**Given name:**____________________ **Family name:**____________________

**Student number:**____________________ **Signature:**____________________

**UNIVERSITY OF TORONTO**
**Faculty of Arts and Science**

**ECO 326H1S Section L0101 (Advanced Economic Theory—Micro)**
Instructor: Martin J. Osborne

**MIDTERM EXAMINATION**
**October 2006**

**Duration: 1 hour 50 minutes**

**No aids allowed**

This examination paper consists of **6** pages and **5** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

**TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS.**

1. (a) [10] Find all the Nash equilibria, in pure and mixed strategies, of the following strategic game.

| | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| $T$ | $1,3$ | $4,2$ | $3,1$ |
| $M$ | $2,2$ | $1,3$ | $0,2$ |
| $B$ | $0,0$ | $1,1$ | $2,4$ |

**Solution:** Player 1's action $B$ is strictly dominated, so the Nash equilibria of the game are the same as the Nash equilibria of the game

| | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| $T$ | $1,3$ | $4,2$ | $3,1$ |
| $M$ | $2,2$ | $1,3$ | $0,2$ |

In this game player 2's action $Z$ is strictly dominated, so the Nash equilibria are the same as the Nash equilibria of the game

| | $X$ | $Y$ |
|---|---|---|
| $T$ | $1,3$ | $4,2$ |
| $M$ | $2,2$ | $1,3$ |

This game has a unique Nash equilibrium, in mixed strategies: $((\frac{1}{2},\frac{1}{2}),(\frac{3}{4},\frac{1}{4}))$. Thus the unique Nash equilibrium of the original game is $((\frac{1}{2},\frac{1}{2},0),(\frac{3}{4},\frac{1}{4},0))$.

(b) [10] Find all the Nash equilibria of the strategic game that differs from the game in part (a) *only* in that player 1's payoff to $(M, X)$ is 1 rather than 2.

**Solution:** As for the game in part (a), we can eliminate the actions $B$ and $Z$. The resulting game is

| | $X$ | $Y$ |
|---|---|---|
| $T$ | $1,3$ | $4,2$ |
| $M$ | $1,2$ | $1,3$ |

The players' best response functions are shown in Figure 1. We see that the pair $((p, 1-p), (q, 1-q))$ is a mixed strategy Nash equilibrium if and only if $\frac{1}{2} \le p \le 1$ and $q = 1$.

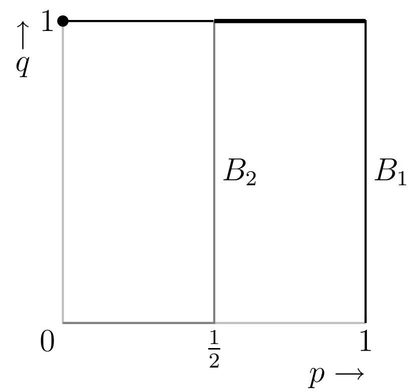


**Figure 1**.

2. Two people have to select one of three alternatives, $A$, $B$, and $C$. Their preferences between the alternatives may differ; neither player is indifferent between any two alternatives. The following method is used to select an alternative: each person submits a *ranking* of the alternatives and the alternative for which the *sum* of the submitted ranks is *smallest* is selected. Each person may submit any ranking she wishes as long as it is strict (i.e. does not contain any ties). (For example, it is possible to rank $A$ first (1), $B$ second (2), and $C$ third (3).) If two or more alternatives are tied for the smallest total ranking, the tied alternative whose name is closest to the start of the alphabet is selected. (For example, if player 1 submits the ranking $ABC$ and player 2 submits the ranking $BAC$ then the total rankings of $A$ and $B$ are equal and $A$ is selected.)

(a) [5] In a model of this situation as a strategic game, what is the set of actions of each player?

**Solution:** Each player's set of actions consists of the six possible rankings of the three alternatives, $ABC$, $ACB$, $BAC$, $BCA$, $CAB$, and $CBA$.

(b) [5] Suppose that player 1 prefers $A$ to $B$ to $C$. Is any action of player 1 weakly dominated?

