# Merger Simulation: Bertand with Linear Demand 

Workshop 2

## U.S. 2010 HMG, p. 20

"In differentiated product industries, some products can be very close substitutes and compete strongly with each other, while other products are more distant substitutes and compete less strongly. For example, one high-end product may compete much more directly with another high-end product than with any lowend product."
"A merger between firms selling differentiated products may diminish competition by enabling the merged firm to profit by unilaterally raising the price of one or both products above the pre-merger level. Some of the sales lost due to the price rise will merely be diverted to the product of the merger partner and, depending on relative margins, capturing such sales loss through merger may make the price increase profitable even though it would not have been profitable prior to the merger."

## Differentiated Goods Mergers

## - Focus is on closeness of substitutes

- Mergers among firms that produce close substitutes cause greater harm
- Market shares and concentration (e.g. HHI or CR4) not informative
- Demand estimated and used as input
- With merger simulation tools, explore how varying the substitution patterns and identity of merging firms affects post-merger equilibrium


## Linear, Differentiated Goods Bertrand <br> Model: Basis for Merger Simulation

- Linear system of demand equations:

$$
\begin{aligned}
& q_{1}=a_{1}-b p_{1}+d p_{2}+e p_{3} \\
& q_{2}=a_{2}+d p_{1}-f p_{2}+g p_{3} \\
& q_{3}=a_{3}+e p_{1}+g p_{2}-h p_{3}
\end{aligned}
$$

- Constant marginal costs (can differ): $c_{i}$
- 3 Bertrand firms: Firms 1 \& 2 propose merging
- Possible cost efficiencies from merger:
- Post-merger marginal costs $=e_{1} c_{1}$ and $=e_{2} c_{2}$

See Sections 23.2.1 and 23.8 of C-W

Firms' Profit Functions

$$
\begin{aligned}
& \pi_{1}=\left(p_{1}-c_{1}\right) q_{1}=\left(p_{1}-c_{1}\right)\left(a_{1}-b p_{1}+d p_{2}+e p_{3}\right) \\
& \pi_{2}=\left(p_{2}-c_{2}\right) q_{2}=\left(p_{2}-c_{2}\right)\left(a_{2}+d p_{1}-f p_{2}+g p_{3}\right) \\
& \pi_{3}=\left(p_{3}-c_{3}\right) q_{3}=\left(p_{3}-c_{3}\right)\left(a_{3}+e p_{1}+g p_{2}-h p_{3}\right)
\end{aligned}
$$

Firms' Pre-Merger FOCs

$$
\begin{aligned}
& -2 b p_{1}+d p_{2}+e p_{3}=-a_{1}-b c_{1} \\
& d p_{1}-2 f p_{2}+g p_{3}=-a_{2}-f c_{2} \\
& e p_{1}+g p_{2}-2 h p_{3}=-a_{3}-h c_{3}
\end{aligned}
$$

What are the unknowns?
How to solve?

## Merged Firm Maximizes Profits

$$
\begin{aligned}
\pi_{1 \& 2}= & \left(p_{1}-e_{1} c_{1}\right) q_{1}+\left(p_{2}-e_{2} c_{2}\right) q_{2} \\
\pi_{1 \& 2}= & \left(p_{1}-e_{1} c_{1}\right)\left(a_{1}-b p_{1}+d p_{2}+e p_{3}\right)+ \\
& \left(p_{2}-e_{2} c_{2}\right)\left(a_{2}+d p_{1}-f p_{2}+g p_{3}\right) \\
\frac{\partial \pi_{1 \& 2}}{\partial p_{1}}= & a_{1}-b p_{1}+d p_{2}+e p_{3}-b\left(p_{1}-e_{1} c_{1}\right)+d\left(p_{2}-e_{2} c_{2}\right) \stackrel{\text { set }}{=} 0 \\
\frac{\partial \pi_{1 \& 2}}{\partial p_{2}}= & d\left(p_{1}-e_{1} c_{1}\right)+a_{2}+d p_{1}-f p_{2}+g p_{3}-f\left(p_{2}-e_{2} c_{2}\right) \stackrel{\text { set }}{=} 0
\end{aligned}
$$

The merged firm has two FOCs?
Takes into account the substitutability of goods 1 and 2?

## Firms' Post-Merger FOCs

$$
\begin{aligned}
-2 b p_{1}+2 d p_{2}+e p_{3} & =-a_{1}-b e_{1} c_{1}+d e_{2} c_{2} \\
2 d p_{1}-2 f p_{2}+g p_{3} & =-a_{2}-f e_{2} c_{2}+d e_{1} c_{1} \\
e p_{1}+g p_{2}-2 h p_{3} & =-a_{3}-h c_{3}
\end{aligned}
$$

What are the unknowns?
How to solve?

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## Bertrand Merger Simulation Spreadsheets

- Interactive work with two different versions of the Bertrand Merger Simulation workbooks
- Demand parameters and pre-merger prices are the inputs
- Demand parameters and pre-merger marginal costs are the inputs

