Classic Oligopoly Models: Bertrand and Cournot

Class 2

Note: There are supplemental readings, including Werden (2008) "Unilateral Competitive Effects of Horizontal Mergers I: Basic Concepts and Models," that complement this lecture

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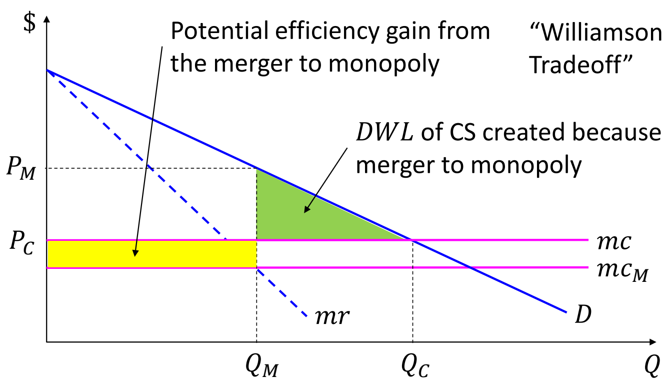

Ashenfelter et al (2013) "In June of 2008 the U.S. Department of Justice approved a joint venture between Miller and Coors, the second and third largest firms in the industry. Despite substantially increasing concentration in an already concentrated industry, the merger was approved by the antitrust authority partially because it was expected to reduce shipping and distribution costs." p. 2

http://www.economist.com/news/business/21665074-ab-inbev-may-combine-sabmiller-flat-market-big-beer-brands-beer-monster

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Oligopoly (a few firms) & Game Theory

- Unlike monopoly, *profit interdependency* for oligopoly: a firm's profits depend on its own behavior and its rivals'; Use game theory
  - Eg: Cellular telephone service in Toronto:
    - Players: Rogers (Fido), Bell, Telus, Wind, Mobilicity
    - Compete on price, services, and advertising
  - To measure market power, add i or j subscript:

$$Lerner\ Index_j = \frac{p_j - mc_j}{p_j}$$

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Basic Elements & Assumptions

- Noncooperative Game Theory: Each player acts in its own self-interest and cannot credibly commit to do otherwise
  - Allows collusion (cartels)
  - Players rational: seek best payoffs
  - ECO316H, ECO326H
- A strategic game:
  - Players
  - Rules: Timing of moves, available actions, info
    - Assume each firm knows *its* profits & rivals' profits for all outcomes
  - Outcomes: Results of all possible actions combos
  - Payoffs: Preferences over outcomes

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Number of Time Periods

- Static game: One period and all players move simultaneously (long run)
- Extensive game (multi-period): Multiple period game where players move sequentially and maybe more than once
- Dynamic game (repeated game): Infinite period game where players repeatedly interact over time

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Two Symmetric 2 by 2 Normal Form Games: Discrete Actions

"Cournot"	Firm 2: $q_2 = 9$	Firm 2: $q_2 = 12$
Firm 1: $q_1 = 9$	$(\$405, \$405)$	$(\$337.50, \$450)$
Firm 1: $q_1 = 12$	$(\$450, \$337.50)$	$(\$360, \$360)$

Each cell shows: (π_1, π_2)

"Bertrand"	Firm 2: $p_2 = \$20$	Firm 2: $p_2 = \$25$
Firm 1: $p_1 = \$20$	$(\$500, \$500)$	$(\$625, \$375)$
Firm 1: $p_1 = \$25$	$(\$375, \$625)$	$(\$562.50, \$562.50)$

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Best Response and NE

- Best response: A strategy s_i is a best response for firm i to its rivals' strategies s_{-i} if for all s_i':
  $\pi_i(s_i, s_{-i}) \geq \pi_i(s_i', s_{-i})$
  - Given the rivals' strategies, a firm could do no better than playing its best response

- Nash equilibrium (NE): A strategy profile $s^* \, (s_1^*, s_2^*, \ldots, s_n^*)$ is a NE if for all s_i' and for all players i:
  $\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i', s_{-i}^*)$
  - Given the rivals' strategies, no one firm has an incentive to change its strategy

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Finding NE (if exist, may be multiple)

- For discrete actions, find NE by checking every possible combination of strategies and asking if any firm would want to change its strategy
  - If no change desired for any firm, then have a NE
    - Otherwise, do not have a NE
- For continuous actions, find NE by looking for intersection of best response functions
  - Each firm playing a best response *given* the other firms' strategies: means none wish to change

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Example: Find any and all NE

	Firm 2: $p_2 = \$5$	Firm 2: $p_2 = \$6$	Firm 2: $p_2 = \$7$
Firm 1: $p_1 = \$9$	$(\$119, \$14)$	$(\$126, \$18)$	$(\$133, \$12)$
Firm 1: $p_1 = \$10$	$(\$120, \$15)$	$(\$128, \$20)$	$(\$136, \$15)$
Firm 1: $p_1 = \$11$	$(\$117, \$16)$	$(\$126, \$22)$	$(\$135, \$18)$

How about in Slide 6?

