

ECO220Y, Term Test #3

February 10, 2017, 9:10 – 11:00 am

U of T E-MAIL: _____@MAIL.UTORONTO.CA

SURNAME
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(FIRST NAME):

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Instructions:

- You have 110 minutes. Keep these test papers and the *Supplement* closed and face up on your desk until the start of the test is announced. You must stay for a minimum of 60 minutes.
- You may use a non-programmable calculator.
- There are 5 questions (most with multiple parts) with varying point values worth a total of 100 points.
- An answer guide for your response ends each question. It lets you know what is expected: e.g. a quantitative analysis, a graph, and/or sentences. Anything requested by the question and/or guide is required.
- Clearly show your work. Make your reasoning clear. Apply your understanding to the specific questions asked. Offer context-specific explanations rather than generic definitions or quotes from class or the book. Show that you can successfully *apply* your understanding to the specific circumstances presented.
- This test has 8 pages plus the *Supplement*. The *Supplement* contains the aid sheets (formula sheets, Normal and *t* tables) as well as graphs, tables, and other information needed to answer the test questions. Anything written on the *Supplement* will *not* be graded. We will only collect these test papers, not the *Supplement*.
- Write your answers clearly, completely and concisely in the designated space provided immediately after each question. **Your entire answer must fit in the designated space provided immediately after each question.** No extra space/pages are possible. You *cannot* use blank space for other questions nor can you write answers on the *Supplement*.
 - **Write in PENCIL and use an ERASER as needed.** This way you can make sure to fit your final answer (including work and reasoning) in the appropriate space.
 - Most questions give more blank space than is needed to answer. Follow the answer guides and avoid excessively long answers.
- For questions with multiple parts (e.g. (a) – (c)), attempt each part even if you had trouble with earlier parts.
- ***Unless otherwise specified, you choose the significance level. If there are no special considerations, you may choose a 5% significance level.***

(1) See *Supplement for Question (1)*: Financial literacy.

(a) [14 pts] After the one-day course, how much higher are marks on the test? Answer using the appropriate method of inference that directly addresses the question and 1 precise sentence interpreting your numeric results.

(b) [6 pts] Another researcher raises concerns about people writing the test twice and suggests comparing test scores of 25 people who didn't take the course with another 25 people who did. What, if anything, should you expect this suggestion to do to the width of the confidence interval estimate? Why? Answer with 2 – 3 sentences.

(2) See *Supplement for Question (2)*: “Asiaphoria Meets Regression to the Mean.”

(a) [6 pts] For Graph #1, what is the interpretation of the OLS intercept and OLS slope? Answer with 2 sentences.

(b) [5 pts] The R-squared value for Graph #1 is notably higher than for Graph #5. What does that mean? Make sure to offer a *context-specific* answer. Answer with 2 – 3 sentences.

(c) [5 pts] Considering Graph #6 and Graph #8, which *substantially* violates the linearity assumption? *Explain*. Answer in 2 – 3 sentences.

(d) [10 pts] In a main table of results – Table 1, which is provided for your reference in the *Supplement* – the authors use OLS regressions to predict the growth rate in one decade with the growth rate in an earlier decade. They use cross-sectional data with over 100 countries. Growth in each decade is measured as an annual percent. If you used only two countries – Bangladesh and Kenya – and focused on adjacent of decades 1990 – 2000 and 2000 – 2010, what is the equation of the OLS line? Clearly define the y-variable and the x-variable. Answer with a quantitative analysis and the equation of the OLS line.

