Economics 326: Advanced Economic Theory: Micro

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Answers to Final Examination

1. The pure Nash equilibria are (M, m) and (B, m).

2. Suppose that the payoff of consumer i to W exceeds her payoff to U, given the other consumers' choices. Then $iN_w > (6 - i)N_u - c$ and hence $(i + 1)N_w > (6 - (i + 1))N_u - c$. Thus in any Nash equilibrium in which i chooses W, all consumers $j > i$ also choose W. That is, in every equilibrium either all consumers chooses U, all consumers choose W, or there is a number i such that consumers 1, ..., i choose U and consumers $i + 1$, ..., 5 choose W.

   All consumers choose U: if consumer 5 switches to W her payoff changes from $5 - c$ to 5. Thus this action profile is not a Nash equilibrium.

   All consumers choose W: if any consumer i switches to U her payoff changes from $5i$ to $(6 - i) - c$. Thus this action profile is a Nash equilibrium.

   Consumers 1, ..., i choose U and consumers $i + 1$, ..., 5 choose W: if consumer i switches to W her payoff changes from $(6 - i)i - c$ to $(5-(i-1))i = (6-i)i$. Thus no such action profile is a Nash equilibrium.

   We conclude that the game has a unique Nash equilibrium, in which all consumers choose W.

3. (a) The game is given in Figure 1.

		Cop: A	Cop: B
Robber	A	$-z, z$	$v_A, -v_A$
	B	$v_B, -v_B$	$\pi v_B - (1 - \pi)z, -\pi v_B + (1 - \pi)z$

Figure 1. The game in Question 3.

   (b) The game has no pure strategy Nash equilibrium. Consider an equilibrium in mixed strategies. Denote the probability the robber assigns to A by p and the probability the cop assigns to A by q.

For an equilibrium we need the robber's expected payoffs to A and B to be the same, or

$$(1-q)v_A = qv_B + (1-q)\pi v_B,$$

which means that $q = 1 - v_B/[v_A + (1-\pi)v_B]$. Denote this probability by q^*. We need also the cop's expected payoffs to A and B to be the same, or

$$-(1-p)v_B = -pv_A - (1-p)\pi v_B,$$

which means that $p = 1 - v_A/[v_A + (1-\pi)v_B]$. Denote this probability by p^*. Thus the game has a unique Nash equilibrium $((p^*, 1-p^*), (q^*, 1-q^*))$.

(c) The robber's expected payoff in the equilibrium of the game in part b is $(1-q)v_A$, where q is the equilibrium probability that the cop chooses A, and is thus equal to $v_Av_B/[v_A + (1-\pi)v_B]$.

Thus if $h \le v_Av_B/[v_A + (1-\pi)v_B]$, a Nash equilibrium of the game is the mixed strategy equilibrium $((p^*, 1-p^*, 0), (q^*, 1-q^*))$.

If $h > v_Av_B/[v_A + (1-\pi)v_B]$, one Nash equilibrium is the strategy pair $((0,0,1), (q^*, 1-q^*))$. (If $h \ge v_B$ then $((0,0,1),(1,0))$ is also a Nash equilibrium, and if $h \ge v_A$ then $((0,0,1),(0,1))$ is also a Nash equilibrium.)

4. (a) The game is shown in Figure 2.

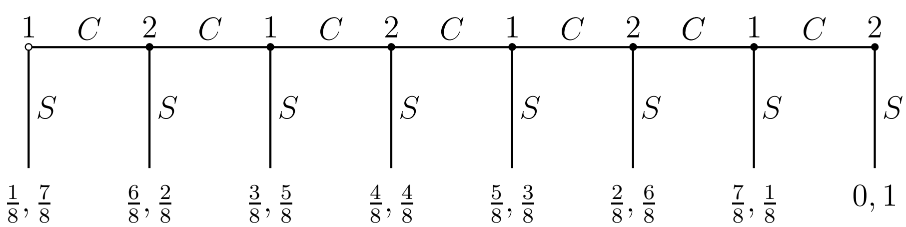

Figure 2. The game in Question 4.

(b) Using backward induction we find that player 1's subgame perfect equilibrium strategy takes a step on her first two turns and shoots thereafter, and player 2's strategy takes a step on her first turn and shoots thereafter. That is, the game has a unique subgame perfect equilibrium in which player 1's strategy is $CCSS$ and player 2's strategy is $CSSS$. The outcome of the equilibrium is that each player's payoff is $\frac{1}{2}$.

