Given name:____________ Family name:____________

Student number:____________ Signature:____________

UNIVERSITY OF TORONTO
Faculty of Arts and Science

PLEASE HAND IN

APRIL/MAY EXAMINATIONS 2005

ECO326H1S Section L0101 (Advanced Economic Theory—Micro)
Instructor: Martin J. Osborne

Duration: 3 hours

No aids allowed

This examination paper consists of **6** pages and **7** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

all

TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS. You may use the last 4 pages of the exam for rough work.

For graders' use:

	Score
1 (12)	
2 (13)	
3 (15)	
Subtotal	

	Score
4 (15)	
5 (15)	
6 (15)	
7 (15)	
Subtotal	

Total (100)	

1. [12] Each of $n \geq 3$ people announces an integer from the set $\{1, \ldots, K\}$. The person whose integer is closest to the average of the announced integers wins \$1. If there is a tie for the integer closest to the average, \$1 is split equally between the people whose

integer is closest to the average. Find all the pure strategy equilibria of the strategic game that models this situation.

Solution: An action profile $(a_1, \ldots, a_n)$ is a Nash equilibrium if and only if $a_1 = a_2 = \cdots = a_n$. (That is, the game has K Nash equilibria, $(1, \ldots, 1)$, $\ldots$, $(K, \ldots, K)$.)

Any such action profile is a Nash equilibrium because any deviation leads the deviating player to lose rather than share the dollar.

In any other action profile at least one player loses and any such player can deviate to the average of the other players' actions and obtain at least a share of the dollar. Thus no other action profile is a Nash equilibrium.

2. [13] Find the range of values of a, b, c, and d, if any, for which the mixed strategy pair $((0, \frac{1}{3}, \frac{2}{3}), (\frac{1}{4}, 0, \frac{3}{4}))$ is a mixed strategy Nash equilibrium of the following strategic game.

		Player 2		
		L	C	R
	T	a, b	$2, 2$	$1, 3$
Player 1	M	$6, c$	$3, d$	$0, 1$
	B	$0, 1$	$4, 3$	$2, 0$

Solution: For the strategy pair to be a mixed strategy Nash equilibrium we need player 1's expected payoffs to M and B to be equal, and to be at least her expected payoff to T, given player 2's mixed strategy. Her expected payoffs to M and B are the same, equal to $\frac{3}{2}$, given player 2's mixed strategy, so we need only

$$\tfrac{1}{4}a + \tfrac{3}{4} \leq \tfrac{3}{2}$$

or $a \leq 3$. We need also player 2's expected payoffs to L and R to be equal, and to be at least her expected payoff to C, given player 1's mixed strategy. These conditions are equivalent to

$$\tfrac{1}{3}c + \tfrac{2}{3} = \tfrac{1}{3} \geq \tfrac{1}{3}d + 2$$

or $c = -1$ and $d \leq -5$.

In summary, the strategy pair is a mixed strategy Nash equilibrium if and only if $a \leq 3$, $c = -1$, and $d \leq -5$. (b may take any value.)

3. A third-price auction with perfect information is a variant of a second-price auction with perfect information in which the price paid by the winner (the player who submits the highest bid) is the third highest of the bids submitted. [That is, $n \geq 3$ players simultaneously submit bids for a single indivisible object. Player i's valuation of the object is v_i, where $v_1 > v_2 > \cdots > v_n$. The highest bid wins; in the event of a tie, the player whose index is smallest wins. (E.g. if players 1 and 2 tie for the highest bid, player 1 wins.)]

Denote by G the strategic game that models this situation.

(a) [7] Either find a (pure strategy) Nash equilibrium of G in which the winner is player 1 and the price is less than v_2 (the second-highest valuation) or show that G has no such equilibrium.

Solution: Any action profile $(b_1, \ldots, b_n)$ with the following properties is such an equilibrium:

- the winning bid is b_1
- the second-highest bid is at least v_2 and is not submitted by player 2
- the third-highest bid is less than v_2 and at least v_j, where j is the player who submits the second-highest bid.

