b) remains an equilibrium if the equilibrium expected payoff of each player is positive. This payoff is

$$\pi^*(-l-c)+(1-\pi^*)(\tfrac{1}{2}v-l-c)$$

where π^* is the equilibrium probability assigned to h by each player. This payoff is equal to

$$-l-c+\tfrac{1}{2}v(1-\pi^*).$$

Thus the mixed strategy Nash equilibrium in (b) remains an equilibrium if and only if

$$c\leq -l+\tfrac{1}{2}v(1-\pi^*)=-l+\tfrac{1}{2}v\left(1-\tfrac{1}{2}v/(h-l)\right).$$

End of examination
Total pages: 6
Total marks: 100