(c) No, the game has no such Nash equilibrium. For a strategy pair in which player 1 shoots on her first move to be a Nash equilibrium, player 2 must shoot at her first move, otherwise player 1 can increase her payoff by taking a step on her first move and shooting on her second move. But if player 2 shoots on her first move, player 1 is better off taking a step on her first move.

5. (a) Not a subgame perfect equilibrium. Player 2's rejection of the offer $(51, 49)$ is not optimal: acceptance yields her 49, while rejection leads her to get at most 50 with at least one period of delay.

   (b) A subgame perfect equilibrium:

   - If player 1 demands less, player 2 accepts her offer and player 1 is worse off.
   - If player 1 rejects an offer of 99 or more she gets at most 100 with one period of delay, for a net payoff of 99.
   - If player 2 offers player 1 more than 99, player 1 accepts her offer and she is worse off. If she offers player 1 less than 99, player 1 rejects her offer, and player 2 gets either 0 at some future date (if she subsequently accepts an offer of player 1), or at most 1 at some future date (if player 1 subsequently accepts an offer of hers).
   - If player 2 rejects any offer then she gets at most 1 at some future date, by the same argument.

   (c) Not a subgame perfect equilibrium. Player 1's acceptance of the offer $(50, 50)$ is not optimal: if she rejects it, she gets 52 with one period of delay, which is worth 51 to her.

6. (a) (N, N) and (B, B).

   (b) i. A Bayesian game that models the situation is defined as follows.

   Players The two players in the original game.

   States The set of states is the set of pairs (c_1, c_2) with $0 \leq c_i \leq 1$ for $i = 1, 2$.

   Actions The set of actions of each player is $\{N, B\}$.

   Signals The set of signals that each player may receive is $[0, 1]$. The signal function of player i is given by $\tau_i(c_1, c_2) = c_i$. (Each player i observes only c_i.)

   Beliefs Each player i believes that c_j is at most any given number c with probability c.

Payoffs The payoffs are derived from the basic game.

ii. The strategy of the other player and the distribution of the other player's value of c generate probabilities for N and B, say q and $1-q$. If the best response of a player of type c is B then

$$q(b-c)-(1-q)c \geq -(1-q)d,$$

or

$$c \leq qb+(1-q)d.$$

Thus the best response of a player of type c' with $c' < c$ is also B.

iii. Consider a strategy pair (s_1, s_2) in which s_i takes the form given, with $c_i^* = c^*$, $i = 1, 2$. For this strategy pair to be a Nash equilibrium of the Bayesian game we need B to be optimal for every type c of each player with $c < c^*$ and N to be optimal for every type c of each player with $c \geq c^*$.

If type c of player i chooses B she obtains the payoff $b-c$ if player j chooses N and $-c$ if player j chooses B. Now, if player j uses the strategy specified, she chooses N if $c_j > c^*$ and B if $c_j \leq c^*$. Player i believes that c_j is uniformly distributed between 0 and 1, so that she believes the probability that $c_j > c^*$ is $1-c^*$ and the probability that $c_j \leq c^*$ is c^*. Thus the expected payoff of type c of player i if she chooses B is

$$(b-c)(1-c^*)-cc^*.$$

Similarly, her expected payoff if she chooses N is $-dc^*$.

We conclude that if $0 < c^* < 1$, then for B to be optimal for every type c of each player with $c < c^*$ and N to be optimal for every type c of each player with $c \geq c^*$ we need

$$(b-c)(1-c^*)-cc^* \geq -dc^* \text{ if } c < c^*$$
$$-dc^* \geq (b-c)(1-c^*)-cc^* \text{ if } c \geq c^*.$$

Thus we need $(b-c^*)(1-c^*)-c^*c^* = -dc^*$, or $b = c^*(1+b-d)$. Given $d > 1$, no value of c^* satisfies this equation.

The strategy pair for $c^* = 0$ is an equilibrium if $0 \geq b-c$ for all c. Given $b < 1$, this inequality is not satisfied for $c = 1$.

The strategy pair for $c^* = 1$ is an equilibrium if $-c \geq -d$ for all c. Given $d > 1$, this inequality is satisfied.

We conclude that the game has a unique Nash equilibrium of the type given, in which $c^* = 1$.