(3) [16 pts] See **Supplement for Question (3)**: “The Value of Postsecondary Credentials in the Labor Market: An Experimental Study,” which you have already seen on Test #2. Regardless of race, is there a difference in callback rates between male and female applicants? To get you started, part of the solution is already completed below. You provide a quantitative measure of the strength of the evidence and fully interpret the results. A full interpretation requires: **(i)** addressing *both* the *percentage point* and *percent* difference in the call back rates, **(ii)** discussing whether the difference is *statistically significant* and/or *economically significant*, and **(iii)** explicitly addressing the issue of *causality*. Answer with a P-value (with work) and 5 – 7 sentences.

$$H_0: p_M - p_F = 0$$

$$H_1: p_M - p_F \neq 0$$

$$\hat{p}_F = \frac{0.092 * 2,620 + 0.090 * 2,680}{2,620 + 2,680} = \frac{482}{5,300} = 0.09094$$

$$\hat{p}_M = \frac{0.066 * 2,456 + 0.077 * 2,728}{2,456 + 2,728} = \frac{372}{5,184} = 0.07176$$

$$\bar{p} = \frac{X_F + X_M}{n_F + n_M} = \frac{482 + 372}{5,300 + 5,184} = 0.08146$$

$$z = \frac{\hat{p}_M - \hat{p}_F}{\sqrt{\frac{\bar{p}(1 - \bar{p})}{n_M} + \frac{\bar{p}(1 - \bar{p})}{n_F}}} = \frac{0.07176 - 0.09094}{\sqrt{\frac{0.08146(1 - 0.08146)}{5,184} + \frac{0.08146(1 - 0.08146)}{5,300}}} = \frac{-0.01918}{0.00534} = -3.59$$

(4) [16 pts] A researcher wishes to prove that a new type of microloan would yield a default rate below 20%. To convince others, the researcher plans to make 100 such loans to a random sample from the pool of borrowers and record whether or not each loan defaults. If the true default rate in the entire population were 15 percent, how powerful is the planned test? Answer with formal hypotheses, two graphs, and a quantitative measure of power.

(5) See *Supplement for Question (5)*: “Medicaid Increases Emergency-Department Use: Evidence from Oregon’s Health Insurance Experiment.”

(a) [8 pts] Under “*Number of Visits*” in Panel A, locate “(2.632)” and “(0.116),” which are highlighted with boldface font. How should you expect these values to differ if there were 100,000 lottery participants instead of 24,646? Explain. Answer with 2 – 3 sentences.

(b) [2 pts] Focusing on those in the *treatment group* with *no visits* to the emergency department prior to the lottery, what fraction had any visit to the emergency department in the roughly 18 months after the lottery? Answer with a number.

(c) [2 pts] Focusing on those in the *treatment group* with *two or more visits* to the emergency department prior to the lottery, on average how many trips to the emergency department did they make in the roughly 18 months after the lottery? Answer with a number.

(d) [10 pts] What is the hypothesis test for the blacked-out cell in Table 2? Compute the missing P-value. Is the result statistically significant? Answer with hypotheses using formal notation, the P-value and 1 sentence.

The pages of this supplement will *not* be graded: write your answers on the test papers. **Supplement: Page 1 of 8**

This *Supplement* contains the aid sheets (formula sheets and Normal and *t* tables) as well as graphs, tables, and other information needed to answer the test questions. For each question directing you to this *Supplement*, make sure to carefully review all relevant materials. Remember, only your answers written on the test papers (in the designated space immediately after each question) will be graded. Any writing on this *Supplement* will *not* be graded.

Sample mean: $\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$ **Sample variance:** $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n-1} = \frac{\sum_{i=1}^n x_i^2}{n-1} - \frac{(\sum_{i=1}^n x_i)^2}{n(n-1)}$ **Sample s.d.:** $s = \sqrt{S^2}$

Sample coefficient of variation: $CV = \frac{s}{\bar{X}}$ **Sample covariance:** $S_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{X})(y_i - \bar{Y})}{n-1} = \frac{\sum_{i=1}^n x_i y_i}{n-1} - \frac{(\sum_{i=1}^n x_i)(\sum_{i=1}^n y_i)}{n(n-1)}$

