This *Supplement* contains the aid sheets (formula sheets and Normal, t and F tables) as well as graphs, tables, and other information needed to answer some of the test questions. For each question directing you to this *Supplement*, make sure to carefully review all relevant materials. Remember, only your answers written on the test papers (in the designated space immediately after each question) will be graded. Any writing on this *Supplement* will *not* be graded.

THE FORMULA SHEETS AND NORMAL, t AND F TABLES ARE ON PAGES 5 THROUGH 10 OF THIS SUPPLEMENT.

Supplement for Question (1): Recall that on February 14, 2014, February 6, 2015, and on February 12, 2016 students in ECO220Y engaged in an experiment like the original participants from Andreoni and Vesterlund's "Which is the Fair Sex: Gender Differences in Altruism" published in 2001 and hereafter abbreviated as A&V (2001).

Each participant divided tokens between her/himself and another randomly selected participant in the room (whose identity would never be revealed). Each person made eight allocation decisions – budgets 1 through 8 shown below – where the number of tokens and the point values to each person (self and other) varied. Both A&V (2001) and ECO220Y (2014, 2015, 2016) *randomized* the order the eight budgets appeared to each participant on the decision sheet. Each point is worth $0.10 to all participants in all cases.

1. Divide 40 tokens: *Hold* _______ @ 1 point each, and *Pass* _______ @ 3 points each.
2. Divide 60 tokens: *Hold* _______ @ 1 point each, and *Pass* _______ @ 2 points each.
3. Divide 75 tokens: *Hold* _______ @ 1 point each, and *Pass* _______ @ 2 points each.
4. Divide 60 tokens: *Hold* _______ @ 1 point each, and *Pass* _______ @ 1 point each.
5. Divide 100 tokens: *Hold* _______ @ 1 point each, and *Pass* _______ @ 1 point each.
6. Divide 60 tokens: *Hold* _______ @ 2 points each, and *Pass* _______ @ 1 point each.
7. Divide 75 tokens: *Hold* _______ @ 2 points each, and *Pass* _______ @ 1 point each.
8. Divide 40 tokens: *Hold* _______ @ 3 points each, and *Pass* _______ @ 1 point each.

We attempted to replicate the original study. One difference is that rather than pay everyone for one randomly selected budget as A&V (2001) did using a research grant, ECO220Y (2014, 2015, 2016) paid randomly selected participants using money students donated right before the session and $20.00 per session (total of $220.00) donated by Prof. Murdock. Nearly all students voluntarily donated $2.00 to a collection jar as suggested by Prof. Murdock. ECO220Y (2014, 2015, 2016) used data from eleven sessions spread over February 14, 2014 (three sessions), February 6, 2015 (five sessions), and on February 12, 2016 (three sessions): 868 participated (334 males and 534 females). Because each participant made eight allocation decisions, 6,944 allocation decisions are observed. Consider the *total* money (CAN dollars) passed to the (anonymous) partner across all eight decisions: `tot_mon_pass`. Also, `male` is a dummy variable equal to one if the participant is male.

Summary of total money passed, by sex:

```
-> male = 0

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
tot_mon_pass |        534    26.18446    12.23994          0       60.5


-> male = 1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
tot_mon_pass |        334    26.97126    16.44741          0       72.5
```

Supplement for Question (1) continues on next page >>>>>

Supplement for Question (1), cont'd:

Regression #1:

```
. regress tot_mon_pass male, robust;

Linear regression                                      Number of obs =     868
                                                       F(  1,   866) =    0.57
                                                       Prob > F      =  0.4513
                                                       R-squared     =  0.0007
                                                       Root MSE      =  14.008

------------------------------------------------------------------------------
             |               Robust
tot_mon_pass |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        male |   .7868006   1.044054     0.75   0.451    -1.262371    2.835972
       _cons |   26.18446   .5297886    49.42   0.000     25.14464    27.22428
------------------------------------------------------------------------------
```

