## ECO410H: Practice Questions 2 - SOLUTIONS

1. (a) The unique Nash equilibrium strategy profile is $s^{*}=(M, M)$.
(b) The unique Nash equilibrium strategy profile is $s^{*}=(R 4, C 3)$.
(c) The two Nash equilibria are $(D, L)$ and $(M, M)$.
2. $a>1$ and $b<2$
3. (a) Each firm will set a price of $\$ 4$ and their profits will be 32 each.
(b) No. The Bertrand paradox is the prediction that if two Bertrand competitors sell homogeneous goods then the equilibrium will be price equals marginal cost: same as in perfect competition despite the fact that this is highly concentrated oligopoly industry. This example seems to relate to differentiated goods, which would explain why a firm does not lose all of its sales when it unilaterally raises its price.
4. This is a discrete problem, which means we cannot use calculus to solve it. Instead we set up the normal form of the game populated by the calculated profits of the firms and find the NE.

$$
\begin{gathered}
\pi_{1}=T R-T C \\
\pi_{1}=q_{1}\left(50-0.05 q_{1}-0.05 q_{2}\right)-50-10 q_{1} \\
\pi_{1}=q_{1}\left(40-0.05\left(q_{1}+q_{2}\right)\right)-50
\end{gathered}
$$

Find profits of Firm 2 by symmetry. Fill in the normal form of the game by calculating profits for each firm in the nine possible contingencies.

|  |  | Firm 2 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 200 | 400 |
|  | 0 | $(-50,-50)$ | $(-50,5950)$ | $(-50,7950)$ |
| Firm 1 | 200 | $(5950,-50)$ | $(3950,3950)$ | $(1950,3950)$ |
|  | 400 | $(7950,-50)$ | $(3950,1950)$ | $(-50,-50)$ |

There are 3 NE: $(200,200),(400,200)$ and $(200,400)$.
5. FALSE. The reason is that Firm 1 would capture all sales simply by setting a price just below $\$ 12$ (for example $\$ 11.99$ ). It would not need to go any lower because Firms 2 and 3 would already be unable to match $\$ 11.99$ and hence would not sell any quantity.
6. (a) Yes. Neither firm has an incentive to deviate given the other players strategy.
(b) There are 2 NE of this game: $\left(p_{1}^{*}, p_{2}^{*}\right)=(c, c)$ and $\left(p_{1}^{*}, p_{2}^{*}\right)=(c+0.01, c+0.01)$. Because we have introduced discreteness in the choice variable (price) we have created a second NE. Both firms charging a price one cent above marginal cost is also a NE because neither firm has an incentive to undercut the other firm by one cent because that would result in zero profits (because price would be equal to marginal cost). Of course if firms were free to cut prices by less than one cent then they would have an incentive to undercut as discussed in Lecture 3 where we considered a continuous price variable.
(c) In that case the choke price of demand is equal to the marginal cost of production. Hence there is no room for a deal between consumers and producers because relative to the maximum willingness of consumers to pay for the good, the costs of producing it are too high. No output would be sold.
(d) Again, by the logic just given no output would be sold.
7. (a) Use these to check your final answers:
i.

$$
q_{1}^{*}=q_{2}^{*}=q_{3}^{*}=\frac{1}{4} \frac{a-c}{b}
$$

ii.

$$
Q=\frac{3}{4} \frac{a-c}{b}
$$

iii.

$$
P=\frac{a+3 c}{4}
$$

iv.

$$
\varepsilon=\frac{\partial Q}{\partial P} \frac{P}{Q}=-\frac{a+3 c}{3(a-c)}
$$

v.

$$
\pi_{i}=\frac{(a-c)^{2}}{16 b}
$$

for $i=1,2,3$
vi.

$$
P S=\frac{6(a-c)^{2}}{32 b}
$$

vii.

$$
C S=\frac{9(a-c)^{2}}{32 b}
$$

viii.