Sample interquartile range: $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{S_{xy}}{s_x s_y} = \frac{\sum_{i=1}^n z_{x_i} z_{y_i}}{n-1}$

Addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **Conditional probability:** $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Complement rules: $P(A^c) = P(A') = 1 - P(A)$ $P(A^c|B) = P(A'|B) = 1 - P(A|B)$

Multiplication rule: $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$

Expected value: $E[X] = \mu = \sum_{all\ x} xp(x)$ **Variance:** $V[X] = E[(X - \mu)^2] = \sigma^2 = \sum_{all\ x} (x - \mu)^2 p(x)$

Covariance: $COV[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] = \sigma_{XY} = \sum_{all\ x} \sum_{all\ y} (x - \mu_X)(y - \mu_Y)p(x, y)$

Laws of expected value:

$E[c] = c$

$E[X + c] = E[X] + c$

$E[cX] = cE[X]$

$E[a + bX + cY] = a + bE[X] + cE[Y]$

Laws of variance:

$V[c] = 0$

$V[X + c] = V[X]$

$V[cX] = c^2V[X]$

$V[a + bX + cY] = b^2V[X] + c^2V[Y] + 2bc * COV[X, Y]$

$V[a + bX + cY] = b^2V[X] + c^2V[Y] + 2bc * SD(X) * SD(Y) * \rho$

where $\rho = CORRELATION[X, Y]$

Laws of covariance:

$COV[X, c] = 0$

$COV[a + bX, c + dY] = bd * COV[X, Y]$

Sampling distribution of \bar{X} :

$\mu_{\bar{X}} = E[\bar{X}] = \mu$

$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

Sampling distribution of \hat{P} :

$\mu_{\hat{P}} = E[\hat{P}] = p$

$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$

$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$:

$\mu_{\hat{P}_2 - \hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$

$\sigma_{\hat{P}_2 - \hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$

$\sigma_{\hat{P}_2 - \hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

Sampling distribution of $(\bar{X}_1 - \bar{X}_2)$, independent samples:

$\mu_{\bar{X}_1 - \bar{X}_2} = E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_1 - \bar{X}_2}^2 = V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$\sigma_{\bar{X}_1 - \bar{X}_2} = SD[\bar{X}_1 - \bar{X}_2] = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Sampling distribution of (\bar{X}_d) , paired ($d = X_1 - X_2$):

$\mu_{\bar{X}_d} = E[\bar{X}_d] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_d}^2 = V[\bar{X}_d] = \frac{\sigma_d^2}{n} = \frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}$

$\sigma_{\bar{X}_d} = SD[\bar{X}_d] = \frac{\sigma_d}{\sqrt{n}} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}}$

The pages of this supplement will *not* be graded: write your answers on the test papers. **Supplement: Page 2 of 8**

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$ for $x = 0, 1, 2, \dots, n$

If X is Binomial ($X \sim B(n, p)$) then $E[X] = np$ and $V[X] = np(1-p)$

If X is Uniform ($X \sim U[a, b]$) then $f(x) = \frac{1}{b-a}$ and $E[X] = \frac{a+b}{2}$ and $V[X] = \frac{(b-a)^2}{12}$

Inference about a population proportion:

z test statistic: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ **CI estimator:** $\hat{P} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Inference about comparing two population proportions:

z test statistic under Null hypothesis of no difference: $z = \frac{\hat{p}_2 - \hat{p}_1}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}}$ **Pooled proportion:** $\bar{P} = \frac{X_1 + X_2}{n_1 + n_2}$

CI estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_2(1-\hat{p}_2)}{n_2} + \frac{\hat{p}_1(1-\hat{p}_1)}{n_1}}$

Inference about the population mean:

t test statistic: $t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$ **CI estimator:** $\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

Inference about a comparing two population means, independent samples, unequal variances:

t test statistic: $t = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Degrees of freedom: $\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1} \left(\frac{s_2^2}{n_2}\right)^2}$