Regression #2:

```
. regress tot_money_passed male;

      Source |       SS       df       MS              Number of obs =     868
-------------+------------------------------           F(  1,   866) =    0.65
       Model |  127.202988     1  127.202988           Prob > F      =  0.4210
    Residual |  169934.285   866  196.228967           R-squared     =  0.0007
-------------+------------------------------           Adj R-squared = -0.0004
       Total |  170061.488   867  196.149352           Root MSE      =  14.008

------------------------------------------------------------------------------
tot_mon_pass |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        male |   .7868006   .9772317     0.81   0.421    -1.131219     2.70482
       _cons |   26.18446    .606193    43.19   0.000     24.99468    27.37424
------------------------------------------------------------------------------
```

Summary of total money passed, by year:

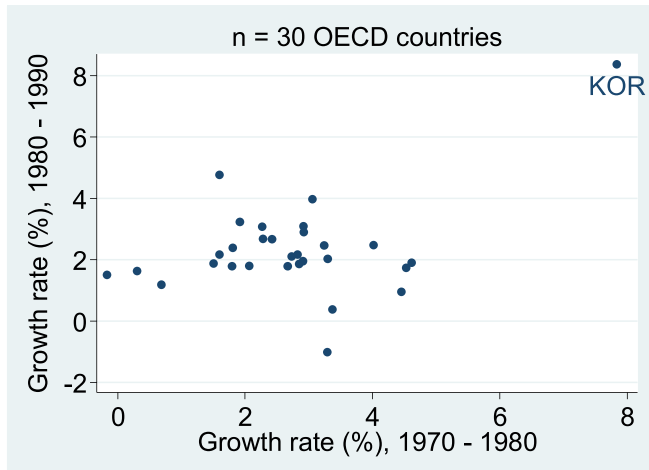
```
-> data_source = ECO220Y, Feb. 14, 2014

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
tot_mon_pass |       200      27.691    13.44333          0       72.5


-> data_source = ECO220Y, Feb. 6, 2015

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
tot_mon_pass |       461    26.52842    14.61084          0       72.5


-> data_source = ECO220Y, Feb. 12, 2016

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
tot_mon_pass |       207    25.23237    13.08705          0         53
```

Supplement for Question (2): Recall the 2014 *NBER* working paper "Asiaphoria Meets Regression to the Mean" (http://www.nber.org/papers/w20573.pdf). Test #1, Nov. 2016 provided a table "Table 1: Little persistence in cross-national growth rates across decades" and a scatter diagram. Recall the dot labeled "KOR," which is South Korea. South Korea is an outlier and the table below shows three possibilities: (1) keep it, (2) drop it, or (3) put a dummy in for it.

Three Regression Specifications
(Dependent variable: *Growth rate (%), 1980 – 1990*)

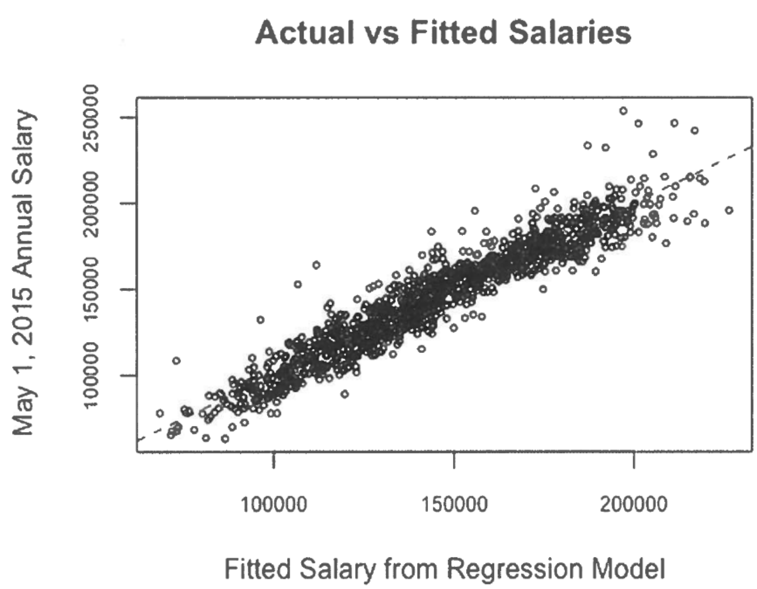
	(1)	(2)	(3)
Growth rate (%), 1970 – 1980	0.4326 (0.1772)	-0.1025 (0.1744)	-0.1025 (0.1744)
KOR	-	-	6.7889 (1.4287)
Intercept	1.1511 (0.5479)	2.3803 (0.4863)	2.3803 (0.4863)
Number of observations	30	29	30
R-squared	0.1755	0.0126	0.5510
SST	69.1815	31.4610	69.1815
SSR	12.1397	0.3975	38.1179
SSE	57.0418	31.0635	31.0635

