Confirm this is your name, student id #, and email. *Signature:* ________________________________________

**NAME:**

*Initial:* ______

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*Initial:* ______

**EMAIL:**

*Initial:* ______

# ECO220Y1Y, Test #4, Prof. Murdock

**March 28, 2025, 8:10 am – 10:00 am**

**Instructions:**

- **Keep ALL pages closed and face up on your desk until we announce the start.**
- There are 8 test pages with 8 questions, with varying numbers of parts, worth a total of 95 points.
- **Questions (1) to (3) have no supplementary material. The rest require the *Supplement*, which will be distributed after the test has begun. You already have the aids sheets, with formulae and statistical tables.**
- You have 110 minutes. You must stay for a minimum of 60 minutes.
- For each question referencing the *Supplement*, carefully review *all* materials. The ***Supplement and aid sheets are NOT collected:*** write your answers on the test papers. At the end, hand in only your Crowdmark test papers.
- Write your answers clearly, completely, and concisely in the designated space provided immediately after each question. An answer guide ends each question to let you know what is expected. For example, a quantitative analysis, a fully labelled graph, and/or sentences. Any answer guide asking for a quantitative analysis *always* automatically means that you must show your work and make your reasoning clear.
    - Marking TAs are instructed to accept all reasonable rounding.
- ***Your entire answer must fit in the designated space provided immediately after each question.*** No extra space/pages are possible. You *cannot* use blank space for other questions, nor can you write answers on the *Supplement*. ***Write in PENCIL and use an ERASER as needed*** so that you can fit your final answer (including work and reasoning) in the appropriate space. We give more blank space than is needed for each answer (with typical handwriting) worth full marks. ***Follow the answer guides and avoid excessively long answers.***

**(1)** [4 pts] A research team has cross-sectional data measuring hourly wages in dollars and each employee's gender. They obtain these OLS results: $\ln\widehat{(wage)}_i = 3.2189 - 0.0764 * woman_i$. *Interpret* -0.0764. Answer with 1 sentence.

**(2)** [5 pts] A research team randomly assigns minutes of preparation time to students. Then they record points earned out of 10 on a multiple-choice quiz. They obtain these OLS results: $\ln\widehat{(points)}_i = 0.6420 + 0.3551 * \ln(prep\_time)_i$. *Interpret* 0.3551. Answer with 1 sentence.

**(3)** [5 pts] Using data for 128 countries in 2023, a research team studies how healthy life expectancy in years relates with real GDP per capita in USD. They obtain these OLS results: $\widehat{life\_expect}_i = 26.7259 + 4.0380 * \ln(real\_GDP\_pc)_i$. *Interpret* 4.0380. Answer with 1 sentence.

**(4)** See ***Supplement for Question (4)***: *The New Geography of Labor Markets.*

**(a)** [7 pts] In the figure, the OLS slope estimate is 14.7. *Interpret* 14.7. In your interpretation, be sure to discuss its size in this context and causality. Answer with 3 – 4 sentences.

**(b)** [7 pts] The standard error of the OLS slope estimate is 0.235. What should you expect to happen to the standard error if you drop the dot in the top right of the figure from the analysis? Explain the *three* reasons why. To organize your explanation of the three reasons, write the relevant formula from the aid sheets. Answer with 3 – 4 sentences.

**(5)** See ***Supplement for Question (5)***: *Reassessing Qualitative Self-Assessments and Experimental Validation.*

**(a)** [8 pts] In Table A.2, is the value -0.09 statistically significant? If yes, at which significance level? Answer with hypotheses in formal notation, a quantitative analysis & 1 sentence.

**(b)** [6 pts] In Table A.2, *interpret* 0.32. In your interpretation, be sure to discuss its size in this context and what its size implies. (*Note:* You do *not* need to mention the source of these data: Falk et al. (2023).) Answer with 2 – 3 sentences.

**(6)** See ***Supplement for Question (6)***: *Happiness in China and India.*

**(a)** [2 pts] In the data used to estimate the regression, what is the *unit of observation*? Answer with a phrase.

**(b)** [14 pts] Create a figure to *interpret* the coefficient estimates in the Stata output. In other words, graphically illustrate the meaning of the OLS coefficient estimates in this context. Answer with a fully labelled graph & show your work.

