

ECO220Y, Term Test #3

January 19, 2018, 9:10 – 11:00 am

U of T E-MAIL: _____@MAIL.UTORONTO.CA

SURNAME
(LAST NAME):

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

GIVEN NAME
(FIRST NAME):

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

UTORID:
(e.g. LIHAO118)

--	--	--	--	--	--	--	--

Instructions:

- You have 110 minutes. Keep these test papers and the *Supplement* closed and face up on your desk until the start of the test is announced. You must stay for a minimum of 60 minutes.
- You may use a non-programmable calculator.
- There are 6 questions (some with multiple parts) with varying point values worth a total of 100 points.
- This test includes these 8 pages plus the *Supplement*. The *Supplement* contains a formula sheet, the Standard Normal table, and necessary materials for some test questions.
 - Anything written on the *Supplement* will *not* be graded. We will only collect these test papers, not the *Supplement*.
- Write your answers clearly, completely and concisely in the designated space provided immediately after each question. An answer guide for your response ends each question to let you know what is expected. Answer guides are underlined so you do not miss them. For example, a quantitative analysis (which shows your work), a fully-labelled graph, and/or sentences.
 - Anything requested by the question and/or answer guide is required.
 - Similarly, limit yourself to the answer guide. For example, if the answer guide does not request sentences, provide only what is requested (e.g. quantitative analysis). Leave yourself time to complete all questions rather than overdoing some questions and running out of time.
 - For questions with multiple parts (e.g. (a) – (d)), **attempt each part** even if you have trouble with earlier parts.
- **Your entire answer must fit in the designated space provided immediately after each question.** No extra space/pages are possible. You *cannot* use blank space for other questions nor can you write answers on the *Supplement*. **Write in PENCIL and use an ERASER as needed.** This way you can make sure to fit your final answer (including work and reasoning) in the appropriate space. Most questions give more blank space than is needed to answer. **Follow the answer guides and avoid excessively long answers.**

(1) See *Supplement for Question (1)*: Canadian young adults aged 20 to 34.

(a) [8 pts] In a random sample of 5 young adults in Canada, what is the chance that more than 3 are living outside the 16 selected census metropolitan areas? Answer with a quantitative analysis.

(b) [8 pts] In a random sample of 1,800 young adults living in Toronto, what is the chance that more than 825 live with their parents? Answer with a quantitative analysis.

(c) [10 pts] What is the chance a random sample of 400 young adults living in Vancouver has **more** people living with their parents than a random sample of 400 young adults living in Hamilton? Answer with a quantitative analysis.

(2) See *Supplement for Question (2): Cars making life-and-death decisions: sacrifice passenger or save pedestrians?*

(a) [4 pts] In Figure 2a the mean is about 74 and the standard deviation is about 29. Are these numbers parameters or statistics? Use correct formal notation to say what each is. Answer with 1 sentence and formal notation.

(b) [3 pts] In Figure 2b the mean is about 59 and the standard deviation is about 35. What fraction of the observations lie within two standard deviations of the mean? Answer with a quantitative analysis and 1 sentence.

(c) [8 pts] Consider opinions in Canada about what AVs should do, with answers from 0 “Protect driver at all costs” to 100 “Minimize casualties on the road.” Suppose that among *all* Canadians, the shape of the distribution is similar to Figure 2a and the mean is 75 with a standard deviation of 30. Draw a fully-labelled graph of the sampling distribution of the sample mean for a sample size of 200. Answer with a quantitative analysis and a fully-labelled graph.

(d) [9 pts] The article is somewhat unclear about the sample size in Study Two: the confidence intervals in Figure 3 are inconsistent with $n = 451$. It must be that each participant assessed *one* of the five scenarios (selected at random) and *not each* of the five, which makes the sample size for each lives-saved scenario about 90 ($\approx 451/5$), not 451. Compute the 95% CI if *all* participants in Study Two had assessed the **1 life-saved** scenario for what AV's *should* do. Is that interval narrower or wider than the corresponding CI in Figure 3? Why? Answer with a quantitative analysis and 1 – 2 sentences.

(e) [10 pts] For Study Two, we can obtain the 95 percent CI estimate of the difference in proportion agreeing “Should AVs sacrifice?” between the 100 lives-saved scenario and 20 lives-saved scenario as [0.004, 0.236]. Offer a full interpretation of that interval, which is both precise and clear to someone that has not read the article. Include discussion of the magnitude of the *point estimate* and *margin of error* (i.e. is each large or small in this context?). Answer with 4 – 5 sentences and do NOT recompute the CI estimate, which already has been computed for you.