Note: Coefficient estimates with standard errors in parentheses.



Supplement for Question (4): Recall the May 26, 2016 "Salary Anomaly Working Group: Analysis & Findings" report and appendix. It analyzes 1,171 Waterloo faculty members who completed an annual performance review on May 1, 2015. This includes 344 females and 827 males. The next page reproduces the regression results – Specifications (1) and (2) – with clarifications in places. The bullet list below describes some of the variables, with others described with the regression results. Categorical (nominal) information is included with dummy variables: the names clearly indicate when the value is 1. The omitted (aka reference) category for each set of dummies is specified in the bullet list. Also, the figure named "Actual vs Fitted Salaries" is reproduced from the original report and it corresponds to Specification (1).

- *Annual salary:* Annual base salary as of May 1, 2015 excluding stipends (CAN dollars)
- *Mean annual merit score:* Average annual merit score for all available years for the faculty member from 2009 through 2014. Each year faculty members receive a merit score on a two-point scale (2.0 is the highest score) that scores overall productivity in research, teaching, and service. A typical score is 1.6.
- *Highest degree earned:* The omitted category is *Bachelor Degree (BA).*
- *Current academic rank:* The omitted category is *Assistant Professor.*
- *UNIT: Academic unit of faculty member:* The omitted category is *Applied Health Sciences.*



- *RANK@HIRE: Academic rank when hired at Waterloo:* The omitted category is *Assistant Professor.*
- *PROFS_RANKS:* Is a dummy variable equal to 1 for those with a rank of Assistant Professor, Associate Professor or Professor. The omitted category is Lecturers and Clinical Lecturers.

Supplement for Question (4) continues on next page >>>>>

Multiple Regression Estimation Results for the Waterloo Case Study (Dependent variable: *Annual salary*)

