Normal Table: Read it, Use it

For ECO220Y1Y, 2024/25 by Jennifer Murdock; Version: August 21, 2024

A massive flood in Grand Forks, North Dakota in 1997 cost billions to clean up. The levee could protect the town even if the river rose to 51 feet. But, beyond that, it would flood the town. The Weather Service knew heavy snowfall would cause high river waters. It forecast levels would rise to 49 feet and so the town did not take extra precautions. Unfortunately, the waters rose to 54 feet. The section "The Importance of Communicating Uncertainty" of the 2012 book *The Signal and the Noise: Why So Many Predictions Fail–But Some Don't* by Nate Silver tells this story.

We cannot perfectly predict water levels: 49 feet is only expected. On page 178, Silver says there was "about a 35 percent chance of the levees being overtopped" and footnote 8 explains that a Normal model gives 35 percent. After working through this supplement, you can find the parameters of the model and draw a fully-labeled graph, like Figure 1, to illustrate your reasoning. (Make sure to solve Exercise 9 on page 9.) Silver concludes with "An oft-told joke: a statistician drowned crossing a river that was only three feet deep *on average*. On average, the flood might be forty-nine feet in the Weather Service's forecast model, but just a little bit higher and the town would be inundated."

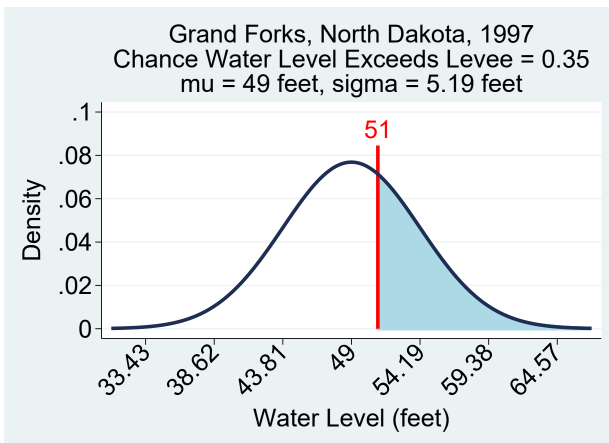


Figure 1: A fully-labeled graph, but you are *not* responsible for ticking the numbers on the *vertical* axis when drawing this graph by hand

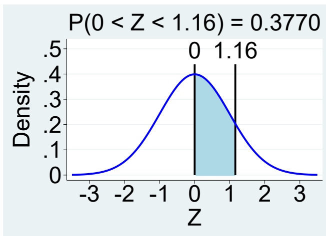
Learn how to use the **one-page Standard Normal table** on page 10. Use this one-page table throughout our course and when solving practice problems because that is what we give you during tests and exams. In contrast, our textbook uses a redundant two-page table (pages B-2 and B-3).

Also, you must remember some skills. To standardize a random variable X with mean μ and standard deviation σ you subtract μ and divide by σ: $Z = \frac{X-\mu}{\sigma}$. Z has mean 0 and standard deviation 1. When X is Normal, $X \sim N(\mu, \sigma^2)$, then Z is Standard Normal, $Z \sim N(0, 1)$. Using the Standard Normal table involves both **standardizing** $Z = \frac{X-\mu}{\sigma}$ and **unstandardizing** $X = \mu + \sigma Z$. Further, some exercises require using some key properties of **linear transformations** (e.g. $Y = a + bX$ where a and b are constants) and **linear combinations** (e.g. $W = a + bX_1 + cX_2$ where a, b, and c are constants) including the **Laws of Expectation** and the **Laws of Variance**, which appear on the first page of our **course aid sheets**. For a linear combination of *independent* random variables the correlation/covariance term is zero for the last Law of Variance. Further, there is an important theorem: any linear combination of independent Normal random variables is also Normally distributed.

Examples A to Q show how to read the one-page Standard Normal table. *Examples R to W* show how to use standardizing and unstandarizing to answer questions with the table. **Exercises 1 to 9** (starting on page 7) let you practice putting it all together with *applications*.

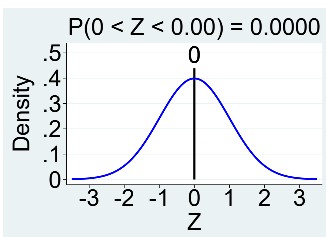
Example A. In the middle of the table, find 0.3770. What does it mean?

It is in row 1.1 and column 0.06, which together are $z = 1.16$: $P(0 < Z < 1.16) = 0.3770$.



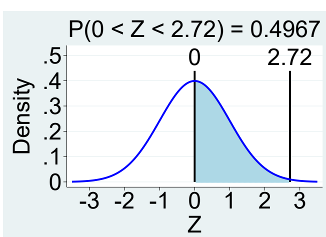
Example B. In the top left of the table, find 0.0000. What does it mean?

It is in row 0.0 and column 0.00, which together are $z = 0.00$: $P(0 < Z < 0.00) = 0.0000$. The probability is the *area* under a density function. For all continuous distributions, the probability of a specific value is zero: if there is no width, there can be no area.



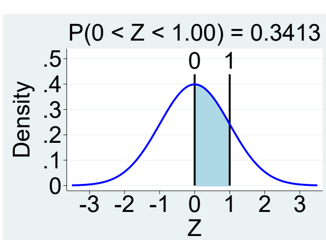
Example C. Near the bottom of the table, find 0.4967. What does it mean?