(3) [6 pts] A volunteer is calling potential donors using three lists: A, B, and C. People on List A have a history of giving to this organization: the expected donation per person contacted is \$26 with a standard deviation of \$55. People on List B have a history of giving to others: the expected donation per person contacted is \$19 with a standard deviation of \$49. People on List C have an unknown history: the expected donation per person contacted is \$8 with a standard deviation of \$31. All three lists are long and a computer pulls people in a random order. If the volunteer contacts 6 people on List A, 7 people on List B, and 12 people on List C, the expected total amount raised is \$385. What is the standard deviation of the total amount raised? Answer with a quantitative analysis.

(4) [9 pts] Recall Ray et al. (2012) "Sex ratios among Canadian liveborn infants of mothers from different countries." The researchers investigate sex selection in favor of males. Recall that the overall chance of a male infant, absent any interference (i.e. no sex selection), is 51.2 percent. For a particular subgroup, suppose the P-value for the hypothesis test to show sex selection in favor of males is 0.952. What does this P-value imply about the sample proportion relative to 51.2 percent male? What should we conclude? Why? Make sure to specify the hypotheses and to discuss the evidence. Answer with hypotheses in formal notation and 2 – 3 sentences.

(5) Consider opinion polls in the GTA that contact a random sample of Torontonians. Suppose Mayor John Tory wishes to prove that the majority of Torontonians favor a proposed expansion of public transit.

(a) [3 pts] What are the implied hypotheses? Answer with hypotheses using formal notation.

(b) [9 pts] Which of these two scenarios provide stronger statistical support in favor of Mayor John Tory's position: (1) Of 1,988 people polled, 1,033 favor the expansion or (2) Of 390 people polled, 226 favor the expansion? Why? (Hint: Compute P-values.) Answer with quantitative analyses and 1 sentence.

(6) See **Supplement for Question (6)**: *ON public sector salary disclosure and Monte Carlo simulations*.

(a) [7 pts] If a sample size of 30 were sufficiently large for the Central Limit Theorem to apply in this case, what would be the expected value of the 99th percentile in Summary #1? Answer with a quantitative analysis.

(b) [4 pts] In a random sample of 30 employees from the population, which would be more surprising: a **sample mean** below \$130,000 or a **sample median** above \$145,000? Explain. Answer with 1 – 2 sentences.

(c) [2 pts] In a random sample of 30 employees from the population, what is the approximate chance of getting a **sample median** above \$170,000? Answer with a quantitative analysis.

The pages of this supplement will *not* be graded: write your answers on the test papers. **Supplement: Page 1 of 6**

This *Supplement* contains a formula sheet, the Standard Normal table, and necessary materials for some test questions. For each question referencing this *Supplement*, carefully review *all* materials, noticing continuations onto the next page.

Sample mean: $\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$ **Sample variance:** $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n-1} = \frac{\sum_{i=1}^n x_i^2}{n-1} - \frac{(\sum_{i=1}^n x_i)^2}{n(n-1)}$ **Sample s.d.:** $S = \sqrt{S^2}$

Sample coefficient of variation: $CV = \frac{s}{\bar{X}}$ **Sample covariance:** $S_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{X})(y_i - \bar{Y})}{n-1} = \frac{\sum_{i=1}^n x_i y_i}{n-1} - \frac{(\sum_{i=1}^n x_i)(\sum_{i=1}^n y_i)}{n(n-1)}$

Sample interquartile range: $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{S_{xy}}{S_x S_y} = \frac{\sum_{i=1}^n z_{x_i} z_{y_i}}{n-1}$

Addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **Conditional probability:** $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Complement rules: $P(A^c) = P(A') = 1 - P(A)$ $P(A^c|B) = P(A'|B) = 1 - P(A|B)$

Multiplication rule: $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$

Expected value: $E[X] = \mu = \sum_{\text{all } x} xp(x)$ **Variance:** $V[X] = E[(X - \mu)^2] = \sigma^2 = \sum_{\text{all } x} (x - \mu)^2 p(x)$

Covariance: $COV[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] = \sigma_{XY} = \sum_{\text{all } x} \sum_{\text{all } y} (x - \mu_X)(y - \mu_Y)p(x, y)$

Laws of expected value:

$E[c] = c$

$E[X + c] = E[X] + c$

$E[cX] = cE[X]$

$E[a + bX + cY] = a + bE[X] + cE[Y]$

Laws of variance:

$V[c] = 0$

$V[X + c] = V[X]$

$V[cX] = c^2V[X]$

$V[a + bX + cY] = b^2V[X] + c^2V[Y] + 2bc * COV[X, Y]$

$V[a + bX + cY] = b^2V[X] + c^2V[Y] + 2bc * SD(X) * SD(Y) * \rho$

where $\rho = CORRELATION[X, Y]$

Laws of covariance:

$COV[X, c] = 0$

$COV[a + bX, c + dY] = bd * COV[X, Y]$

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$ for $x = 0, 1, 2, \dots, n$

If X is Binomial ($X \sim B(n, p)$) **then** $E[X] = np$ **and** $V[X] = np(1-p)$

If X is Uniform ($X \sim U[a, b]$) **then** $f(x) = \frac{1}{b-a}$ **and** $E[X] = \frac{a+b}{2}$ **and** $V[X] = \frac{(b-a)^2}{12}$

Sampling distribution of \bar{X} :

$\mu_{\bar{X}} = E[\bar{X}] = \mu$

$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

Sampling distribution of \hat{P} :

$\mu_{\hat{P}} = E[\hat{P}] = p$

$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$

$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$:

$\mu_{\hat{P}_2 - \hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$

$\sigma_{\hat{P}_2 - \hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$

$\sigma_{\hat{P}_2 - \hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

Inference about a population proportion:

z test statistic: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ **CI estimator:** $\hat{P} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Inference about comparing two population proportions:

CI estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2} \sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

Supplement for Question (1): Consider Table 1 and the excerpt below from “Census in Brief: Young adults living with their parents in Canada in 2016” released on August 2, 2017 (<http://www12.statcan.gc.ca/census-recensement/2016/as-sa/98-200-x/2016008/98-200-x2016008-eng.cfm>). Also consider Table 2 below, which gives the population size of young adults in each of the 16 selected census metropolitan areas (CMAs) in Table 1 as well as for the entire country (Canada).

EXCERPT: For the 35 census metropolitan areas (CMAs), the percentage of young adults aged 20 to 34 who lived with their parents was 36.2% in 2016, slightly above the national average. Among rural areas (regions located outside CMAs and census agglomerations), the share of young adults living with their parents was lower, at 32.2%. Among the 35 CMAs of the country, Toronto (47.4%) and Oshawa (47.2%) had the largest shares of young adults living with their parents—nearly one in two. Toronto and Oshawa were followed by five other CMAs located in Ontario: Hamilton (44.5%), Windsor (43.0%), Barrie (40.8%), St. Catharines–Niagara (40.7%) and Brantford (39.5%). Two CMAs located in British Columbia were ranked 8th and 9th: Abbotsford–Mission (39.1%) and Vancouver (38.6%).

Table 1. Proportion (percentage) of young adults aged 20 to 34 living with their parents, Canada, and 16 selected census metropolitan areas, 2016.

Region	Proportion (percentage)
Toronto	47.4
Oshawa	47.2
Hamilton	44.5
Windsor	43.0
Barrie	40.8
St. Catharines–Niagara	40.7
Brantford	39.5
Abbotsford–Mission	39.1
Vancouver	38.6
Kitchener–Cambridge–Waterloo	34.9
Winnipeg	34.8
Canada	34.7
Montréal	33.1
Ottawa–Gatineau	33.0
Calgary	28.5
Edmonton	26.8
Québec	23.8

Note: Included are all CMAs with a total population of 500,000 or more and smaller CMAs that had a proportion higher than the proportion for all CMAs (36.2%): Oshawa, Windsor, Barrie, St. Catharines–Niagara, Brantford and Abbotsford–Mission.
Source: Statistics Canada, Census of Population, 2016.

Table 2. Population size of young adults aged 20 to 34, Canada, and 16 selected census metropolitan areas, 2016.

Region	Population
Toronto	1,256,140
Oshawa	70,945
Hamilton	141,520
Windsor	60,670
Barrie	38,155
St. Catharines–Niagara	70,520
Brantford	24,040
Abbotsford–Mission	35,375
Vancouver	529,845
Kitchener–Cambridge–Waterloo	111,955
Winnipeg	165,755
Canada	6,858,075
Montréal	821,555
Ottawa–Gatineau	267,055
Calgary	317,575
Edmonton	313,730
Québec	155,900

Note: The total population of 20 to 34 year olds for the 16 selected CMAs above is 4,380,735. Hence, the total population of 20 to 34 year olds for the rest of Canada is 2,477,340 (= 6,858,075 – 4,380,735).
Source: Statistics Canada, Census of Population, 2016, Data tables: catalogue number 98-400-X2016001.

