Given name:____________________ Family name:____________________

Student number:____________________ Signature:___________________________

UNIVERSITY OF TORONTO
Faculty of Arts and Science

PLEASE HAND IN

DECEMBER EXAMINATIONS 2006

ECO326H1F Sections L0101 and L5101 (Advanced Economic Theory—Micro)
Instructor: Martin J. Osborne and Ettore Damiano

Duration: 3 hours

No aids allowed

This examination paper consists of **22** pages and **7** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Please check the box corresponding to the section in which you are registered:

- ☐ **Section L0101 (Osborne): answer questions 1–6.**
- ☐ **Section L5101 (Damiano): answer questions 1–5 and 7.**

TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS. You may use the last 3 pages of the exam for rough work.

For graders' use:

	Score
1 (17)	
2 (18)	
3 (17)	
Subtotal	

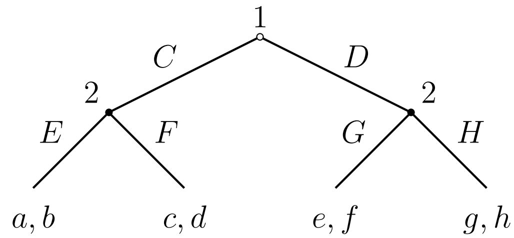
	Score
4 (10)	
5 (16)	
6 (16)	
7 (16)	
Subtotal	

Total (100)	

1. (a) [8] Either give an example of a strategic game with the following properties, or argue that no such game exists.
   - The game has exactly one Nash equilibrium.
   - If one action of one of the players is deleted, the resulting game has exactly one Nash equilibrium, and each player is *better off* in this equilibrium than in the equilibrium of the original game.

Question continues on next page

(b) [9] Consider an extensive game with the structure shown in the figure below. Assume that neither player is indifferent between any two terminal histories. Are there any values for the payoffs such that the game has a Nash equilibrium in which both players are better off than they are in any subgame perfect equilibrium?



Circle one: Yes No

Reason (required for credit!):

2. Consider the following two-player strategic game. Assume that the four numbers a_1, b_1, c_1, and d_1 are all different, and the four numbers a_2, b_2, c_2, d_2 are all different. Further assume that $a_1 > c_1$.

	A	B
A	a_1, a_2	b_1, b_2
B	c_1, c_2	d_1, d_2

(a) [4] Find conditions on the payoffs for which the game has a unique mixed strategy Nash equilibrium and in this equilibrium each player assigns positive probability to each of her actions.

Question continues on next page

(b) Suppose that a_1 decreases slightly, but remains larger than c_1. All other payoffs remain the same. How does this change affect

i. [4] the equilibrium probability player 1 assigns to A?

ii. [4] the equilibrium probability player 2 assigns to A?

Question continues on next page

iii. [6] the equilibrium payoff of player 1?

3. Two players use the following variant of the ultimatum game to split \$$c$. Player 1 starts by naming an amount \$$p$, which can be any nonnegative number. Then player 2 either takes \$$p$ from player 1, or gives \$$p$ to player 1. If player 2 chooses to take \$$p$ from player 1, the two players proceed to divide the \$$c$ by playing an ultimatum game in which player 1 is the proposer. If instead player 2 decides to give \$$p$ to player 1, the two player proceed to divide the \$$c$ by playing an ultimatum game in which player 2 is the proposer. Both players care about the total amount of money they have at the end of the game. (Suppose, for example, that player 1 names \$$p$, player 2 takes this amount, and player 1 then offers y to player 2. In this case, if player 2 accepts y then player 1's payoff is $c - y - p$ and player 2's payoff is $y + p$, and if player 2 rejects y then player 1's payoff is $-p$ and player 2's payoff is p.)

   (a) [5] Model this scenario as an extensive game with perfect information. (A diagram is sufficient.)

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(b) [12] Find all subgame perfect equilibria of the game.

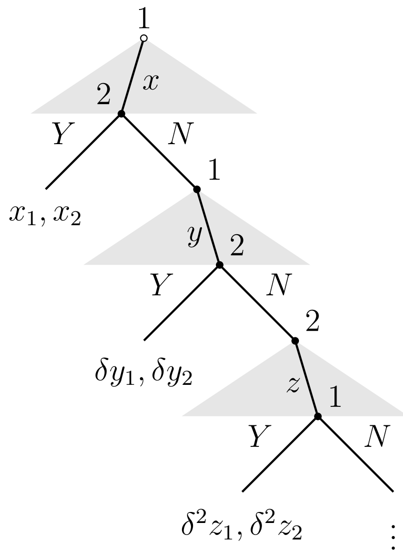

Figure 1. The first three periods of the variant of the bargaining game of alternating offers in Problem 4.

4. Consider the following variant of the bargaining game of alternating offers. The size of the pie is $1 and both players discount future payoffs with the discount factor $0 < \delta < 1$. As in the original bargaining game of alternating offers, first player 1 makes an offer, which player 2 can accept or reject. But then if player 2 rejects player 1's offer, *player 1* makes *another* offer, which player 2 can again accept or reject. If player 2 rejects this second offer of player 1, she makes a counteroffer, which player 1 can accept or reject. If player 1 rejects the counteroffer, then play continues as if we were at the start of the game: player 1 gets the opportunity to make offers for two periods before player 2 can make a counteroffer. The first three periods of the game are shown in Figure 1.