	Specification (1)		***Specification (2)***	
	Estimate	Std. Error	Estimate	Std. Error
Male	-	-	2904.59	700.82
Mean annual merit score, 2009 through 2014	26097.98	1791.94	25821.67	1699.22
Number of previous Outstanding Performance Awards	3664.14	685.64	3587.37	651.23
Years since hire until 2015 (i.e. 2015 minus the year hired)	2226.66	140.71	2252.47	133.39
Years since hire until 2015 squared	-15.84	3.47	-16.60	3.29
LAG: Lag of years between earning highest degree and hire at Waterloo	829.88	147.10	858.08	138.79
Highest degree earned:				
Doctoral (PhD)	10084.27	3378.91	8564.38	3251.73
Master's or equivalent	1616.14	3519.85	146.71	3386.44
Professional	7339.61	4825.99	3533.10	4448.90
Graduate Licence	-7700.33	7201.54	-10290.55	6873.11
Current academic rank:				
Professor	15811.02	1598.82	15141.57	1525.42
Associate Professor	7666.75	1171.28	7325.93	1118.28
Clinical Lecturer	9038.29	11059.44	9675.22	10520.73
Lecturer	-7349.55	4462.49	-6553.49	4262.62
UNIT: Academic unit of faculty member:				
School of Accounting & Finance	15244.60	5091.68	14049.27	4838.09
Economics	5084.50	5922.80	4167.66	5631.41
Psychology	-15413.75	6812.83	-16971.48	6468.86
Chemical Engineering	-7227.35	7405.35	-4803.26	7035.98
Electrical & Computer Engineering	13516.82	6268.18	10629.27	5986.53
School of Computer Science	7520.51	5183.75	5114.92	4948.56
School of Optometry	10792.42	12693.56	11736.54	12002.76
School of Pharmacy	21043.85	2811.67	21177.10	2668.87
Faculty of Environment	-12225.70	8596.71	-13716.05	8169.56
Arts (excluding units already listed above)	-10388.70	4735.48	-11526.09	4515.81
Engineering (excluding units already listed above)	14752.95	4697.43	14007.87	4502.53
Mathematics (excluding units already listed above)	6532.34	4439.02	5383.34	4239.10
Science (excluding units already listed above)	-1135.99	5025.57	-1950.10	4779.45
RANK@HIRE: Academic rank when hired at Waterloo:				
Professor	10079.46	3681.30	10097.90	3495.13
Associate Professor	5319.26	2096.70	4952.56	1996.99
Clinical Lecturer	4242.50	11932.31	5743.84	11349.63
Lecturer	-2210.70	1641.43	-2101.91	1630.66
Interaction terms between UNIT and PROFS_RANKS:				
*UNIT=School of Accounting & Finance * PROFS_RANKS*	18910.82	5572.08	19887.39	5284.41
*UNIT=Economics * PROFS_RANKS*	7780.52	6408.68	7679.05	6089.70
*UNIT=Psychology * PROFS_RANKS*	16166.87	7150.68	17518.53	6793.48
*UNIT=Chemical Engineering * PROFS_RANKS*	19640.16	7753.66	15814.88	7365.46
*UNIT=Electrical & Computer Engineering * PROFS_RANKS*	3970.11	6512.00	5431.13	6200.93
*UNIT=School of Computer Science * PROFS_RANKS*	16368.08	5508.11	17615.55	5244.71
*UNIT=School of Optometry * PROFS_RANKS*	4233.38	12696.43	4146.50	12024.74
*UNIT=Faculty of Environment * PROFS_RANKS*	9908.77	8779.14	10683.69	8341.98
*UNIT=Arts * PROFS_RANKS*	5092.17	4996.96	5963.93	4769.95
*UNIT=Engineering * PROFS_RANKS*	779.42	4960.85	679.95	4740.64
*UNIT=Mathematics * PROFS_RANKS*	5397.36	4733.94	5372.86	4506.28
*UNIT=Science * PROFS_RANKS*	2055.99	5280.47	1765.49	5016.45
Interaction terms between LAG and RANK@HIRE:				
*LAG * RANK@HIRE=Professor*	889.20	228.67	889.46	216.39
*LAG * RANK@HIRE=Associate Professor*	410.74	237.82	444.89	225.81
*LAG * RANK@HIRE=Lecturer*	-335.24	181.71	-383.49	172.40
Intercept	46748.06	4592.89	47757.05	4401.09
Number of observations	1,171		1,171	

Sample mean: $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ **Sample variance:** $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{X})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$ **Sample s.d.:** $s = \sqrt{s^2}$

Sample coefficient of variation: $CV = \frac{s}{\bar{X}}$ **Sample covariance:** $s_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{X})(y_i - \bar{Y})}{n-1} = \frac{\sum_{i=1}^{n} x_i y_i}{n-1} - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(n-1)}$

Sample interquartile range: $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{s_{xy}}{s_x s_y} = \frac{\sum_{i=1}^{n} z_{x_i} z_{y_i}}{n-1}$

Addition rule: $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ **Conditional probability:** $P(A|B) = \frac{P(A\ and\ B)}{P(B)}$

Complement rules: $P(A^C) = P(A') = 1 - P(A)$ $P(A^C|B) = P(A'|B) = 1 - P(A|B)$

Multiplication rule: $P(A\ and\ B) = P(A|B)P(B) = P(B|A)P(A)$

Expected value: $E[X] = \mu = \sum_{all\ x} xp(x)$ **Variance:** $V[X] = E[(X-\mu)^2] = \sigma^2 = \sum_{all\ x}(x-\mu)^2 p(x)$

Covariance: $COV[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = \sigma_{XY} = \sum_{all\ x}\sum_{all\ y}(x-\mu_X)(y-\mu_Y)p(x,y)$