$$
T S=\frac{15(a-c)^{2}}{32 b}
$$

ix.

$$
L_{i}=\frac{a-c}{a+3 c}
$$

(b) In a homogeneous Cournot model, the demand faced by each firm is not the market demand but a residual demand (after subtracting the output of the other firms). For the symmetric Cournot model, we worked out in class that $L_{i}=-\frac{1}{n \varepsilon}$ where $n$ is the number of firms and $\varepsilon$ is the market demand elasticity (notice there is no $i$ subscript on $\varepsilon$ ). You can verify that $L_{i}=-\frac{1}{n \varepsilon}$ does hold using your answers for the previous parts.
(c) Each of the three firms would get one-third of a monopolist's profit: $\pi_{i}=\frac{1}{3} \frac{(a-c)^{2}}{4 b}=\frac{(a-c)^{2}}{12 b}$, which is clearly greater than $\pi_{i}=\frac{(a-c)^{2}}{16 b}$. The remaining firm's best response is given by its FOC (i.e. its best response function): $q_{1}=\frac{a-b\left(q_{2}+q_{3}\right)-c}{2 b}$. Hence Firm 1 would produce $q_{1}=\frac{a-b \frac{a-c}{3 b}-c}{2 b}=\frac{a-c}{3 b}$, which is much more than one-third the monopolist's output $\left(\frac{a-c}{6 b}\right)$. Hence, even though all three firms would be better off if each produced one-third of the monopolist's output, they each have a strong profit incentive to deviate and produce more, which is why the Cournot NE results in higher outputs and lower industry profits.
8. (a) There are no fixed costs: both terms in the cost function vary with output. Marginal costs are not constant, but rather vary with the output level: $C^{\prime}\left(q_{i}\right)=18+2 q_{i}$. There are negative returns to scale (diseconomies of scale) because average costs per unit are increasing with the output level: $A C\left(q_{i}\right)=C\left(q_{i}\right) / q_{i}=18+q_{i}$.
(b)

$$
\begin{gathered}
\max _{q_{1}} \pi_{1}=\left(150-q_{1}-q_{2}-q_{3}\right) q_{1}-18 q_{1}-q_{1}^{2} \\
\frac{\partial \pi_{1}}{\partial q_{1}}=132-4 q_{1}-q_{2}-q_{3} \stackrel{\text { set }}{=} 0
\end{gathered}
$$

In the symmetric equilibrium $q_{1}=q_{2}=q_{3}=q$. Substituting this into the first-order condition and solving, we find that each firm produces $q=22$. Thus, $P=\$ 84$ and $\pi_{1}=\pi_{2}=\pi_{3}=\$ 968$.
(c) Denote the output of the newly merged firm as $q^{*}$. Clearly it should use both firm's production assets because there are diseconomies of scale. Further, given that both of the merging firms have the same cost function the diseconomies will be minimized by producing half the output at each of the original two firm's facilities. The merged firm's costs are:

$$
C^{*}\left(q^{*}\right)=2 C\left(\frac{q^{*}}{2}\right)=2\left[18\left(\frac{q^{*}}{2}\right)+\left(\frac{q^{*}}{2}\right)^{2}\right]=18 q^{*}+\frac{q^{* 2}}{2}
$$

Hence the merged firm's optimization problem is:

$$
\begin{gathered}
\max _{q^{*}} \pi^{*}=\left(150-q^{*}-q_{3}\right) q^{*}-18 q^{*}-\frac{q^{* 2}}{2} \\
\frac{\partial \pi^{*}}{\partial q^{*}}=132-3 q^{*}-q_{3} \stackrel{\text { set }}{=} 0
\end{gathered}
$$

Firm 3's optimization problem is:

$$
\begin{gathered}
\max _{q_{3}} \pi_{3}=\left(150-q^{*}-q_{3}\right) q_{3}-18 q_{3}-q_{3}^{2} \\
\frac{\partial \pi_{3}}{\partial q_{3}}=132-q^{*}-4 q_{3} \stackrel{\text { set }}{=} 0
\end{gathered}
$$

