ECO410H: Practice Questions 2

1. Consider the following games. For each, find all of the NE.

(a)

	L	M	R
U	(5,5)	(2,6)	(1,8)
M	(6,2)	(4,4)	(2,3)
D	(8,1)	(3,2)	(0,0)

(b)

	C1	C2	C3	C4
R1	(10,7)	(8,8)	(0,6)	(2,6)
R2	(6,5)	(2,3)	(5,1)	(7,4)
R3	(0,4)	(5,8)	(3,7)	(5,10)
R4	(4,6)	(9,8)	(6,9)	(1,1)

(c)

	L	M	R
U	(5,4)	(0,1)	(0,6)
M	(4,1)	(1,2)	(1,1)
D	(5,6)	(0,3)	(4,4)

2. For the following game, give conditions on a and b such that there are no NE.

	C1	C2	C3
R1	(1,1)	(4,0)	(3,-1)
R2	(a,b)	(2,2)	(2,1)
R3	(-1,3)	(-1,1)	(1,1)

3. In a duopoly market Firm A and Firm B choose price. Here is the normal form of the game showing their profits (Firm A, Firm B).

	Firm B			
	\$3	\$4	\$5	\$6
\$3	24, 24	30, 25	36, 20	42, 12
\$4	25, 30	32, 32	41, 30	48, 24
\$5	20, 36	30, 41	40, 40	50, 36
\$6	12, 42	24, 48	36, 50	48, 48

   (a) What is the static Bertrand NE?

   (b) Is this an example of the Bertrand paradox? Explain.

4. Consider a Cournot duopoly. Market demand is: $P = 50 - 0.05Q$. The cost function for each firm is: $C(q) = 50 + 10q$. Each firm must choose a quantity of 0, 200 or 400 for technological reasons. In other words, it is not possible for a firm to produce 350 units or any other quantity aside from the three just listed. Find any and all static Nash Equilibrium.

5. TRUE/FALSE EXPLAIN Consider a market with only three Bertrand competitors that all sell identical goods. If Firms 1, 2, and 3 have constant marginal costs c_1 = \$10, c_2 = \$12, and c_3 = \$14 respectively, then the market price would be \$10 and the firms have no market power.

6. Consider a homogeneous goods Bertrand model but where price is a discrete variable rather than a continuous variable. Suppose that the two Bertrand competitors can only charge prices in dollars and cents. In other words firms cannot charge a price of \$1.562, which would require a new coin worth less than a penny that does not exist. Suppose market demand is: $Q = \alpha - P$. Suppose firms are symmetric and have cost functions: $C(q_i) = cq_i$. Further, suppose that marginal cost is measured in dollars and cents and $\alpha > c + 0.01$.

   (a) Is $(p_1^*, p_2^*) = (c, c)$ a NE of this game?

   (b) List all NE of this game. (Hint: There are 2 NE in total.)

   (c) What would be the outcome of this game if $\alpha = c$?

   (d) What would be the outcome of this game if $\alpha < c$?

7. Suppose market demand is $P = a - bQ$, there are *three* perfectly symmetric Cournot competitors and each has marginal costs equal to c.

   (a) Derive (solve for while showing all of your work) each of the following:

      i. Cournot NE
      ii. Market output
      iii. Market price
      iv. Elasticity of market demand at the market price
      v. Each firm's profits
      vi. Producer surplus
      vii. Consumer surplus
      viii. Total surplus
      ix. Lerner index for each firm

   (b) Why isn't the negative of the inverse of your answer for part 7(a)iv equal to your answer to part 7(a)ix? What is the relationship between these?

   (c) If instead of Cournot oligopoly there was a monopolist the market price and quantity would be $P = \frac{a+c}{2}$ and $Q = \frac{a-c}{2b}$. Verify that all three Cournot firms would have higher profits if each produced one-third of the monopolist's output compared to the Cournot NE. If the other two firms are each producing one-third the monopolist's output, what is the remaining firm's best response? What does this mean about the stability of the one-third-each arrangement?

