Economics 326: Advanced Economic Theory—Micro

Winter 2000 Martin J. Osborne

Answers to Final Examination

1. (a) The game is:

   Players The n people.

   Actions Each person's actions are *Drive* and *Bus*.

   Preferences Each person's preferences are represented by the payoff function that assigns the negative of her travel time to each action profile.

   (b) The unique Nash equilibrium is the action profile in which every player drives.

   This action profile is a Nash equilibrium because if any player switches to the bus her travel time increases from $50 + 2n$ minutes to $50 + 2(n-1) + m = 51 + 2n$ minutes.

   No other action profile is a Nash equilibrium: Suppose k people drive, where $k < n$, and consider a person who takes the bus. By switching to driving she increases the travel time of every vehicle by 2 minutes, but saves herself 3 minutes, and is thus better off.

2. Firm 1's best response to a_2 maximizes $2a_1 - a_1^2 - 4a_1a_2$, or $a_1(2 - a_1 - 4a_2)$. Thus firm 1's best response is $b_1(a_2) = 1 - 2a_2$. [If advertising expenditures are, sensibly, restricted to be nonnegative, then $b_1(a_2) = \max\{0, 1 - 2a_2\}$.]

   Firm 2's best response to a_1 maximizes $4a_2 - a_2^2 - 8a_1a_2$, or $a_2(4 - a_2 - 8a_1)$. Thus firm 1's best response is $b_2(a_1) = 2 - 4a_1$. [Or $\max\{0, 2 - 4a_1\}$.]

   A Nash equilibrium is a pair (a_1^*, a_2^*) such that $a_1^* = 1 - 2a_2^*$ and $a_2^* = 2 - 4a_1^*$, so that the unique Nash equilibrium is $(a_1^*, a_2^*) = (\frac{3}{7}, \frac{2}{7})$. [If advertising expenditures are, sensibly, restricted to be nonnegative, then there are two more Nash equilibria: $(0, 2)$ and $(1, 0)$.]

3. (a) Player i's action of bidding v_i weakly dominates an action of bidding less than v_i. Let $b_i < v_i$.

- If the highest of the other players' bids is at most v_i then the only possible difference between bidding b_i and bidding v_i is that bidding b_i may lead player i to lose rather than win; if she wins then her payoff is nonnegative and is the same regardless of her bid.
- If the highest of the other players' bids is more than v_i then player i loses when she bids b_i and also when she bids v_i.

(b) Player i's action of bidding v_i does not weakly dominate an action of bidding more than v_i. Let $b_i > v_i$. If the highest of the other players' bids is between v_i and b_i and the lowest of these bids is less than v_i then a bid of v_i generates a payoff of 0, while a bid of b_i leads player i to win and obtain a positive payoff.

(c) Any action profile in which player 1's bid b_1 satisfies $v_2 \leq b_1 \leq v_1$, every other player's bid is at most b_1, and all players' bids are at least v_2 is a Nash equilibrium.

4. (a)
   - There is no Nash equilibrium in which no candidate or one candidate enters, because in each case another candidate can enter and at least tie for first place.
   - Any action pair (x_1, x_2) in which x_1 and x_2 are positions is a Nash equilibrium. In every such pair, the players tie for first place. In each case, if a player changes to another position she still ties for first place.

   (b)
   - In any Nash equilibrium every candidate who chooses a position must be tied for first place, otherwise some candidate loses, and can do better by staying out of the competition.
   - The action profile in which no candidate enters is not a Nash equilibrium because any candidate can enter and win outright.
   - Any action profile in which a single candidate enters is not a Nash equilibrium because an additional candidate who enters ties for first place.
   - If two candidates enter then
     - if their positions are the same a third candidate can enter and win outright
     - if their positions are adjacent a third candidate can enter at one of the other positions and win outright
     - if their positions are on a diagonal any candidate who enters loses—so such an action profile is a **Nash equilibrium**.

- If three candidates enter then they must do so at the same position (otherwise they do not tie for first place), in which case the fourth candidate can enter at another position and win outright.
- If four candidates enter then
  - if their positions are the same then any one of them can move to a different position and win outright
  - if two of them are at one position and two at another position and these positions are on a diagonal then no player can increase her probability of winning by moving, so any such action profile is a **Nash equilibrium**.
  - if two of them are at one position and two at another position and these positions are adjacent then any player can increase her probability of winning by moving to an unoccupied position
  - if one candidate is at each of the positions then no candidate can increase her probability of winning by moving to a different position, so this action profile is a **Nash equilibrium**.

In summary, there are three types of Nash equilibrium:

- Two candidates enter, at two positions on a diagonal of the square.
- Each of the four candidates enters at a different position.
- Each of the four candidates enters; two enter at one position and two enter at the diagonally opposite position.

