**Given name:**____________________ **Family name:**____________________

**Student number:**____________________ **Signature:**__________________________

**UNIVERSITY OF TORONTO**
**Faculty of Arts and Science**

**DECEMBER EXAMINATIONS 2004**

**ECO 326 H Section L0101 and Section L0501**
**(Advanced Economic Theory—Micro)**
Instructor: Section L0101: Martin J. Osborne. Section L0501: Ettore Damiano

**Duration: 3 hours**

**No aids allowed**

This examination paper consists of **22** pages and **6** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

**TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS.** You may use the last 4 pages of the exam for rough work. The preceding three pages contain supplementary information to which you may refer during the exam.

For graders' use:

| | Score |
|---|---|
| 1 (5) | |
| 2 (15) | |
| 3 (20) | |
| Subtotal | |

| | Score |
|---|---|
| 4 (20) | |
| 5 (20) | |
| 6 (20) | |
| Subtotal | |

| | |
|---|---|
| Total (100) | |

1. [5] Find all the Nash equilibria in pure strategies of the following game.

| | $l$ | $m$ | $r$ |
|---|---|---|---|
| $T$ | $0,3$ | $2,1$ | 0,2 |
| $M$ | $1,1$ | $4,2$ | 1,2 |
| $B$ | $0,2$ | $4,3$ | 2,1 |

2. [15] Each of five consumers must choose to adopt one of two possible operating systems, $W$ and $U$. For $i = 1, 2, 3, 4, 5$, consumer $i$ receives the payoff $iN_w$ if she adopts $W$ and the payoff $(6 - i)N_u - c$ if she adopts $U$, where $N_w$ is the total number of consumers (including $i$) that choose $W$, $N_u$ is the total number of consumers (including $i$) that choose $U$, and $c$ is a small positive cost a consumer must pay if she adopts $U$. Find all pure strategy Nash equilibria of the game in which all consumers simultaneously choose the operating system to adopt.

*Space for answer continues on next page*

3. Consider a game between a robber (player 1) and a cop (player 2). The robber must attempt a robbery at one of two locations, A and B. The cop must choose to guard one of these two locations in an attempt to foil the robber.

   - If the cop and the robber choose different locations, the robbery certainly succeeds.
   - If the cop and the robber both choose location A, the robbery certainly fails.
   - If the cop and the robber both choose location B, the robbery succeeds with probability $\pi$ and fails with probability $1 - \pi$.

   The value to the robber of a successful robbery is $v_A$ at location A and $v_B$ at location B. The value to the robber of an unsuccessful robbery is $-z$. The robber's payoff is the expected value of a robbery; the cop's payoff is the negative of the robber's payoff.

   (a) [4] Model this situation as a strategic game.

*Question continues on next page*

(b) [8] Assume $z = 0$ and $v_A > \pi v_B$. Find all the Nash equilibria of the game (pure and/or mixed).

*Question continues on next page*

(c) [8] Suppose that the robber has a third option, to stay home ($H$), under which her payoff is $h$ no matter what the cop does and the cop's payoff is 0. Continue to assume that $z = 0$ and $v_A > \pi v_B$. For each value of $h$, find a Nash equilibrium of the game that models this situation. [*Note that you are asked only to find one equilibrium in this case, not all equilibria.*]

4. Two people are initially 8 paces apart. Each person holds a balloon and has a gun loaded with a single bullet. The people take actions *alternately*, starting with person 1. If the people are more than one pace apart, the person whose turn it is to act can *either* move one pace closer to the other person *or* shoot at the other person's balloon. If the people are one pace apart, the only option of the person whose turn it is to act is to shoot at the other person's balloon. The probability that a person who shoots succeeds in bursting the other person's balloon depends on the distance between the people. The probability of a person's shot bursting the other person's balloon when the people are $m$ paces apart is $1 - (m - 1)/8$. Once one person has shot her single bullet, the game ends. The payoff of the person who shoots is the probability she bursts the other person's balloon and the payoff of the other person is the probability that the shooter misses.

   (a) [7] Model this situation as an extensive game with perfect information.

*Question continues on next page*

(b) [6] Find the subgame perfect equilibria of the game. (Be sure to describe each player's equilibrium strategy completely.)

*Question continues on next page*

(c) [7] Does the game have any *Nash equilibrium* in which player 1 shoots on her first move? If so, specify such a Nash equilibrium. If not, argue why the game has no such Nash equilibrium.

5. Consider the following variant of the bargaining game of alternating offers. The size of the pie is \$100. Neither player discounts future payoffs (i.e. both discount factors are equal to 1), but in any period that Player 1 rejects an offer she has to pay a penalty of \$1 to a third party, and in any period that Player 2 rejects an offer she has to pay a penalty of \$2 to a third party. Which, if any, of the following strategy pairs are subgame perfect equilibria of this game?

   (a) [7] Player 1 always proposes $(50, 50)$ and accepts any offer $(x, 100 - x)$ in which $x \geq 50$, and Player 2 always proposes $(50, 50)$ and accepts any offer $(x, 100 - x)$ in which $100 - x \geq 50$.

*Question continues on next page*

(b) [7] Player 1 always proposes $(100, 0)$ and accepts any offer $(x, 100 - x)$ in which $x \geq 99$, and Player 2 always proposes $(99, 1)$ and accepts any offer $(x, 100 - x)$ in which $100 - x \geq 0$.

*Question continues on next page*

(c) [6] Player 1 always proposes $(52, 48)$ and accepts any offer $(x, 100 - x)$ in which $x \geq 50$, and Player 2 always proposes $(50, 50)$ and accepts any offer $(x, 100 - x)$ in which $100 - x \geq 48$.

6. Consider the strategic game given below, where $0 < b$, $c_1 < d$, and $c_2 < d$.

| | $N$ | $B$ |
|---|---|---|
| $N$ | $0, 0$ | $-d, b - c_2$ |
| $B$ | $b - c_1, -d$ | $-c_1, -c_2$ |

(a) [2] Find the Nash equilibria of the game when $b < c_1$ and $b < c_2$.

*Question continues on next page*

(b) Suppose that the numbers in the cells of the table are Bernoulli payoffs and that player 1 observes only $c_1$ and player 2 observes only $c_2$. Assume that each player $i$ believes that $c_j$, for $j \neq i$, is distributed uniformly between 0 and 1, independently of $c_i$. (That is, player $i$ believes that the probability that $c_j$ is at most any given number $c$ is exactly $c$.) Assume also that $b < 1 < d$.

i. [4] Model this situation as a Bayesian game.

*Question continues on next page*

ii. [7] Show that if the best response of a player of type $c$ to the other player's strategy is $B$ then the best response of every type $c' < c$ of that player is also $B$.

*Question continues on next page*

iii. [7] Deduce that for $i = 1, 2$, in any Nash equilibrium of the Bayesian game the strategy of player $i$ takes the form

$$s_i(c) = \begin{cases} B & \text{if } c \leq c_i^* \\ N & \text{if } c > c_i^*. \end{cases}$$

Find a number $c^*$ for which the strategy pair $(s_1, s_2)$ in which $c_1^* = c_2^* = c^*$ is a Nash equilibrium of the Bayesian game.

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You may use the next four pages for rough work.

**For rough work (will not be graded)**

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**End of examination**
**Total pages: 22**
**Total marks: 100**