Circle one: Yes No

Reason (required for credit!):

**Solution:** Yes: if fact, all rankings except $ABC$ and $ACB$ are weakly dominated by $ABC$. For example, player 1's switching from $CAB$ to $ABC$ either does not affect the outcome or causes it to change from $C$ to either $A$ or $B$, depending on player 2's action. (In addition, $CAB$ is weakly dominated by $ACB$, $CBA$ is weakly dominated by $BAC$ and $ACB$, and $BCA$ is weakly dominated by $BAC$.)

The outcome as a function of the actions of the players is given as follows:

| | $ABC$ | $ACB$ | $BAC$ | $BCA$ | $CAB$ | $CBA$ |
|---|---|---|---|---|---|---|
| $ABC$ | $A$ | $A$ | $A$ | $B$ | $A$ | $A$ |
| $ACB$ | $A$ | $A$ | $A$ | $A$ | $A$ | $C$ |
| $BAC$ | $A$ | $A$ | $B$ | $B$ | $A$ | $B$ |
| $BCA$ | $B$ | $A$ | $B$ | $B$ | $C$ | $B$ |
| $CAB$ | $A$ | $A$ | $A$ | $C$ | $C$ | $C$ |
| $CBA$ | $A$ | $C$ | $B$ | $B$ | $C$ | $C$ |

(c) [5] Are there any preferences for the players such that the action pair in which each player submits a ranking equal to her true preferences is not a Nash equilibrium?

Circle one: Yes No

Reason (required for credit!):

**Solution:** Yes. Suppose player 1 prefers $A$ to $B$ to $C$ and submits the ranking $ABC$. Suppose that player 2 prefers $C$ to $B$ to $A$. Then if she submits the ranking $CBA$ the outcome is $A$, whereas if she submits the ranking $BCA$ the outcome is $B$, which is better for player 2.

(d) [5] Find a Nash equilibrium of the game in the case that player 1's true ranking is $ABC$ and player 2's true ranking is $BAC$.

**Solution:** The Nash equilibria are $(ACB, ABC)$, $(ACB, ACB)$, $(CAB, ACB)$, $(ACB, BAC)$, $(CAB, BAC)$, $(ACB, BCA)$, and $(ACB, CAB)$. The outcome in each case is that $A$ is chosen. (Note: the question asks you only to find one Nash equilibrium.)

3. [20] Consider the example of Cournot's model of duopoly (two firms) in which the inverse demand function is linear, given by

$$P(Q) = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha, \end{cases}$$

the cost function of firm 1 is $C_1(q_1) = cq_1$, and the cost function of firm 2 is $C_2(q_2) = q_2^2$. Assume that $c < (3/4)\alpha$.

Find the Nash equilibrium (equilibria?) of the strategic game. (Carefully present the steps in your argument. Throughout your analysis, ignore the case in which the total output is so high that the price is zero—assume that the price is always given by $P(Q) = \alpha - Q$.)

**Solution:** First find the firms' best response functions.

Firm 1's payoff function, and hence its best response function, is the same as one derived in class. Thus its best response function is

$$b_1(q_2) = \tfrac{1}{2}(\alpha - c - q_2)$$

where this function is nonnegative.

Firm 2's payoff function is

$$\pi_2(q_1, q_2) = q_2(\alpha - q_1 - q_2) - q_2^2 = q_2(\alpha - q_1 - 2q_2),$$

a quadratic that is zero when $q_2 = 0$ and when $q_2 = \frac{1}{2}(\alpha - q_1)$. Thus firm 2's best response function is

$$b_2(q_1) = \tfrac{1}{4}(\alpha - q_1).$$

A Nash equilibrium is a pair $(q_1^*, q_2^*)$ such that

$$q_1^* = b_1(q_2^*) \text{ and } q_2^* = b_2(q_1^*),$$

or

$$q_1^* = \tfrac{1}{2}(\alpha - c - q_2^*) \text{ and } q_2^* = \tfrac{1}{4}(\alpha - q_1^*).$$

Solving these two equations simultaneously we obtain

$$(q_1^*, q_2^*) = (\tfrac{3}{7}\alpha - \tfrac{4}{7}c, \tfrac{1}{7}\alpha + \tfrac{1}{7}c).$$

(We have $q_1^* > 0$ because $c < \frac{3}{4}\alpha$.)

4. Consider a model that differs from Hotelling's two-candidate model of electoral competition *only* in the candidates' preferences. Assume that each candidate cares not about winning *per se*, but about the position of the winner. Denote candidate $i$'s favorite position by $x_i^*$ for $i = 1, 2$. Then, for example, candidate $i$ prefers the outcome in which the other candidate adopts the position $x_i^*$ and wins to one in which $i$ adopts a position different from $x_i^*$ and wins.