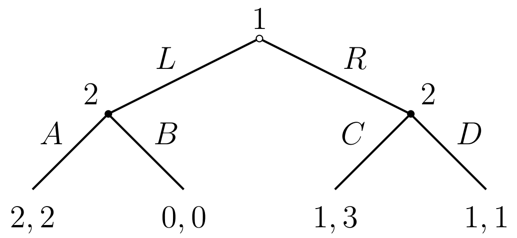
In such an action profile, player 1 wins and pays less than v_2. Denote by p^* the price player 1 pays. If the player who submits the second-highest bid changes her bid then either the outcome does not change or, if her bid exceeds b_1, she wins and pays the price p^*, which is at least her valuation (by the third condition). If any other player deviates either the outcome does not change or, if the deviant's bid exceeds b_1, the deviant wins and pays a price equal to the original second-highest bid, which is at least v_2 and hence at least equal to the deviant's valuation.

(Note that you are asked only to find *one* equilibrium. An example of an action profile that satisfies the conditions is $(b_1, \ldots, b_n) = (v_1, v_n, v_3, v_2, v_n, \ldots, v_n)$.)

(b) [8] Either find a (pure strategy) Nash equilibrium of G in which the winner is player n (who has the lowest valuation) or show that G has no such equilibrium.

Solution: Consider an action profile in which the winner is player n. Player n's bid b_n must be the highest, and the third-highest bid must be at most v_n, otherwise player n's payoff is negative so that she can do better by bidding 0. But now consider a deviation by the player submitting the second-highest bid. If she bids more than b_n then she wins and the price she pays is at most v_n, so her payoff increases. Hence no such action profile is a Nash equilibrium.

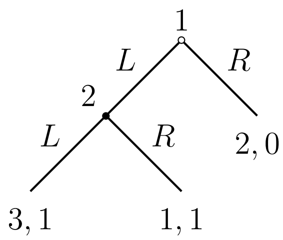
4. (a) [7] Consider the extensive game with perfect information shown below. Does this game have a Nash equilibrium for which the outcome differs from the outcome of any subgame perfect equilibrium?



Solution: The game has a unique subgame perfect equilibrium, (L, AC), with the outcome (L, A). It has a Nash equilibrium with a different outcome: (R, BC), with the outcome (R, C).

(b) [8] Is it possible for an extensive game with perfect information and no simultaneous moves to have two subgame perfect equilibria s and s' with the property that every player prefers the outcome of s to the outcome of s'? Either give an example of such a game or prove that no such game exists.

Solution: The game in the following figure has two subgame perfect equilibria, (L, L) and (R, R), with the payoffs $(3, 1)$ and $(2, 0)$.



5. [15] Consider a variant of Bertrand's duopoly game in which firm 2 is in the industry but firm 1 chooses whether or not to enter. The total demand at the price p is $D(p) = \alpha - p$ if $p \leq \alpha$ and zero otherwise. Each firm's unit cost is constant, equal to $c < \alpha$.

First firm 1 first chooses whether to enter or not. If it enters, both firms simultaneously choose prices and their payoffs are as in Bertrand's original model, except that firm 1 pays the entry cost $f > 0$. If firm 1 does not enter, its output is zero and firm 2 chooses a price p_2; firm 1's payoff is 0 (it does not pay the entry cost) and firm 2's payoff is $(p_2 - c)D(p_2)$.

Find the subgame perfect equilibrium (equilibria?) of the extensive game that models this situation. *Be sure to specify the equilibrium strategies fully.* Do not specify only the equilibrium outcome!

Solution: The unique Nash equilibrium of the subgame that follows the challenger's entry is (c, c), by an argument like that given in class. The challenger's profit is $-f < 0$ in this equilibrium. By choosing to stay out the challenger obtains the profit of 0, so in any subgame perfect equilibrium the challenger stays out. After the history in which the challenger stays out, the incumbent chooses its price p_2 to maximize its profit $(p_2 - c)(\alpha - p_2)$.

Thus for any value of $f > 0$ the whole game has a unique subgame perfect equilibrium, in which the strategies are:

Challenger

- at the start of the game: stay out
- after the history in which the challenger enters: choose the price c

Incumbent

- after the history in which the challenger enters: choose the price c
- after the history in which the challenger stays out: choose the price p_2 that maximizes $(p_2 - c)(\alpha - p_2)$ (namely $(\alpha + c)/2$).