It is in row 2.7 and column 0.02, which together are $z = 2.72$: $P(0 < Z < 2.72) = 0.4967$.



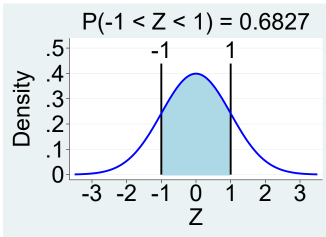
Example D. What is the probability that Z is between 0 and 1?

For $z = 1.00$, look at row 1.0 and column 0.00 to get: $P(0 < Z < 1.00) = 0.3413$.



Example E. What is the probability that Z is within one standard deviation of its mean?

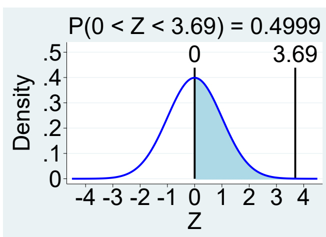
Z has a mean of zero and a standard deviation of one. The distribution is symmetric about 0. Hence, the question is $P(-1 < Z < 1) = ?$. The table says that $P(0 < Z < 1) = 0.3413$ and symmetry implies $P(0 < Z < 1) = P(-1 < Z < 0)$. Hence, $P(-1 < Z < 1) = 2 * 0.3413 = 0.6826$. (Actually, with software and without the rounding to the fourth decimal place in the table, it is 0.6827 as shown below.)



Remember the Empirical Rule (pages 272 - 273 of our textbook)? For a sample from a Normal population (i.e. Bell shaped), about 68.3% of the observations should lie within one standard deviation of the mean. Hence, if you forget the numbers in the Empirical Rule, you now know how to find 68.3%, 95.4% and 99.7% from the table.

Example F. In the very bottom right of the table, find 0.4999. What does it mean?

It is in row 3.6 and column 0.09, which together are $z = 3.69$: $P(0 < Z < 3.69) = 0.4999$.

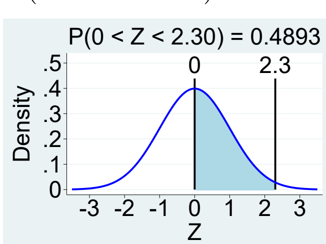


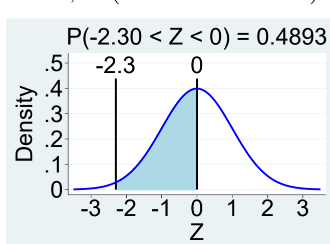
What is $P(0 < Z < \infty)$? No need to use the table. $P(Z > 0) = 0.5$ because this distribution is symmetric around zero. The total area under any density function is 1. Since the Standard Normal is centered at 0, half the area is above zero and half the area is below zero.

What is $P(Z > 3.69)$? The table says $P(0 < Z < 3.69) = 0.4999$ and we know $P(Z > 0) = 0.5$. Hence $P(Z > 3.69) = P(Z > 0) - P(0 < Z < 3.69) = 0.5 - 0.4999 = 0.0001$. There is a 1 in 10,000 chance of being 3.69 or more standard deviations above the mean.

Example G. What is $P(-2.30 < Z < 0)$?

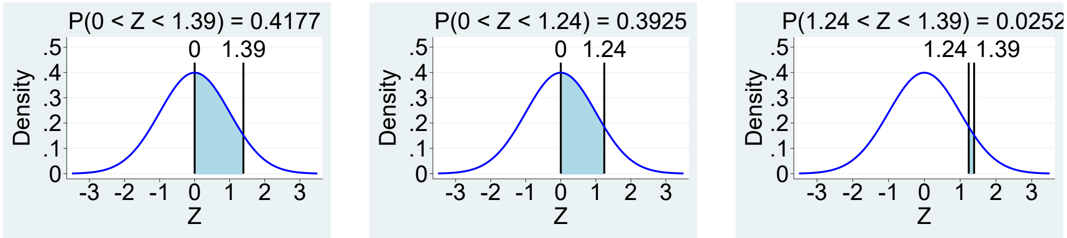
Because of symmetry: $P(-2.30 < Z < 0) = P(0 < Z < 2.30)$. From the table $P(0 < Z < 2.30) = 0.4893$. Hence, $P(-2.30 < Z < 0) = 0.4893$.





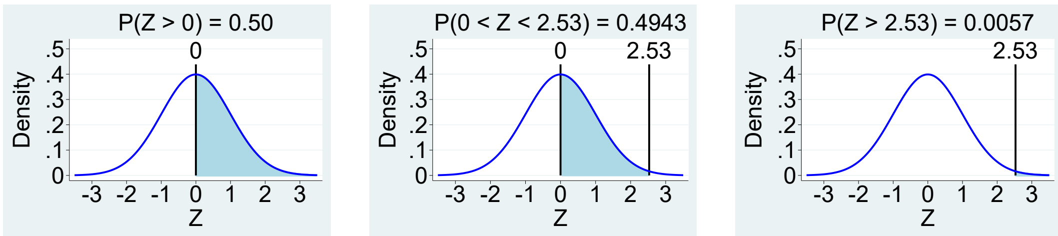
Example H. What is $P(1.24 < Z < 1.39)$?