There are two FOCs and two unknowns. Solving for $q^{*}$ and $q_{3}$ yields $q^{*}=36$ and $q_{3}=24$. Hence $P=\$ 90, \pi^{*}=\$ 1,944$ and $\pi_{3}=\$ 1,152$. It pays for the two firms to merge because the merged firms' profits are more than double those of the original firms before the merger.
(d) If the firms' costs are $C(q)=18 q$, it makes no difference whether the merged firm uses both original firms' assets or just those of one firm because this cost function implies constant returns to scale. Similar calculations to those above yield that before the merger $q_{1}=q_{2}=q_{3}=33, P=\$ 51$, and $\pi_{1}=\pi_{2}=\pi_{3}=\$ 1,089$, whereas after the merger $q^{*}=q_{3}=44, P=\$ 62$, and $\pi^{*}=\pi_{3}=\$ 1,936$. With constant marginal costs, it therefore does not pay for the two firms to merge. (Note that Firm 3 benefits in both cases.)
9. (a) No. Clearly they are differentiated. As one firm increases its price it does lead some customers to substitute to the other good, but NOT all consumers switch. Each firm is facing a downward sloping demand.
(b) Yes. Each faces the same demand for its good and the same costs.
(c)

$$
\begin{gathered}
\pi_{1}\left(p_{1}, p_{2}\right)=T R_{1}-T C_{1} \\
\pi_{1}\left(p_{1}, p_{2}\right)=p_{1} * q_{1}-2-q_{1}
\end{gathered}
$$

Write in terms of the choice variables $p_{1}$ and $p_{2}$ :

$$
\begin{gathered}
\pi_{1}\left(p_{1}, p_{2}\right)=p_{1} *\left(25-5 p_{1}+2 p_{2}\right)-2-\left(25-5 p_{1}+2 p_{2}\right) \\
\frac{\partial}{\partial p_{1}} \pi_{1}\left(p_{1}, p_{2}\right)=\left(25-5 p_{1}+2 p_{2}\right)+p_{1} *(-5)+5
\end{gathered}
$$

Set derivative equal to zero:

$$
\left(25-5 p_{1}+2 p_{2}\right)+p_{1} *(-5)+5=0
$$

Solve for $p_{1}$ to find best response function:

$$
\begin{aligned}
& 30+2 p_{2}=10 p_{1} \\
& p_{1}\left(p_{2}\right)=\frac{15+p_{2}}{5}
\end{aligned}
$$

(d) Firm 1's best response (profit maximizing response) is given by its best response function. No need to solve the profit maximization problem all over again. Plugging in $p_{1}(1)=$ $\frac{15+1}{5}=3.2$ we see that Firm 1 should charge a price of $\$ 3.20$ for Good 1 .
(e) Plugging in $p_{1}(2)=\frac{15+2}{5}=3.4$ we see that Firm 1 should charge a price of $\$ 3.40$ for Good 1.
(f) Plugging in $p_{1}(1)=\frac{15+3}{5}=3.6$ we see that Firm 1 should charge a price of $\$ 3.60$ for Good 1.
(g) Plugging in $p_{1}(1)=\frac{15+4}{5}=3.8$ we see that Firm 1 should charge a price of $\$ 3.60$ for Good 1.
(h) Firm 1 responds to its competitor's price increase by increasing its own price.
(i)

$$
\begin{gathered}
\pi_{2}\left(p_{1}, p_{2}\right)=T R_{2}-T C_{2} \\
\pi_{2}\left(p_{1}, p_{2}\right)=p_{2} * q_{2}-2-q_{2}
\end{gathered}
$$

