ECO220Y, Term Test #4: SOLUTIONS

March 31, 2017, 9:10 – 11:00 am

(1) (a) Be unchanged. [The number of observations would go up, but there is no reason to expect an upward or downward change in the standard deviation, which measures the variability among males. By chance it may be higher or lower due to sampling error, but we have no reason to *expect* that a larger sample size would yield either direction of change in particular. If you thought "go down," please note that that misconception is contradicted by the **"Summary of total money passed, by year,"** which shows no relationship between the sample size each year and the summary statistics (including the s.d.).]

(b) $H_0: (p_M - p_F) = 0$

$H_1: (p_M - p_F) \neq 0$

$$z = \frac{\hat{P}_M - \hat{P}_F}{\sqrt{\frac{\bar{P}(1-\bar{P})}{n_M} + \frac{\bar{P}(1-\bar{P})}{n_F}}} = \frac{\frac{50}{334} - \frac{34}{534}}{\sqrt{0.09677(1-0.09677)\left(\frac{1}{334}+\frac{1}{534}\right)}} = \frac{0.1497 - 0.0637}{0.0206} = 4.18$$

$$\bar{P} = \frac{X_M + X_F}{n_M + n_F} = \frac{50 + 34}{334 + 534} = 0.09677$$

This difference is highly statistically significant at all conventional significance levels including 1%: in fact, the P-value is less than 0.0001. The difference, 15% of males keeping all tokens for themselves versus only 6% of females is economically significant: this is a whopping 9 percentage point difference. Males are more than twice as likely as females to keep all the tokens for themselves. [The reason that the average generosity is basically the same between the sexes is because males are also more likely to be extremely generous.]

(c) $H_0: (\mu_M - \mu_F) = 0$

$H_1: (\mu_M - \mu_F) \neq 0$

$$t = \frac{(\bar{X}_M - \bar{X}_F) - \Delta_0}{\sqrt{\frac{s_M^2}{n_M} + \frac{s_F^2}{n_F}}} = \frac{(26.97126 - 26.18446) - 0}{\sqrt{\frac{16.44741^2}{334} + \frac{12.23994^2}{534}}} = \frac{0.787}{1.044} = 0.75$$

(d) $H_0: (\mu_M - \mu_F) = 0$

$H_1: (\mu_M - \mu_F) \neq 0$

$$t = \frac{(\bar{X}_M - \bar{X}_F) - \Delta_0}{\sqrt{\frac{s_p^2}{n_M} + \frac{s_p^2}{n_F}}} = \frac{(26.97126 - 26.18446) - 0}{\sqrt{\frac{196.2289}{334} + \frac{196.2289}{534}}} = \frac{0.787}{0.977} = 0.81$$

$$s_p^2 = \frac{(n_M - 1)s_M^2 + (n_F - 1)s_F^2}{n_M + n_F - 2} = \frac{(334-1)16.44741^2 + (534-1)12.23994^2}{334 + 534 - 2} = 196.2289$$

(e) Define $yr2015$ to be a dummy variable = 1 for students that participated on Feb. 6, 2015. Define $yr2016$ to be a dummy variable = 1 for students that participated on Feb. 12, 2016. We can make the omitted category (aka the reference category) students that participated Feb. 14, 2014 ($yr2014$). We can define tmp to be an interval variable measuring the total money a student passed to the partner over all eight decisions.

Model: $tmp_i = \alpha + \delta_1 yr2015_i + \delta_2 yr2016_i + \varepsilon_i$

OLS estimate: $\widehat{tmp}_i = 27.691 - 1.163 * yr2015_i - 2.459 * yr2016_i$

[Note: If you select 2016 as the omitted category: $\widehat{tmp}_i = 25.232 + 2.459 * yr2014_i + 1.296 * yr2015_i$. If you select 2015 as the omitted category: $\widehat{tmp}_i = 26.528 + 1.163 * yr2014_i - 1.296 * yr2016_i$.]

(f) No, there is no statistically significant difference at any conventional significance level: 1%, 5% or 10%.

(2) (a) The SST measures the total variability of the y-variable, which is growth rate (%) from 1980 – 1990, around its mean across the 30 member nations of the OECD. Because Specifications (1) and (3) include the same 30 observations, the SST is equal. However, because Specification (2) excludes an outlier – South Korea – which had much higher growth than the other 29 OECD countries during the 1980's, the SST drops considerably. Put simply, growth in the 1980's is much less variable among the 29 OECD countries excluding South Korea, which all had much more modest growth than South Korea.