Inference about a comparing two population means, independent samples, assuming equal variances:

t test statistic: $t = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$ **Degrees of freedom:** $\nu = n_1 + n_2 - 2$

Pooled variance: $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$

Inference about a comparing two population means, paired data: (n is number of pairs and $d = X_1 - X_2$)

t test statistic: $t = \frac{\bar{d} - \Delta_0}{s_d/\sqrt{n}}$ **CI estimator:** $\bar{X}_d \pm t_{\alpha/2} \frac{s_d}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

SIMPLE REGRESSION:

Model: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ **OLS line:** $\hat{y}_i = b_0 + b_1 x_i$ $b_1 = \frac{s_{xy}}{s_x^2} = r \frac{s_y}{s_x}$ $b_0 = \bar{Y} - b_1 \bar{X}$

Residuals: $e_i = y_i - \hat{y}_i$ **Standard deviation of residuals:** $s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^n (e_i - 0)^2}{n-2}}$

$SST = \sum_{i=1}^n (y_i - \bar{Y})^2 = SSR + SSE$ $SSR = \sum_{i=1}^n (\hat{y}_i - \bar{Y})^2$ $SSE = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \hat{y}_i)^2$

$s_y^2 = \frac{SST}{n-1}$ $MSE = \frac{SSE}{n-2}$ $Root\ MSE = \sqrt{\frac{SSE}{n-2}}$

Coefficient of determination: $R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} = (r)^2$

THE NORMAL AND T TABLES ARE ON THE LAST TWO PAGES OF THIS SUPPLEMENT.

Supplement for Question (1): A researcher randomly selects 25 young adults with substantial debt and has each take a financial literacy test with 50 multiple-choice questions each worth 2 points. The score, out of 100, is recorded but the test-takers are not told their results or allowed to see which questions were answered correctly. The same 25 people then participate in a one-day course designed to improve financial literacy. They retake the financial literacy test. Unsurprisingly, the scores on the first and second test are positively correlated: the coefficient of correlation is 0.6387. Also, here are STATA summaries of the scores on the first and second test.

```
. summarize test_1 test_2;
```

Variable	Obs	Mean	Std. Dev.	Min	Max
test_1	25	53.04	17.72117	24	94
test_2	25	60.8	15.29706	36	96

