ECO220Y, Term Test #3 SOLUTIONS

February 5, 2016, 9:10 – 11:00 am

(1)

For the subset making $300K or less:

$\mu_{\bar{X}} = E[\bar{X}] = \mu = \$125.3419K$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}} = \frac{29.96436}{\sqrt{2000}} = \$0.670K$

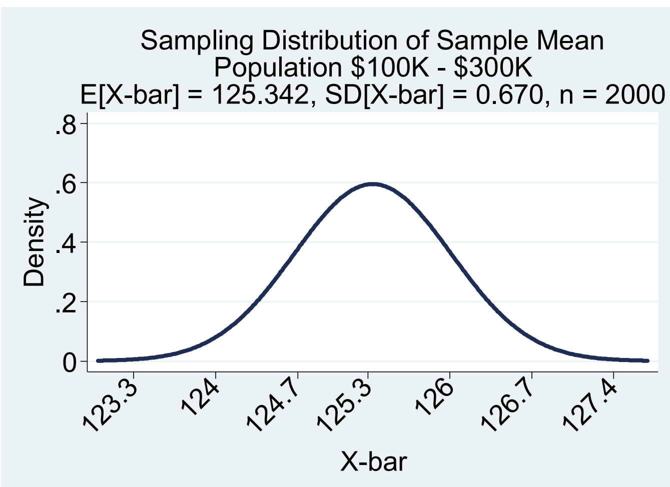
A sample size of 2,000 is surely sufficiently large such that the Central Limit Theorem (CLT) applies: the sampling distribution of $\bar{X}$ will be Normal even though the population is extremely positively skewed.

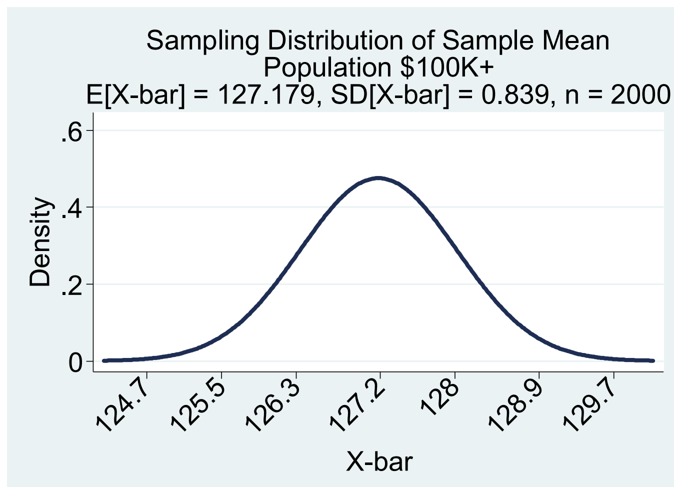
For the full population:

$\mu_{\bar{X}} = E[\bar{X}] = \mu = \$127.1793K$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}} = \frac{37.52571}{\sqrt{2000}} = \$0.839K$

A sample size of 2,000 is surely sufficiently large such that the Central Limit Theorem (CLT) applies: the sampling distribution of $\bar{X}$ will be Normal even though the population is *very extremely* positively skewed.





While both are Normal (n = 2,000 is surely sufficiently large), there are two differences in these sampling distributions. The second one (above) is centered about $2,000 higher and is more spread out. This is because the population mean and especially the population s.d. are higher when the long tail of ON employees making $300K+ are included. (Note: Students do *not* need to label the heights (e.g. 0.4) of the density function but otherwise the graphs must be fully-labelled.)

(2) (a)

$H_0: \mu = 0.07$

$H_1: \mu < 0.07$

The critical value (c.v.) for $\alpha = 0.01$ and $\nu = n - 1 = 14$ is -2.624 (which the textbook calls t^*). The rejection region is any t statistic that is less than (or equal) -2.624. Once we have the data we compute $t = \frac{\bar{X}-\mu_0}{s/\sqrt{15}} = \frac{\bar{X}-0.07}{s/\sqrt{15}}$ and compare this test statistic with the rejection region.

Example #1: $\bar{X} = 0.06$ and $s = 0.01$ which implies $t = \frac{0.06-0.07}{0.01/\sqrt{15}} = -3.87$ which leads to the conclusion that the fleet is in compliance at a 1% significance level.

