ECO220Y, Term Test #3

February 5, 2016, 9:10 – 11:00 am

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Instructions:

- You have 110 minutes. Keep these test papers closed on your desk until the start of the test is announced.
- You may use a non-programmable calculator.
- There are 5 questions (some with multiple parts) with varying point values worth a total of 84 points.
- Write your answers clearly, completely and concisely in the designated space provided immediately after each question. No extra space/pages are possible. You *cannot* use blank space for other questions nor can you write answers on the *Supplement*. Your entire answer must fit in the designated space provided immediately after each question.
    - **Write in pencil and use an eraser as needed.** This way you can make sure to fit your final answer (including work and reasoning) in the appropriate space.
    - **Most questions give more blank space than is needed to answer.** Follow the answer guides and avoid excessively long answers.
- Clearly show your work. Make your reasoning clear. **Unless otherwise specified, you choose the significance level. (If there are no special considerations, you may choose a 5% significance level.)**
- Apply your understanding to the specific questions asked. Offer context-specific explanations rather than generic definitions or quotes from class or the book. Show that you can successfully *apply* your understanding to the specific circumstances presented.
- A guide for your response ends each question. The guide lets you know what is expected: e.g. a quantitative analysis, a graph, and/or sentences. If the question and/or guide ask for a fully-labeled graph, it is required.
- For questions with multiple parts (e.g (a) – (c)), attempt each part even if you had trouble with earlier parts.
- This test has 8 pages plus the *Supplement*. The *Supplement* contains the aid sheets (formula sheets, Standard Normal table, and Student t table) as well as graphs, tables, and other information needed to answer the test questions. Anything written on the *Supplement* will *not* be graded. You must write your answers in the designated space provided immediately after each question.

(1) [16 pts] See the *Supplement for Question (1)*. For the subset of the population making $300K or less, find and graph the sampling distribution of the sample mean for a sample size of 2,000 employees. Make it clear why the graph looks the way it does. How would the sampling distribution of the sample mean for 2,000 employees randomly selected from the full population look different from the one you drew? *Explain*. Answer with an analysis, a fully-labelled graph and 2 – 4 sentences.

Page Pts:

(2) [20 pts] Canadian emissions regulations require that light-duty vehicles manufactured in 2009 or later have overall fleet average emissions of no more than 0.07 grams per mile of nitrogen oxide (NOx) (http://laws-lois.justice.gc.ca/eng/regulations/sor-2003-2/page-5.html#h-23). A researcher tests emissions for a random sample of 15 light-duty vehicles manufactured in 2012 in a particular fleet.

Page Pts:

(a) [12 pts] If the researcher is hired by the manufacturer to prove to regulators at a 1% significance level that the fleet is in compliance, how should the hypotheses be written? What is the rejection region? Illustrate with two specific examples of samples, each with a sample mean of 0.06 (you choose the s.d.'s), that would yield different conclusions. Answer with formal hypotheses, the rejection region, two specific examples and 2 – 3 sentences.

(b) [8 pts] If the researcher is hired by a journalist to prove to the public at a 5% significance level that the fleet is not in compliance (i.e. an emissions scandal), how should the hypotheses be written? What is the rejection region? Illustrate with one specific example of a sample (you choose the mean and s.d.) where there is some evidence of a scandal but it doesn't meet the burden of proof. Answer with formal hypotheses, the rejection region, one specific example and 1 – 2 sentences.

Page Pts:

(3) [16 pts] See the *Supplement for Question (3)*.

Page Pts:

(a) [8 pts] How strong is the evidence that mortality within one year of admission declined from 1987 to 1994 for patients receiving CABG surgery? (Mortality means death.) The test statistic is -11.15. (You do *not* need to compute this number yourself, even though you have all the information needed.) Is the decline statistically significant? Significant? Answer with formal hypotheses and 2 – 3 sentences.

(b) [8 pts] How strong is the evidence that the rate of readmission with AMI within one year of admission changed from 1987 to 1994 for patients receiving CABG surgery? The test statistic is 2.19. (Again, you do *not* need to compute this number yourself, even though you have all the information needed.) What is the P-value? Is the difference statistically significant? Significant? Answer with formal hypotheses, a quantitative analysis and 1 – 2 sentences.