$P(1.24 < Z < 1.39) = P(0 < Z < 1.39) - P(0 < Z < 1.24) = 0.4177 - 0.3925 = 0.0252.$



Example I. What is $P(Z > 2.53)$?

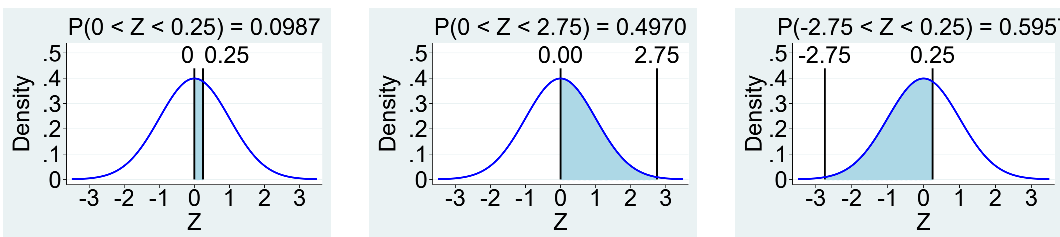
$P(Z > 2.53) = P(Z > 0) - P(0 < Z < 2.53) = 0.5 - 0.4943 = 0.0057.$



Example J. What is $P(-2.75 < Z < 0.25)$?

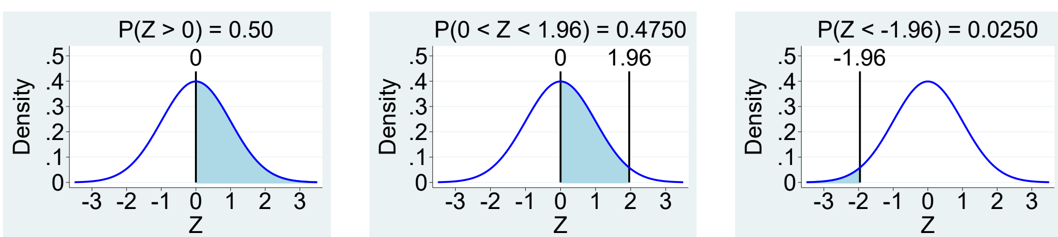
$P(-2.75 < Z < 0.25) = P(0 < Z < 0.25) + P(-2.75 < Z < 0)$

$= P(0 < Z < 0.25) + P(0 < Z < 2.75) = 0.0987 + 0.4970 = 0.5957.$



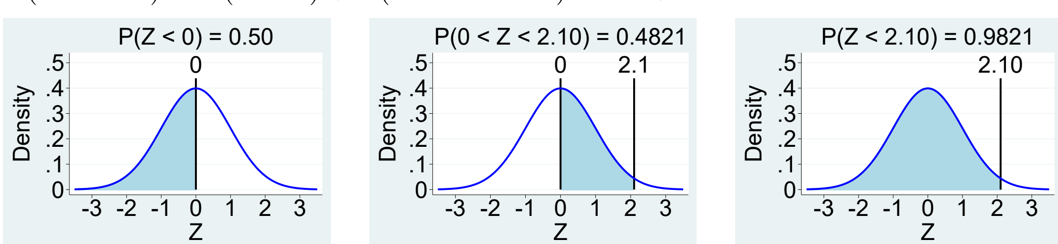
Example K. What is $P(Z < -1.96)$?

$P(Z < -1.96) = P(Z > 1.96) = P(Z > 0) - P(0 < Z < 1.96) = 0.5 - 0.4750 = 0.0250.$



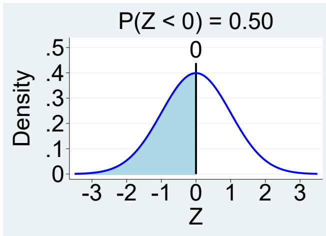
Example L. What is $P(Z < 2.10)$?

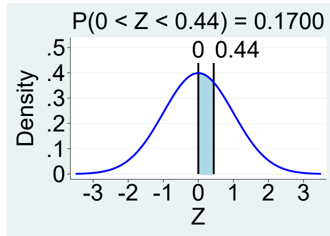
$P(Z < 2.10) = P(Z < 0) + P(0 < Z < 2.10) = 0.5 + 0.4821 = 0.9821.$

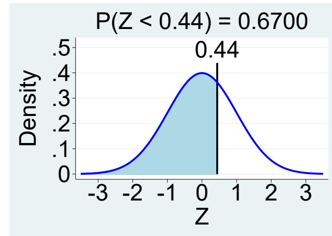


Example M. What is the 67th percentile of the Standard Normal distribution?

In formal notation, find ? in $P(Z < \ ?) = 0.67$. The 67th percentile is above 0 because the median of the Standard Normal is zero. $P(Z < \ ?) = 0.67$ means $P(0 < Z < \ ?) = 0.17$ because $P(Z < 0) = 0.5$ (and 0.67 minus 0.5 is 0.17). From the table, $? = 0.44$ in $P(0 < Z < \ ?) = 0.17$. Hence, $P(Z < 0.44) = 0.67$. The 67th percentile is 0.44.

