Economics 326: Advanced Economic Theory—Micro

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Answers to Term Test 2

1. (a) Player i's action of bidding v_i weakly dominates an action of bidding less than v_i. Let $b_i < v_i$.
   - If the highest of the other players' bids is at most v_i then the only possible difference between bidding b_i and bidding v_i is that bidding b_i may lead player i to lose rather than win; if she wins then her payoff is nonnegative and is the same regardless of her bid.
   - If the highest of the other players' bids is more than v_i then player i loses when she bids b_i and also when she bids v_i.

   (b) Player i's action of bidding v_i does not weakly dominate an action of bidding more than v_i. Let $b_i > v_i$. If the highest of the other players' bids is between v_i and b_i and the lowest of these bids is less than v_i then a bid of v_i generates a payoff of 0, while a bid of b_i leads player i to win and obtain a positive payoff.

   (c) Any action profile in which player 1's bid b_1 satisfies $v_2 \leq b_1 \leq v_1$, every other player's bid is at most b_1, and all players' bids are at least v_2 is a Nash equilibrium. [You were asked only to find *one* of these equilibria.]

2. (a)
   - There is no Nash equilibrium in which no candidate or one candidate enters, because in each case another candidate can enter and at least tie for first place.
   - Any action pair (x_1, x_2) in which x_1 and x_2 are positions is a Nash equilibrium. In every such pair, the players tie for first place. In each case, if a player changes to another position she still ties for first place.

   (b) i. If two candidates enter then
   - if their positions are the same a third candidate can enter and win outright
   - if their positions are adjacent a third candidate can enter at one of the other positions and tie for first place

- if their positions are on a diagonal any candidate who enters loses—so such an action profile is a **Nash equilibrium**.

ii. If four candidates enter then

- if their positions are the same then any one of them can move to a different position and win outright
- if two of them are at one position and two at another position and these positions are on a diagonal then no player can increase her probability of winning by moving, so any such action profile is a **Nash equilibrium**.
- if two of them are at one position and two at another position and these positions are adjacent then any player can increase her probability of winning by moving to an unoccupied position
- if one candidate is at each of the positions then no candidate can increase her probability of winning by moving to a different position, so this action profile is a **Nash equilibrium**.

[Note that you were asked only to find **one** equilibrium.]

3. Player 1's action B is strictly dominated (by T), so the Nash equilibria of the game are the same as the Nash equilibria of the game

	X	Y	Z
T	$1,3$	$4,2$	$3,1$
M	$2,2$	$1,3$	$0,2$

In this case player 2's action Z is strictly dominated, so the Nash equilibria are the same as the Nash equilibria of the game

	X	Y
T	$1,3$	$4,2$
M	$2,2$	$1,3$

This game has a unique Nash equilibrium, in mixed strategies: $((\frac{1}{2},\frac{1}{2}),(\frac{3}{4},\frac{1}{4}))$. Thus the unique Nash equilibrium of the original game is $((\frac{1}{2},\frac{1}{2},0),(\frac{3}{4},\frac{1}{4},0))$.

4. (a) Denote by p_i the probability with which each witness with cost c_i reports the crime, for $i = 1, 2$. For each witness with cost c_1 to report with positive probability less than one, we need

$$\begin{aligned} v - c_1 &= v \cdot \Pr\{\text{at least one other person calls}\} \\ &= v\left(1 - (1-p_1)(1-p_2)^2\right), \end{aligned}$$

or

$$c_1 = v(1-p_1)(1-p_2)^2. \tag{1}$$

Similarly, for each witness with cost c_2 to report with positive probability less than one, we need

$$\begin{aligned} v - c_2 &= v \cdot \Pr\{\text{at least one other person calls}\} \\ &= v\left(1-(1-p_1)^2(1-p_2)\right), \end{aligned}$$

or

$$c_2 = v(1-p_1)^2(1-p_2). \tag{2}$$

Dividing (1) by (2) we obtain

$$1 - p_2 = c_1(1-p_1)/c_2.$$

Substituting this expression for $1-p_2$ into (1) we get

$$p_1 = 1 - \left(\frac{c_1}{v} \cdot \left(\frac{c_2}{c_1}\right)^2\right)^{1/3}.$$

Similarly,

$$p_2 = 1 - \left(\frac{c_2}{v} \cdot \left(\frac{c_1}{c_2}\right)^2\right)^{1/3}.$$

For these two numbers to be probabilities, we need each of them to be nonnegative and at most one, which requires

$$\frac{c_2^2}{v} < c_1 < (vc_2)^{1/2}.$$

(b) In this case the game has a Nash equilibrium in which each player with cost c_1 calls with positive probability and each player with cost c_2 does not call. For such a strategy profile to be a Nash equilibrium we need each player with cost c_1 to be indifferent between calling and not calling, or

$$v - c_1 = vp,$$

where p is the probability she calls. Thus $p = 1 - c_1/v$. Each player with cost c_2 prefers not to call because her payoff is then positive (one of the other players may call), whereas her payoff from calling is zero.