Economics 326: Advanced Economic Theory—Micro

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Answers to Term Test 1

1. (a) No action of either player is strictly dominated.

   (b) The actions T and M of player 1 are both weakly dominated by B. The action R of player 2 is weakly dominated by C.

   (c) The Nash equilibria of the game are (T, L), (M, L), (B, C), and (B, R).

   (d) No Nash equilibrium is strict.

2. (a) The game is:

   Players The n people.

   Actions Each person's actions are *Drive* and *Bus*.

   Preferences Each person's preferences are represented by the payoff function that assigns the negative of her travel time to each action profile.

   (b) The unique Nash equilibrium is the action profile in which every player drives.

   This action profile is a Nash equilibrium because if any player switches to the bus her travel time increases from $50 + 2n$ minutes to $50 + 2(n - 1) + m = 51 + 2n$ minutes.

   No other action profile is a Nash equilibrium: Suppose k people drive, where $k < n$, and consider a person who takes the bus. By switching to driving she increases the travel time of every vehicle by 2 minutes, but saves herself 3 minutes, and is thus better off.

   (c) In the Nash equilibrium each player's travel time is $50 + 2n$. If every player were to take the bus, each player's travel time would be $50 + m = 52$. Given $n \geq 2$, every player is better off in the outcome in which everyone takes the bus than she is in the Nash equilibrium.

3. To find the best response function of country 1, solve

$$\max_{t_1}[t_1(t_2 - t_1 + 2)].$$

We find $b_1(t_2) = \frac{1}{2}(t_2 + 2)$. Similarly the best response function of player 2 is $b_2(t_1) = \frac{1}{2}(t_1 + 8)$. A Nash equilibrium is a pair (t_1^*, t_2^*) such that $t_1^* = b_1(t_2^*)$ and $t_2^* = b_2(t_1^*)$. These two equations have a unique solution, $(t_1, t_2) = (4, 6)$. Thus the game has a unique Nash equilibrium, $(t_1^*, t_2^*) = (4, 6)$.

4. (a) Yes, the pair of prices (c, c) is a Nash equilibrium. For this action pair, each firm's profit is 0. If either firm lowers its price its profit is negative. If either firm raises its price its profit remains 0.

   (b) First note that if firm 1 serves the whole market at the price p its profit is $(\alpha - p)(p - c)$ and if firm 2 serves the whole market at the price p its profit is $(\alpha - p)p - c(\alpha - p) - (\alpha - p)^2 = (\alpha - p)(2p - c - \alpha)$. These two functions are shown in Figure 1.

   Now suppose that $p_1 = p_2 > c$. Let $\overline{p} = p_1 = p_2$. Firm 1's profit is $(\alpha - \overline{p})(\overline{p} - c)$ and firm 2's profit is zero (by the assumption about the way in which demand is split when the firms' prices are the same).

   From Figure 1 we see that firm 1's profit is positive if $c < \overline{p} < \alpha$. Thus if it raises its price its profit decreases (to 0). If it lowers its price to p_1' then its profit changes to $(\alpha - p_1')(p_1' - c)$ and we see from Figure 1 that this is less than $(\alpha - \overline{p})(\overline{p} - c)$ if $\overline{p} \leq \frac{1}{2}(\alpha + c)$.

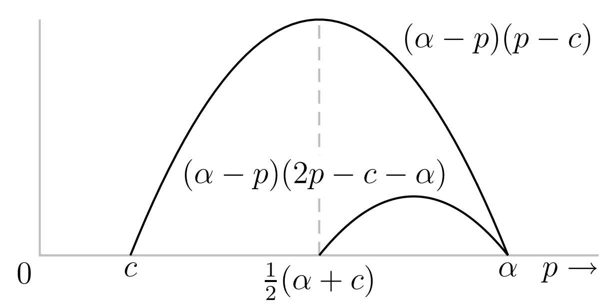

Figure 1. The functions $(\alpha - p)(p - c)$ and $(\alpha - p)(2p - c - \alpha)$ in Problem 4.

   Thus firm 1 cannot profitably deviate from $(\overline{p}, \overline{p})$ if $c < \overline{p} \leq \frac{1}{2}(c + \alpha)$.

   Now consider firm 2. If it raises its price, its profit remains zero. If it lowers its price to p_2', its profit becomes $(\alpha - p_2')(2p_2' - c - \alpha)$. From the figure we see that for $p_2' < \alpha$ this payoff is nonpositive if $p_2' \leq \frac{1}{2}(c + \alpha)$.

   We conclude that every pair (p_1, p_2) with $p_1 = p_2 = \overline{p}$ and $c < \overline{p} \leq \frac{1}{2}(c + \alpha)$ is a Nash equilibrium.

(c) If $p_1 > p_2 > c$ then firm 1's profit is zero and it can increase this profit by reducing its price to a level just below p_2. Thus no pair (p_1, p_2) with $p_1 > p_2 > c$ is a Nash equilibrium.

If $p_2 > p_1 > c$ then firm 1's profit is $(p_1 - c)(\alpha - p_1)$. If firm 1 raises its price its profit thus increases if $p_1 < \frac{1}{2}(\alpha + c)$; if it lowers its price its profit increases if $p_1 > \frac{1}{2}(\alpha + c)$. So for an equilibrium we need $p_1 = \frac{1}{2}(\alpha + c)$. If, in this case, firm 2 lowers its price to just below p_1 its profit is just less than $(\alpha - \frac{1}{2}(\alpha + c))(2(\frac{1}{2}(\alpha + c) - c - \alpha) = 0$, so that firm 2 cannot profitably deviate.

We conclude that any pair (p_1, p_2) with $p_1 = \frac{1}{2}(\alpha + c)$ and $p_2 \geq p_1$ is a Nash equilibrium.

(d) The game has no Nash equilibrium in which either firm's price is less than c, because the profit of the firm with the lower price is negative and this firm can raise it to c and increase its profit to 0.

If $p_1 = c$ and $p_2 > c$, firm 1's profit is zero and it can increase this profit by raising its price. Thus no such pair of prices is a Nash equilibrium.

If $p_1 > c$ and $p_2 = c$, we see from the figure that firm 2's profit is negative. Firm 2 can increase its profit to 0 by raising its price to p_1, so no such pair of prices is a Nash equilibrium.