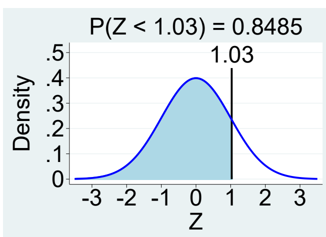


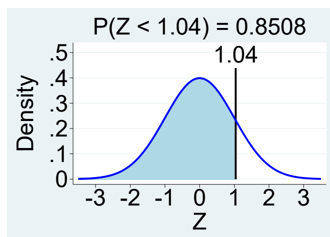


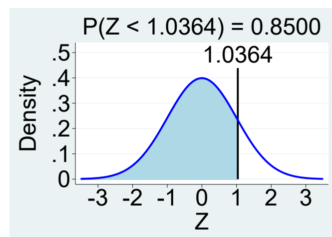


Example N. What is the 85th percentile of the Standard Normal distribution?

This is just like the previous example, but we cannot get an exact answer using the table. $P(0 < Z < 1.03) = 0.3485$ and $P(0 < Z < 1.04) = 0.3508$. Hence, ? in $P(Z < \ ?) = 0.85$ is somewhere between 1.03 and 1.04. You can interpolate between values in the table but, simply using the nearest (0.3508 giving 1.04) is perfectly acceptable. With Excel you can get the exact values. Hence, the 85th percentile is approximately 1.04.

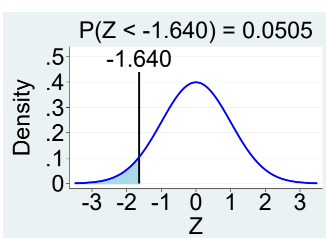


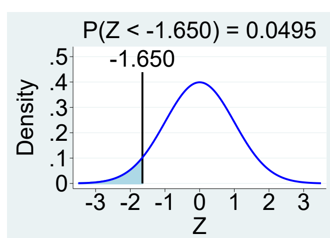


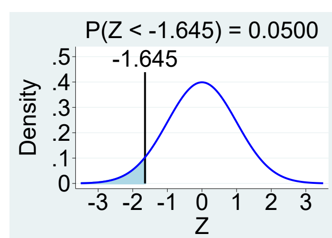


Example O. What is the 5th percentile of the Standard Normal distribution?

$P(Z < \ ?) = 0.05$. The 5th percentile is below 0 because the median is 0. From the table $P(0 < Z < 1.645) = 0.45$ implying $P(Z < -1.645) = 0.05$. We did interpolate between 1.64 and 1.65, because we will use 1.645 a lot. The 5th percentile is about -1.645.

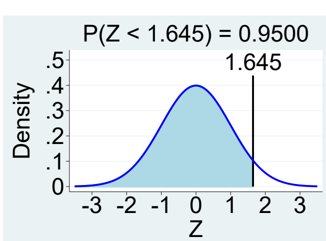






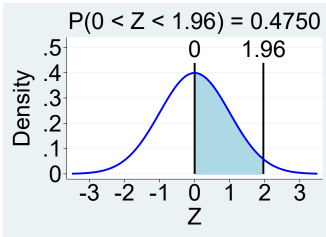
Example P. What is the 95th percentile of the Standard Normal distribution?

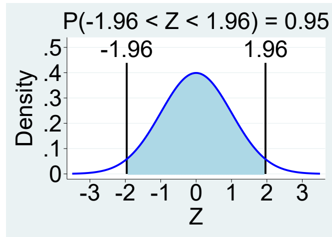
From the previous example, $P(Z < 1.645) = 0.95$. The 95th percentile is about 1.645.



Example Q. What is the value of ? in $P(-? < Z < \ ?) = 0.95$?

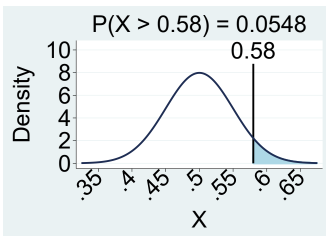
Because the upper and lower bounds are the same except for the sign, the center point must be zero. $P(0 < Z < \ ?) = 0.95/2$ and from the table $P(0 < Z < 1.96) = 0.4750$. Hence, $P(-1.96 < Z < 1.96) = 0.95$. Can you see the link to the Empirical Rule?





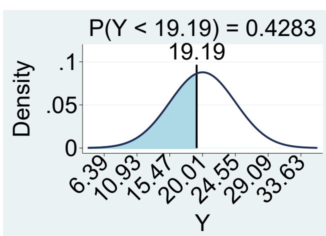
Example R. A random variable X is Normally distributed with $\mu_X = 0.5$ and $\sigma_X = 0.05$. What is the chance X is bigger than 0.58? Illustrate the answer with a graph.

It asks $P(X > 0.58 \mid \mu_X = 0.5, \sigma_X = 0.05) = \ ?$. Standardize to use the table. $P(X > 0.58) = P(Z > \frac{0.58-0.5}{0.05}) = P(Z > 1.6) = 0.5 - 0.4452 = 0.0548$.



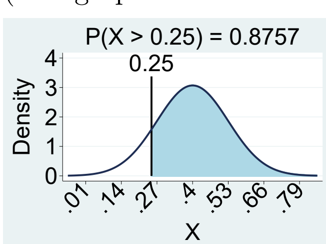
Example S. A random variable Y is Normally distributed with $\mu_Y = 20.01$ and $\sigma_Y = 4.54$. What is the chance Y is less than 19.19? Illustrate the answer with a graph.