Example #2: $\bar{X} = 0.06$ and $s = 0.02$ which implies $t = \frac{0.06-0.07}{0.02/\sqrt{15}} = -1.94$ which fails to prove that the fleet is in compliance (fail to reject the null) at a 1% significance level.

(b)

$H_0: \mu = 0.07$

$H_1: \mu > 0.07$

The c.v. for $\alpha = 0.05$ and $\nu = n - 1 = 14$ is 1.761. The rejection region is any t statistic greater than 1.761.

Once we have the data we compute $t = \frac{\bar{X}-\mu_0}{s/\sqrt{15}} = \frac{\bar{X}-0.07}{s/\sqrt{15}}$ and compare this test statistic with the rejection region.

Example #1: $\bar{X} = 0.075$ and $s = 0.02$ which implies $t = \frac{0.075-0.07}{0.02/\sqrt{15}} = 0.97$ which fails to prove that the fleet is out of compliance (fail to reject the null) at a 5% significance level even though there is some evidence that the fleet is out of compliance (the sample mean is 0.075, which is above 0.07).

(3) (a)

$H_0: p_{1994} - p_{1987} = 0$

$H_1: p_{1994} - p_{1987} < 0$

$z = -11.15$ This is extremely strong evidence: the lower death rate is highly statistically significant (the P-value is basically zero). In addition to being highly statistically significant, the difference is definitely significant: a change in the death rate from 12.2% to 10.7% is a decline of 1.5 percentage points (economically significant).

(b)

$H_0: p_{1994} - p_{1987} = 0$

$H_1: p_{1994} - p_{1987} \neq 0$

$z = 2.19$ This is fairly strong evidence: the P-value is $= 2 * (0.5 - 0.4857) = 0.0286$, which meets a 5% significance level but not a 1% significance level. However, a 0.1 percentage point difference in the readmission rate is tiny and not economically significant. (The huge sample sizes in this example mean that even tiny differences will be statistically significant). Hence this difference is not significant.

(4)

$$(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$$

$\hat{P}_2 = \frac{11}{30} = 0.3667$ and $\hat{P}_1 = \frac{19}{30} = 0.6333$

$$(0.3667 - 0.6333) \pm 1.96\sqrt{\frac{0.3667(1-0.3667)}{30} + \frac{0.6333(1-0.6333)}{30}}$$

$(-0.2667) \pm 1.96 * 0.1244$

-0.267 ± 0.244

$LCL = -0.511$ and $UCL = -0.023$. We are 95% confident that among *all* potential people asked to self-report, signing at the top will *cause* the percent that cheat to be between 2.3 and 51.1 percentage points lower compared to signing at the bottom. The point estimate of the difference is 26.7%, which is a big difference in the percent of people cheating, with a margin of error of 24.4% (a big margin of error). Having people sign at the top potentially causes a huge decrease in cheating but the big margin of error (due to the relatively small sample sizes) means that the true effect could be fairly modest (lower confidence limit is only a 2.3 percentage point decline in cheating).

(5) (a)

$H_0: \mu = 0.32$

$H_1: \mu > 0.32$

(b) A Type I error would be rejecting the null hypothesis and inferring that a potential source is profitable when in fact it is not profitable. Spartan would be concerned about this kind of mistake because setting up a supply chain likely involves substantial fixed costs: it does not want to realize after the fact that it should have never started sourcing from a particular location. Spartan can lower the chances of this kind of error by choosing a high burden of proof, which corresponds to a low significance level such as 1% or even 0.1% instead of the standard 5% alpha.

(c) A Type II error would be failing to conclude that a profitable source is profitable: a missed opportunity. If profitable sources are scarce or the missed opportunity is particularly good, these kinds of mistakes would negatively affect Sparton's profitability. Sparton can lower the risk of Type II errors by collecting a bigger sample size for each source: in other words, more than 10 batches of coal ash. As the sample size increases, the amount of sampling error declines: it will get a much clearer positive picture of sources that will turn out to profitable and this will lower the chance of inconclusive results of failing to reject the null because there is simply too much sampling error.