   A strategy profile is a subgame perfect equilibrium of this game if and only if it satisfies the one deviation property. Find the subgame perfect equilibrium (equilibria?) of the game.

   (a) [5] Player 1 always proposes
   $$\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$$
   and accepts any offer $(y, 1-y)$ in which $y \geq \delta/(1+\delta)$, and player 2 always proposes
   $$\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right)$$
   and accepts any offer $(x, 1-x)$ in which $1 - x \geq \delta/(1+\delta)$.

Space for answer continues on next page

Question continues on next page

(b) [5] Refer to a proposal of player 1 at the start of the game or after player 1 has rejected an offer of player 2 as a "first round" offer, and to a proposal of player 1 in a period following a rejection by player 2 of a first round offer as a "second round" offer.

The strategy pair to consider is:

- Player 1's first round offer is always

$$(\frac{1-\delta^2}{1-\delta^3}, \frac{\delta^2(1-\delta)}{1-\delta^3}),$$

her second round offer is always

$$(1 - \frac{\delta(1-\delta)}{1-\delta^3}, \frac{\delta(1-\delta)}{1-\delta^3}),$$

and she accepts every offer $(y, 1-y)$ of player 2 in which $y \geq \frac{\delta(1-\delta^2)}{1-\delta^3}$.

- Player 2 always proposes

$$(\frac{\delta(1-\delta^2)}{1-\delta^3}, \frac{1-\delta}{1-\delta^3}),$$

accepts any first round offer $(x, 1-x)$ of player 1 for which $1-x \geq \frac{\delta^2(1-\delta)}{1-\delta^3}$, and accepts any second round offer $(z, 1-z)$ of player 1 for which $1-z \geq \frac{\delta(1-\delta)}{1-\delta^3}$.

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5. Two people have to choose the type of computer operating system to use. There are two options, W and M. Each person is one of two possible types: type w, who has a preference for W, all other things equal, and type m, who has a preference for M. Each person's payoff depends mainly on the operating systems chosen by the *other* person, but depends partly on her own preference. Specifically, a person who chooses Y obtains the payoff

$$0.1U + 0.9V,$$

where U is 1 if the operating system Y matches the person's type (W for w and M for m) and 0 otherwise, and V is 1 if the operating system Y matches the *other* person's *action* and 0 otherwise.

Each person knows her own type but not the type of any other person. She believes that the type of any other given person is W with probability α and M with probability $1-\alpha$ (where $0 \leq \alpha \leq 1$).

(a) [5] In the Bayesian game that models this situation, what is the set of states and what is the signal function of each person?

Question continues on next page

(b) [11] Find the Nash equilibria of the Bayesian game in which the action chosen by type w of player 1 is the same as the action chosen by type w of player 2 and the action chosen by type m of player 1 is the same as the action chosen by type m of player 2. (The character of the set of such equilibria depends on the value of α.)

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For students in Section L0101 (Osborne) only.
DO NOT ANSWER THIS QUESTION IF YOU ARE IN SECTION L5101 (Damiano)!

6. [16] Consider Bertrand's duopoly game when the demand function is given by $D(p) = \alpha - p$ if $p \leq \alpha$, and $D(p) = 0$ for $p > \alpha$, and the cost function of each firm i is given by $C_i(q_i) = f + cq_i$ for $q_i > 0$, and $C_i(0) = 0$, where f is positive and less than the maximum of $(p - c)(\alpha - p)$ with respect to p. Assume that firm 1 gets all the demand when both firms charge the same price. Either find a Nash equilibrium of the game or show that no Nash equilibrium exists.

For students in Section L5101 (Damiano) only.
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7. A university professor must decide whether or not to write a letter of recommendation for a student who is applying for graduate school. The professor knows the student's ability while the graduate school admission committee only knows that there is a 50% chance the student is good and a 50% chance the student is outsdanding. If the professor does not write a letter of recommendation, the student will not be admitted to graduate school and the payoffs to both the professor and the admission committee are 0. If the professor writes the letter of recommendation, the admission committee must decide whether to accept or reject the student's application. The admission committee receives a payoff of 1 from admitting an outstanding student and a payoff of -2 from admitting a good student. The payoff from rejecting a student is 0, regardless of the student's quality. The professor incurs a cost of $c > 0$ from writing a letter of recommendation and receives a reward of $h > 0$ if an outstanding student is accepted, a reward of $g > 0$ if a good student is accepted, and no reward if the student is not accepted. The professor's payoff is the difference between the reward received and the cost of writing a letter.

   (a) [5] Model this scenario as an extensive game with imperfect information.

Question continues on next page

(b) [11] Assume that $h > g > c$. Find all weak sequential equilibria in which the professor always writes a letter of recommendation if the student is outstanding.

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You may use the next three pages for rough work.

For rough work (will not be graded)

For rough work (will not be graded)

End of examination
Total pages: 22
Total marks: 100