**(c)** [4 pts] In the t column of the Stata output, *interpret* 5.10. In other words, in plain English and in this context, say what you can conclude. Answer with 1 sentence.

**(d)** [2 pts] What is the numeric value of $t_{\alpha/2}$ needed to compute the two erased numbers in the Stata output? (Note: You are *not* asked to compute the two erased numbers.) Make your reasoning clear.

**(7)** See ***Supplement for Question (7)***: *Asiaphoria Meets Regression to the Mean.*

**(a)** [4 pts] Is the number .01629 in the Stata output large or small in this context? *Explain*. Answer with 1 – 2 sentences.

**(b)** [4 pts] Give an example of a *real-life question* – in PLAIN ENGLISH (like an interpretation) – that would require using the formula below from our aid sheets. Your question should be answerable using the formula below and what is given in the *Supplement*, but you should *not* answer it *nor* plug into the formula. Answer with **ONE** *question*.

$t = \frac{b_1 - \beta_{10}}{s.e.(b_1)}$ (Note: $\beta_{10}$ is read as "Beta one naught," and not as "Beta ten")

**(c)** [4 pts] Give an example of a *real-life question* – in PLAIN ENGLISH (like an interpretation) – that would require using the formula below from our aid sheets. Your question should be answerable using the formula below and what is given in the *Supplement*, but you should *not* answer it *nor* plug into the formula. Answer with **ONE** *question*.

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{\frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}}$$

**(8)** See ***Supplement for Question (8)***: *How Work-from-Home Rates Covary with Individual Characteristics.*

**(a)** [6 pts] Contrary to first appearances, **Column (1)** in Table 2 shows a *multiple* regression. In plain English, explain what is being controlled for and *why* it makes sense to do so. (Use your own words. Do *not* copy from the *Supplement*.) Answer with 2 – 3 sentences.

**(b)** [4 pts] According to **Column (3)** in Table 2, how does a male who lives with a child under 14 compare with a male who does not live with a child under 14? Answer by filling in the four blanks below.

According to **Column (3)**, compared to a male who does not live with a child under 14, a male who does live with a child under 14 works _____ [#] ________________________ [percent / percentage points] ________ [more / less] days from home. This difference is _____________________________________________________ [not significant / not statistically significant, but economically significant / not economically significant, but statistically significant / significant].

**(c)** [9 pts] In **Columns (4) and (5)** in Table 2, *interpret* the results in the cells with 16.5 and 11.4, which are in boldface. (Do *not* be repetitive and do *not* copy language from the *Supplement*. Instead, write a coherent short paragraph in your own words.) Answer with 3 – 4 sentences.

***Supplement for Questions (1), (2) and (3):*** N/A (all information given with these questions)

***Supplement for Question (4):***
Consider a 2025 *NBER Working Paper* titled "The New Geography of Labor Markets."

The figure to the right is titled "Employers in areas with high housing prices have a much greater share of distant employees."

*Notes:* 50 miles is 80.5 km. These are binned data from the US and a zipcode is a small geographic area like a postal code in Canada.


***Supplement for Question (5):*** From a 2025 *NBER Working Paper* titled "Reassessing Qualitative Self-Assessments and Experimental Validation," consider the excerpt and table below. It discusses a paper and data that we have studied in workshops. Neither the wording of the survey questions for people to self-assess themselves nor the design of experiments to elicit measures of personality traits are needed to answer the test questions. Two numbers in Table A.2 are in **boldface** for easy reference.

> ***Excerpt (p. 4, Online Appendix):*** Table A.2 presents the correlations between qualitative self-assessments and incentivized elicitations using the replication dataset from Falk et al. (2023). As in our sample, qualitative self-assessments are often statistically significantly correlated with incentivized elicitations other than those they are intended to proxy for.