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Initial Oligopoly Assumptions

- Consumers are price takers
- Market demand is downward sloping
- Each firm has a single choice variable
- Homogeneous products [note: will be relaxed]
- No capacity constraints
- Market equilibrium is static NE
  - No entry, exit, product repositioning, etc.

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In Class Exercise

- Teams complete a worksheet
- Market demand is P = 10 – 0.5Q
- Each firm sets its <u>price</u> to the nearest 10 cents
  - E.g. $2.60 but not $2.65
- Marginal costs are $1; there are no fixed costs
- Homogenous products
  - Consumers buy from firm with lowest price: any higher-priced firm sells zero units
  - If any firms are tied for lowest price, sales divided evenly
- Deadline: 5 minutes
- Publically reveal results

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Illustrate the Bertrand Paradox

- Bertrand duopoly, homogenous goods
  - Price is continuous: any price possible
  - Costs: $C_1(q_1) = cq_1$ and $C_2(q_2) = cq_2$
  - What is NE in Bertrand game: (p_1^*, p_2^*)?
    - Is each firm playing its best response?
    - Profits at NE? Both firms better at (p^M, p^M)?
    - What is the best response if rival plays p^M?
  - If $C_1(q_1) = F_1 + cq_1$ and $C_2(q_2) = F_2 + cq_2$?
  - If $C_1(q_1) = c_1 q_1$ and $C_2(q_2) = c_2 q_2$ if $c_1 < c_2$?

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Escape the Paradox

- But no paradox if goods differentiated, capacity constrained, interactions repeated
  - E.g. Linear differentiated goods Bertrand model
    - $q_1 = a_1 - b_{11}p_1 + b_{12}p_2$
    - $q_2 = a_2 - b_{22}p_2 + b_{21}p_1$
    - Costs: $C_1(q_1) = c_1 q_1$ and $C_2(q_2) = c_2 q_2$
    - Why allow for asymmetric marginal costs?
  - Before working with the linear model analytically and graphically, consider a general model

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$$\pi_j(p_1, \dots, p_N) = p_j q_j(p_1, \dots, p_N) - C_j(q_j)$$

$$\frac{\partial \pi_j}{\partial p_j} = q_j(p_1, \dots, p_N) + (p_j - mc_j)\frac{\partial q_j(p_1, \dots, p_N)}{\partial p_j} \overset{set}{=} 0$$

$$(p_j - mc_j)\frac{\partial q_j}{\partial p_j} = -q_j$$

$$\frac{p_j - mc_j}{p_j} = -\frac{1}{\frac{\partial q_j}{\partial p_j}\frac{p_j}{q_j}}$$

$$\frac{p_j - mc_j}{p_j} = -\frac{1}{\varepsilon_j}$$

What is name of LHS?

Do rival firms' prices affect Firm j's price?

In general, is ε_j constant? If not, what could affect it?

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Find Bertrand NE: $(p_1^*, \dots, p_N^*)$

- Start with Firm i's profit function: $\pi_i(p_1, \dots, p_N) = TR_i(p_1, \dots, p_N) - TC_i(p_1, \dots, p_N)$
- Take partial derivative of $\pi_i(p_1, \dots, p_N)$ wrt choice variable p_i and set equal to 0
- Solve for p_i: Firm i's best response function
- Repeat for other firms (or use symmetry)
- Find intersection of all best response functions, which is the NE