It asks $P(Y < 19.19 \mid \mu_Y = 20.01, \sigma_Y = 4.54) = \ ?$. Standardize to use the table. $P(Y < 19.19) = P(Z < \frac{19.19-20.01}{4.54}) = P(Z < -0.181) \approx P(Z < -0.18) = 0.5 - 0.0714 = 0.4286$. (The graph shows an exact area from software.)



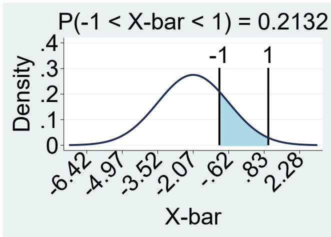
Example T. A random variable X is Normally distributed with $\mu_X = 0.40$ and $\sigma_X = 0.13$. What is the chance X is bigger than 0.25? Illustrate the answer with a graph.

It asks $P(X > 0.25 \mid \mu_X = 0.40, \sigma_X = 0.13) = \ ?$. Standardize to use the table. $P(X > 0.25) = P(Z > \frac{0.25-0.4}{0.13}) = P(Z > -1.154) \approx P(Z > -1.15) = 0.5 + 0.3749 = 0.8749$. (The graph shows the exact area from software.)



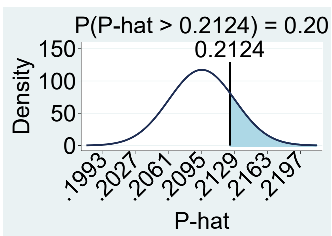
Example U. A random variable $\bar{X}$ is Normally distributed with $\mu_{\bar{X}} = -2.07$ and $\sigma_{\bar{X}} = 1.45$. What is the chance $\bar{X}$ is between -1 and 1? Illustrate the answer with a graph.

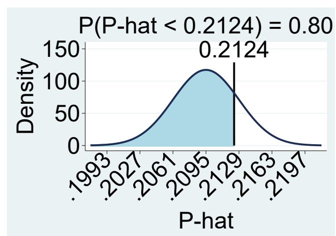
It asks $P(-1 < \bar{X} < 1 \mid \mu_{\bar{X}} = -2.07, \sigma_{\bar{X}} = 1.45) = ?$. Standardize to use the table. $P(-1 < \bar{X} < 1) = P(\frac{-1--2.07}{1.45} < Z < \frac{1--2.07}{1.45}) = P(0.738 < Z < 2.117) \approx P(0.74 < Z < 2.12) = 0.4830 - 0.2704 = 0.2126$. (The graph shows the exact area from software.)



Example V. A random variable $\hat{P}$ is Normally distributed with $\mu_{\hat{P}} = 0.2095$ and $\sigma_{\hat{P}} = 0.0034$. What is the cut-off value such that there is a 20 percent chance of being above this value? In other words, what is the 80th percentile? Illustrate the answer with a graph.

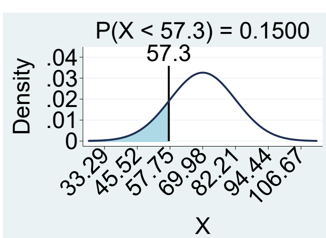
It asks $P(\hat{P} > ? \mid \mu_{\hat{P}} = 0.2095, \sigma_{\hat{P}} = 0.0034) = 0.20$. Use the table to get that $P(Z > 0.84) = 0.20$. Next, un-standardize. $0.84 = \frac{\hat{P}-0.2095}{0.0034}$ so $\hat{P} = 0.2095 + 0.0034 * 0.84 = 0.2124$. (Either graph below is an acceptable answer.)





Example W. A random variable X is Normally distributed with $\mu_X = 69.98$ and $\sigma_X = 12.23$. What is the cut-off value such that there is a 15 percent chance of being below this value? In other words, what is the 15th percentile? Illustrate the answer with a graph.

It asks $P(X < ? \mid \mu_X = 69.98, \sigma_X = 12.23) = 0.15$. Use the table to get that $P(Z < -1.04) = 0.15$. Next, un-standardize. $-1.04 = \frac{x-69.98}{12.23}$ so $x = 69.98 - 12.23 * 1.04 = 57.3$.



Work through these exercises and check your understanding. Practice *applying* the skills. After you have completed each exercise, check *your answers* against the suggested solutions on pages 11 to 14.

Exercise 1. Consider 2022 and *all* people aged 25 to 54 years in Canada. According to Stats Canada, 67.1% were born in Canada. The proportion 0.671 is a parameter because it describes a population. For a random sample of 1,126 such people, a random variable named $\hat{P}$ is the sample proportion (a statistic) born in Canada. $\hat{P}$ is Normally distributed with mean 0.671 and standard deviation 0.014: $\hat{P} \sim N(\mu_{\hat{P}} = 0.671, \sigma_{\hat{P}} = 0.014)$. What is the probability that the sample proportion is above 0.70? Illustrate the answer with a fully-labeled graph.

Exercise 2. Suppose that for Canada in 2023 women's foot lengths are Normally distributed with a mean of 24.21cm and a standard deviation of 0.94cm. If women with foot lengths between 23.2cm and 23.7cm wear a size 7 shoe, what is the probability a randomly selected women is a size 7? Illustrate the answer with a fully-labeled graph.