(b) The R-squared measures what fraction of the variation in growth rates in the 1990's across OECD countries can be explained by variation in their growth rates in the 1980's. In Specification (2), which excludes South Korea, the R-squared is basically zero, which is consistent with the scatter diagram that, but for South Korea, shows a scatter of points with no clear up or down pattern. In Specification (3) it is very high because the dummy for South Korea can now single-handedly explain much of the variation in growth rates in the 1990's because the very high growth of South Korea caused more than half of the total variation in growth among the 30 OECD countries (which is what Part (a) pointed out) and the dummy for South Korea controls for everything special about that observation.

(3)

$H_0: \beta = \delta = \gamma = 0$

$H_1: Not\ all\ of\ the\ slope\ coefficients\ are\ zero$

$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)} = \frac{0.0881/3}{(1-0.0881)/(100-3-1)} = 3.09$$

The multiple regression results are statistically significant overall at the 5% level, but not the 1% level.

(4) (a) The $22,000 difference does not hold anything else fixed, whereas the $2,904.59 difference does. The multiple regression coefficient that says males earn $2,904.59 more than females (with a s.e. of $700.82) is *after controlling for* academic merit scores, previous Outstanding Performance Awards, years employed at Waterloo, highest degree earned, lag between highest degree and hire, current academic rank, academic unit, rank when hired, interaction effects between academic unit and rank, and interaction effects between lag and rank when hired. The sex ratios vary considerably across academic units as does the pay: female faculty are scarce in the STEM fields where pay is high. Further, female faculty members may be more junior on average as many disciplines were male dominated historically and senior faculty typically earn high salaries. [A third obvious and plausible explanation is that the sex-ratios are not equal across the types of faculty ranks: the professorial positions are more male dominated than the lecturer positions, but lecturer positions are worse paid.] However, once we have controlled for these differences we hope to get a better estimate of the pure sex-based differences.

(b) $H_0: \beta_3 = 2000$

$H_1: \beta_3 > 2000$

$t = \frac{b_3 - 2000}{se(b_3)} = \frac{3587.37 - 2000}{651.23} = 2.44$. The P-value is about 0.0073 using the Normal approximation to the Student t (or you may say it is between 0.005 and 0.01 using the Student t table).

(c) $\widehat{salary} = 47757.05 + 2904.59 * 1 + 25821.67 * 1.75 + 3587.37 * 1 + 2252.47(2015 - 2010) - 16.60 * (2015 - 2010)^2 + 8564.38 + 5114.92 + 17615.55 * 1 = \$141{,}579$

(d) $\frac{\partial salary}{\partial years} = 2252.47 - 33.20 * years$. Hence, the point estimates of the slopes are 2086.47 and 1090.47, respectively. [Hence, salary is increasing with years of service but at a decreasing rate.]

(e) $b_M \pm t_{\frac{\alpha}{2}} * se(b_M) = 2904.59 \pm 1.646 * 700.82 = 2904.59 \pm 1153.55$. [NOTE: Using 1.645, instead of 1.646, is also OK.] Hence the Lower Confidence Limit (LCL) is $1,751 and the Upper Confidence Limit (UCL) is $4,058. We are 90% confident that male faculty members earn between $1,751 to $4,058 more than female faculty members even when they have the same merit, same number of Outstanding Performance Awards, have worked at Waterloo for the same number of years, have the same lag between their degree and hire, have the same highest degree, have the same current academic rank, work within the same academic unit, and were hired at the same rank. This is a fairly wide interval -- the margin of error is $1,154 and we are only 90% confident (the margin of error would be even bigger for a higher confidence level) – especially since Waterloo is giving all female faculty a raise using these results: the raise may be substantially too big or substantially too small.

(f) If salary were logged then $b_1 * 100$ measures the percent boost in salary associated with being male. Given that a typical salary appears to be roughly $160,000, we would expect a value of b_1 that is roughly 0.02: males have salaries that are $2,904.59 higher than females after controlling for the other explanatory variables and that is roughly 2% higher (100*2,905/160,000).