

ECO220Y1Y, Test #3, Prof. Murdock

March 4, 2022, 9:10 – 11:00 am

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Instructions:

- You have 110 minutes. Keep these test papers and the *Supplement* closed and face up on your desk until the start of the test is announced. You must stay for a minimum of 60 minutes.
- You may use a **non-programmable calculator**.
- There are 9 questions (some with multiple parts) with varying point values worth a total of 95 points.
- This test includes these 10 pages plus the *Supplement*. The *Supplement* contains the aid sheets and statistical tables (Standard Normal and Student t). **The Supplement will NOT be collected:** write your answers on these test papers. When we announce the end of the test, hand these test papers to us (you keep the *Supplement*).
- Write your answers clearly, completely, and concisely in the designated space provided immediately after each question. An answer guide ends each question to let you know what is expected. For example, a quantitative analysis, a fully labelled graph, and/or sentences. Any answer guide asking for a quantitative analysis *always* automatically means that you must show your work and make your reasoning clear.
 - Anything requested by the question and/or the answer guide is required. Similarly, limit yourself to the answer guide. For example, if the answer guide does not request sentences, provide only what is requested (e.g. quantitative analysis).
 - Marking TAs are instructed to accept all reasonable rounding.
- **Your entire answer must fit in the designated space provided immediately after each question.** No extra space/pages are possible. You *cannot* use blank space for other questions, nor can you write answers on the *Supplement*. **Write in PENCIL and use an ERASER as needed** so that you can fit your final answer (including work and reasoning) in the appropriate space. Questions give more blank space than is needed for an answer (with typical handwriting) worth full marks. **Follow the answer guides and avoid excessively long answers.**

(1) Economists study differences in risk tolerance between women and men. For example, in “Gender Differences in Job Search and the Earnings Gap: Evidence from Business Majors” Cortés et al. (2021) measure risk tolerance on a scale from 1 to 6, with 6 being the highest tolerance for risk. They define *high risk* as those with a score of 5 or higher.

(a) [2 pts] To assess if there is a statistically significant difference between women and men in the fraction categorized as high risk, what are the relevant hypotheses? Answer with hypotheses in formal notation.

(b) [4 pts] Continuing from Part (a), explain what a Type I error is *in this context*. Answer with 1 precise sentence.

(c) [4 pts] Continuing from Part (a), explain what a Type II error is *in this context*. Answer with 1 precise sentence.

(2) [5 pts] To prove that less than one-third of the population supports the government with a sample of 1,000 citizens and a significance level of 5 percent, the power is 0.73 if, among all citizens, 30 percent of support the government. (Hence, β is 0.27.) List FOUR *different* modifications that would *increase* the power. Answer with a bullet list.

(3) [12 pts] Recall Table 2A from Karlan and List (2007) “Does Price Matter in Charitable Giving? Evidence from a Large-Scale Natural Field Experiment.” For the point estimate highlighted by a box, what is the associated 99% confidence interval estimate? What is the *interpretation* of that CI estimate? Answer with a quantitative analysis & 2 – 3 sentences.

TABLE 2A—MEAN RESPONSES
(Mean and standard errors)