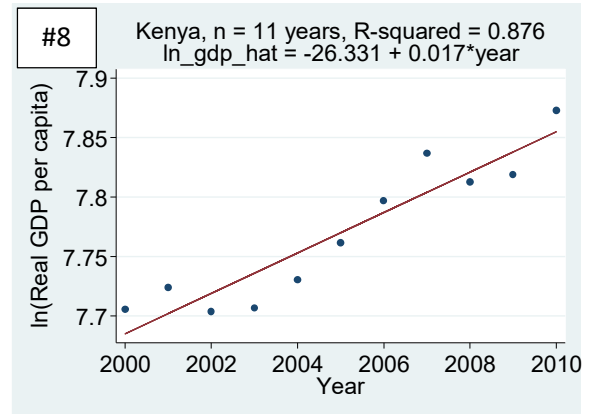
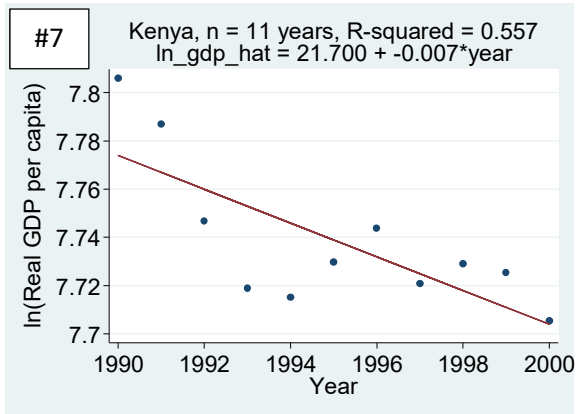
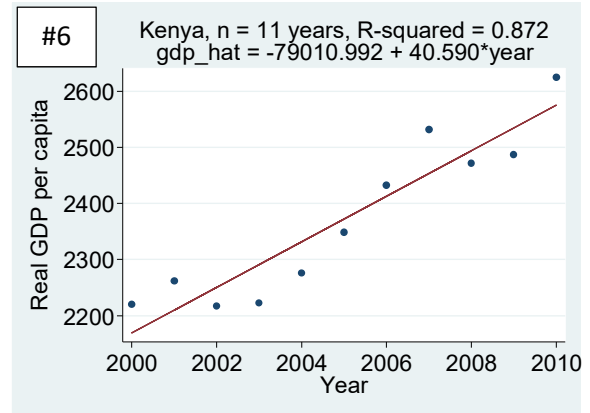
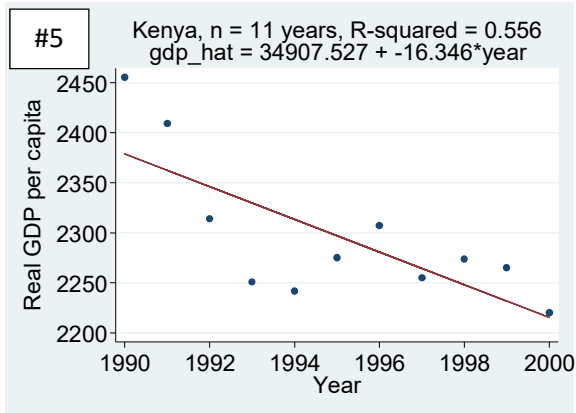
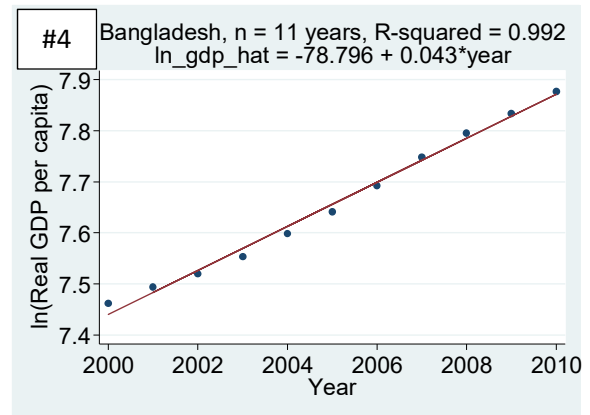
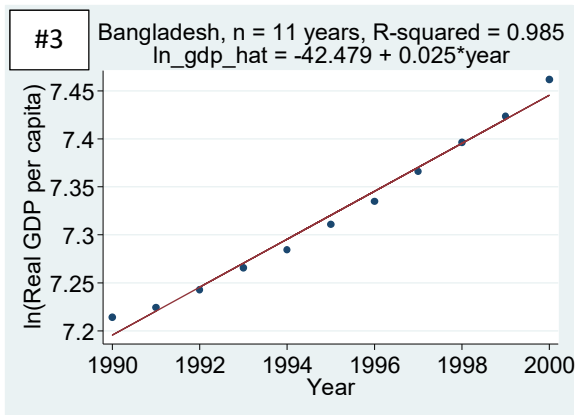
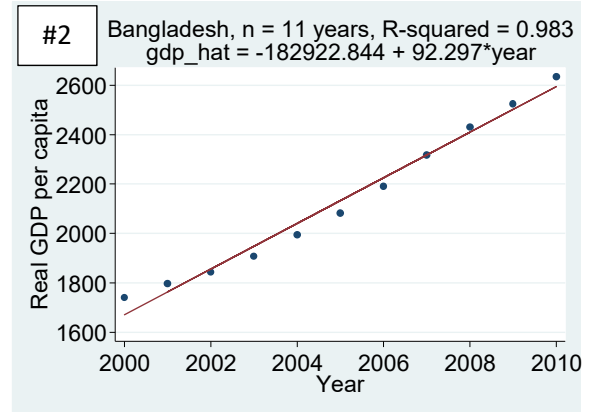
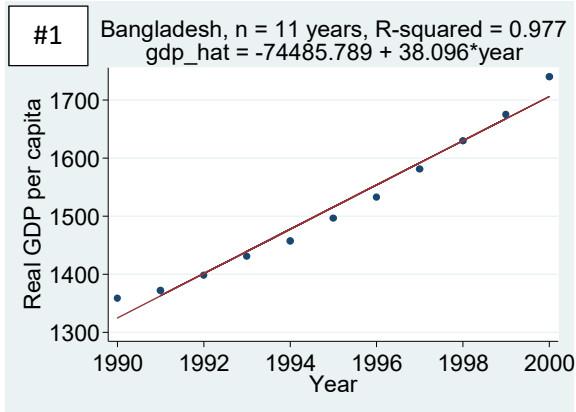
Supplement for Question (2): Recall the 2014 NBER working paper “Asiaphoria Meets Regression to the Mean” (<http://www.nber.org/papers/w20573.pdf>). The original Table 1 is updated below to use the most recent PWT 9.0 data released on June 9, 2016 (DOI: 10.15141/S5J01T).¹ **Note:** Table 1 is just to remind you about the paper. The test questions ask you about *Graphs #1 through #8, which are on the next page.*