Supplement for Question (2): Consider a 2016 article “The social dilemma of autonomous vehicles” in the magazine *Science*. “Dilemma” means a difficult problem and “autonomous vehicles” (AVs) are cars that drive themselves, using sensors and computer algorithms (programs) rather than a human driver. Consider the excerpts and figures.

EXCERPT (from Abstract): Autonomous vehicles (AVs) should reduce traffic accidents, but they will sometimes have to choose between two evils, such as running over pedestrians or sacrificing themselves and their passenger to save the pedestrians. [See Figures 1a and 1b.] Defining the algorithms that will help AVs make these moral decisions is a huge challenge. We found that participants in six Amazon Mechanical Turk studies [online surveys] approved of utilitarian AVs (that is, AVs that sacrifice their passengers for the greater good) and would like others to buy them, but they would themselves prefer to ride in AVs that protect their passengers at all costs.



Figure 1a. Car decides between killing one pedestrian or sacrificing (i.e. killing) its own passenger to save 1 life.



Figure 1b. Car decides between killing ten pedestrians or sacrificing (i.e. killing) its own passenger to save 10 lives.

EXCERPT (from page 1574): In *Study One* (n = 182 participants), participants were asked to rate which was the most moral way to program AVs, on a scale from 0 (protect the passenger at all costs) to 100 (minimize the number of casualties on the road). [“Casualties” means deaths.] Figure 2a shows that participants overwhelmingly expressed a moral preference for utilitarian AVs programmed to minimize the number of casualties (median = 85). However, Figure 2b shows that participants were less certain that AVs would be programmed in a utilitarian manner (median = 70).

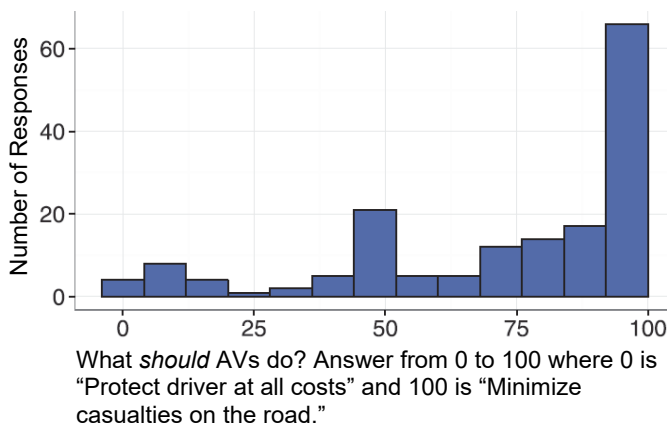


Figure 2a. Participants’ replies in Study One about their opinions on what AVs *should* do.

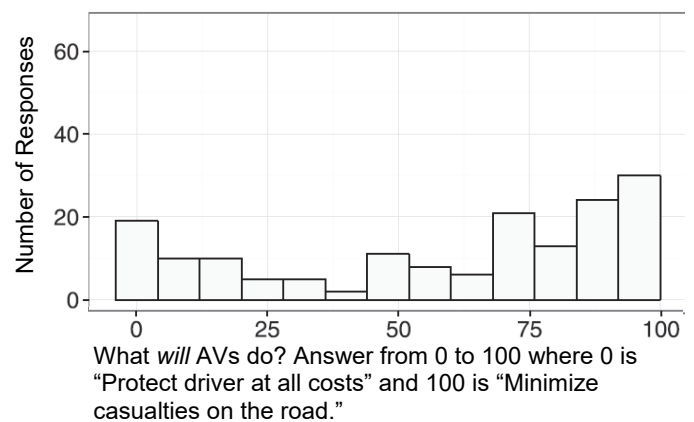


Figure 2b. Participants’ replies in Study One about their opinions on what AVs *will* do.

>>> **Supplement for Question (2), continued:**

EXCERPT (from page 1574): In Study Two (n = 451 participants), participants were presented with dilemmas that varied the number of pedestrians' lives that could be saved, from 1 to 100. Participants did not think that AVs should sacrifice [kill] their passenger when only one pedestrian could be saved, with an average approval rate of 23% [and 95% CI of 14 to 32], but their moral approval increased with the number of lives that could be saved. [Figure 3 shows that the approval rate jumped to 54% for 2 lives saved, 66% for 5 lives saved, 74% for 20 lives saved, and 86% for 100 lives saved.]