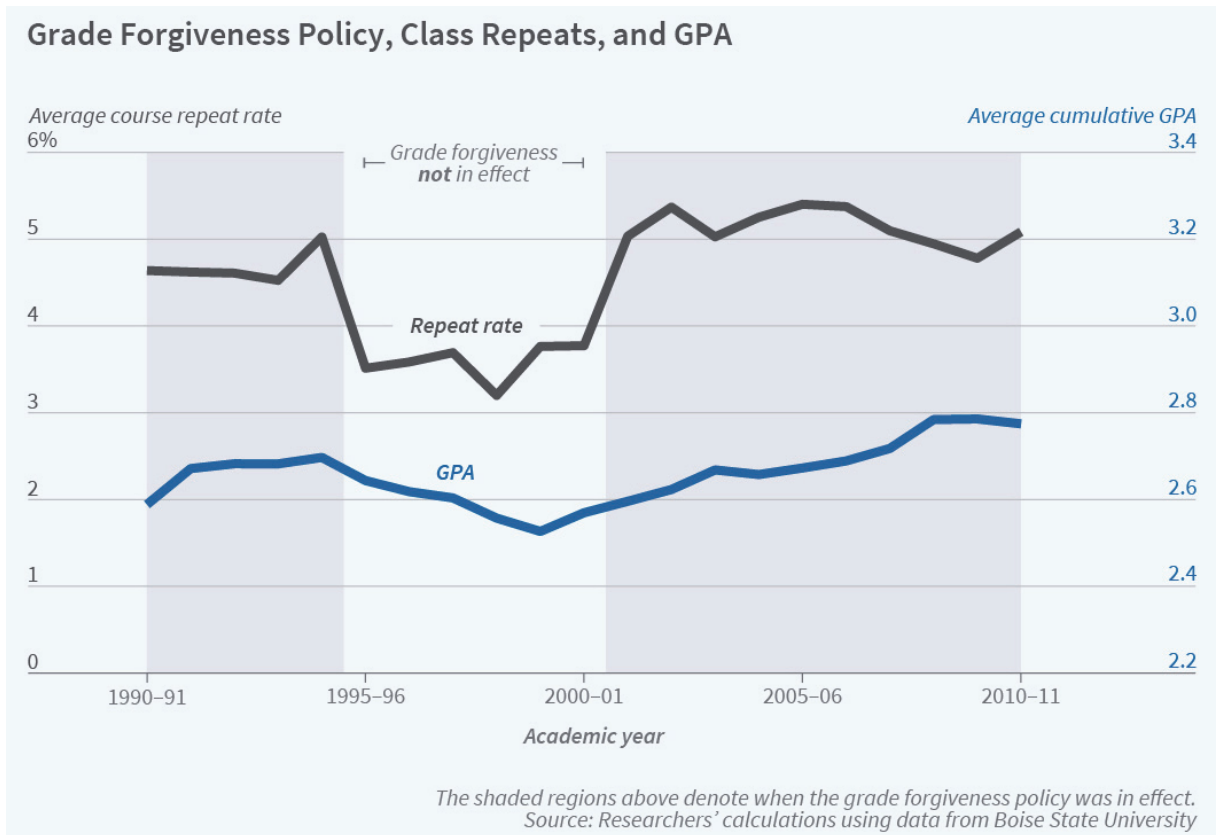
| | Control | Treatment | Match ratio | | |
|--------------------------------------|-------------------|-------------------|--------------------------|-------------------|-------------------|
| | | | 1:1 | 2:1 | 3:1 |
| Implied price of \$1 of public good: | 1.00 | 0.36 | 0.50 | 0.33 | 0.25 |
| <i>Panel A</i> | (1) | (2) | (3) | (4) | (5) |
| Response rate | 0.018 (0.001) | 0.022 (0.001) | 0.021 (0.001) | 0.023 (0.001) | 0.023 (0.001) |
| Dollars given, unconditional | 0.813 (0.063) | 0.967 (0.049) | 0.937 (0.089) | 1.026 (0.089) | 0.938 (0.077) |
| Dollars given, conditional on giving | 45.540 (2.397) | 43.872 (1.549) | 45.143 (3.099) | 45.337 (2.725) | 41.252 (2.222) |
| Observations | 16,687 | 33,396 | 11,133 | 11,134 | 11,129 |

(4) [11 pts] Recall Karlan and List (2007). Imagine a similar field experiment in Ontario and Quebec. The PivotTable to the right summarizes the results. For Ontario, compute the P-value to assess if there a statistically significant difference in the response rates between the control and treatment groups. Include a fully labelled graph illustrating the P-value. Answer with hypotheses in formal notation, a quantitative analysis & a fully labelled graph.