   Suppose that $x_1^* < m$ and $x_2^* > m$, where $m$ is the median of the voters' favorite positions. Suppose that for each candidate, the further a position is from her favorite position, the less she likes it. Finally, suppose that if the candidates tie, then the outcome is the compromise given by the average of their positions.

   For each of the pairs of positions in parts (a) and (b), determine whether there are any preferences for the citizens under which the pair is a Nash equilibrium of the strategic game.

   (a) [10] $(x_1^*, x_2^*)$

**Solution:** Suppose that the distribution of the citizens' preferred positions is continuous. (I intended this condition to hold, though it is not explicitly stated in the question.)
There are three cases. Suppose that when the positions are $(x_1^*, x_2^*)$, candidate 1 wins outright. Then candidate 2 can induce an outcome that she prefers by choosing the position $m$ rather than $x_2^*$, in which case she wins and the outcome is $m$ rather than $x_1^*$. Thus $(x_1^*, x_2^*)$ is not a Nash equilibrium. Similarly, if candidate 2 wins outright when the candidates' positions are $(x_1^*, x_2^*)$ then candidate 1 can induce an outcome she prefers by deviating to $m$. Finally, if the candidates tie when the positions are $(x_1^*, x_2^*)$ then either candidate can induce an outcome she prefers by deviating to a position slightly closer to $m$, which induces that outcome rather than the compromise $\frac{1}{2}(x_1^* + x_2^*)$. Thus in no case is $(x_1^*, x_2^*)$ a Nash equilibrium.
If the distribution of the citizens' preferences is not continuous, it is possible that $(x_1^*, x_2^*)$ is a Nash equilibrium. For example, if $x_1^*$ is the favorite position of half the citizens and $x_2^*$ is the favorite position of the remaining citizens, then $(x_1^*, x_2^*)$ is a Nash equilibrium. (If you specified this distribution and gave a complete argument, you obtained full marks.)

(b) [10] $(m, m)$

**Solution:** If either candidate deviates from $m$ then she loses, so that the outcome does not change. Thus $(m, m)$ is a Nash equilibrium.

5. Each of three people chooses a positive integer (1, 2, 3, ...). If all three people choose the *same* integer, each person's payoff is $\frac{1}{3}$. If all three choose different integers, the payoff of the player who chooses the *smallest* integer is 1 and the payoffs of the others are 0. If two people choose the same integer and the third person chooses a different integer, the third person's payoff is 1 and the others' payoffs are 0.

(One interpretation is that an integer is a way of dressing. Everyone wants to dress differently from everyone else, and, in situations where people dress differently, everyone wants to be as "cool" as possible (cool = small integer).)

(a) [4] Find all the pure strategy Nash equilibria of the strategic game that models this situation.

**Solution:** An action profile is a Nash equilibrium if and only if either (i) exactly one player chooses 1 or (ii) exactly two players choose 1.

(b) [16] Does the game have a ***symmetric*** mixed strategy Nash equilibrium in which each player assigns positive probability only to 1 and 2? If so, find such an equilibrium. If not, argue why no such equilibrium exists.

Circle one: Yes No

Reason (required for credit!):

**Solution:** No, the game does not have such an equilibrium. Suppose that two of the players assign probability $p$ to 1 and probability $1 - p$ to 2. Then if the remaining player chooses 2 her payoff is 1 with probability $p^2$, $\frac{1}{3}$ with probability $(1-p)^2$, and 0 with the remaining probability. If she chooses 3 then her payoff is 1 with probability $p^2 + (1-p)^2$ and 0 with the remaining probability. Thus her expected payoff to 3 exceeds her expected payoff to 2, so that any best response to the other players' strategies assigns probability 0 to 2.

(There *is* a value of $p$ (namely $\frac{1}{2}$) such that the payoffs of a player to the actions 1 and 2 are equal when each of the other players chooses 1 with probability $p$ and 2 with probability $1-p$, but for this value of $p$ (and in fact for any value of $p$) the player's expected payoff to the action 3 exceeds her expected payoff to the action 2.)

**End of examination**
**Total pages: 6**
**Total marks: 100**