Write in terms of the choice variables $p_{1}$ and $p_{2}$ :

$$
\begin{gathered}
\pi_{2}\left(p_{1}, p_{2}\right)=p_{2} *\left(25-5 p_{2}+2 p_{1}\right)-2-\left(25-5 p_{2}+2 p_{1}\right) \\
\frac{\partial}{\partial p_{2}} \pi_{2}\left(p_{1}, p_{2}\right)=\left(25-5 p_{2}+2 p_{1}\right)+p_{2} *(-5)+5
\end{gathered}
$$

Set derivative equal to zero:

$$
\left(25-5 p_{2}+2 p_{1}\right)+p_{2} *(-5)+5=0
$$

Solve for $p_{2}$ to find best response function:

$$
\begin{aligned}
& 30+2 p_{1}=10 p_{2} \\
& p_{2}\left(p_{1}\right)=\frac{15+p_{1}}{5}
\end{aligned}
$$

(j) Because the firms are symmetric. In the future, not need to solve separately for the best response function for each firm if the firms are symmetric. For symmetric firms, you only need solve the profit maximization problem once because you know the answer will be the same for all of the symmetric firms.
(k) Graph required.
(l) Solve for the intersection of the best response functions:

$$
\begin{gathered}
p_{1}\left(p_{2}\right)=\frac{15+p_{2}}{5} \\
p_{1}\left(\frac{15+p_{1}}{5}\right)=\frac{15+\frac{15+p_{1}}{5}}{5} \\
p_{1}=\frac{75+15+p_{1}}{25} \\
24 p_{1}=90 \\
p_{1}^{*}=\$ 3.75 \\
p_{2}\left(p_{1}\right)=\frac{15+p_{1}}{5} \\
p_{2}\left(p_{1}^{*}\right)=\frac{15+3.75}{5} \\
p_{2}^{*}=\$ 3.75
\end{gathered}
$$

This corresponds to the following quantities:

$$
\begin{aligned}
& q_{1}=25-5 * 3.75+2 * 3.75=13.75 \\
& q_{2}=25-5 * 3.75+2 * 3.75=13.75
\end{aligned}
$$

(m)

$$
\begin{aligned}
& L_{1}=\frac{3.75-1}{3.75}=0.73 \\
& L_{2}=\frac{3.75-1}{3.75}=0.73
\end{aligned}
$$

This shows that these firms have substantial market power: $73 \%$ of the price charged to consumers is pure mark-up.
(n) In this case we have allowed for heterogenous goods that are not perfect substitutes for each other. Hence neither firm can capture the entire market by slightly undercutting the other.
(o) This is a two-good monopolist problem. Monopolist would take into account the fact that

Goods 1 and 2 are substitutes and set prices accordingly.

$$
\begin{gathered}
\pi\left(p_{1}, p_{2}\right)=T R-T C \\
\pi\left(p_{1}, p_{2}\right)=p_{1} * q_{1}+p_{2} * q_{2}-\left(2+q_{1}+2+q_{2}\right) \\
\pi\left(p_{1}, p_{2}\right)=p_{1} *\left(25-5 p_{1}+2 p_{2}\right)+p_{2} *\left(25-5 p_{2}+2 p_{1}\right)-\left(2+\left(25-5 p_{1}+2 p_{2}\right)+2+\left(25-5 p_{2}+2 p_{1}\right)\right)
\end{gathered}
$$

Two choice variables means need to take a derivate with respect to each (two equations and two unknowns):

$$
\frac{\partial}{\partial p_{1}} \pi\left(p_{1}, p_{2}\right)=25-5 p_{1}+2 p_{2}-5 p_{1}+2 p_{2}+5-2
$$

Set partial with respect to $p_{1}$ to zero:

$$
\begin{gathered}
25-5 p_{1}+2 p_{2}-5 p_{1}+2 p_{2}+5-2=0 \\
28-10 p_{1}+4 p_{2}=0 \\
\frac{\partial}{\partial p_{2}} \pi\left(p_{1}, p_{2}\right)=2 p_{1}+25-5 p_{2}+2 p_{1}-5 p_{2}-2+5
\end{gathered}
$$