| Count of group Row Labels | Column Labels | | |
|------------------------------|---------------|--------------|--------------|
| | Ontario | Quebec | Grand Total |
| Control | 6648 | 10029 | 16677 |
| Did not give | 6541 | 9808 | 16349 |
| Gave | 107 | 221 | 328 |
| Treatment | 13594 | 19777 | 33371 |
| Did not give | 13316 | 19330 | 32646 |
| Gave | 278 | 447 | 725 |
| Grand Total | 20242 | 29806 | 50048 |

(5) This figure and excerpt from *The NBER Digest* (<https://www.nber.org/digest-2022-02>) describe the paper “A Second Chance at Success? Effects of College Grade Forgiveness Policies on Student Outcomes.”

Excerpt: Grade forgiveness policies allow students to retake courses in which they are dissatisfied with their grades; only the most recent grade is included in their grade point average. To study the impact of such policies, the researchers looked at transcript data from all entering cohorts at Boise State University between 1990 and 2017. This window presented a unique research opportunity because starting in



1988 the university offered a grade forgiveness option for all courses. In 1995, it switched to a policy of averaging the first-attempt grade and the repeat-attempt grades, and then, in 2001, it returned to grade forgiveness. It is possible to estimate the effects of grade forgiveness by comparing data from the three periods.

(a) [8 pts] Comparing **2000-01** with **2010-11**, approximate and *interpret* the difference for **Repeat rate**? Next, again comparing 2000-01 with 2010-11, approximate and *interpret* the difference for **GPA**? Answer with 2 – 4 sentences.

(b) [2 pts] To assess if GPAs are higher in 2010-11 than in 2000-01, what are the hypotheses? Answer with hypotheses in formal notation.

(6) [8 pts] Recall the 2021 article in *The New York Times* “Archaeologists vs. Computers: A Study Tests Who’s Best at Sifting the Past; When it came to the tedious task of categorizing pottery fragments, a deep-learning model was found to be just as accurate as four human experts, and far more efficient.” Table 6 summarizes some results from the original research paper. What is the 95% confidence interval estimate for Human B? What is the *interpretation* of the CI estimate? Answer with a quantitative analysis & 1 precise sentence.

Table 6. Accuracy of pottery fragment categorization, by classifier type, for sample of 2,407 fragments

| Classifier | # of successes |
|--------------------|----------------|
| Human A | 1,918 |
| Human B | 1,772 |
| Human C | 2,092 |
| Human D | 2,005 |
| Machine (Computer) | 1,986 |

(7) Recall Dizon-Ross (2019) “Parents’ Beliefs about Their Children’s Academic Ability: Implications for Educational Investments.” An excerpt of Table C.25 is to the right. It uses 39 randomly selected primary schools in the Machinga and Balaka districts in Malawi. Chichewa is the local language. We add the two shaded rows in boldface using the replication data.

(a) [5 pts] In the first shaded row, what is the interpretation of 21.6732? Answer with 2 sentences.

| Summary Statistics | | |
|--|----------------|----------------|
| | Mean | SD |
| <i>Academic Performance (Average Achievement Scores)</i> | | |
| Overall score | 46.8 | 17.5 |
| Math score | 44.9 | 20.2 |
| English score | 44.2 | 20.1 |
| Chichewa score | 51.2 | 22.5 |
| (Math – English) score | 0.71 | 19.5 |
| (Math – Chichewa) score | -6.3588 | 21.6732 |
| <i>Respondent’s Beliefs about Child’s Academic Performance</i> | | |
| Believed Overall score | 62.4 | 16.5 |
| Believed Math score | 64.7 | 19.0 |
| Believed English score | 55.3 | 20.9 |
| Believed Chichewa score | 66.8 | 19.4 |
| Believed (Math – English) score | 9.48 | 21.5 |
| Believed (Math – Chichewa) score | -2.0251 | 19.5205 |
| Sample size (number of kids) | 5,268 | |

(b) [7 pts] In the second shaded row, what is the margin of error for a 90% confidence level? Is the margin of error large or small in this context? Explain. Answer with a quantitative analysis & 2 sentences.