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Firm 1's Best Response Function, Linear Bertrand Duopoly Model

$$\pi_1(p_1, p_2) = TR_1(p_1, p_2) - TC_1(p_1, p_2)$$

$$\pi_1(p_1, p_2) = p_1 q_1(p_1, p_2) - c_1 q_1(p_1, p_2)$$

$$\pi_1(p_1, p_2) = (p_1 - c_1)(a_1 - b_{11}p_1 + b_{12}p_2)$$

$$\frac{\partial \pi_1}{\partial p_1} = a_1 - b_{11}p_1 + b_{12}p_2 - b_{11}(p_1 - c_1) \overset{set}{=} 0$$

$$a_1 - 2b_{11}p_1 + b_{12}p_2 + b_{11}c_1 = 0$$

$$p_1 = \frac{a_1 + b_{11}c_1 + b_{12}p_2}{2b_{11}}$$

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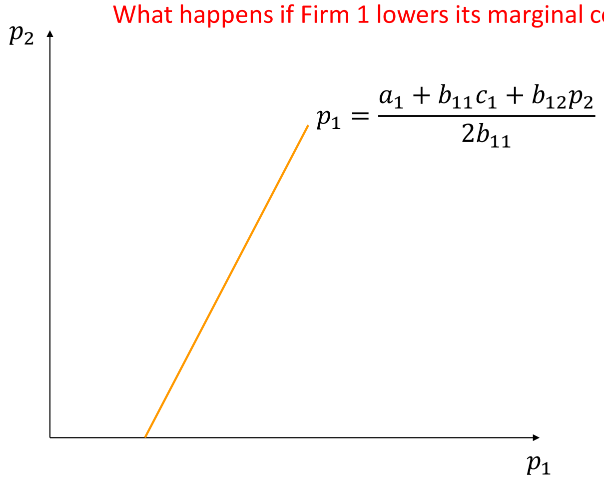
What happens if Firm 1 lowers its marginal costs?



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Firm 2's Best Response Function

$$\pi_2(p_1, p_2) = TR_2(p_1, p_2) - TC_2(p_1, p_2)$$

$$\pi_2(p_1, p_2) = p_2 q_2(p_1, p_2) - c_2 q_2(p_1, p_2)$$

$$\pi_2(p_1, p_2) = (p_2 - c_2)(a_2 - b_{22}p_2 + b_{21}p_1)$$

$$\frac{\partial \pi_2}{\partial p_2} = a_2 - b_{22}p_2 + b_{21}p_1 - b_{22}(p_2 - c_2) \overset{set}{=} 0$$

$$a_2 - 2b_{22}p_2 + b_{21}p_1 + b_{22}c_2 = 0$$

$$p_2 = \frac{a_2 + b_{22}c_2 + b_{21}p_1}{2b_{22}}$$

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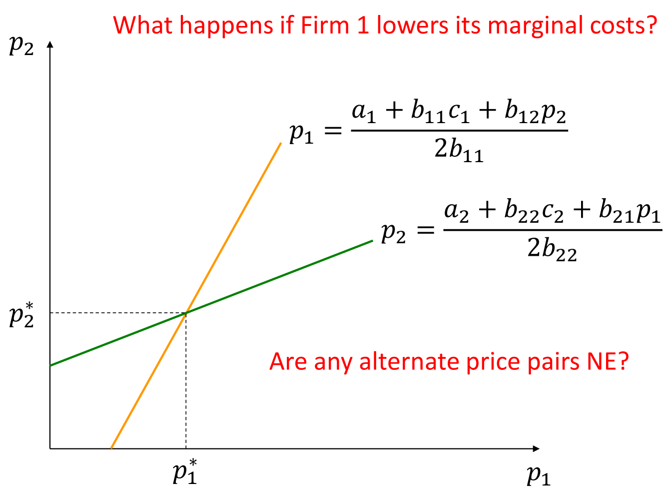

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Classic Cournot Model

- Firms choose quantity (continuous, non-neg.)
- Use initial oligopoly assumptions
  - Including homogeneous goods: $Q = \sum_{j=1}^{N} q_j$
- $\pi_i(q_1, \ldots, q_N) = P(Q)q_i - C_i(q_i)$
- First solve general case, then explore linear Cournot model:
  - Market demand: $P = a - bQ$
  - Costs: $C_1(q_1) = c_1 q_1$ and $C_2(q_2) = c_2 q_2$

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$$\pi_j(q_1, \ldots, q_N) = P(Q)q_j - C_j(q_j)$$

$$\frac{\partial \pi_j}{\partial q_j} = P(Q) + \frac{\partial P(Q)}{\partial Q} q_j - mc_j \overset{set}{=} 0$$

$$P(Q) - mc_j = -\frac{\partial P(Q)}{\partial Q} q_j$$

$$\frac{P(Q) - mc_j}{P(Q)} = -\frac{1}{\frac{\partial Q}{\partial P(Q)} \frac{P(Q)}{Q}} \frac{q_j}{Q}$$

$$\frac{P(Q) - mc_j}{P(Q)} = -\frac{s_j}{\varepsilon}$$

If symmetric ($mc_j = c$ for all j) then obtain:

$$\frac{P(Q) - c}{P(Q)} = -\frac{1}{N\varepsilon}$$

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Find Cournot NE: $(q_1^*, \dots, q_N^*)$

