## ECO410H: Practice Questions 1 - SOLUTIONS

1. (a) The cross price elasticity of good 1 with respect to a change in price of good 3 is $\gamma$. This is shown by rewriting this constant elasticity demand as:

$$
\begin{gathered}
\varepsilon_{13}=\frac{\partial Q_{1}}{\partial P_{3}} \frac{P_{3}}{Q_{1}} \\
Q_{1}=e^{\alpha} P_{1}^{\beta} P_{2}^{\lambda} P_{3}^{\gamma} e^{\rho X} \\
\frac{\partial Q_{1}}{\partial P_{3}}=\gamma e^{\alpha} P_{1}^{\beta} P_{2}^{\lambda} \gamma P_{3}^{\gamma-1} e^{\rho X}=\frac{Q_{1}}{P_{3}} \gamma \\
\varepsilon_{13}=\frac{Q_{1}}{P_{3}} \gamma \frac{P_{3}}{Q_{1}}=\gamma
\end{gathered}
$$

(b) The demand shifters are the prices of the two potentially related goods (goods 2 and 3) and population size. Note that own-price $P_{1}$ is NOT a demand shifter.
(c) This result indicates a serious problem with the data or the econometric analysis because a positive estimate of $\beta$ indicates an upward sloping demand curve (positive price elasticity): for the same good, consumers purchase a larger quantity at a higher price. The data analysis must be invalid.
(d) The result indicates that demand is estimated to be inelastic: a $10 \%$ increase in price is associated with a $5.6 \%$ decrease in quantity demanded.
(e) These results indicate that both goods 2 and 3 are estimated to be substitutes for good 1 . Good 3 is a closer substitute because a $10 \%$ increase in the price of good 3 yields a $2.4 \%$ increase in the quantity of good 1 demanded whereas a $10 \%$ increase in the price of good 2 yields only a $1.2 \%$ increase in the quantity of good 1 demanded.
(f) Given that $\hat{\rho}$ is positive an increase in population would cause the estimated demand curve to shift right (outward).
2. Elzinga and Mills argue that the Lerner Index - defined as (Price - marginal cost)/Price - does not necessarily reflect a lack of competition and that there are many factors that affect the size of the Lerner Index. In markets where production technologies are characterized by high fixed costs (IRTS), high price-cost margins are necessary for even zero profits. Further, a high Lerner Index for a firm in the industry may simply reflect a marginal cost advantage it has achieved relative to its competitors or that it has successfully differentiated its product relative to its rivals making its own demand less elastic and allowing it to charge a higher mark-up. In fact, firms facing strong rivals have an incentive to constantly seek to achieve cost advantages over their rivals and to differentiate their products, which means a high Lerner index could actually be caused by intense competition rather than a lack of competition.
3. (a) To visualize, consider the inverse demand curve, which is what is typically graphed: $P=$ $a-\frac{1}{S} Q$. Hence an increase in $S$ would cause the demand curve to rotate out (i.e. the vertical intercept stays the same but the slope becomes more shallow and the horizontal intercept increases). This would correspond to an increase in the size of the market: i.e. if there was an increase in population that added more (identical) consumers. (Recall that
market demand is the horizontal sum of individual consumer demands.) $S$ refers to the size of the market.
(b) Continuing with the solution to the previous part, $a$ is the vertical intercept and is the maximum willingness to pay. The vertical intercept of the inverse demand curve is sometimes called the "choke price" because it is the (lowest) price the chokes off all demand (i.e. quantity demanded is zero). As $a$ increases demand shifts up.
(c) Solve the monopolist's profit maximization problem:

$$
\begin{gathered}
\pi=T R-T C \\
\pi=P * Q-(F+c Q) \\
\pi=\left(a-\frac{1}{S} Q\right) Q-(F+c Q) \\
\frac{\partial \pi}{\partial Q}=-\frac{1}{S} Q+\left(a-\frac{1}{S} Q\right)-c \stackrel{\text { set }}{=} 0 \\
Q^{M}=\frac{S(a-c)}{2} \\
P^{M}=\frac{a+c}{2} \\
\pi^{M}=\frac{S(a-c)^{2}}{4}
\end{gathered}
$$

(d)

$$
\begin{gathered}
\varepsilon=\frac{\partial Q}{\partial P} \frac{P}{Q} \\
\varepsilon=-S \frac{a-\frac{1}{S} Q}{Q} \\
\varepsilon=\frac{Q-a S}{Q}=1-\frac{a S}{Q} \\
\varepsilon^{M}=1-\frac{a S}{\frac{S(a-c)}{2}} \\
\varepsilon^{M}=-\frac{a+c}{a-c}
\end{gathered}
$$

(e) Lerner Index $=\frac{P^{M}-c}{P^{M}}=\frac{a-c}{a+c}$. It tells what fraction of the price is mark-up over marginal costs.
(f) Verified. This is true regardless of the specific functional forms used (in this example linear).
(g) In this simple static model nothing happens if the monopolist's fixed costs $F$ change. By definition fixed costs are a scalar (constant) so they do not affect the FOC (first order condition) that determines the monopolist's price and quantity: the derivative of a constant is zero. The monopolist will not pass through any fixed cost savings (or increases) to its customers.
(h) Changes in a monopolist's marginal costs affect its pricing decisions. For the specific
functional forms assumed in this example - linear demand and constant marginal costs - the formula $P^{M}=\frac{a+c}{2}$ shows the relationship between the monopolist's price and its marginal costs. If marginal costs decrease by $20 \%$ then $P^{M 1}=\frac{a+0.8 c}{2}=P^{M}-0.1 c$ so the price declines by only $10 \%$ of marginal costs. Hence the monopolist does pass through some of its marginal costs savings to its customers in the form of lower prices but not one-to-one (i.e. some of the marginal cost savings are kept by the monopolist in the form of higher profits).
(i) Yes, both fixed costs and marginal costs affect a firm's profitability and hence both affect a firm's decision about entering (or exiting) an industry.
4. (a) An elasticity of -1.25 means that if the price of beer goes up by $1 \%$ then the quantity demanded will decrease by $1.25 \%$. It is elastic because a $1 \%$ increase in price causes more than a $1 \%$ decrease in the quantity demanded.
(b) If the elasticity of demand is the same no matter what the price then this is a constant elasticity demand. It can be written as $\ln (Q)=\alpha-1.25 \ln (P)$ or $Q=e^{\alpha} P^{-1.25}$ or $Q=\frac{e^{\alpha}}{P^{1.25}}$.
(c) Nothing. We have not been told anything about the supply side. If the supply side is perfect competition then there is no market power: the Lerner Index is zero. If the supply side is monopoly then there is a lot of market power: a large Lerner Index (for monopoly $80 \%$ ( $-1 /-1.25$ ) of price would be mark-up over marginal costs). Of course the supply side could be oligopoly and then we would get something in between. We cannot make an inference about the Lerner Index with the information given.