(8) Recall Taubman et al. (2013) “Medicaid Increases Emergency-Department Use: Evidence from Oregon’s Health Insurance Experiment.”

| Table 2. Emergency-department use | | | | | | | |
|---|--------|--|-----------------------------|---------|-------------------------------------|-----------------------------|---------|
| | | <i>Percent with any visits¹</i> | | | <i>Number of visits²</i> | | |
| | N | Percent in Control Group | Effect of Medicaid Coverage | P-value | Mean Value in Control Group | Effect of Medicaid Coverage | P-value |
| Panel A: Overall | | | | | | | |
| All visits | 24,646 | 34.5 | 7.0 (2.4) | 0.003 | 1.022 (2.632) | 0.408 (0.116) | <0.001 |
| <p><i>Notes:</i> We report the estimated effect of Medicaid on emergency department use over our study period (March 10, 2008 – September 30, 2009). We report the sample size, the control mean of the dependent variable (with standard deviation for continuous outcomes in parentheses), the estimated effect of Medicaid coverage (with standard error in parentheses), and the p-value of the estimated effect. Sample consists of individuals in Portland-area zip codes (N=24,646).</p> <p>¹ For the percent-with-any-visits measures, the estimated effects of Medicaid coverage are shown in percentage points.</p> <p>² The number-of-visits measures are unconditional, including those with no visits.</p> | | | | | | | |

(a) [4 pts] In Table 2, what is the interpretation of 34.5? Answer with 1 precise sentence.

(b) [5 pts] In Table 2, what is “(0.116)”? What are the FOUR factors that affect its size? Answer with 1 – 2 sentences.

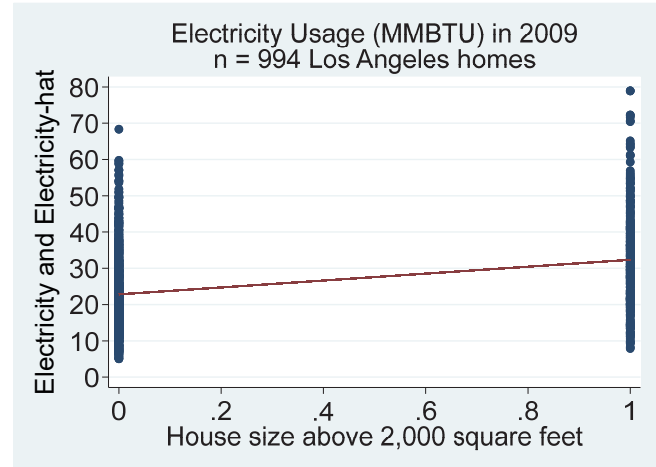
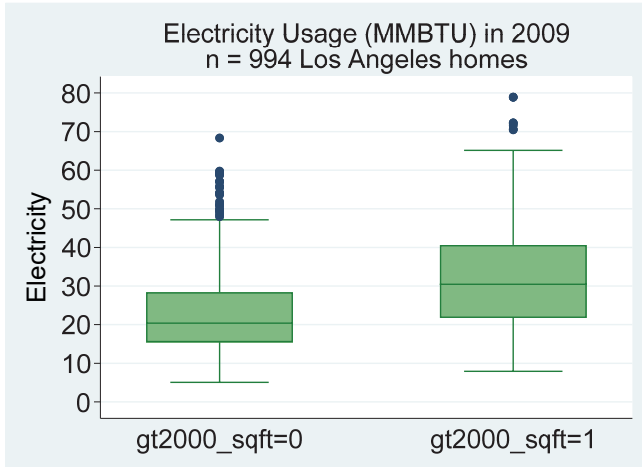
For your easy visual reference, the same background from the previous page is repeated below.