8. An industry consists of three firms with identical costs $C(q_i) = 18q_i + q_i^2$ where $i = 1, 2, 3$. Market demand is $Q = 150 - P$.

   (a) Are there fixed costs? Constant marginal costs? Economies of scale?

   (b) Derive the Cournot NE (price, output and profits).

   (c) Would it pay for Firm 1 and Firm 2 to merge, if, after the merger, there is a Cournot duopoly equilibrium? (Hint: Carefully consider whether the merged firm would produce using both original firms' plants or just that of one firm.)

   (d) What happens if their costs are $C(q) = 18q$ instead?

9. Suppose an industry consists of two firms: Firm 1 and Firm 2. These firms are Bertrand competitors. Firm 1 makes and sells Good 1 and Firm 2 makes and sells Good 2. Here is demand for each of these goods:

$$q_1 = 25 - 5p_1 + 2p_2$$

$$q_2 = 25 - 5p_2 + 2p_1$$

Here are the production and sales costs for these two firms:

$$C(q_1) = 2 + q_1$$

$$C(q_2) = 2 + q_2$$

(a) Are the goods produced by Firms 1 and 2 homogeneous?

(b) Are these firms symmetric?

(c) Find the best response function of Firm 1. (Note: For this part and all parts of this question, derive all of the answers yourself. In other words, do not use any final results you may find from lectures or other sources. Being able to figure out results for a set of assumptions is a skill you are required to develop in this course.)

(d) What price should Firm 1 set for Good 1 if it observes a price of \$1 for Good 2?

(e) What price should Firm 1 set for Good 1 if it observes a price of \$2 for Good 2?

(f) What price should Firm 1 set for Good 1 if it observes a price of \$3 for Good 2?

(g) What price should Firm 1 set for Good 1 if it observes a price of \$4 for Good 2?

(h) How does the price Firm 1 sets change with increases in the price of its competitor's good?

(i) Find the best response function of Firm 2.

(j) Why are the solutions to 9c and 9i the same up to the identity of the firm?

(k) Graph the best response functions in the same graph and point out the NE in the graph.

(l) What is the NE?

(m) What is the Lerner Index for Good 1? Good 2? What is the interpretation? Do the firms have market power?

(n) Explain why the Bertrand Paradox of zero market power does not apply in this case?

(o) Suppose these two firms merged. Of course the new firm would set the prices of Goods 1 and 2 to maximize its profits. What would p_1 and p_2 be in this case? (Assume that there are no cost synergies associated with producing Goods 1 and 2 together.)

(p) Explain intuitively why the price of the merged firm is higher than those set by the Bertrand competitors.

(q) Find the total profits of the merged firm and compare with the combined profits of the Bertrand competitors. Which is higher?

(r) Is $(p_1^*, p_2^*) = (\$4.67, \$4.67)$ a NE in the Bertrand competition? Explain why or why not.

10. Market demand is $P = 1000 - 5Q$ and three firms are engaged in Cournot competition. Currently Firms 1, 2 and 3 are selling quantities of 20, 20, and 60 respectively. Firms have constant marginal costs and some fixed costs. Firms 1 and 2 propose merging. They anticipate some fixed costs savings with the merger.

    (a) If this merger were allowed what would the price be? How much would the merger increase price?

    (b) The total surplus pre-merger is 47,000. The total surplus post-merger would be 47,558.5. Using a total surplus standard, would this merger substantially lessen competition? Why would this merger increase total surplus?

11. Consider a Bertrand duopoly where each firm produces a single differentiated good. Suppose that Firms 1 and 2 have constant marginal costs equal to c_1 and c_2, respectively. Suppose the system of demand equations for goods 1 and 2 is:

$$ln(q_1) = 10 - 2.5ln(p_1) + ln(p_2)$$

$$ln(q_2) = 11 + 1.5ln(p_1) - 2ln(p_2)$$

    (a) Compute the Lerner Index for each good assuming the market is in equilibrium.

    (b) Suppose you observe Firm 1 charging a price of \$10 and Firm 2 charging a price of \$14. Compute each firm's marginal cost.