- Firm i's profit function:
  $$\pi_i(q_1, \dots, q_N) = P(Q)q_i - C_i(q_i)$$
- Take partial derivative of $\pi_i(q_1, \dots, q_N)$ wrt choice variable q_i and set equal to 0
- Solve for q_i: Firm i's best response function
- Repeat for other firms (or use symmetry)
- Find intersection of all best response functions, which is the NE

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Linear Cournot Duopoly Model

Firm 1's best response function:	Firm 2's best response function:
$\pi_1(q_1, q_2) = P(Q)q_1 - C_1(q_1)$	$\pi_2(q_1, q_2) = P(Q)q_2 - C_2(q_2)$
$\pi_1 = (a - bQ - c_1)q_1$	$\pi_2 = (a - bQ - c_2)q_2$
$\pi_1 = (a - bq_1 - bq_2 - c_1)q_1$	$\pi_2 = (a - bq_1 - bq_2 - c_2)q_2$
$\frac{\partial \pi_1}{\partial q_1} = a - bq_1 - bq_2 - c_1 - bq_1$	$\frac{\partial \pi_2}{\partial q_2} = a - bq_1 - bq_2 - c_2 - bq_2$
$a - 2bq_1 - bq_2 - c_1 \overset{set}{=} 0$	$a - 2bq_2 - bq_1 - c_2 \overset{set}{=} 0$
$q_1 = \frac{a - c_1 - bq_2}{2b}$	$q_2 = \frac{a - c_2 - bq_1}{2b}$

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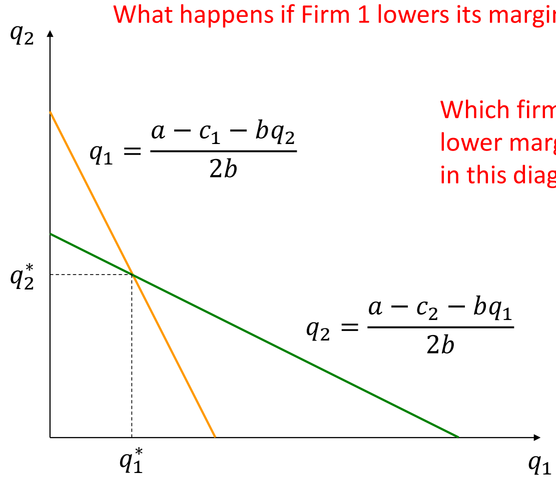
What happens if Firm 1 lowers its marginal costs?



Which firm has lower marginal costs in this diagram?

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Solve for NE Algebraically

$$q_1 = \frac{a - c_1 - bq_2}{2b} \qquad q_2 = \frac{a - c_2 - bq_1}{2b}$$

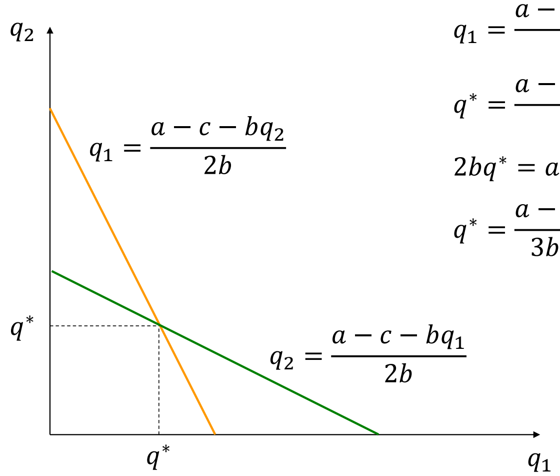
$$q_1 = \frac{a - c_1 - b\frac{a - c_2 - bq_1}{2b}}{2b}$$