(4) [14 pts] See the *Supplement for Question (4)*. How big of a difference does it make to cheating to have people sign at the top rather than the bottom? *Fully interpret* your results. Answer with a quantitative analysis (using the best approach to fully address the question asked) and 2 – 3 sentences.

Page Pts:

(5) [18 pts] With a random sample of 10 batches of coal ash from each potential source, Sparton Resources checks if there is sufficient evidence to conclude it is a profitable source. To profitably exploit a source requires an average concentration of at least 0.32 pounds of uranium oxide per tonne of ash.

Page Pts:

(a) [2 pts] What is the formal hypothesis test? Answer with formal hypotheses.

(b) [8 pts] What would a Type I error be? (*Apply* the concept to Sparton.) Why may Sparton worry about Type I errors? How can it lower the risk of Type I errors as it reviews potential sources? Why will your suggestion lower the risk of Type I errors? Answer with 3 – 4 sentences.

(c) [8 pts] What would a Type II error be? (*Apply* the concept to Sparton.) Why may Sparton worry about Type II errors? How can it lower the risk of Type II errors as it reviews potential sources (assuming that it does *not* wish to increase the chance of Type I errors)? Why will your suggestion lower the risk of Type II errors? Answer with 3 – 4 sentences.

Page Pts:

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This *Supplement* contains the aid sheets (formula sheets, Standard Normal table, and Student t table) as well as graphs, tables, and other information needed to answer the test questions. For each question directing you to this *Supplement*, make sure to carefully review all relevant materials. Remember, only your answers written on the test papers (in the designated space immediately after each question) will be graded. Any writing on this *Supplement* will *not* be graded.

Sample mean: $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ **Sample variance:** $s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{X})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$ **Sample s.d.:** $s = \sqrt{s^2}$

Sample coefficient of variation: $CV = \frac{s}{\bar{X}}$ **Sample covariance:** $s_{xy} = \frac{\sum_{i=1}^{n}(x_i-\bar{X})(y_i-\bar{Y})}{n-1} = \frac{\sum_{i=1}^{n} x_i y_i}{n-1} - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(n-1)}$

Sample interquartile range: $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{s_{xy}}{s_x s_y} = \frac{\sum_{i=1}^{n} z_{x_i} z_{y_i}}{n-1}$

Addition rule: $P(A \text{ } or \text{ } B) = P(A) + P(B) - P(A \text{ } and \text{ } B)$ **Conditional probability:** $P(A|B) = \frac{P(A \text{ } and \text{ } B)}{P(B)}$

Complement rules: $P(A^C) = P(A') = 1 - P(A)$ $P(A^C|B) = P(A'|B) = 1 - P(A|B)$

Multiplication rule: $P(A \text{ } and \text{ } B) = P(A|B)P(B) = P(B|A)P(A)$

Expected value: $E[X] = \mu = \sum_{all\ x} xp(x)$ **Variance:** $V[X] = E[(X-\mu)^2] = \sigma^2 = \sum_{all\ x}(x-\mu)^2 p(x)$

Covariance: $COV[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = \sigma_{XY} = \sum_{all\ x}\sum_{all\ y}(x-\mu_X)(y-\mu_Y)p(x,y)$

Laws of expected value:

$E[c] = c$
$E[X+c] = E[X] + c$
$E[cX] = cE[X]$
$E[a+bX+cY] = a + bE[X] + cE[Y]$

Laws of variance:

$V[c] = 0$
$V[X+c] = V[X]$
$V[cX] = c^2 V[X]$
$V[a+bX+cY] = b^2V[X] + c^2V[Y] + 2bc * COV[X,Y]$
$V[a+bX+cY] = b^2V[X] + c^2V[Y] + 2bc * SD(X) * SD(Y) * \rho$
where $\rho = CORRELATION[X,Y]$

Laws of covariance:

$COV[X,c] = 0$
$COV[a+bX, c+dY] = bd * COV[X,Y]$

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$ for $x = 0,1,2,\ldots,n$

If *X* is Binomial $(X \sim B(n,p))$ **then** $E[X] = np$ **and** $V[X] = np(1-p)$

If *X* is Uniform $(X \sim U[a,b])$ **then** $f(x) = \frac{1}{b-a}$ **and** $E[X] = \frac{a+b}{2}$ **and** $V[X] = \frac{(b-a)^2}{12}$