Set partial with respect to $p_{2}$ to zero:

$$
\begin{gathered}
2 p_{1}+25-5 p_{2}+2 p_{1}-5 p_{2}-2+5=0 \\
28-10 p_{2}+4 p_{1}=0
\end{gathered}
$$

Now have two equations and two unknowns ( $p_{1}$ and $p_{2}$ ):

$$
\begin{aligned}
& 28-10 p_{1}+4 p_{2}=0 \\
& 28-10 p_{2}+4 p_{1}=0
\end{aligned}
$$

Solve to find: $p_{1}^{*}=\$ 4.67$ and $p_{2}^{*}=\$ 4.67$. This corresponds to the following quantities:

$$
\begin{aligned}
& q_{1}=25-5 * 4.67+2 * 4.67=11 \\
& q_{2}=25-5 * 4.67+2 * 4.67=11
\end{aligned}
$$

(p) The merged firm takes into account the fact that Goods 1 and 2 are substitutes. When the price of Good 1 is lowered this leads some consumers to substitute from Good 2 to Good 1. The merged firm takes into account this cannibalization, which leads the monopolist to set higher prices. In contrast a Bertrand competitor only worries about its own profits and not the profits of its rivals. Hence a Bertrand competitor has more incentive than the merged firm to lower its prices because it will be stealing business from its rival and not cannibalizing its own sales. Another way to say this is that when a Bertrand competitor lowers its prices that has a negative externality on its rivals. The merged firm internalizes this effect, which removes this externality.
(q) Total profits of merged firm:

$$
\pi\left(p_{1}, p_{2}\right)=4.67 * 11+4.67 * 11-(2+11+2+11)=76.74
$$

Total profits of the Bertrand competitors:

$$
\begin{gathered}
\pi_{1}\left(p_{1}, p_{2}\right)=3.75 * 13.75-2-13.75=35.8125 \\
\pi_{2}\left(p_{1}, p_{2}\right)=3.75 * 13.75-2-13.75=35.8125 \\
35.8125+35.8125=71.63
\end{gathered}
$$

As expected, the merged firm has higher profits than the combined Bertrand competitors.
(r) No. Each firm would have an incentive to lower its price. Hence the merged firm outcome, while it does have higher profits, cannot be sustained. The logic is the same as for the Prisoners' Dilemma: both prisoners would be better off if they denied, but they each cannot resist the incentive to deviate and confess, which in the end is worse for both of them.
10. (a)

$$
\begin{gathered}
\pi_{i}=T R_{i}-T C_{i} \\
\pi_{i}=(1000-5 Q) q_{i}-c_{i} q_{i}-F_{i} \\
\pi_{i}=\left(1000-5 \sum_{j=1}^{3} q_{j}\right) q_{i}-c_{i} q_{i}-F_{i} \\
\frac{\partial \pi_{i}}{\partial q_{i}}=1000-5 \sum_{j=1}^{3} q_{j}-5 q_{i}-c_{i} \stackrel{\text { set }}{=} 0
\end{gathered}
$$

FOC's for each of the three firms:

$$
\begin{aligned}
& 1000-10 q_{1}-5 q_{2}-5 q_{3}-c_{1}=0 \\
& 1000-5 q_{1}-10 q_{2}-5 q_{3}-c_{2}=0 \\
& 1000-5 q_{1}-5 q_{2}-10 q_{3}-c_{3}=0
\end{aligned}
$$

Back-out each firm's marginal costs given the initial quantities:

$$
\begin{gathered}
1000-10 * 20-5 * 20-5 * 60-c_{1}=0 \\
c_{1}=400 \\
1000-5 * 20-10 * 20-5 * 60-c_{2}=0 \\
c_{2}=400 \\
1000-5 * 20-5 * 20-10 * 60-c_{3}=0 \\
c_{3}=200
\end{gathered}
$$

Post-merger what is the predicted price?