<u>Laws of expected value</u>:

$E[c] = c$

$E[X + c] = E[X] + c$

$E[cX] = cE[X]$

$E[a + bX + cY] = a + bE[X] + cE[Y]$

<u>Laws of variance</u>:

$V[c] = 0$

$V[X + c] = V[X]$

$V[cX] = c^2 V[X]$

$V[a + bX + cY] = b^2V[X] + c^2V[Y] + 2bc * COV[X,Y]$

$V[a + bX + cY] = b^2V[X] + c^2V[Y] + 2bc * SD(X) * SD(Y) * \rho$

where $\rho = CORRELATION[X,Y]$

<u>Laws of covariance</u>:

$COV[X,c] = 0$

$COV[a + bX, c + dY] = bd * COV[X,Y]$

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$ for $x = 0,1,2,\dots,n$

If *X* is Binomial $(X \sim B(n,p))$ **then** $E[X] = np$ **and** $V[X] = np(1-p)$

If *X* is Uniform $(X \sim U[a,b])$ **then** $f(x) = \frac{1}{b-a}$ **and** $E[X] = \frac{a+b}{2}$ **and** $V[X] = \frac{(b-a)^2}{12}$

<u>Sampling distribution of $\bar{X}$</u>:

$\mu_{\bar{X}} = E[\bar{X}] = \mu$

$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

<u>Sampling distribution of $\hat{P}$</u>:

$\mu_{\hat{P}} = E[\hat{P}] = p$

$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$

$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

<u>Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$</u>:

$\mu_{\hat{P}_2 - \hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$

$\sigma_{\hat{P}_2 - \hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$

$\sigma_{\hat{P}_2 - \hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

<u>Sampling distribution of $(\bar{X}_1 - \bar{X}_2)$, independent samples</u>:

$\mu_{\bar{X}_1 - \bar{X}_2} = E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_1 - \bar{X}_2}^2 = V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$\sigma_{\bar{X}_1 - \bar{X}_2} = SD[\bar{X}_1 - \bar{X}_2] = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

<u>Sampling distribution of $(\bar{X}_d)$, paired $(d = X_1 - X_2)$</u>:

$\mu_{\bar{X}_d} = E[\bar{X}_d] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_d}^2 = V[\bar{X}_d] = \frac{\sigma_d^2}{n} = \frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}$

$\sigma_{\bar{X}_d} = SD[\bar{X}_d] = \frac{\sigma_d}{\sqrt{n}} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}}$

Inference about a population proportion:

***z* test statistic:** $z = \frac{\hat{P}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ **CI estimator:** $\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

Inference about comparing two population proportions:

***z* test statistic under Null hypothesis of no difference:** $z = \frac{\hat{P}_2-\hat{P}_1}{\sqrt{\frac{\bar{P}(1-\bar{P})}{n_1}+\frac{\bar{P}(1-\bar{P})}{n_2}}}$ **Pooled proportion:** $\bar{P} = \frac{X_1+X_2}{n_1+n_2}$

CI estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

Inference about the population mean:

***t* test statistic:** $t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$ **CI estimator:** $\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

Inference about a comparing two population means, independent samples, unequal variances:

***t* test statistic:** $t = \frac{(\bar{X}_1-\bar{X}_2)-\Delta_0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Degrees of freedom: $\nu = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2+\frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$

Inference about a comparing two population means, independent samples, assuming equal variances:

***t* test statistic:** $t = \frac{(\bar{X}_1-\bar{X}_2)-\Delta_0}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$ **Degrees of freedom:** $\nu = n_1 + n_2 - 2$

Pooled variance: $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

Inference about a comparing two population means, paired data: (n is number of pairs and $d = X_1 - X_2$)

***t* test statistic:** $t = \frac{\bar{d}-\Delta_0}{s_d/\sqrt{n}}$ **CI estimator:** $\bar{X}_d \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

SIMPLE REGRESSION:

Model: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ **OLS line:** $\hat{y}_i = b_0 + b_1 x_i$ $b_1 = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x}$ $b_0 = \bar{Y} - b_1\bar{X}$