Table A.2: Correlations between Qualitative Self-Assessments and Incentivized Elicitations in Falk et al. (2023)

| | | *Incentivized Elicitations* | | | | | |
|---|---|---|---|---|---|---|---|
| | | Risk Tolerance | Impatience | Altruism | Trust | Reciprocity | Punishment |
| *Qualitative Self-Assessment* | Risk Tolerance | **0.32** | -0.02 | -0.03 | 0.09 | -0.10 | 0.03 |
| | Impatience | 0.09 | 0.08 | 0.08 | 0.06 | -0.00 | 0.01 |
| | Altruism | -0.02 | 0.13 | 0.39 | 0.16 | 0.24 | -0.01 |
| | Trust | 0.03 | 0.19 | 0.17 | 0.27 | 0.23 | **-0.09** |
| | Reciprocity | -0.03 | 0.12 | 0.11 | 0.23 | 0.22 | -0.05 |
| | Punishment | 0.05 | -0.00 | 0.00 | 0.05 | 0.15 | 0.17 |

*Notes:* Data from Falk et al. (2023), n = 360 for measures of reciprocity and punishment, and n = 382 for all other domains.

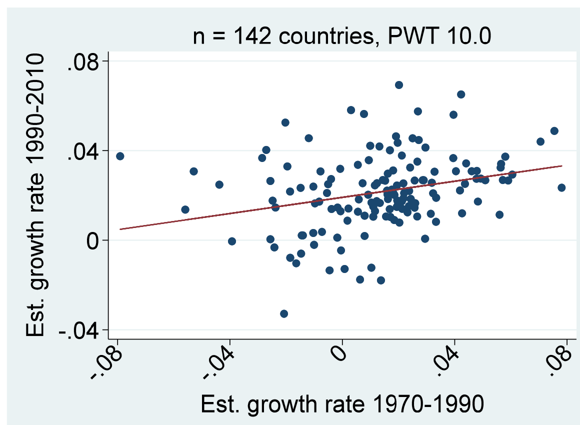
***Supplement for Question (6):*** Recall the data from the 2024 World Happiness Report. Each year from 2006 through 2023, an independent sample of 1,000+ people in India answer the Cantril ladder survey question to assess happiness on a 0 to 10 scale. In China it is the same except that they did not run the survey in 2022. The variable named `trend` is 1 in 2006, 2 in 2007, 3 in 2008, and so on. The other variables are self-explanatory given the descriptive variable names. See the Stata regression output below. One number is in **boldface** in the Stata output below for easy reference and two numbers are intentionally erased.

```
      Source |       SS           df       MS      Number of obs   =        35
-------------+----------------------------------   F(3, 31)        =     37.29
       Model |  10.5522064         3  3.51740215   Prob > F        =    0.0000
    Residual |  2.92441995        31  .094336127   R-squared       =    0.7830
-------------+----------------------------------   Adj R-squared   =    0.7620
       Total |  13.4766264        34  .396371365   Root MSE        =    .30714

------------------------------------------------------------------------------
     cantril |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       india |   .6896452   .2158244     3.20   0.003     .2494684    1.129822
       trend |   .0759614   .0148996     5.10   0.000
 indiaXtrend |  -.1558129   .0204134    -7.63   0.000    -.1974462   -.1141795
       _cons |   4.472334    .154165    29.01   0.000     4.157912    4.786756
------------------------------------------------------------------------------
```

***Supplement for Question (7):*** Recall "Asiaphoria Meets Regression to the Mean" (Pritchett and Larry Summers (2014)) and the PWT 10.0 data. The variables `r_1970_90` and `r_1990_10` are the estimated annual growth rate of real GDP per capita from 1970 to 1990 and from 1990 to 2010, respectively, for each of 142 countries.

One number is in **boldface** in the Stata output below for easy reference.




```
. summarize r_1990_10 r_1970_90

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
   r_1990_10 |        142    .0215697    .0169006  -.0328348   .0693116
   r_1970_90 |        142    .0135592    .0261719   -.079102   .0780859


. regress r_1990_10 r_1970_90;

      Source |       SS           df       MS      Number of obs   =       142
-------------+----------------------------------   F(1, 140)       =     11.79
       Model |  .00312832          1  .00312832    Prob > F        =    0.0008
    Residual |  .037145489       140  .000265325   R-squared       =    0.0777
-------------+----------------------------------   Adj R-squared   =    0.0711
       Total |  .040273808       141  .00028563    Root MSE        =    .01629

------------------------------------------------------------------------------
   r_1990_10 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   r_1970_90 |   .1799742   .0524136     3.43   0.001     .0763498    .2835987
       _cons |   .0191294   .0015406    12.42   0.000     .0160835    .0221753
------------------------------------------------------------------------------
```

***Supplement for Question (8):*** Consider a 2023 paper "The Evolution of Work from Home" published in the *Journal of Economic Perspectives*. It analyzes survey data from the United States. Consider Table 2 below and an excerpt from the article where the authors discuss the results in Table 2. In Table 2 some results are in **boldface** for easy reference.