6. [15] Describe the unique subgame perfect equilibrium of the bargaining game of alternating offers in which each player's discount factor is δ and show that it is indeed a subgame perfect equilibrium. You may use the fact that a strategy pair in the bargaining game of alternating offers is a subgame perfect equilibrium if and only if neither player can increase her payoff by changing her action at the start of any subgame in which she is the first-mover, *given* the other player's strategies *and* the rest of her own strategy. *Be sure to give a complete argument!*

Solution: Denote by s^* the following strategy:

- player 1 always proposes x^* and accepts a proposal y if and only if $y_1 \geq y_1^*$
- player 2 always proposes y^* and accepts a proposal x if and only if $x_2 \geq x_2^*$,

where

$$\begin{aligned} x^* &= \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right) \\ y^* &= \left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right). \end{aligned}$$

I argue that s^* is a subgame perfect equilibrium of the bargaining game of alternating offers. The game has two types of subgame: one in which the first move is an offer, and one in which the first move is a response to an offer.

First consider a subgame in which the first move is an offer. Suppose the offer is made by player 1, and fix player 2's strategy to be s_2^*. If player 1 uses the strategy s_1^*, her payoff is x_1^*. If she deviates from s_1^* in the first period of the subgame, she is worse off by the following arguments.

- If she offers player 2 more than x_2^* in the first period, then player 2 accepts her proposal, and her payoff is less than x_1^*.
- If she offers player 2 less than x_2^* in the first period, then player 2 rejects her proposal and proposes (y_1^*, y_2^*). Player 1 accepts this proposal, obtaining the payoff δy_1^*, which is less than x_1^*.

A symmetric argument shows that player 2 cannot profitably deviate in the first period of a subgame that starts with her making an offer.

Now consider a subgame in which the first move is a response to an offer. Suppose that the responder is player 1, and fix player 2's strategy to be s_2^*. Denote by (y_1, y_2) the offer to which player 1 is responding. Player 1's strategy s_1^* calls for her to accept the proposal if and only if $y_1 \geq y_1^*$. If she rejects the proposal, she proposes x^*, which player 2 accepts, so that her payoff is $\delta_1 x_1^*$, which is equal to y_1^*. Thus no deviation in the first period of the subgame increases player 1's payoff. A symmetric argument shows that player 2 cannot profitably deviate in the first period of a subgame in which she responds to a proposal.

7. Consider the following Bayesian game. There are two players, 1 and 2, and three states, α, β, and γ. Player 1 has two actions, T and B, and player 2 has two actions, L and

R. Player 1 receives one of two possible signals: if the state is α or β she receives the signal A, while if the state is γ she receives the signal B. Player 2 receives different signals in each of the three states. If player 1 receives the signal A, she believes the state is α with probability $\frac{2}{3}$ an β with probability $\frac{1}{3}$. The players' payoffs in the three states are given below.

	L	R
T	$3, 1$	$1, 0$
B	$1, 2$	$3, 1$

State α

	L	R
T	$4, 2$	$1, 1$
B	$1, 0$	$4, 1$

State β

	L	R
T	$0, 0$	$4, 1$
B	$2, 1$	$0, 0$

State γ

(a) [3] Indicate in the figure the information structure of the game.

Solution: Player 1 cannot distinguish between states α and β, but can distinguish between these two states and γ. Player 2 can distinguish between all three states.

(b) [12] Find the pure strategy Nash equilibrium (equilibria?) of the game.

Solution: In state γ, both players know the payoffs. Thus in this state each player's action is a best response to the other player's action, and hence either player 1 chooses B and player 2 chooses L or player 1 chooses T and player 2 chooses R.

Does the game have a Nash equilibrium in which player 1 chooses T if she gets the signal A? If she does so, player 2 chooses L in both state α and state β, and player 1's expected payoff is $\frac{2}{3} \cdot 3 + \frac{1}{3} \cdot 4 = \frac{10}{3}$. If she deviates and chooses B when she gets the signal A, her expected payoff is 1.

Thus the game has a Nash equilibrium in which player 1 chooses T if she gets the signal A and player 2 chooses L in states α and β.

Does the game have a Nash equilibrium in which player 1 chooses B if she gets the signal A? If she does so, then player 2 chooses L in state α and R in state β, and player 1's expected payoff is $\frac{2}{3} \cdot 1 + \frac{1}{3} \cdot 4 = 2$. If she deviates and chooses A when she gets the signal A, her expected payoff is $\frac{2}{3} \cdot 3 + \frac{1}{3} \cdot 1 = \frac{7}{3}$. Thus she is better off deviating, so that the game has no Nash equilibrium in which chooses B if she gets the signal A.

In conclusion, the game has two pure strategy Nash equilibria:

- $((T, B), (L, L, L))$
- $((T, T), (L, L, R))$.

End of examination
Total pages: 6
Total marks: 100