Recall Taubman et al. (2013) “Medicaid Increases Emergency-Department Use: Evidence from Oregon’s Health Insurance Experiment.”

| Table 2. Emergency-department use | | | | | | | |
|---|--------|--------------------------------------|-----------------------------|---------|-------------------------------|-----------------------------|---------|
| | | Percent with any visits ¹ | | | Number of visits ² | | |
| | N | Percent in Control Group | Effect of Medicaid Coverage | P-value | Mean Value in Control Group | Effect of Medicaid Coverage | P-value |
| Panel A: Overall | | | | | | | |
| All visits | 24,646 | 34.5 | 7.0 (2.4) | 0.003 | 1.022 (2.632) | 0.408 (0.116) | <0.001 |
| <p><i>Notes:</i> We report the estimated effect of Medicaid on emergency department use over our study period (March 10, 2008 – September 30, 2009). We report the sample size, the control mean of the dependent variable (with standard deviation for continuous outcomes in parentheses), the estimated effect of Medicaid coverage (with standard error in parentheses), and the p-value of the estimated effect. Sample consists of individuals in Portland-area zip codes (N=24,646).</p> <p>¹ For the percent-with-any-visits measures, the estimated effects of Medicaid coverage are shown in percentage points.</p> <p>² The number-of-visits measures are unconditional, including those with no visits.</p> | | | | | | | |

(c) [7 pts] There are two columns labelled “Effect of Medicaid Coverage.” What does 0.408 measure? Next, explain in which way the estimate of 0.408 represents a *bigger* effect of Medicaid coverage than the estimate of 7.0 in Table 2. Answer with 3 – 4 sentences.

(9) Recall Levinson (2016) “How Much Energy Do Building Energy Codes Save? Evidence from California Houses.” The figures below use a subset of those data and a dummy variable for homes greater than 2,000 square feet.



(a) [4 pts] For $\widehat{electricity}_i = a + b \text{gt}2000_sqft_i$, roughly what are the values of a and b ? Also, if $lt2000_sqft_i$ is a dummy variable equal to 1 for houses *less than* 2,000 square feet, then for $\widehat{electricity}_i = b_0 + b_1 lt2000_sqft_i$, roughly what are the values of b_0 and b_1 ? Answer with approximate values for a , b , b_0 , and b_1 .

(b) [7 pts] Which is the best estimate of the s_e (the root MSE): 0.015, 0.384, or 11.739? Why? What are its units of measurement? In this context, is the value of the s_e large or small? Explain. Answer with 3 – 4 sentences.

The pages of this *Supplement* will NOT be graded: write your answers on the test papers. **Supplement: Page 1 of 4**

Sample mean: $\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$ **Sample variance:** $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n-1} = \frac{\sum_{i=1}^n x_i^2}{n-1} - \frac{(\sum_{i=1}^n x_i)^2}{n(n-1)}$ **Sample s.d.:** $S = \sqrt{S^2}$

Sample coefficient of variation: $CV = \frac{s}{\bar{x}}$ **Sample covariance:** $S_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{X})(y_i - \bar{Y})}{n-1} = \frac{\sum_{i=1}^n x_i y_i}{n-1} - \frac{(\sum_{i=1}^n x_i)(\sum_{i=1}^n y_i)}{n(n-1)}$

Sample interquartile range: $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{S_{xy}}{S_x S_y} = \frac{\sum_{i=1}^n z_{x_i} z_{y_i}}{n-1}$

Addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **Conditional probability:** $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Complement rules: $P(A^c) = P(A') = 1 - P(A)$ $P(A^c|B) = P(A'|B) = 1 - P(A|B)$

Multiplication rule: $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$

Expected value: $E[X] = \mu = \sum_{all\ x} xp(x)$ **Variance:** $V[X] = E[(X - \mu)^2] = \sigma^2 = \sum_{all\ x} (x - \mu)^2 p(x)$

Covariance: $COV[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] = \sigma_{XY} = \sum_{all\ x} \sum_{all\ y} (x - \mu_X)(y - \mu_Y)p(x, y)$

