Economics 326: Advanced Economic Theory—Micro

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Answers to Midterm Test

1. (a) No action of either player is strictly dominated.
   (b) X and Y are weakly dominated for player 1 and Y and Z are weakly dominated for player 2.
   (c) The players' best responses are indicated in the table below. The Nash equilibria are (X, Y), (Y, Y), and (Z, X). The equilibrium (Z, X) is a strict equilibrium.

	X	Y	Z
X	$2, 2^*$	$0^*, 2^*$	$3, 1$
Y	$3, 0^*$	$0^*, 0^*$	$4, 0^*$
Z	$4^*, 2^*$	$0^*, 1$	$5^*, 1$

2. (a) A strategic game that models the situation is:

	X	Y
X	$5, -5$	$10, -10$
Y	$1, -1$	$-20, 20$

   (b) The game has a unique Nash equilibrium, (X, X).

3. First find the best response functions.

   Player 1's payoff function is a quadratic, with maximizer $a_1 = a_2$. Thus the best response function of player 1 is

$$b_1(a_2) = a_2.$$

   Player 2's payoff function is a quadratic, with maximizer $1 - a_1$. Thus player 2's best response function is

$$b_2(a_1) = 1 - a_1.$$

   Now solve for the Nash equilibria. Any Nash equilibrium (a_1^*, a_2^*) satisfies

$$\begin{aligned} a_1^* &= a_2^* \\ a_2^* &= 1 - a_1^*. \end{aligned}$$

   Solving these equations we obtain $(a_1^*, a_2^*) = (\frac{1}{2}, \frac{1}{2})$.

4. First find the firms' best response functions.

   Firm 1's payoff function, and hence its best response function, is the same as one studied in the book. Thus its best response function is

$$b_1(q_2) = \tfrac{1}{2}(\alpha - c - q_2)$$

   where this function is nonnegative.

   Firm 2's payoff function is

$$\pi_2(q_1, q_2) = q_2(\alpha - q_1 - q_2) - q_2^2 = q_2(\alpha - q_1 - 2q_2),$$

   a quadratic that is zero when $q_2 = 0$ and when $q_2 = \frac{1}{2}(\alpha - q_1)$. Thus firm 2's best response function is

$$b_2(q_1) = \tfrac{1}{4}(\alpha - q_1)$$

   A Nash equilibrium is a pair (q_1^*, q_2^*) such that

$$q_1^* = b_1(q_2^*) \text{ and } q_2^* = b_2(q_1^*),$$

   or

$$q_1^* = \tfrac{1}{2}(\alpha - c - q_2^*) \text{ and } q_2^* = \tfrac{1}{4}(\alpha - q_1^*).$$

   Solving these two equations simultaneously we obtain

$$(q_1^*, q_2^*) = (\tfrac{3}{7}\alpha - \tfrac{4}{7}c, \tfrac{1}{7}\alpha + \tfrac{1}{7}c).$$

   (We have $q_1^* > 0$ because $c < \frac{3}{4}\alpha$.)

5. (a) Firm i's payoff function is

$$\begin{cases} (p_i - c)(R_i(p_i) + D(p_i)) & \text{if } p_i < p_j \\ (p_i - c)(R_i(p_i) + \frac{1}{2}D(p_i)) & \text{if } p_i = p_j \\ (p_i - c)R_i(p_i) & \text{if } p_i > p_j. \end{cases}$$

   (b) The pair (c, c) of prices is not a Nash equilibrium: The profit of each firm is 0, and either firm can make its profit positive by increasing its price, because $R_i(c) > 0$ for $i = 1, 2$.

   (c) Consider a pair of actions (p, p).

   - If $p < c$ then each firm's profit is negative; each firm can increase its profit to 0 by raising its price to c.

- If $p > c$ then firm i's profit is $(p-c)(R_i(p) + \frac{1}{2}D(p))$. If firm i lowers its price slightly this profit becomes approximately $(p-c)(R_i(p) + D(p))$. Thus if $D(p) > 0$, (p, p) is not a Nash equilibrium. If $D(p) = 0$ then (p, p) can be a Nash equilibrium only if p maximizes $(p-c)R_i(p)$ for $i = 1$ and $i = 2$. However, this condition is not sufficient for (p, p) to be a Nash equilibrium, because either firm may still find it profitable to lower its price significantly, reducing its revenue from regular customers but possibly increasing its revenue from customers who compare prices.

We conclude that whether or not there is a Nash equilibrium of the form (p, p) depends on the relative sizes of D, R_1, and R_2. If D is small relative to R_1 and R_2 then there may be a Nash equilibrium in which each firm charges its monopoly price to its regular customers; if D is large relative to R_1 and R_2 then there is no equilibrium of the form (p, p).

6. If a single candidate enters, then either of the remaining candidates can enter and either win outright or tie for first place. Thus there is no Nash equilibrium in which a single candidate enters.

   In any Nash equilibrium in which more than one candidate enters, all the candidates that enter tie for first place, since if they do not then some candidate loses, and hence can do better by staying out of the race.

   If two candidates enter, then by the argument in the text for the case in which there are two candidates, each takes the position m. But then the third candidate can enter and win outright. Thus there is no Nash equilibrium in which two candidates enter.

   If all three candidates enter and choose the same position, each candidate receives one third of the votes. If the common position is equal to m then any candidate can win outright (obtaining close to one-half of the votes) by moving slightly to one side of m. If the common position is different from m then any candidate can win outright (obtaining more than one-half of the votes) by moving to m. Thus there is no Nash equilibrium in which all three candidates enter and choose the same position.

   If all three candidates enter and do not all choose the same position then they all tie for first place, by the second argument. At least one candidate (i) does not share her position with any other candidate and (ii) is an extremist (her position is not between the positions of the other candidates). This candidate can move slightly closer to the other candidates and win outright. Thus there is no Nash equilibrium in which all three candidates enter and not all of them choose the same position.

We conclude that the game has no Nash equilibrium.