# Merger Simulation: Cournot with Linear Demand 

Workshop 1

## Merger Simulation Methods

- "Where sufficient data are available, the Agencies may construct economic models designed to quantify the unilateral price effects resulting from the merger. These models often include independent price responses by non-merging firms. They also can incorporate merger-specific efficiencies." (p. 21, U.S. 2010 HMG)


## Symmetric Cournot

- The symmetric Cournot model has appealing and simple results, no need for software
- For example, relationship between market power and number of firms: $\frac{P(Q)-c}{P(Q)}=-\frac{1}{N \varepsilon}$
- If you assume a linear demand, it is easy to solve for the market price and quantity even if there are many Cournot competitors (using symmetry)
- $Q^{*}=\frac{N}{N+1} \frac{a-c}{b}$ and $P^{*}=\frac{a+N c}{N+1}$ where $N$ is \# of firms


## Symmetric Cournot: Highly Limited

Werden and Froeb (2011) "Courts in antitrust cases occasionally have excluded economic testimony because it was premised on a model not sufficiently grounded in the facts of the case. The leading example is Concord Boat, in which the appeals court declared that an expert's economic model "should not be admitted if it does not apply to the specific facts of the case" and excluded the opinions on which the plaintiff's case rested because they were based on a symmetric Cournot model when the competing firms were highly asymmetric in both their sales and the appeal of their products to customers." p. 26
[Note: This article, "Choosing Among Tools for Assessing Unilateral Merger Effects," is supplemental reading for Class 4.]

## Concentration: Beyond $N$

- In a symmetric industry, the number of firms $(N)$ tells concentration
- Asymmetric markets?

- Herfindahl-Hirschman Index (HHI):

$$
H H I=\sum_{i=1}^{N} s_{i}^{2}
$$

- E.g. $H H I=39^{2}+$ $26^{2}+7^{2}+6^{2}=2,282$ - Why didn't add $22^{2}$ ?
- Minimum HHI possible? Maximum?


## Market Concentration:

Measurement Issues

- Using sales, two ways to measure shares:

$$
s_{j}=\frac{q_{j}}{Q} \quad \text { or } \quad s_{j}=\frac{p_{j} q_{j}}{\sum_{i=1}^{N} p_{i} q_{i}}
$$

- Pie: "market shares of U.S. beer sales, by dollars"
- Will it makes a difference in Cournot?
- Also, may measure capacity shares
- Four-firm concentration ratio: $C R 4=\sum_{i=1}^{4} s_{i}$ for four largest firms


## Cournot Made HHI Famous

- With symmetry: $H H I=\sum_{i=1}^{N} s_{i}^{2}=\sum_{i=1}^{N}\left(\frac{1}{N}\right)^{2}=\frac{1}{N}$
- Class 2 found $L=-\frac{1}{N \varepsilon}$
- Rewrite as $L=-\frac{H H I}{\varepsilon}$
- With asymmetry found $L_{j}=-\frac{s_{j}}{\varepsilon}$
- Find weighted average Lerner Index:
$L_{\text {w.a. }}=\sum_{i=1}^{N} s_{i} \frac{P-c_{i}}{P}=-\sum_{i=1}^{N} s_{i} \frac{s_{i}}{\varepsilon}=-\frac{1}{\varepsilon} \sum_{i=1}^{N} s_{i}^{2}=-\frac{H H I}{\varepsilon}$


## Linear Cournot Model: <br> Basis for Merger Simulation

- Market demand: $P=a-b Q$
- Constant marginal costs (can differ): $c_{i}$
- 5 Cournot competitors in the market
- Firms 1 and 2 propose merging
- Possible cost efficiencies from merger:
- Post-merger marginal costs $=e c_{1}$
- Range of values for $e$ ? Interpretation?


## Firms' Profit Functions

$$
\begin{aligned}
& \pi_{1}=\left(P-c_{1}\right) q_{1}=\left(a-b q_{1}-b q_{2}-b q_{3}-b q_{4}-b q_{5}-c_{1}\right) q_{1} \\
& \pi_{2}=\left(P-c_{2}\right) q_{2}=\left(a-b q_{1}-b q_{2}-b q_{3}-b q_{4}-b q_{5}-c_{2}\right) q_{2} \\
& \pi_{3}=\left(P-c_{3}\right) q_{3}=\left(a-b q_{1}-b q_{2}-b q_{3}-b q_{4}-b q_{5}-c_{3}\right) q_{3} \\
& \pi_{4}=\left(P-c_{4}\right) q_{4}=\left(a-b q_{1}-b q_{2}-b q_{3}-b q_{4}-b q_{5}-c_{4}\right) q_{4} \\
& \pi_{5}=\left(P-c_{5}\right) q_{5}=\left(a-b q_{1}-b q_{2}-b q_{3}-b q_{4}-b q_{5}-c_{5}\right) q_{5}
\end{aligned}
$$

## Firms' Pre-Merger FOCs

$$
\begin{aligned}
& 2 b q_{1}+b q_{2}+b q_{3}+b q_{4}+b q_{5}=a-c_{1} \\
& b q_{1}+2 b q_{2}+b q_{3}+b q_{4}+b q_{5}=a-c_{2} \\
& b q_{1}+b q_{2}+2 b q_{3}+b q_{4}+b q_{5}=a-c_{3} \\
& b q_{1}+b q_{2}+b q_{3}+2 b q_{4}+b q_{5}=a-c_{4} \\
& b q_{1}+b q_{2}+b q_{3}+b q_{4}+2 b q_{5}=a-c_{5}
\end{aligned}
$$

What are the unknowns?
How to solve?

## Firms' Post-Merger FOCs

$$
\begin{aligned}
& 2 b q_{1 \& 2}+b q_{3}+b q_{4}+b q_{5}=a-e c_{1} \\
& b q_{1 \& 2}+2 b q_{3}+b q_{4}+b q_{5}=a-c_{3} \\
& b q_{1 \& 2}+b q_{3}+2 b q_{4}+b q_{5}=a-c_{4} \\
& b q_{1 \& 2}+b q_{3}+b q_{4}+2 b q_{5}=a-c_{5}
\end{aligned}
$$

## Cournot Merger Simulation Spreadsheets

- Interactive work with two different versions of the Cournot Merger Simulation workbooks
- Demand parameters and pre-merger quantities are the inputs
- Demand parameters and pre-merger marginal costs are the inputs