Laws of expected value:

$E[c] = c$

$E[X + c] = E[X] + c$

$E[cX] = cE[X]$

$E[a + bX + cY] = a + bE[X] + cE[Y]$

Laws of variance:

$V[c] = 0$

$V[X + c] = V[X]$

$V[cX] = c^2 V[X]$

$V[a + bX + cY] = b^2 V[X] + c^2 V[Y] + 2bc * COV[X, Y]$

$V[a + bX + cY] = b^2 V[X] + c^2 V[Y] + 2bc * SD(X) * SD(Y) * \rho$

where $\rho = CORRELATION[X, Y]$

Laws of covariance:

$COV[X, c] = 0$

$COV[a + bX, c + dY] = bd * COV[X, Y]$

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$ for $x = 0, 1, 2, \dots, n$

If X is Binomial ($X \sim B(n, p)$) then $E[X] = np$ and $V[X] = np(1-p)$

If X is Uniform ($X \sim U[a, b]$) then $f(x) = \frac{1}{b-a}$ and $E[X] = \frac{a+b}{2}$ and $V[X] = \frac{(b-a)^2}{12}$

Sampling distribution of \bar{X} :

$\mu_{\bar{X}} = E[\bar{X}] = \mu$

$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

Sampling distribution of \hat{P} :

$\mu_{\hat{P}} = E[\hat{P}] = p$

$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$

$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$:

$\mu_{\hat{P}_2 - \hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$

$\sigma_{\hat{P}_2 - \hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$

$\sigma_{\hat{P}_2 - \hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

Sampling distribution of $(\bar{X}_1 - \bar{X}_2)$, independent samples:

$\mu_{\bar{X}_1 - \bar{X}_2} = E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_1 - \bar{X}_2}^2 = V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$\sigma_{\bar{X}_1 - \bar{X}_2} = SD[\bar{X}_1 - \bar{X}_2] = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Sampling distribution of (\bar{X}_d) , paired ($d = X_1 - X_2$):

$\mu_{\bar{X}_d} = E[\bar{X}_d] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_d}^2 = V[\bar{X}_d] = \frac{\sigma_d^2}{n} = \frac{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}{n}$

$\sigma_{\bar{X}_d} = SD[\bar{X}_d] = \frac{\sigma_d}{\sqrt{n}} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}{n}}$

Inference about a population proportion:

z test statistic: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ **CI estimator:** $\hat{P} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Inference about comparing two population proportions:

z test statistic under Null hypothesis of no difference: $z = \frac{\hat{p}_2 - \hat{p}_1}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}}$ **Pooled proportion:** $\bar{P} = \frac{X_1 + X_2}{n_1 + n_2}$

CI estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_2(1-\hat{p}_2)}{n_2} + \frac{\hat{p}_1(1-\hat{p}_1)}{n_1}}$

Inference about the population mean:

t test statistic: $t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$ **CI estimator:** $\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

Inference about a comparing two population means, independent samples, unequal variances:

t test statistic: $t = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Degrees of freedom: $\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1} \left(\frac{s_2^2}{n_2}\right)^2}$

Inference about a comparing two population means, independent samples, assuming equal variances:

t test statistic: $t = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$ **Degrees of freedom:** $\nu = n_1 + n_2 - 2$

Pooled variance: $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$

Inference about a comparing two population means, paired data: (n is number of pairs and $d = X_1 - X_2$)

t test statistic: $t = \frac{\bar{d} - \Delta_0}{s_d/\sqrt{n}}$ **CI estimator:** $\bar{X}_d \pm t_{\alpha/2} \frac{s_d}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

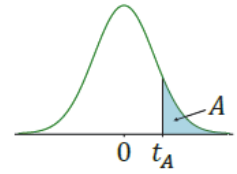
SIMPLE REGRESSION:

Model: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ **OLS line:** $\hat{y}_i = b_0 + b_1 x_i$ $b_1 = \frac{s_{xy}}{s_x^2} = r \frac{s_y}{s_x}$ $b_0 = \bar{Y} - b_1 \bar{X}$

Residuals: $e_i = y_i - \hat{y}_i$ **Standard deviation of residuals:** $s_e = \text{Root MSE} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^n (e_i - \bar{e})^2}{n-2}}$

$SST = \sum_{i=1}^n (y_i - \bar{Y})^2 = SSR + SSE$ $SSR = \sum_{i=1}^n (\hat{y}_i - \bar{Y})^2$ $SSE = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \hat{y}_i)^2$

$s_y^2 = \frac{SST}{n-1}$ **Coefficient of determination:** $R^2 = (r)^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$



Critical Values of Student t Distribution:

| ν | $t_{0.10}$ | $t_{0.05}$ | $t_{0.025}$ | $t_{0.01}$ | $t_{0.005}$ | $t_{0.001}$ | $t_{0.0005}$ | ν | $t_{0.10}$ | $t_{0.05}$ | $t_{0.025}$ | $t_{0.01}$ | $t_{0.005}$ | $t_{0.001}$ | $t_{0.0005}$ |
|-------|------------|------------|-------------|------------|-------------|-------------|--------------|----------|------------|------------|-------------|------------|-------------|-------------|--------------|
| 1 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 318.3 | 636.6 | 38 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.33 | 31.60 | 39 | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 3.313 | 3.558 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.21 | 12.92 | 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 | 41 | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 3.301 | 3.544 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 | 42 | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 | 43 | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 3.291 | 3.532 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 | 44 | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 | 45 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 3.281 | 3.520 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 | 46 | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 | 3.515 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 | 47 | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 3.273 | 3.510 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 | 48 | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 | 3.505 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 | 49 | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 3.265 | 3.500 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 | 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 | 51 | 1.298 | 1.675 | 2.008 | 2.402 | 2.676 | 3.258 | 3.492 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 | 52 | 1.298 | 1.675 | 2.007 | 2.400 | 2.674 | 3.255 | 3.488 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 | 53 | 1.298 | 1.674 | 2.006 | 2.399 | 2.672 | 3.251 | 3.484 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 | 54 | 1.297 | 1.674 | 2.005 | 2.397 | 2.670 | 3.248 | 3.480 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 | 55 | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 | 3.245 | 3.476 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 | 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 | 65 | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 | 3.220 | 3.447 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 | 70 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 | 75 | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 | 3.202 | 3.425 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 | 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 | 90 | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 | 3.402 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 | 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 | 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 | 140 | 1.288 | 1.656 | 1.977 | 2.353 | 2.611 | 3.149 | 3.361 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 | 160 | 1.287 | 1.654 | 1.975 | 2.350 | 2.607 | 3.142 | 3.352 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 | 180 | 1.286 | 1.653 | 1.973 | 2.347 | 2.603 | 3.136 | 3.345 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 | 200 | 1.286 | 1.653 | 1.972 | 2.345 | 2.601 | 3.131 | 3.340 |
| 31 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 3.375 | 3.633 | 250 | 1.285 | 1.651 | 1.969 | 2.341 | 2.596 | 3.123 | 3.330 |
| 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 | 300 | 1.284 | 1.650 | 1.968 | 2.339 | 2.592 | 3.118 | 3.323 |
| 33 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 3.356 | 3.611 | 400 | 1.284 | 1.649 | 1.966 | 2.336 | 2.588 | 3.111 | 3.315 |
| 34 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 | 500 | 1.283 | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| 35 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 3.340 | 3.591 | 750 | 1.283 | 1.647 | 1.963 | 2.331 | 2.582 | 3.101 | 3.304 |
| 36 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 | 1000 | 1.282 | 1.646 | 1.962 | 2.330 | 2.581 | 3.098 | 3.300 |
| 37 | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 3.326 | 3.574 | ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |

Degrees of freedom: ν