Exercise 3. According to the 2023 World Happiness report, for a random sample of 1,001 people completing a 2022 survey in Egypt asking the Cantril ladder question – people self-assess happiness on scale from 0 to 10 – the sample mean reply is 4.170 with a sample standard deviation of 1.88. Suppose that among *all* people in Egypt, the population mean is 4.388 and the population standard deviation is 1.929. A random variable named $\bar{X}$ is the sample mean (a statistic). For a random sample of $n = 1,001$, $\bar{X}$ is Normally distributed with a mean of 4.388 and a standard deviation of 0.061. What is the chance the sample mean is as low as 4.170, which means that low or even lower? (*Big hint*: It asks $P(\bar{X} < 4.170 \mid \mu_{\bar{X}} = 4.388, \sigma_{\bar{X}} = 0.061) = ?$) Illustrate the answer with a fully-labeled graph.

Exercise 4. A final exam is out of 120 points. Point earned are Normally distributed with a mean of 69 points and a standard deviation of 15 points. What is the 1st percentile? Illustrate the answer with a fully-labeled graph.

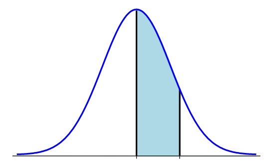
Exercise 5. Continue with the previous exercise. To compute the adjusted percentage marks, the professor adds six points to each students' score and then divides by 120 and multiplies by 100. For example, a student with 66 points out of 120 obtains an adjusted mark of 60 percent ($= 100 \times (66 + 6)/120$). What percent of students obtained an adjusted score of at least a C- on the final exam? (Recall that a mark of 60% is required for a C-.) Illustrate the answer with a fully-labeled graph.

Exercise 6. A 2021 *NBER Working Paper* titled "CEO Stress, Aging, and Death" investigates, among other things, how the extreme demands of being a CEO can cause them to look older. They have data for 463 CEOs. The biological age of this group in 2006 is approximately Normally distributed with a mean of 55.54 years and a standard deviation of 6.55 years. Given this, what is the 92nd percentile? Illustrate the answer with a fully-labeled graph.

Exercise 7. Continue with the previous exercise. What is the interquartile range of age? Illustrate the answer with a fully-labeled graph.

Exercise 8. Due to disruptions from the pandemic, as of 2023, the most recent PISA test scores are from 2018. This is a standardized international test that assesses fifteen-year-old students' skills in math, reading, and science. The scores are Normally distributed across students within each country. The mean math score in Japan is 527 with a standard deviation of 86. The mean math score in Germany is 500 with a standard deviation of 95. Consider comparing the scores between a randomly selected student in Japan with a randomly selected student in Germany. What is the probability that the German student's score is higher? Illustrate the answer with a fully-labeled graph.

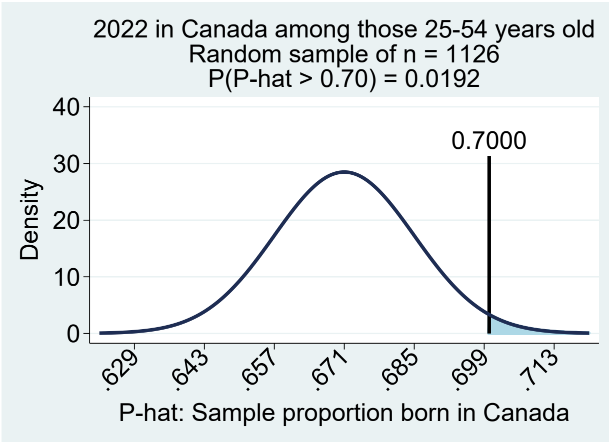
Exercise 9. Finally, return to the flood in Grand Forks, North Dakota that opened this supplement. Again suppose the levee could withstand up to 51 feet and that The Weather Service predicted water levels would rise to 49 feet. If there is a 30 percent chance that the flood waters overtake the levee then what is the standard deviation of the predicted water level? Illustrate the answer with a fully-labeled graph.



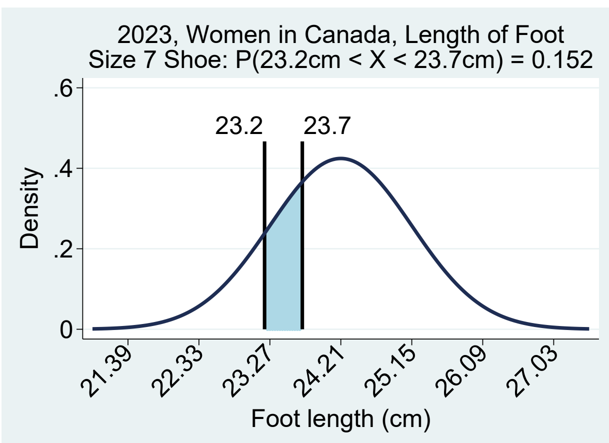
The Standard Normal Distribution:

	Second decimal place in z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999