Coefficient of determination: $R^2 = (r)^2$ **Residuals:** $e_i = y_i - \hat{y}_i$

Standard deviation of residuals: $s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i-0)^2}{n-2}}$ **Standard error of slope:** $s.e.(b_1) = s_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$

Inference about the population slope:

***t* test statistic:** $t = \frac{b_1 - \beta_{10}}{s.e.(b_1)}$ **CI estimator:** $b_1 \pm t_{\alpha/2} s.e.(b_1)$ **Degrees of freedom:** $\nu = n - 2$

Standard error of slope: $s.e.(b_1) = s_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$

Prediction interval for y at given value of x (x_g):

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \textbf{or} \quad \hat{y}_{x_g} \pm t_{\alpha/2} \sqrt{(s.e.(b_1))^2 (x_g - \bar{X})^2 + \frac{s_e^2}{n} + s_e^2}$$

Degrees of freedom: $\nu = n - 2$

Confidence interval for predicted mean at given value of x (x_g):

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{\frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \textbf{or} \quad \hat{y}_{x_g} \pm t_{\alpha/2} \sqrt{(s.e.(b_1))^2 (x_g - \bar{X})^2 + \frac{s_e^2}{n}}$$

Degrees of freedom: $\nu = n - 2$

SIMPLE & MULTIPLE REGRESSION:

Model: $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + \varepsilon_i$

$$SST = \sum_{i=1}^{n} (y_i - \bar{Y})^2 = SSR + SSE \quad SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{Y})^2 \quad SSE = \sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$s_y^2 = \frac{SST}{n-1} \quad MSE = \frac{SSE}{n-k-1} \quad Root\ MSE = \sqrt{\frac{SSE}{n-k-1}} \quad MSR = \frac{SSR}{k}$$

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \quad Adj.R^2 = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)} = \left(R^2 - \frac{k}{n-1}\right)\left(\frac{n-1}{n-k-1}\right)$$

Residuals: $e_i = y_i - \hat{y}_i$ **Standard deviation of residuals:** $s_e = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i - 0)^2}{n-k-1}}$

Inference about the overall statistical significance of the regression model:

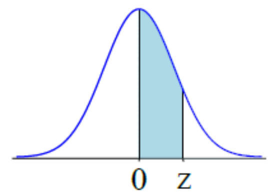
$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)} = \frac{(SST-SSE)/k}{SSE/(n-k-1)} = \frac{SSR/k}{SSE/(n-k-1)} = \frac{MSR}{MSE}$$

Numerator degrees of freedom: $\nu_1 = k$ **Denominator degrees of freedom:** $\nu_2 = n - k - 1$

Inference about the population slope for explanatory variable j:

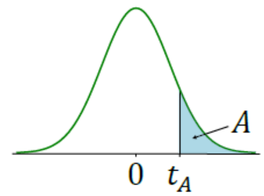
***t* test statistic:** $t = \frac{b_j - \beta_{j0}}{s_{b_j}}$ **CI estimator:** $b_j \pm t_{\alpha/2} s_{b_j}$ **Degrees of freedom:** $\nu = n - k - 1$

Standard error of slope: $s.e.(b_j) = s_{b_j}$ (for multiple regression, must be obtained from technology)



Normal Probabilities:

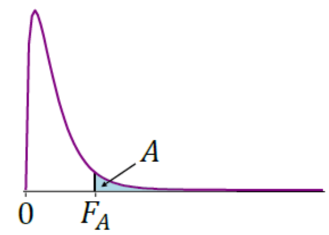
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999



Critical Values of t:

ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
1	3.078	6.314	12.706	31.821	63.657	38	1.304	1.686	2.024	2.429	2.712
2	1.886	2.920	4.303	6.965	9.925	39	1.304	1.685	2.023	2.426	2.708
3	1.638	2.353	3.182	4.541	5.841	40	1.303	1.684	2.021	2.423	2.704
4	1.533	2.132	2.776	3.747	4.604	41	1.303	1.683	2.020	2.421	2.701
5	1.476	2.015	2.571	3.365	4.032	42	1.302	1.682	2.018	2.418	2.698
6	1.440	1.943	2.447	3.143	3.707	43	1.302	1.681	2.017	2.416	2.695
7	1.415	1.895	2.365	2.998	3.499	44	1.301	1.680	2.015	2.414	2.692
8	1.397	1.860	2.306	2.896	3.355	45	1.301	1.679	2.014	2.412	2.690
9	1.383	1.833	2.262	2.821	3.250	46	1.300	1.679	2.013	2.410	2.687
10	1.372	1.812	2.228	2.764	3.169	47	1.300	1.678	2.012	2.408	2.685
11	1.363	1.796	2.201	2.718	3.106	48	1.299	1.677	2.011	2.407	2.682
12	1.356	1.782	2.179	2.681	3.055	49	1.299	1.677	2.010	2.405	2.680
13	1.350	1.771	2.160	2.650	3.012	50	1.299	1.676	2.009	2.403	2.678
14	1.345	1.761	2.145	2.624	2.977	51	1.298	1.675	2.008	2.402	2.676
15	1.341	1.753	2.131	2.602	2.947	52	1.298	1.675	2.007	2.400	2.674
16	1.337	1.746	2.120	2.583	2.921	53	1.298	1.674	2.006	2.399	2.672
17	1.333	1.740	2.110	2.567	2.898	54	1.297	1.674	2.005	2.397	2.670
18	1.330	1.734	2.101	2.552	2.878	55	1.297	1.673	2.004	2.396	2.668
19	1.328	1.729	2.093	2.539	2.861	60	1.296	1.671	2.000	2.390	2.660
20	1.325	1.725	2.086	2.528	2.845	65	1.295	1.669	1.997	2.385	2.654
21	1.323	1.721	2.080	2.518	2.831	70	1.294	1.667	1.994	2.381	2.648
22	1.321	1.717	2.074	2.508	2.819	75	1.293	1.665	1.992	2.377	2.643
23	1.319	1.714	2.069	2.500	2.807	80	1.292	1.664	1.990	2.374	2.639
24	1.318	1.711	2.064	2.492	2.797	90	1.291	1.662	1.987	2.368	2.632
25	1.316	1.708	2.060	2.485	2.787	100	1.290	1.660	1.984	2.364	2.626
26	1.315	1.706	2.056	2.479	2.779	120	1.289	1.658	1.980	2.358	2.617
27	1.314	1.703	2.052	2.473	2.771	140	1.288	1.656	1.977	2.353	2.611
28	1.313	1.701	2.048	2.467	2.763	160	1.287	1.654	1.975	2.350	2.607
29	1.311	1.699	2.045	2.462	2.756	180	1.286	1.653	1.973	2.347	2.603
30	1.310	1.697	2.042	2.457	2.750	200	1.286	1.653	1.972	2.345	2.601
31	1.309	1.696	2.040	2.453	2.744	250	1.285	1.651	1.969	2.341	2.596
32	1.309	1.694	2.037	2.449	2.738	300	1.284	1.650	1.968	2.339	2.592
33	1.308	1.692	2.035	2.445	2.733	400	1.284	1.649	1.966	2.336	2.588
34	1.307	1.691	2.032	2.441	2.728	500	1.283	1.648	1.965	2.334	2.586
35	1.306	1.690	2.030	2.438	2.724	750	1.283	1.647	1.963	2.331	2.582
36	1.306	1.688	2.028	2.434	2.719	1000	1.282	1.646	1.962	2.330	2.581
37	1.305	1.687	2.026	2.431	2.715	∞	1.282	1.645	1.960	2.326	2.576

Degrees of freedom: ν



Critical Values of F: $A = 0.10$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	11	12
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.91	1.89
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.74	1.71
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.63	1.60
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55

Critical Values of F: $A = 0.05$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	11	12
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75

Critical Values of F: $A = 0.01$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	11	12
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.96	9.89
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

Numerator degrees of freedom: ν_1; Denominator degrees of freedom: ν_2