$$2bq_1 = \frac{a}{2} - c_1 + \frac{c_2}{2} + \frac{b}{2}q_1$$

$$4bq_1 = a - 2c_1 + c_2 + bq_1$$

$$q_1^* = \frac{a - 2c_1 + c_2}{3b} \qquad q_2^* = \frac{a - 2c_2 + c_1}{3b}$$

Symmetric Cournot Duopoly: A Special Case



$$q_1 = \frac{a - c_1 - bq_2}{2b}$$

$$q^* = \frac{a - c - bq^*}{2b}$$

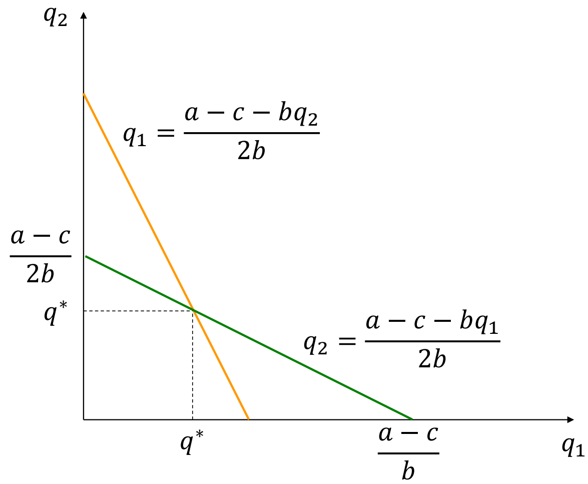
$$2bq^* = a - c - bq^*$$

$$q^* = \frac{a - c}{3b}$$

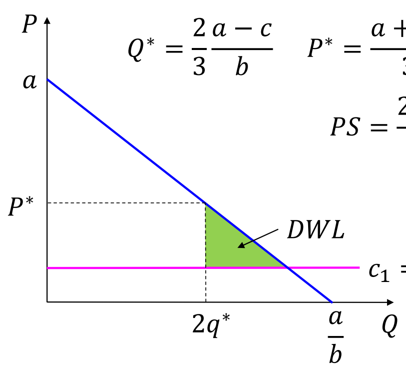
Linear Demand: Comparing Outcomes

- $P = a - bQ$
- Symmetric ($c_1 = c_2 = c$) Cournot market output, $Q = q_1 + q_2$, is between that of a perfectly competitive market and a monopoly

Market Structure	Market Output (Q)
Perfect Competition	$\frac{a - c}{b}$
Cournot Duopoly	$\frac{2}{3}\frac{a - c}{b}$
Monopoly	$\frac{1}{2}\frac{a - c}{b}$



Welfare, Efficiency of Symmetric Cournot Duopoly



$$Q^* = \frac{2}{3}\frac{a-c}{b} \qquad P^* = \frac{a+2c}{3} \qquad \pi_1 = \pi_2 = \frac{(a-c)^2}{9b}$$

$$PS = \frac{2(a-c)^2}{9b} \qquad CS = \frac{2(a-c)^2}{9b}$$

$$TS = \frac{4(a-c)^2}{9b}$$

$$DWL = \frac{(a-c)^2}{18b}$$

Three Firms in Cournot Industry

$$\pi_1(q_1, q_2, q_3) = P(Q)q_1 - C_1(q_1)$$

$$\pi_1 = (a - bQ - c_1)q_1$$

$$\pi_1 = (a - bq_1 - bq_2 - bq_3 - c_1)q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = a - bq_1 - bq_2 - bq_3 - c_1 - bq_1$$

$$a - 2bq_1 - bq_2 - bq_3 - c_1 \overset{set}{=} 0$$

$$q_1 = \frac{a - c_1 - bq_2 - bq_3}{2b}$$

However, if symmetric:

$$q^* = \frac{a - c - bq^* - bq^*}{2b}$$

$$q^* = \frac{a-c}{4b}$$

$$Q^* = \frac{3}{4}\frac{a-c}{b}$$

Bertrand versus Cournot

- Bertrand: Firms choose price
  - Output (capacity) more flexible than price
    - Strategic variable is price
    - Output adjusted to clear the market: meet demand at chosen price
  - E.g. software, streaming movies (Netflix)
- Cournot: Firms choose quantity
  - Price more flexible than output (capacity)
    - Strategic variable is quantity
    - Price adjusted to clear the market
  - E.g. commodities (milk, wheat, chemicals, steel, etc.)

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Slope of Best Response

- Strategic substitutes: Best response functions have negative slope
  - Cournot, linear demand, constant mc
- Strategic complements: Best response functions have positive slope
  - Bertrand, linear demand (differentiated goods), constant mc
- Implications for merger analysis

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Looking Ahead

- Workshop 1, Tuesday, 11:10 – 1:00, this room
  - Bring a laptop with Excel
  - Download the merger simulation spreadsheets from our course site ahead of time
    - http://homes.chass.utoronto.ca/~murdockj/eco410/
  - Interactive parts and there are also slides posted
- Also, Assignment #1 given on Tuesday

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