Given name:____________________ **Family name:**____________________

Student number:____________________ **Signature:**__________________________

UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL/MAY EXAMINATIONS 2000

ECO 326H1 S (Advanced Economic Theory—Micro)
Instructor: Martin J. Osborne

Duration: 3 hours

No aids allowed

This examination paper consists of **18** pages and **10** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS. The last three pages of the exam may be used for rough work.

For graders' use:

	Score
1 (7)	
2 (10)	
3 (13)	
4 (14)	
5 (10)	
Subtotal	

	Score
6 (12)	
7 (7)	
8 (9)	
9 (10)	
10 (8)	
Subtotal	

Total (100)	

1. A group of n people use the same road to commute to work at the same time. Each person can drive her own car or take a bus. The bus is large enough to accommodate all n people; it runs even if no one takes it, and takes the same amount of time to drive along the road as does a car. When k cars (and the bus) use the road, each vehicle's travel time is $50 + 2k$ minutes. A person who takes the bus takes m extra minutes (because she needs to get to the bus stop). Each person cares only about the time she spends commuting.

   (a) [3] Model this situation as a strategic game. (*Note*: n may be *any* integer.)

   (b) [4] Find the Nash equilibrium (equilibria?) of the game for $m = 3$.

2. [10] Each of two firms chooses its advertising budget a_i. If the budgets chosen are (a_1, a_2) then the profit of firm 1 is

$$2a_1 - a_1^2 - 4a_1a_2,$$

while the profit of firm 2 is

$$4a_2 - 8a_1a_2 - a_2^2.$$

Find the Nash equilibrium (equilibria?) of the strategic game that models this situation.

3. A *lowest-price sealed-bid auction* is a variant of a second-price auction in which the price paid by the winner (the player who submits the highest bid) is the *lowest* of the bids submitted. [That is, $n \geq 2$ players simultaneously submit bids for a single indivisible object. Player i's valuation of the object is v_i, where $v_1 > v_2 > \cdots > v_n$. The highest bid wins; in the event of a tie, the player whose index is smallest wins. (E.g. if players 1 and 2 tie for the highest bid, player 1 wins.)]

   (a) [4] Does player i's action of bidding v_i weakly dominate an action of bidding less than v_i?

Question continues on next page

(b) [4] Does player i's action of bidding v_i weakly dominate an action of bidding more than v_i?

(c) [5] Find a Nash equilibrium of the game.

4. Consider a variant of Hotelling's model of electoral competition in which the set of possible positions consists of the four corners of a square in two dimensional space (instead of consisting of the set of all points on a line).

   Each citizen's favorite position is one of the four possible positions. Each of these positions is the favorite position of exactly 25% of the citizens.

   As in Hotelling's model, a citizen votes for the candidate whose position is closest to her favorite position; the votes of citizens for whom the two most desirable candidates are equally distant are divided equally between these candidates.

   The players are the candidates. Each candidate has **five** possible actions: she may take one of the four possible positions, **or** stay out of the competition. She prefers to stay out than to lose, but prefers to tie for first place with any number of other candidates than to stay out.

   (a) [5] Find all the Nash equilibria when there are two candidates.

Question continues on next page

(b) [9] Find all the Nash equilibria when there are four candidates.

5. [10] Find all the Nash equilibria, in pure and mixed strategies, of the following strategic game.

	X	Y	Z
T	$1,3$	$4,2$	$3,1$
M	$2,2$	$1,3$	$0,2$
B	$0,0$	$1,1$	$2,4$

6. [12] Each of two sellers has available one indivisible unit of a good. Seller 1 posts the price p_1 and seller 2 posts the price p_2. Each of two buyers would like to obtain one unit of the good; they simultaneously decide which seller to approach. If both buyers approach the same seller, each trades with probability $\frac{1}{2}$; the disappointed buyer does not subsequently have the option to trade with the other seller. Each buyer's preferences are represented by the expected value of a payoff function that assigns the payoff 0 to not trading and the payoff $1 - p$ to purchasing one unit of the good at the price p. (Neither buyer values more than one unit.)

   For any pair (p_1, p_2) of prices with $0 < p_i < 1$ for $i = 1, 2$, and $2p_1 - 1 < p_2 < \frac{1}{2}(1 + p_1)$ find the Nash equilibria (in pure and in mixed strategies) of the strategic game that models this situation.

Space for answer continues on next page

7. (a) [3] Represent in a figure the following extensive game.

Players 1, 2, and 3.

Terminal histories (A, D), (A, E), B, (C, F), (C, G).

Player function $P(\varnothing) = 1$, $P(A) = 3$, and $P(C) = 2$.

Preferences Player 1 prefers B to (A, D) to (C, G) to (A, E) to (C, F); player 2 prefers (C, F) to (A, E) to (C, G) to (A, D) to B; and player 3 prefers (C, G) to (C, F), and prefers both of these to all other terminal histories, between which she is indifferent.

Question continues on next page

(b) [4] Person 1's set of actions is A_1 and person 2's set of actions is A_2. Each person cares about both her own action and the other player's action. Assume that for every action of player 1, player 2 has a *unique* optimal action.

Compare two games: the strategic game in which player 1's set of actions is A_1 and player 2's set of actions is A_2, and the extensive game with perfect information in which first player 1 chooses an action in A_1, then player 2, after observing player 1's action, chooses an action in A_2 (so that every terminal history has length 2).

Is player 1's payoff in a Nash equilibrium of the strategic game *necessarily* at least as large as her payoff in a subgame perfect equilibrium of the extensive game, or vice versa? Or is there no *necessary* relationship between the equilibrium payoffs?

8. [9] Two individuals are involved in a synergistic relationship. If both individuals devote more effort to the relationship, they are both better off. For any given effort of individual j, the return to individual i's effort first increases, then decreases. Specifically, an effort level is a nonnegative number, and individual i's preferences (for $i = 1, 2$) are represented by the payoff function $a_i(c + a_j - 2a_i)$, when a_i is i's effort level, a_j is the other individual's effort level, and $c > 0$ is a constant.

   First player 1 chooses an effort level, *then* player 2 does so (after observing player 1's effort level). Find the subgame perfect equilibrium (equilibria?) of the extensive game that models this situation. (Be sure to specify the equilibrium **STRATEGIES**, not only the equilibrium outcome.)

9. [10] Person A can exert either low effort or high effort; low effort results in the output x_L, while high effort results in the output x_H. *After* having exerted effort, she negotiates with person B how to split the output with person B. The negotiation takes the form of an ultimatum game in which person B is the proposer. [That is, B proposes a division of the output between A and B, and then A either accepts or rejects this proposal. If A rejects the proposal neither player obtains any output.]

   Person A's payoff is $x - L$ if she exerts low effort and $x - H$ if she exerts high effort, where x is the amount of output she obtains and $0 < L < H$.

   Find the subgame perfect equilibrium (equilibria?) of the extensive game that models this situation. (Be sure to specify the equilibrium **strategies**!)

10. [8] Find the subgame perfect equilibrium of Stackelberg's duopoly game when the inverse demand function is given by $P(Q) = \alpha - Q$ for all $Q \leq \alpha$ (with $P(Q) = 0$ for $Q > \alpha$), firm 1's cost function is $C_1(q_1) = q_1$, and firm 2's cost function is $C_2(q_2) = q_2^2$. Specify both the equilibrium **strategies** and the equilibrium outcome.

End of questions. You may use the following three pages for rough work.

For rough work (will not be graded)

For rough work (will not be graded)

For rough work (will not be graded)

End of examination
Total pages: 18
Total marks: 100