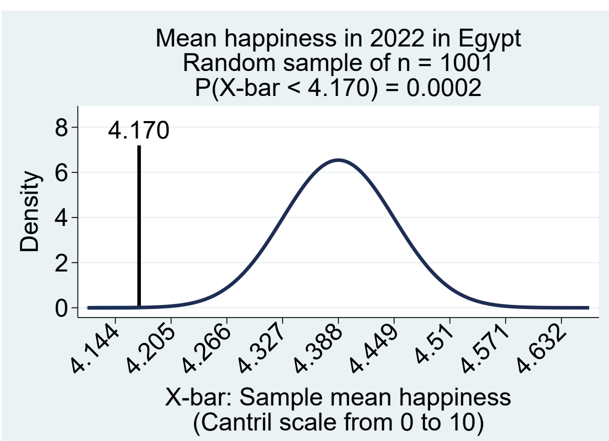
A1. It asks $P(\hat{P} > 0.70 \mid \mu_{\hat{P}} = 0.671, \sigma_{\hat{P}} = 0.014) =?$. $P(Z > \frac{0.70-0.671}{0.014}) = P(Z > 2.071) \approx P(Z > 2.07) = 0.5 - 0.4808 = 0.0192$. Hence, there is a 1.9% chance that the sample proportion is above 70% even though the population proportion is only 67.1%.



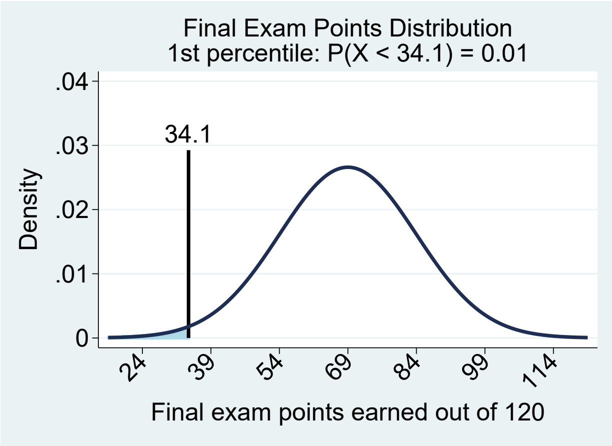
A2. It asks $P(23.2 < X < 23.7 \mid \mu_X = 24.21, \sigma_X = 0.94) =?$. $P(\frac{23.2-24.21}{0.94} < Z < \frac{23.7-24.21}{0.94}) \approx P(-1.07 < Z < -0.54) = 0.3577 - 0.2054 = 0.152$. There is a 15.2% chance a randomly selected woman is a size 7.



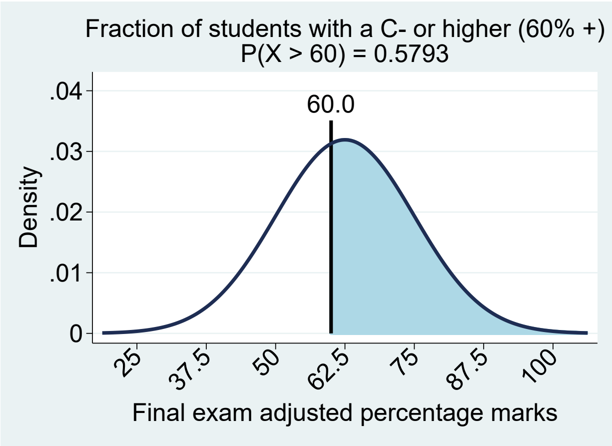
A3. It asks $P(\bar{X} < 4.170 \mid \mu_{\bar{X}} = 4.388, \sigma_{\bar{X}} = 0.061) =?)$. $P(Z < \frac{4.170-4.388}{0.061}) = P(Z < -3.574) \approx P(Z < -3.57) = 0.5 - 0.4998 = 0.0002$. There is only a 0.02% chance of such a low sample mean happiness.



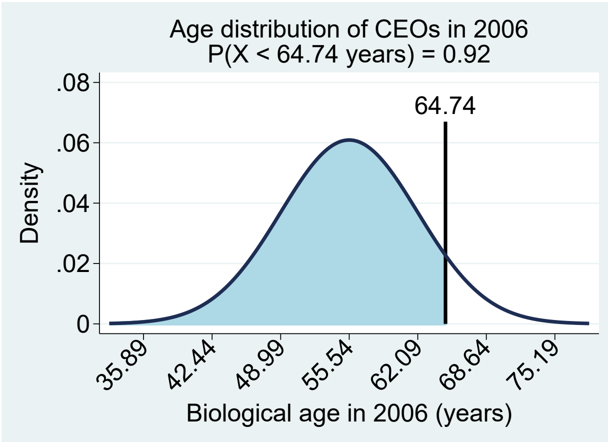
A4. It asks $P(X < ? \mid \mu_X = 69, \sigma_X = 15) = 0.01$. Use the table to get that $P(Z < -2.33) \approx 0.01$. Next, un-standardize. $-2.33 = \frac{x-69}{15}$ so $x = 69 - 2.33 * 15 = 34.1$. The 1st percentile score is 34.1 points out of 120 on the final exam.



