

Given name:\_\_\_\_\_ Family name:\_\_\_\_\_

Student number:\_\_\_\_\_ Signature:\_\_\_\_\_

**UNIVERSITY OF TORONTO**  
**Faculty of Arts and Science**

**ECO 326 H Section L0101 (Advanced Economic Theory—Micro)**

Instructor: Martin J. Osborne

**TERM TEST 2**  
**November 2004**

**Duration: 55 minutes**

**No aids allowed**

This examination paper consists of 14 pages and 4 questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

**TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS.**

For graders' use:

	Score
1 (25)	
2 (25)	
3 (25)	
4 (25)	
<b>Total (100)</b>	

1. A *lowest-price sealed-bid auction* is a variant of a second-price auction in which the price paid by the winner (the player who submits the highest bid) is the *lowest* of the bids submitted. [That is,  $n \geq 2$  players simultaneously submit bids for a single indivisible object. Player  $i$ 's valuation of the object is  $v_i$ , where  $v_1 > v_2 > \dots > v_n$ . The highest bid wins; in the event of a tie, the player whose index is smallest wins. (E.g. if players 1 and 2 tie for the highest bid, player 1 wins.)]
  - (a) [8] Does player  $i$ 's action of bidding  $v_i$  weakly dominate an action of bidding less than  $v_i$ ?

*Question continues on next page*

- (b) [8] Does player  $i$ 's action of bidding  $v_i$  weakly dominate an action of bidding more than  $v_i$ ?

*Question continues on next page*

- (c) [9] Find a Nash equilibrium of the game. [Note: you are *not* asked to find all equilibria.]

2. Consider a variant of Hotelling's model of electoral competition in which the set of possible positions consists of the four corners of a square in two dimensional space (instead of consisting of the set of all points on a line) and each player has the option of not running as a candidate.

Each citizen's favorite position is one of the four possible positions. Each of these positions is the favorite position of exactly 25% of the citizens.

As in Hotelling's model, a citizen votes for the candidate whose position is closest to her favorite position; the votes of citizens for whom the two most desirable candidates are equally distant are divided equally between these candidates.

The players are the candidates. Each candidate has **FIVE** possible actions: she may take one of the four possible positions, **or** stay out of the competition. She prefers to stay out than to lose, but prefers to tie for first place with any number of other candidates than to stay out.

- (a) [10] Find all the Nash equilibria when there are two candidates. (Remember that one possible action for a candidate is to stay out of the competition.)

*Space for answer continues on next page*

*Question continues on next page*

- (b) Suppose there are four candidates.
- i. [7] Does the game have any Nash equilibrium in which exactly **two** candidates enter the competition? If not, argue why not. If so, find one such equilibrium.

- ii. [8] Does the game have any Nash equilibrium in which exactly **four** candidates enter the competition? If not, argue why not. If so, find one such equilibrium.



3. [25] Find all the Nash equilibria, in pure and mixed strategies, of the following strategic game.

	$X$	$Y$	$Z$
$T$	1, 3	4, 2	3, 1
$M$	2, 2	1, 3	0, 2
$B$	0, 0	1, 1	2, 4

*Space for answer continues on next page*



4. Consider a variant of the crime-reporting model in which there are 4 witnesses, 2 of whom incur the cost  $c_1$  to report the crime and 2 of whom incur the cost  $c_2$ , where  $0 < c_1 < v$  and  $0 < c_2 < v$ .
- (a) [15] Find conditions on  $c_1$  and  $c_2$  for which the game has a mixed strategy Nash equilibrium in which every witness's strategy assigns positive probabilities to both reporting and not reporting. Express the equilibrium probability of each witness reporting the crime as a function of  $c_1$ ,  $c_2$ , and  $v$ .

*Space for answer continues on next page*

*Question continues on next page*

(b) [10] For  $0 < c_1 < c_2 = v$ , find a Nash equilibrium in which some players randomize.

Space for rough work (will not be graded)

**End of examination**

**Total pages: 14**

**Total marks: 100**