Sampling distribution of $\bar{X}$:

$\mu_{\bar{X}} = E[\bar{X}] = \mu$
$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$
$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

Sampling distribution of $\hat{P}$:

$\mu_{\hat{P}} = E[\hat{P}] = p$
$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$
$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$:

$\mu_{\hat{P}_2-\hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$
$\sigma_{\hat{P}_2-\hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$
$\sigma_{\hat{P}_2-\hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

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Inference about a population proportion:

***z* test statistic:** $z = \frac{\hat{P}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ **CI estimator:** $\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

Inference about comparing two population proportions:

***z* test statistic under Null hypothesis of no difference:** $z = \frac{\hat{P}_2-\hat{P}_1}{\sqrt{\frac{\bar{P}(1-\bar{P})}{n_1}+\frac{\bar{P}(1-\bar{P})}{n_2}}}$ **Pooled proportion:** $\bar{P} = \frac{X_1+X_2}{n_1+n_2}$

CI estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}+\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

Inference about the population mean:

***t* test statistic:** $t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$ **CI estimator:** $\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

Supplement for Question (1): Recall the salary data for ON public sector employees with salaries of $100,000 or more (http://www.fin.gov.on.ca/en/publications/salarydisclosure/pssd/). Consider the 98,942 employees in the 2014 disclosure that make $300,000 or less. A STATA summary shows the distribution of salaries (measured in $1,000s).

```
                              Salary
-------------------------------------------------------------
      Percentiles      Smallest
 1%     100.2091            100
 5%     100.9725            100
10%     102.0857            100       Obs               98942
25%     105.7196            100       Sum of Wgt.       98942

50%     115.1083                      Mean           125.3419
                        Largest       Std. Dev.      29.96436
75%     132.4765       299.9739
90%     162.9707            300       Variance        897.863
95%     187.7392            300       Skewness       2.382879
99%      254.231            300       Kurtosis       10.12159
```

The next STATA summary shows all 99,692 employees in the 2014 disclosure (including those making over $300,000).

```
                              Salary
-------------------------------------------------------------
      Percentiles      Smallest
 1%       100.21            100
 5%     100.9801            100
10%     102.1043            100       Obs               99692
25%     105.7759            100       Sum of Wgt.       99692

50%     115.3164                      Mean           127.1793
                        Largest       Std. Dev.      37.52571
75%      133.217        772.547
90%     165.0992       903.9706       Variance       1408.179
95%      193.125        915.851       Skewness       4.790958
99%     284.0477           1714       Kurtosis       65.52245
```

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Supplement for Question (3): Consider an academic article "Is More Information Better? The Effects of "Report Cards" on Health Care Providers" in a prominent journal (*Journal of Political Economy*). Here is the abstract:

> **ABSTRACT** Health care report cards—public disclosure of patient health outcomes at the level of the individual physician or hospital or both—may address important informational asymmetries in markets for health care, but they may also give doctors and hospitals incentives to decline to treat more difficult, severely ill patients. Whether report cards are good for patients and for society depends on whether their financial and health benefits outweigh their costs in terms of the quantity, quality, and appropriateness of medical treatment that they induce. Using national data on Medicare patients at risk for cardiac surgery, we find that cardiac surgery report cards in New York and Pennsylvania led both to selection behavior by providers and to improved matching of patients with hospitals. On net, this led to higher levels of resource use and to worse health outcomes, particularly for sicker patients. We conclude that, at least in the short run, these report cards decreased patient and social welfare.

In the section describing their data, the authors present the following table of basic descriptive statistics about the sample of patients used in their study. Note that AMI means "acute myocardial infarction," which most people would simply call a "heart attack" and CABG means "coronary artery bypass graft," which most people would simply call "heart bypass surgery."