$$
\begin{gathered}
\pi_{1,2}=T R_{1,2}-T C_{1,2} \\
\pi_{1,2}=\left(1000-5 q_{1,2}-5 q_{3}\right) q_{1,2}-400 q_{1,2}-F_{1,2} \\
\frac{\partial \pi_{1,2}}{\partial q_{1,2}}=1000-5 q_{1,2}-5 q_{3}-5 q_{1,2}-400 \stackrel{\text { set }}{=} 0 \\
600-10 q_{1,2}-5 q_{3}=0 \\
\pi_{3}=T R_{3}-T C_{3} \\
\pi_{3}=\left(1000-5 q_{1,2}-5 q_{3}\right) q_{3}-200 q_{3}-F_{3} \\
\frac{\partial \pi_{3}}{\partial q_{3}}=1000-5 q_{1,2}-5 q_{3}-5 q_{3}-200 \stackrel{\text { set }}{=} 0 \\
800-5 q_{1,2}-10 q_{3}=0 \\
-2 *\left(600-10 q_{1,2}-5 q_{3}=0\right) \\
800-5 q_{1,2}-10 q_{3}=0 \\
-400+15 q_{1,2}=0 \\
q_{1,2}=26.67 \\
q_{3}=66.67
\end{gathered}
$$

Hence the post-merger price is $P=1000-5(26.67+66.67)=\$ 533.33$. The pre-merger price is $P=1000-5(20+20+60)=\$ 500.00$. Hence the price to consumers would increase by $\$ 33.33$ with this merger.
(b) This merger does not substantially lessen competition under a TS standard, which is the opposite answer we would obtain using a price standard (Part (a) showed that the price to consumers would increase with this merger). Even though there are no marginal cost savings, the merger increases TS because production is shifted from the cost inefficient plants (Firms 1 and 2) to a cost efficient plant (Firm 3): Firm 3's marginal costs of production are half that of Firms 1 and 2 (200 versus 400). With the merger Firms 1 and 2 restrict output. Because quantities are strategic substitutes Firm 3 responds by increasing its output. The increase in TS has nothing to do with the fixed costs savings mentioned in Part (a) because fixed costs do not alter TS. (Although, some sources do include fixed costs in PS, many distinguish between profits (in which fixed costs enter) and PS.)
$T S=C S+P S$
Pre-merger $C S=0.5 *(1000-500) * 100=25,000$
Post-merger $C S=0.5 *(1000-533.33) * 93.34=21,779.49$
Pre-merger $P S=(500-400) * 20+(500-400) * 20+(500-200) * 60=22,000$
Post-merger $P S=(533.33-400) * 26.67+(533.33-200) * 66.67=25,779$
Pre-merger $T S=25,000+22,000=47,000$
Post-merger $T S=21,779.49+25,779=47,558.5$
11. (a) We derived the FOC's in lecture, which can be written as:

$$
\begin{aligned}
& \frac{p_{1}-c_{1}}{p_{1}}=-\frac{1}{\varepsilon_{1}} \\
& \frac{p_{2}-c_{2}}{p_{2}}=-\frac{1}{\varepsilon_{2}}
\end{aligned}
$$

Using these and the own-price elasticities from the constant elasticity demands, obtain a Lerner Index for good 1 of 0.4 ( $40 \%$ of the price is mark-up over marginal costs) and a Lerner Index for good 2 of 0.5 ( $50 \%$ of the price is mark-up over marginal costs).
(b) Solve $\frac{p_{1}-c_{1}}{p_{1}}=-\frac{1}{\varepsilon_{1}}$ for $c_{1} \cdot \frac{10-c_{1}}{10}=-\frac{1}{-2.5}$ implies $c_{1}=\$ 6$. Solve $\frac{p_{2}-c_{2}}{p_{2}}=-\frac{1}{\varepsilon_{2}}$ for $c_{2}$. $\frac{14-c_{2}}{14}=-\frac{1}{-2}$ implies $c_{2}=\$ 7$.

