Given name:_____ **Family name:**_____

Student number:_____ **Signature:**_____

UNIVERSITY OF TORONTO
Faculty of Arts and Science

ECO 326 H Section L0101 (Advanced Economic Theory—Micro)
Instructor: Martin J. Osborne

MIDTERM EXAMINATION
February 2005

Duration: 1 hour 50 minutes

No aids allowed

This examination paper consists of **16** pages and **6** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS.

For graders' use:

	Score
1 (8)	
2 (18)	
3 (18)	
4 (18)	
5 (18)	
6 (20)	
Total (100)	

1. Person 1 has an apple and person 2 has an orange. Each person can either retain her piece of fruit or give it to the other person. Person 1 does not like apples. She is indifferent between having no fruit and having an apple, and between having an orange and having both an orange and an apple, and prefers having an orange to having an apple. Person 2's preferences differ from person 1's only in that the roles of apples and oranges are interchanged.

   (a) [3] Model this situation as a strategic game.

Question continues on next page

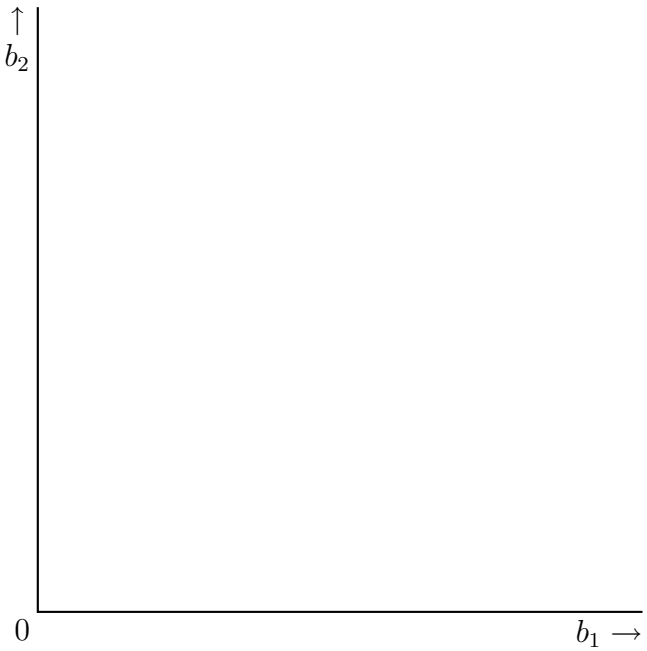
(b) [3] Find the Nash equilibrium (equilibria?) in pure strategies.

(c) [2] Is any action of either player weakly dominated?

2. [18] Two people simultaneously contribute to a public good. The cost of providing the good is c. Each person chooses how much to contribute (a nonnegative number). The good is provided if and only if the sum of the contributions is at least c. If the sum of the contributions is less than c then the contributions are refunded. For $i = 1$, 2, person i's payoff is $v_i - b_i$ if the good is provided and 0 otherwise, where b_i is her contribution and $v_i \geq 0$ is a fixed number. Assume that $v_1 + v_2 \geq c$.

   Find the Nash equilibria of the strategic game that models this situation *and show these equilibria in a diagram using the axes on the following page.*

Space for answer continues on next page


↑
b_2
0
$b_1 \rightarrow$

3. [18] Find the Nash equilibrium (equilibria?) of Cournot's oligopoly game when there are two firms, the inverse demand function is given by

$$P(Q) = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha \end{cases}$$

and the cost function of each firm i is given by $C_i(q_i) = q_i^2$.

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4. [18] Consider a variant of Hotelling's model of electoral competition in which a candidate needs to obtain at least $\frac{2}{3}$ of the votes to win, and one option for each candidate is to stay out of the race.

   Assume there are two candidates and that each candidate regards the outcome when neither candidate obtains $\frac{2}{3}$ of the votes as equivalent to her losing. Each candidate prefers to win than to stay out of the race than to lose. [A tie for first place is not possible.]

   Find the set of Nash equilibria of the strategic game that models this situation.

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5. Consider the strategic game in Figure 1. Assume that $0 < \pi < 1$ and $v_A > \pi v_B > 0$.

	A	B
A	$0, 0$	$v_A, -v_A$
B	$v_B, -v_B$	$\pi v_B, -\pi v_B$

Figure 1. The game in Question 5.

(a) [9] Find the mixed strategy Nash equilibria of the game.

Question continues on next page

(b) [9] Suppose that player 1 has an additional action, C, that yields her the payoff h and yields player 2 the payoff 0, regardless of player 2's action. For each value of h with $0 \leq h < v_B$ find a mixed strategy Nash equilibrium of this new game. [*Note that you are asked only to find one equilibrium for each value of h, not all equilibria.*]

6. Consider a variant of the crime-reporting model in which there are 3 witnesses, 2 of whom incur the cost c_1 to report the crime and attach the value v_1 to the police being notified, and 1 of whom incurs the cost c_2 to report the crime and attaches the value v_2 to the police being notified, where $c_1 > 0$, $c_2 > 0$, $v_1 > 0$, and $v_2 > 0$.

   (a) [12] Find conditions on c_1, c_2, v_1, and v_2 for which the game has a mixed strategy Nash equilibrium in which every witness's strategy assigns positive probabilities to both reporting and not reporting. Express the equilibrium probability of each witness reporting the crime as a function of c_1, c_2, v_1, and v_2.

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Question continues on next page

(b) [8] Find conditions on c_1, c_2, v_1, and v_2 for which the game has a mixed strategy Nash equilibrium in which each witness with cost c_1 and value v_1 assigns positive probabilities to both reporting and not reporting whereas the witness with cost c_2 and value v_2 does not report (i.e. reports with probability 0).

Space for rough work (will not be graded)

Space for rough work (will not be graded)

End of examination
Total pages: 16
Total marks: 100