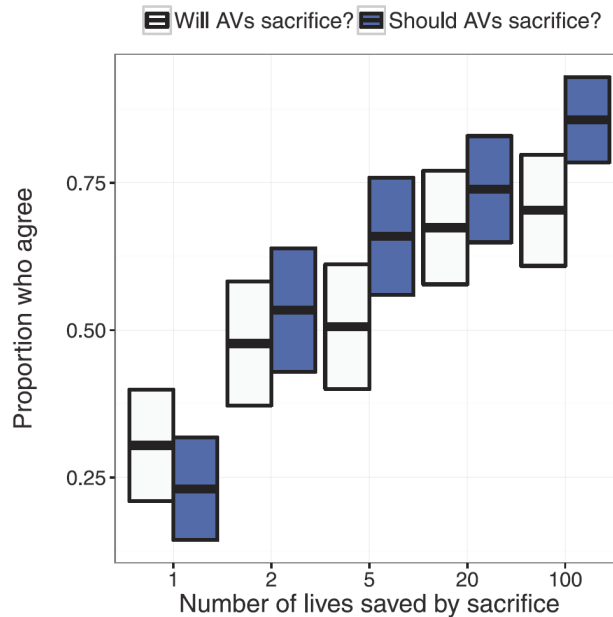
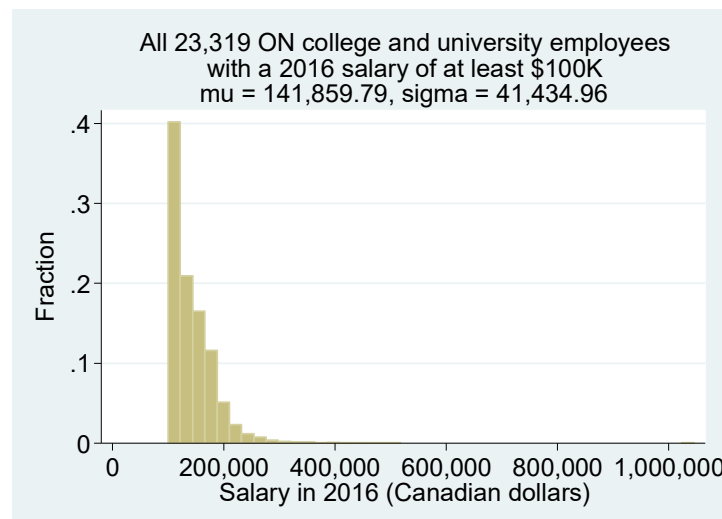


Figure 3. Participants' replies in Study Two. Boxes show the 95% Confidence Intervals. As noted in the legend above, the lighter boxes correspond to proportion agreeing that AVs *will* sacrifice [kill] their passengers and the darker boxes correspond to proportion agreeing that AVs *should* sacrifice [kill] their passengers.

Supplement for Question (6): Recall the Ontario public disclosure of salaries for the university and college sectors. It includes *all* public sector employees in the university or college sectors earning \$100,000 CAN or more in the 2016 calendar year. The figure below summarizes the population.



Supplement for Question (6) continues on next page >>>

The pages of this supplement will *not* be graded: write your answers on the test papers. **Supplement: Page 6 of 6**

>>> **Supplement for Question (6), continued:** STATA Summary #1 summarizes the results from a simulation to obtain the simulated sampling distribution of the sample mean for a samples size of 30 ($n = 30$) using 25,000 simulation draws ($m = 25,000$). STATA Summary #2 summarizes the results from a simulation to obtain the simulated sampling distribution of the sample median for a samples size of 30 ($n = 30$) using 25,000 simulation draws ($m = 25,000$).

Summary #1:

mean salary (X-bar)				

	Percentiles	Smallest		
1%	126168.4	118707.2		
5%	130250.1	118851.1		
10%	132490.3	118940.7	Obs	25000
25%	136567.6	119282.7	Sum of Wgt.	25000
50%	141363.4		Mean	141855.2
		Largest	Std. Dev.	7648.528
75%	146573.8	182770.5		
90%	151701.5	184092.3	Variance	5.85e+07
95%	155099.3	184923.1	Skewness	.4703425
99%	162225.1	187983.8	Kurtosis	3.691849

Summary #2:

median salary (sample median)				

	Percentiles	Smallest		
1%	112798	106756.2		
5%	117496.4	108101.4		
10%	120311.1	108323.6	Obs	25000
25%	125661.4	108381.2	Sum of Wgt.	25000
50%	131697.8		Mean	132097
		Largest	Std. Dev.	9218.706
75%	138094.4	168874.5		
90%	144274.5	169152.1	Variance	8.50e+07
95%	148060.8	171889.6	Skewness	.2672109
99%	155023.4	178330.6	Kurtosis	2.98711