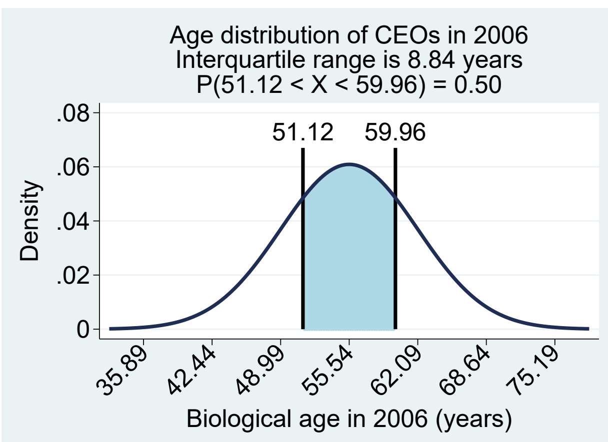
A5. If X is the points earned and Y is the adjusted percentage mark, then $Y = 100\frac{X+6}{120}$, which is a linear transformation of X. Using the Laws of Expected Value, $E[Y] = E[\frac{100}{120}(X+6)] = E[\frac{5}{6}X+5] = \frac{5}{6}E[X]+5 = \frac{5}{6}69+5 = 62.5$. Using the Laws of Variance, $V[Y] = V[\frac{100}{120}(X+6)] = V[\frac{5}{6}X+5] = (\frac{5}{6})^2V[X] = \frac{25}{36}15^2 = 156.25$. Hence, $\mu_Y = 62.5$ and $\sigma_Y = 12.5$. Further, given that X is Normal, the linearly transformed variable Y is also Normal. It asks $P(Y > 60 \mid \mu_Y = 62.5, \sigma_X = 12.5) = ?$. Standardize to use the table. $P(Y > 60) = P(Z > \frac{60-62.5}{12.5}) = P(Z > -0.2) = 0.5 + 0.0793 = 0.5793$. Hence 57.9% of students have earned a mark of C- (60%) or higher on the final exam after the adjustment.



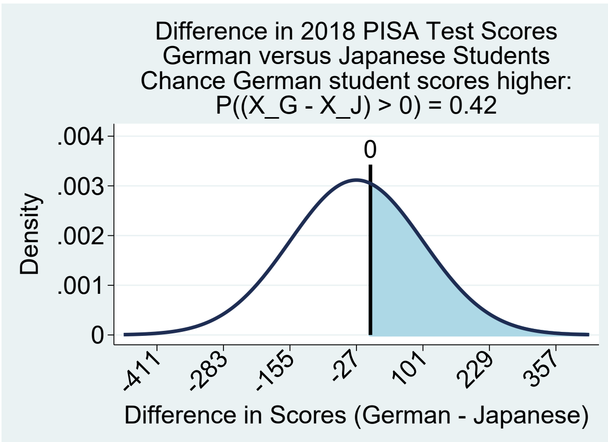
A6. It asks $P(X < \ ? \mid \mu_X = 55.54, \sigma_X = 6.55) = 0.92$. Use the table to get that $P(Z < 1.41) \approx 0.92$. Next, un-standardize. $1.41 = \frac{x-55.54}{6.55}$ so $x = 55.54 + 6.55 * 1.41 = 64.78$. Alternatively, use the table to get that $P(Z < 1.40) \approx 0.92$ and un-standardize for $1.40 = \frac{x-55.54}{6.55}$ so $x = 55.54 + 6.55 * 1.40 = 64.71$. The 92nd percentile is a CEO who is about 65 years old. (The graph shows the exact percentile from software.)



A7. $P(X < \ ? \mid \mu_X = 55.54, \sigma_X = 6.55) = 0.25$ and from the table $P(Z < -0.675) \approx 0.25$ and un-standardize for $-0.675 = \frac{x-55.54}{6.55}$ so $x = 55.54 - 6.55 * 0.675 = 51.12$. $P(X < \ ? \mid \mu_X = 55.54, \sigma_X = 6.55) = 0.75$ and from the table $P(Z < 0.675) \approx 0.75$ and un-standardize for $0.675 = \frac{x-55.54}{6.55}$ so $x = 55.54 + 6.55 * 0.675 = 59.95$. Hence, the interquartile range is 8.84 years.



A8. It asks $P((X_G - X_J) > 0 \mid \mu_{X_G} = 500, \sigma_{X_G} = 95, \mu_{X_J} = 527, \sigma_{X_J} = 86) =?$. The random variables X_G and X_J are each Normal and are independent, hence this linear combination (the difference) is also Normal. Using the Laws of Expected Value, $E[X_G - X_J] = E[X_G] - E[X_J] = 500 - 527 = -27$. By definition, $\mu_{X_G - X_J} = E[X_G - X_J]$. Using the Laws of Variance, $V[X_G - X_J] = V[X_G] + V[X_J] = 95^2 + 86^2 = 16,421$. By definition, $\sigma_{X_G - X_J} = \sqrt{V[X_G - X_J]}$ and $\sqrt{16,421} \approx 128$. Standardize to use the table. $P((X_G - X_J) > 0) = P(Z > \frac{0 - -27}{128}) = P(Z > 0.21) = 0.42$. Despite the mean score being lower in Germany, there is a 42% chance that a randomly selected German student scored higher than a randomly selected Japanese student.



A9. We know $P(X > 51 \mid \mu = 49,\ \sigma = ?) = 0.30$. From the table: $P(Z > 0.525) \approx 0.30$. Given $\frac{51-49}{\sigma} = 0.525$ solve for $\sigma \approx 3.8$ feet. Using software, $\sigma = 3.814$.