> ***Excerpt (pp. 33-34):*** We find that people with children work from home at higher rates. Table 2 develops this point more fully, drawing on data from the Survey of Working Arrangements and Attitudes (Barrero et al. 2020–2023). All specifications control for five-year age bins and month fixed effects. The coefficient on "Female" in Column (2) says that full days worked from home (as a percent of all paid workdays) are 1.0 percentage points higher for women than men, the omitted group. The other coefficient in Column (2) says that full days worked from home are 4.5 percentage points higher for workers who live with children under 14. Column (3) adds a term to capture the interaction between "Female" and "Children under 14." The –2.3 coefficient on this term is statistically significant at the 5 percent level. In other words, living with children is associated with a larger marginal increase in work-from-home intensity for men than women. The coefficient on the main effect for "Children under 14" is now 5.5 percentage points. Finally, when we add controls for the worker's education, industry, and occupation in Columns (4) and (5), we continue to find higher work-from-home rates among those who live with children. The results in Columns (4) and (5) also confirm that education is a powerful predictor of work-from-home intensity. (Here, the omitted group is persons with no postsecondary education.) However, the coefficients on the main and interaction effects for women are no longer statistically significant.

**Table 2.** How Work-from-Home Rates Covary with Individual Characteristics

| | Full days worked at home as percent of paid workdays | | | | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| 1(Female) | 0.9* | 1.0** | 1.9*** | -0.2 | 0.6 |
| | (0.5) | (0.5) | (0.6) | (0.6) | (0.6) |
| 1(Lives with child under 14) | | 4.5*** | 5.5*** | 2.6*** | 1.6** |
| | | (0.5) | (0.7) | (0.7) | (0.7) |
| 1(Female) × 1(Lives with child under 14) | | | -2.3** | -0.0 | 0.5 |
| | | | (1.0) | (0.9) | (0.9) |
| 1(one to three years of college) | | | | 7.0*** | 5.1*** |
| | | | | (0.7) | (0.6) |
| 1(four-year college degree) | | | | **16.5*** ** | **11.4*** ** |
| | | | | **(0.6)** | **(0.7)** |
| 1(Graduate degree) | | | | 19.1*** | 13.4*** |
| | | | | (0.7) | (0.8) |
| Industry and occupation fixed effects | | | | | Y |
| $n$ | 48,244 | 48,244 | 48,244 | 48,244 | 48,244 |
| $R^2$ | 0.01 | 0.01 | 0.01 | 0.04 | 0.11 |

*Notes:* We use data from the Survey of Working Arrangements and Attitudes (Barrero et al. 2020–2023) covering October 2021 to October 2022 (inclusive) and regress full days worked at home as percent of paid workdays on indicators for sex, for whether the respondent lives with a child under 14, and education categories. All columns include monthly survey wave fixed effects, and fixed effects for five-year age bins (e.g. 25 to 29, 30 to 34, etc.). 1(. . .) denotes the indicator function. The sample includes respondents who worked during the reference week, pass our attention-check questions, and have non-missing data on occupation and industry of the current or most recent job. We report standard errors in parentheses. *$p$ < 0.1, **$p$ < 0.05, ***$p$ < 0.01.

**Sample mean:** $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ **Sample variance:** $s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{X})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$ **Sample s.d.:** $s = \sqrt{s^2}$

**Sample coefficient of variation:** $CV = \frac{s}{\bar{X}}$ **Sample covariance:** $s_{xy} = \frac{\sum_{i=1}^{n}(x_i-\bar{X})(y_i-\bar{Y})}{n-1} = \frac{\sum_{i=1}^{n} x_i y_i}{n-1} - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(n-1)}$

**Sample interquartile range:** $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{s_{xy}}{s_x s_y} = \frac{\sum_{i=1}^{n} z_{x_i} z_{y_i}}{n-1}$

**Addition rule:** $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ **Conditional probability:** $P(A|B) = \frac{P(A\ and\ B)}{P(B)}$