5. Player 1's action B is strictly dominated (by T), so the Nash equilibria of the game are the same as the Nash equilibria of the game

	X	Y	Z
T	$1,3$	$4,2$	$3,1$
M	$2,2$	$1,3$	$0,2$

In this case player 2's action Z is strictly dominated, so the Nash equilibria are the same as the Nash equilibria of the game

	X	Y
T	$1,3$	$4,2$
M	$2,2$	$1,3$

This game has a unique Nash equilibrium, in mixed strategies: $((\frac{1}{2},\frac{1}{2}),(\frac{3}{4},\frac{1}{4}))$. Thus the unique Nash equilibrium of the original game is $((\frac{1}{2},\frac{1}{2},0),(\frac{3}{4},\frac{1}{4},0))$.

6. The game is given in the following figure.

		Buyer 2: Seller 1	Buyer 2: Seller 2
Buyer 1	Seller 1	$\frac{1}{2}(1-p_1), \frac{1}{2}(1-p_1)$	$1-p_1, 1-p_2$
	Seller 2	$1-p_2, 1-p_1$	$\frac{1}{2}(1-p_2), \frac{1}{2}(1-p_2)$

Given $2p_1 - 1 < p_2 < \frac{1}{2}(1+p_1)$, a buyer's expected payoff to choosing each seller is the same when

$$\tfrac{1}{2}(1-p_1)\pi + (1-p_1)(1-\pi) = (1-p_2)\pi + \tfrac{1}{2}(1-p_2)(1-\pi),$$

where π is the probability that the other buyer chooses seller 1, or when

$$\pi = \frac{1-2p_1+p_2}{2-p_1-p_2}.$$

The players' best response functions are shown in Figure 1. We see that the game has three mixed strategy equilibria: two pure equilibria in which the buyers approach different sellers, and one mixed strategy equilibrium in which each buyer approaches seller 1 with probability $(1-2p_1+p_2)/(2-p_1-p_2)$.

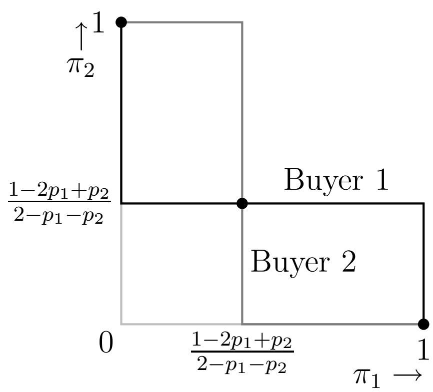
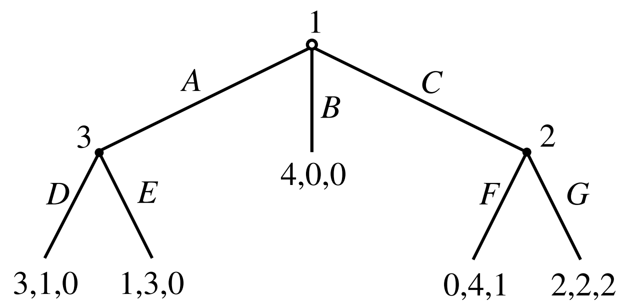


Figure 1. The players' best response functions in the game in Problem 6. The probability with which buyer i approaches seller 1 is π_i.

7. (a) The game is shown in the following diagram.



(b) In the extensive game one option for player 1 is to choose a Nash equilibrium action in the strategic game. If she does so, then player 2's response is her action in the Nash equilibrium. Thus in any subgame perfect equilibrium of the extensive game player 1's payoff is at least as high as it is in any of the Nash equilibria of the strategic game.

8. In a subgame perfect equilibrium player 2's strategy is her best response function to a_1. Thus for any value of a_1 player 2's action a_2 maximizes $a_2(c + a_1 - 2a_2)$, and is thus equal to $\frac{1}{4}(c + a_1)$.

   Player 1's action at the beginning of the game thus maximizes $a_1(c + \frac{1}{4}(c + a_1) - 2a_1)$, or $\frac{1}{4}a_1(5c - 7a_1)$. Thus player 1's subgame perfect equilibrium strategy is $a_1 = 5c/14$.

   Thus the game has a unique subgame perfect equilibrium, in which player 1's strategy is $a_1 = 5c/14$ and player 2's strategy specifies the action $\frac{1}{4}(c + a_1)$ after the history a_1.

   The outcome is that $a_1 = 5c/14$ and $a_2 = 19c/56$.

9. In the unique subgame perfect equilibrium person A chooses low effort, and for each effort level she chooses person B proposes that A's share be zero; A accepts all offers.

10. Firm 2 chooses q_2 to solve

$$\max_{q_2} (\alpha - q_1 - q_2)q_2 - q_2^2,$$

   so that $q_2 = (\alpha - q_1)/4$.

   Firm 1 subsequently chooses q_1 to solve

$$\max_{q_1} (\alpha - q_1 - (\alpha - q_1)/4)q_1 - q_1,$$

   so that $q_1 = \frac{1}{2}\alpha - \frac{2}{3}$.

The equilibrium strategies are: $q_1 = \frac{1}{2}\alpha - \frac{2}{3}$ for firm 1, and $q_2 = (\alpha - q_1)/4$ for firm 2.

The equilibrium outcome is that $q_1 = \frac{1}{2}\alpha - \frac{2}{3}$ and $q_2 = \frac{1}{8}\alpha + \frac{1}{6}$.