Period 1	Period 2	Correlation	Rank Correlation	Regression Coefficient	R-squared	N
PANEL A: Adjacent decades						
1970 – 80	1980 – 90	0.3442	0.3182	0.3231	0.1185	142
1980 – 90	1990 – 00	0.3124	0.4109	0.2518	0.0976	142
1990 – 00	2000 – 10	0.1736	0.2539	0.1440	0.0301	142
<i>Source:</i> Calculations based on PWT 9.0.						

Supplement for Question (2) continues on next page >>>>

¹ Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer (2015), “The Next Generation of the Penn World Table” *American Economic Review*, 105(10), 3150-3182, available for download at www.ggdc.net/pwt.

Supplement for Question (2), cont'd: Graph #1 through Graph #8 below use the PWT 9.0 data. Real GDP per capita is “Real GDP at constant 2011 national prices (in mil. 2011US\$)” divided by “Population (in millions).”



Supplement for Question (3): Recall the 2016 academic article “The Value of Postsecondary Credentials in the Labor Market: An Experimental Study” by Deming et al in the journal *American Economic Review* (DOI: 10.1257/aer.20141757). This paper uses a type of field experiment called a resume audit study. Consider the excerpt and Table 3 below.

EXCERPT (p. 779): We draw upon a vast online bank of actual resumes of job seekers to construct fictitious, but realistic, resumes that randomly vary the fictitious job applicant’s characteristics including postsecondary institution. We use these resumes in applying to job vacancies in five major US metropolitan areas posted on a large, nationally-recognized job search website.

TABLE 3—SUMMARY STATISTICS FOR THE RESUMES USED IN THE AUDIT STUDY

	Callback rate	Number of resumes
Total	0.082	10,484
<i>By city</i>		
Chicago	0.082	2,036
Los Angeles	0.115	1,580
Miami	0.058	2,480
New York City	0.083	2,284
San Francisco Bay Area	0.083	2,104
<i>By occupation and degree requirements</i>		
AA, accounting/finance	0.045	1,084
AA, customer service/sales	0.125	2,920
BA, accounting/finance	0.044	1,928
BA, customer service/sales	0.104	2,172
Licensed practical nurse	0.057	804
Pharmacy technician	0.070	200
Medical assistant (administrative)	0.046	1,016
Medical assistant (clinical)	0.078	360
<i>By race and gender</i>		
White female	0.092	2,620
White male	0.066	2,456
Nonwhite female	0.090	2,680
Nonwhite male	0.077	2,728
<i>By average salary (business jobs only)</i>		
less than \$35,000	0.105	2,497
\$35,000 to \$49,999	0.109	2,468
\$50,000 to \$64,999	0.080	1,254
\$65,000 or more	0.048	1,448
No salary data	0.048	437

Note: The callback rate is the share of resumes that received a personalized callback (by phone or email) from a potential employer.