**Complement rules:** $P(A^C) = P(A') = 1 - P(A)$ $P(A^C|B) = P(A'|B) = 1 - P(A|B)$

**Multiplication rule:** $P(A\ and\ B) = P(A|B)P(B) = P(B|A)P(A)$

**Expected value:** $E[X] = \mu = \sum_{all\ x} xp(x)$ **Variance:** $V[X] = E[(X-\mu)^2] = \sigma^2 = \sum_{all\ x}(x-\mu)^2 p(x)$

**Covariance:** $COV[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = \sigma_{XY} = \sum_{all\ x}\sum_{all\ y}(x-\mu_X)(y-\mu_Y)p(x,y)$

**Laws of expected value:**

$E[c] = c$
$E[X+c] = E[X] + c$
$E[cX] = cE[X]$
$E[a+bX+cY] = a + bE[X] + cE[Y]$

**Laws of variance:**

$V[c] = 0$
$V[X+c] = V[X]$
$V[cX] = c^2V[X]$
$V[a+bX+cY] = b^2V[X] + c^2V[Y] + 2bc * COV[X,Y]$
$V[a+bX+cY] = b^2V[X] + c^2V[Y] + 2bc * SD(X) * SD(Y) * \rho$
where $\rho = CORRELATION[X,Y]$

**Laws of covariance:**

$COV[X,c] = 0$
$COV[a+bX, c+dY] = bd * COV[X,Y]$

**Combinatorial formula:** $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$ for $x = 0,1,2,\ldots,n$

**If $X$ is Binomial** $(X \sim B(n,p))$ **then** $E[X] = np$ **and** $V[X] = np(1-p)$

**If $X$ is Uniform** $(X \sim U[a,b])$ **then** $f(x) = \frac{1}{b-a}$ **and** $E[X] = \frac{a+b}{2}$ **and** $V[X] = \frac{(b-a)^2}{12}$

**Sampling distribution of $\bar{X}$:**

$\mu_{\bar{X}} = E[\bar{X}] = \mu$
$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$
$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

**Sampling distribution of $\hat{P}$:**

$\mu_{\hat{P}} = E[\hat{P}] = p$
$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$
$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

**Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$:**

$\mu_{\hat{P}_2-\hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$
$\sigma_{\hat{P}_2-\hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$
$\sigma_{\hat{P}_2-\hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

**Sampling distribution of $(\bar{X}_1 - \bar{X}_2)$, independent samples:**

$\mu_{\bar{X}_1-\bar{X}_2} = E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2$
$\sigma_{\bar{X}_1-\bar{X}_2}^2 = V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$
$\sigma_{\bar{X}_1-\bar{X}_2} = SD[\bar{X}_1 - \bar{X}_2] = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

**Sampling distribution of $(\bar{X}_d)$, paired ($d = X_1 - X_2$):**

$\mu_{\bar{X}_d} = E[\bar{X}_d] = \mu_1 - \mu_2$
$\sigma_{\bar{X}_d}^2 = V[\bar{X}_d] = \frac{\sigma_d^2}{n} = \frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}$
$\sigma_{\bar{X}_d} = SD[\bar{X}_d] = \frac{\sigma_d}{\sqrt{n}} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}}$

**Inference about a population proportion:**

***z* test statistic:** $z = \frac{\hat{P} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ **CI estimator:** $\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

**Inference about comparing two population proportions:**

***z* test statistic under Null hypothesis of no difference:** $z = \frac{\hat{P}_2 - \hat{P}_1}{\sqrt{\frac{\bar{P}(1-\bar{P})}{n_1} + \frac{\bar{P}(1-\bar{P})}{n_2}}}$ **Pooled proportion:** $\bar{P} = \frac{X_1 + X_2}{n_1 + n_2}$

**CI estimator:** $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

**Inference about the population mean:**

***t* test statistic:** $t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$ **CI estimator:** $\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

**Inference about a comparing two population means, independent samples, unequal variances:**

***t* test statistic:** $t = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

**Degrees of freedom:** $\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$

**Inference about a comparing two population means, independent samples, assuming equal variances:**

***t* test statistic:** $t = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$ **Degrees of freedom:** $\nu = n_1 + n_2 - 2$

**Pooled variance:** $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

**Inference about a comparing two population means, paired data:** ($n$ is number of pairs and $d = X_1 - X_2$)

***t* test statistic:** $t = \frac{\bar{d} - \Delta_0}{s_d/\sqrt{n}}$ **CI estimator:** $\bar{X}_d \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

**SIMPLE REGRESSION:**

**Model:** $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ **OLS line:** $\hat{y}_i = b_0 + b_1 x_i$ $b_1 = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x}$ $b_0 = \bar{Y} - b_1\bar{X}$

**Coefficient of determination:** $R^2 = (r)^2$ **Residuals:** $e_i = y_i - \hat{y}_i$

**Standard deviation of residuals:** $s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i - 0)^2}{n-2}}$ **Standard error of slope:** $s.e.(b_1) = s_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$

**Inference about the population slope:**

***t* test statistic:** $t = \frac{b_1 - \beta_{10}}{s.e.(b_1)}$ **CI estimator:** $b_1 \pm t_{\alpha/2} s.e.(b_1)$ **Degrees of freedom:** $\nu = n - 2$

**Standard error of slope:** $s.e.(b_1) = s_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$

**Prediction interval for $y$ at given value of $x$ ($x_g$):**

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \textbf{or} \quad \hat{y}_{x_g} \pm t_{\alpha/2} \sqrt{\left(s.e.(b_1)\right)^2 \left(x_g - \bar{X}\right)^2 + \frac{s_e^2}{n} + s_e^2}$$

**Degrees of freedom:** $\nu = n - 2$

**Confidence interval for predicted mean at given value of $x$ ($x_g$):**

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{\frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \textbf{or} \quad \hat{y}_{x_g} \pm t_{\alpha/2} \sqrt{\left(s.e.(b_1)\right)^2 \left(x_g - \bar{X}\right)^2 + \frac{s_e^2}{n}}$$

**Degrees of freedom:** $\nu = n - 2$

**SIMPLE & MULTIPLE REGRESSION:**

**Model:** $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + \varepsilon_i$

$$SST = \sum_{i=1}^{n} (y_i - \bar{Y})^2 = SSR + SSE \quad SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{Y})^2 \quad SSE = \sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$s_y^2 = \frac{SST}{n-1} \quad MSE = \frac{SSE}{n-k-1} \quad Root\ MSE = \sqrt{\frac{SSE}{n-k-1}} \quad MSR = \frac{SSR}{k}$$

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \quad Adj.R^2 = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)} = \left(R^2 - \frac{k}{n-1}\right)\left(\frac{n-1}{n-k-1}\right)$$

**Residuals:** $e_i = y_i - \hat{y}_i$ **Standard deviation of residuals:** $s_e = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i - 0)^2}{n-k-1}}$

**Inference about the overall statistical significance of the regression model:**

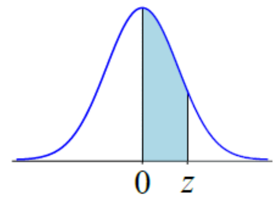
$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)} = \frac{(SST-SSE)/k}{SSE/(n-k-1)} = \frac{SSR/k}{SSE/(n-k-1)} = \frac{MSR}{MSE}$$

**Numerator degrees of freedom:** $\nu_1 = k$ **Denominator degrees of freedom:** $\nu_2 = n - k - 1$

**Inference about the population slope for explanatory variable j:**

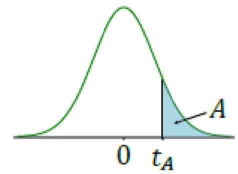
***t* test statistic:** $t = \frac{b_j - \beta_{j0}}{s_{b_j}}$ **CI estimator:** $b_j \pm t_{\alpha/2} s_{b_j}$ **Degrees of freedom:** $\nu = n - k - 1$

**Standard error of slope:** $s.e.(b_j) = s_{b_j}$ (for multiple regression, must be obtained from technology)




**The Standard Normal Distribution:**

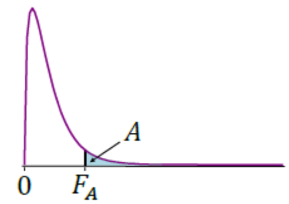
| | *Second decimal place in z* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |




**Critical Values of Student $t$ Distribution:**

| $\nu$ | $t_{0.10}$ | $t_{0.05}$ | $t_{0.025}$ | $t_{0.01}$ | $t_{0.005}$ | $t_{0.001}$ | $t_{0.0005}$ | $\nu$ | $t_{0.10}$ | $t_{0.05}$ | $t_{0.025}$ | $t_{0.01}$ | $t_{0.005}$ | $t_{0.001}$ | $t_{0.0005}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 318.3 | 636.6 | 38 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.33 | 31.60 | 39 | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 3.313 | 3.558 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.21 | 12.92 | 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 | 41 | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 3.301 | 3.544 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 | 42 | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 | 43 | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 3.291 | 3.532 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 | 44 | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 | 45 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 3.281 | 3.520 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 | 46 | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 | 3.515 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 | 47 | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 3.273 | 3.510 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 | 48 | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 | 3.505 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 | 49 | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 3.265 | 3.500 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 | 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 | 51 | 1.298 | 1.675 | 2.008 | 2.402 | 2.676 | 3.258 | 3.492 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 | 52 | 1.298 | 1.675 | 2.007 | 2.400 | 2.674 | 3.255 | 3.488 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 | 53 | 1.298 | 1.674 | 2.006 | 2.399 | 2.672 | 3.251 | 3.484 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 | 54 | 1.297 | 1.674 | 2.005 | 2.397 | 2.670 | 3.248 | 3.480 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 | 55 | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 | 3.245 | 3.476 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 | 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 | 65 | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 | 3.220 | 3.447 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 | 70 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 | 75 | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 | 3.202 | 3.425 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 | 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 | 90 | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 | 3.402 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 | 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 | 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 | 140 | 1.288 | 1.656 | 1.977 | 2.353 | 2.611 | 3.149 | 3.361 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 | 160 | 1.287 | 1.654 | 1.975 | 2.350 | 2.607 | 3.142 | 3.352 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 | 180 | 1.286 | 1.653 | 1.973 | 2.347 | 2.603 | 3.136 | 3.345 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 | 200 | 1.286 | 1.653 | 1.972 | 2.345 | 2.601 | 3.131 | 3.340 |
| 31 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 3.375 | 3.633 | 250 | 1.285 | 1.651 | 1.969 | 2.341 | 2.596 | 3.123 | 3.330 |
| 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 | 300 | 1.284 | 1.650 | 1.968 | 2.339 | 2.592 | 3.118 | 3.323 |
| 33 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 3.356 | 3.611 | 400 | 1.284 | 1.649 | 1.966 | 2.336 | 2.588 | 3.111 | 3.315 |
| 34 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 | 500 | 1.283 | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| 35 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 3.340 | 3.591 | 750 | 1.283 | 1.647 | 1.963 | 2.331 | 2.582 | 3.101 | 3.304 |
| 36 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 | 1000 | 1.282 | 1.646 | 1.962 | 2.330 | 2.581 | 3.098 | 3.300 |
| 37 | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 3.326 | 3.574 | $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |

Degrees of freedom: $\nu$




**The $F$ Distribution:**

| $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 15 | 20 | 30 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu_2$ | **Critical Values of $F$ Distribution for $A = 0.10$:** | | | | | | | | | | | | | | | |
| 5 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 | 3.30 | 3.28 | 3.27 | 3.24 | 3.21 | 3.17 | 3.10 |
| 10 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 | 2.32 | 2.30 | 2.28 | 2.24 | 2.20 | 2.16 | 2.06 |
| 15 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 | 2.06 | 2.04 | 2.02 | 1.97 | 1.92 | 1.87 | 1.76 |
| 20 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 | 1.94 | 1.91 | 1.89 | 1.84 | 1.79 | 1.74 | 1.61 |
| 30 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 | 1.82 | 1.79 | 1.77 | 1.72 | 1.67 | 1.61 | 1.46 |
| 40 | 2.84 | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 | 1.76 | 1.74 | 1.71 | 1.66 | 1.61 | 1.54 | 1.38 |
| 60 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 | 1.71 | 1.68 | 1.66 | 1.60 | 1.54 | 1.48 | 1.29 |
| 120 | 2.75 | 2.35 | 2.13 | 1.99 | 1.90 | 1.82 | 1.77 | 1.72 | 1.68 | 1.65 | 1.63 | 1.60 | 1.55 | 1.48 | 1.41 | 1.19 |
| $\infty$ | 2.71 | 2.30 | 2.08 | 1.94 | 1.85 | 1.77 | 1.72 | 1.67 | 1.63 | 1.60 | 1.57 | 1.55 | 1.49 | 1.42 | 1.34 | 1.00 |
| $\nu_2$ | **Critical Values of $F$ Distribution for $A = 0.05$:** | | | | | | | | | | | | | | | |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.70 | 4.68 | 4.62 | 4.56 | 4.50 | 4.36 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.94 | 2.91 | 2.85 | 2.77 | 2.70 | 2.54 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.51 | 2.48 | 2.40 | 2.33 | 2.25 | 2.07 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.31 | 2.28 | 2.20 | 2.12 | 2.04 | 1.84 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.13 | 2.09 | 2.01 | 1.93 | 1.84 | 1.62 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.04 | 2.00 | 1.92 | 1.84 | 1.74 | 1.51 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.95 | 1.92 | 1.84 | 1.75 | 1.65 | 1.39 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.87 | 1.83 | 1.75 | 1.66 | 1.55 | 1.25 |
| $\infty$ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.79 | 1.75 | 1.67 | 1.57 | 1.46 | 1.00 |
| $\nu_2$ | **Critical Values of $F$ Distribution for $A = 0.01$:** | | | | | | | | | | | | | | | |
| 5 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 | 10.1 | 9.96 | 9.89 | 9.72 | 9.55 | 9.38 | 9.02 |
| 10 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.77 | 4.71 | 4.56 | 4.41 | 4.25 | 3.91 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.73 | 3.67 | 3.52 | 3.37 | 3.21 | 2.87 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.29 | 3.23 | 3.09 | 2.94 | 2.78 | 2.42 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.91 | 2.84 | 2.70 | 2.55 | 2.39 | 2.01 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.73 | 2.66 | 2.52 | 2.37 | 2.20 | 1.80 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.56 | 2.50 | 2.35 | 2.20 | 2.03 | 1.60 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.40 | 2.34 | 2.19 | 2.03 | 1.86 | 1.38 |
| $\infty$ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.25 | 2.18 | 2.04 | 1.88 | 1.70 | 1.00 |
| $\nu_2$ | **Critical Values of $F$ Distribution for $A = 0.001$:** | | | | | | | | | | | | | | | |
| 5 | 47.2 | 37.1 | 33.2 | 31.1 | 29.8 | 28.8 | 28.2 | 27.6 | 27.2 | 26.9 | 26.6 | 26.4 | 25.9 | 25.4 | 24.9 | 23.8 |
| 10 | 21.0 | 14.9 | 12.6 | 11.3 | 10.5 | 9.93 | 9.52 | 9.20 | 8.96 | 8.75 | 8.59 | 8.45 | 8.13 | 7.80 | 7.47 | 6.76 |
| 15 | 16.6 | 11.3 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 | 6.08 | 5.94 | 5.81 | 5.54 | 5.25 | 4.95 | 4.31 |
| 20 | 14.8 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 | 5.08 | 4.94 | 4.82 | 4.56 | 4.29 | 4.00 | 3.38 |
| 30 | 13.3 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 | 4.24 | 4.11 | 4.00 | 3.75 | 3.49 | 3.22 | 2.59 |
| 40 | 12.6 | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 | 3.87 | 3.75 | 3.64 | 3.40 | 3.14 | 2.87 | 2.23 |
| 60 | 12.0 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 | 3.54 | 3.42 | 3.32 | 3.08 | 2.83 | 2.55 | 1.89 |
| 120 | 11.4 | 7.32 | 5.78 | 4.95 | 4.42 | 4.04 | 3.77 | 3.55 | 3.38 | 3.24 | 3.12 | 3.02 | 2.78 | 2.53 | 2.26 | 1.54 |
| $\infty$ | 10.83 | 6.91 | 5.42 | 4.62 | 4.10 | 3.74 | 3.47 | 3.27 | 3.10 | 2.96 | 2.84 | 2.74 | 2.51 | 2.27 | 1.99 | 1.00 |

Numerator degrees of freedom: $\nu_1$; Denominator degrees of freedom: $\nu_2$