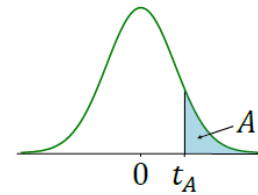
Supplement for Question (5): Recall the paper discussed in class and homework: “Medicaid Increases Emergency-Department Use: Evidence from Oregon’s Health Insurance Experiment” published in *Science* in January 2014. (DOI: 10.1126/science.1246183).

Recall that the *control group* includes the losing lottery participants who do not gain Medicaid health insurance coverage: in other words, continue to have no health insurance. In contrast, the *treatment group* includes the winning lottery participants who gain Medicaid health insurance coverage. To measure the effect of Medicaid coverage, the paper compares these two groups in terms of how much they rely on the emergency department in the approximately 18 months after the lottery. Next, the paper’s abstract, a description of Table 2 [**emphasis added**], and a copy of Table 2 (with one cell intentionally blacked out) are reproduced.

Abstract: In 2008, Oregon initiated a limited expansion of a Medicaid program for uninsured, low-income adults, drawing names from a waiting list by lottery. This lottery created a rare opportunity to study the effects of Medicaid coverage using a randomized controlled design. Using the randomization provided by the lottery and emergency department records from Portland-area hospitals, we study the emergency department use of about 25,000 lottery participants over approximately 18 months after the lottery. We find that Medicaid coverage significantly increases overall emergency use by 0.41 visits per person, or 40 percent relative to an average of 1.02 visits per person in the control group. We find increases in emergency-department visits across a broad range of types of visits, conditions, and subgroups, including increases in visits for conditions that may be most readily treatable in primary care settings.

Description of Table 2 (p. 265): We report the estimated effect of Medicaid on emergency department use over our study period (March 10, 2008 – September 30, 2009) in the entire sample and in subpopulations based on pre-randomization emergency department use. For each subpopulation, we report the sample size, **the control mean of the dependent variable (with standard deviation for continuous outcomes in parentheses), the estimated effect of Medicaid coverage (with standard error in parentheses), and the p-value of the estimated effect.** Sample consists of individuals in Portland-area zip codes (N=24,646) or specified subpopulation.

Table 2. Emergency-department use							
		Percent with any visits ¹			Number of visits ²		
	N	Mean Value in Control Group	Effect of Medicaid Coverage	p-value	Mean Value in Control Group	Effect of Medicaid Coverage	p-value
Panel A: Overall							
All visits	24,646	34.5	7.0 (2.4)	0.003	1.022 (2.632)	0.408 (0.116)	<0.001
Panel B: By emergency department use in the pre-randomization period							
No visits	16,930	22.5	6.7 (2.9)	0.019	0.418 (1.103)	0.261 (0.084)	0.002
One visit	3,881	47.2	9.2 (6.0)	0.127	1.115 (1.898)	0.652 (0.254)	0.010
Two+ visits	3,835	72.2	7.1 (5.6)	0.206	3.484 (5.171)	0.380 (0.648)	■
¹ For the percent-with-any-visits measures, the estimated effects of Medicaid coverage are shown in percentage points.							
² The number-of-visits measures are unconditional, including those with no visits.							



Critical Values of t :

ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
1	3.078	6.314	12.706	31.821	63.657	38	1.304	1.686	2.024	2.429	2.712
2	1.886	2.920	4.303	6.965	9.925	39	1.304	1.685	2.023	2.426	2.708
3	1.638	2.353	3.182	4.541	5.841	40	1.303	1.684	2.021	2.423	2.704
4	1.533	2.132	2.776	3.747	4.604	41	1.303	1.683	2.020	2.421	2.701
5	1.476	2.015	2.571	3.365	4.032	42	1.302	1.682	2.018	2.418	2.698
6	1.440	1.943	2.447	3.143	3.707	43	1.302	1.681	2.017	2.416	2.695
7	1.415	1.895	2.365	2.998	3.499	44	1.301	1.680	2.015	2.414	2.692
8	1.397	1.860	2.306	2.896	3.355	45	1.301	1.679	2.014	2.412	2.690
9	1.383	1.833	2.262	2.821	3.250	46	1.300	1.679	2.013	2.410	2.687
10	1.372	1.812	2.228	2.764	3.169	47	1.300	1.678	2.012	2.408	2.685
11	1.363	1.796	2.201	2.718	3.106	48	1.299	1.677	2.011	2.407	2.682
12	1.356	1.782	2.179	2.681	3.055	49	1.299	1.677	2.010	2.405	2.680
13	1.350	1.771	2.160	2.650	3.012	50	1.299	1.676	2.009	2.403	2.678
14	1.345	1.761	2.145	2.624	2.977	51	1.298	1.675	2.008	2.402	2.676
15	1.341	1.753	2.131	2.602	2.947	52	1.298	1.675	2.007	2.400	2.674
16	1.337	1.746	2.120	2.583	2.921	53	1.298	1.674	2.006	2.399	2.672
17	1.333	1.740	2.110	2.567	2.898	54	1.297	1.674	2.005	2.397	2.670
18	1.330	1.734	2.101	2.552	2.878	55	1.297	1.673	2.004	2.396	2.668
19	1.328	1.729	2.093	2.539	2.861	60	1.296	1.671	2.000	2.390	2.660
20	1.325	1.725	2.086	2.528	2.845	65	1.295	1.669	1.997	2.385	2.654
21	1.323	1.721	2.080	2.518	2.831	70	1.294	1.667	1.994	2.381	2.648
22	1.321	1.717	2.074	2.508	2.819	75	1.293	1.665	1.992	2.377	2.643
23	1.319	1.714	2.069	2.500	2.807	80	1.292	1.664	1.990	2.374	2.639
24	1.318	1.711	2.064	2.492	2.797	90	1.291	1.662	1.987	2.368	2.632
25	1.316	1.708	2.060	2.485	2.787	100	1.290	1.660	1.984	2.364	2.626
26	1.315	1.706	2.056	2.479	2.779	120	1.289	1.658	1.980	2.358	2.617
27	1.314	1.703	2.052	2.473	2.771	140	1.288	1.656	1.977	2.353	2.611
28	1.313	1.701	2.048	2.467	2.763	160	1.287	1.654	1.975	2.350	2.607
29	1.311	1.699	2.045	2.462	2.756	180	1.286	1.653	1.973	2.347	2.603
30	1.310	1.697	2.042	2.457	2.750	200	1.286	1.653	1.972	2.345	2.601
31	1.309	1.696	2.040	2.453	2.744	250	1.285	1.651	1.969	2.341	2.596
32	1.309	1.694	2.037	2.449	2.738	300	1.284	1.650	1.968	2.339	2.592
33	1.308	1.692	2.035	2.445	2.733	400	1.284	1.649	1.966	2.336	2.588
34	1.307	1.691	2.032	2.441	2.728	500	1.283	1.648	1.965	2.334	2.586
35	1.306	1.690	2.030	2.438	2.724	750	1.283	1.647	1.963	2.331	2.582
36	1.306	1.688	2.028	2.434	2.719	1000	1.282	1.646	1.962	2.330	2.581
37	1.305	1.687	2.026	2.431	2.715	∞	1.282	1.645	1.960	2.326	2.576

Degrees of freedom: ν