TABLE A2
DESCRIPTIVE STATISTICS ON ELDERLY MEDICARE BENEFICIARIES WITH AMI AND ELDERLY MEDICARE BENEFICIARIES RECEIVING CABG SURGERY

	With AMI		Receiving CABG Surgery	
	1987	1994	1987	1994
Total hospital expenditures one year prior to admission	$2,690 (6,493)	$2,977 (7,464)	$4,431 (7,188)	$3,771 (7,586)
Total days in hospital one year prior to admission	4.21 (11.48)	4.22 (13.48)	4.97 (8.63)	3.39 (8.05)
Total hospital expenditures one year after admission	$14,634 (13,381)	$18,959 (19,060)	$30,226 (13,857)	$34,474 (22,460)
CABG within one year of admission (1=yes)	9.2%	16.2%	100%	100%
Readmission with AMI within one year of admission	5.8%	5.5%	1.1%	1.2%
Readmission with heart failure within one year of admission	9.0%	9.4%	6.1%	6.6%
Mortality within one year of admission	40.2%	32.9%	12.2%	10.7%
Age	76.0	76.4%	71.39	72.54
Gender (1=female)	49.8%	48.7%	34.2%	34.7%
Race (1=black)	5.5%	5.9%	2.4%	3.4%
Rural residence	30.0%	30.9%	28.1%	29.0%
Sample size	218,641	229,215	88,457	146,986

NOTE.—Hospital expenditures are in 1995 dollars. For full sample 1987–94, the sample size is 1,770,452 for AMI patients and 967,882 for CABG patients. Standard deviations are in parentheses.

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Supplement for Question (4): Consider a September 29, 2015 article in *The New York Times*: "Behaviorists Show the U.S. How to Improve Government Operations" (http://www.nytimes.com/2015/09/30/business/behaviorists-show-the-us-how-to-improve-government-operations.html) that cites an academic article "Signing at the beginning makes ethics salient and decreases dishonest self-reports in comparison to signing at the end" by business school professors (including one at Rotman) in an academic journal. Here is the article's abstract:

> **ABSTRACT** Many written forms required by businesses and governments rely on honest reporting. Proof of honest intent is typically provided through signature at the end of, e.g., tax returns or insurance policy forms. Still, people sometimes cheat to advance their financial self-interests—at great costs to society. We test an easy-to-implement method to discourage dishonesty: signing at the beginning rather than at the end of a self-report, thereby reversing the order of the current practice. Using laboratory and field experiments, we find that signing before–rather than after–the opportunity to cheat makes ethics salient when they are needed most and significantly reduces dishonesty.

In one experiment 60 people completed math puzzles. Participants were then asked to self-report, using a form, how well they did on the puzzles. The 60 people were randomly divided into two groups: one group had a form where they signed on the top and the other group had a form where they signed at the bottom. Here is a quote from the results section of the article: "The percentage of participants who cheated by overstating their performance on the math puzzles task was lower in the signature-at-the-top condition (37%, 11 of 30) than in the signature-at-the-bottom condition (63%, 19 of 30)."

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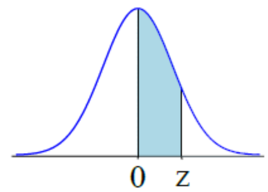

Normal Probabilities:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999

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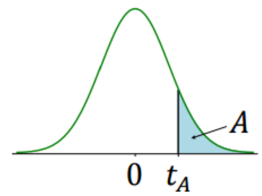

Critical Values of t:

ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
1	3.078	6.314	12.706	31.821	63.657	38	1.304	1.686	2.024	2.429	2.712
2	1.886	2.920	4.303	6.965	9.925	39	1.304	1.685	2.023	2.426	2.708
3	1.638	2.353	3.182	4.541	5.841	40	1.303	1.684	2.021	2.423	2.704
4	1.533	2.132	2.776	3.747	4.604	41	1.303	1.683	2.020	2.421	2.701
5	1.476	2.015	2.571	3.365	4.032	42	1.302	1.682	2.018	2.418	2.698
6	1.440	1.943	2.447	3.143	3.707	43	1.302	1.681	2.017	2.416	2.695
7	1.415	1.895	2.365	2.998	3.499	44	1.301	1.680	2.015	2.414	2.692
8	1.397	1.860	2.306	2.896	3.355	45	1.301	1.679	2.014	2.412	2.690
9	1.383	1.833	2.262	2.821	3.250	46	1.300	1.679	2.013	2.410	2.687
10	1.372	1.812	2.228	2.764	3.169	47	1.300	1.678	2.012	2.408	2.685
11	1.363	1.796	2.201	2.718	3.106	48	1.299	1.677	2.011	2.407	2.682
12	1.356	1.782	2.179	2.681	3.055	49	1.299	1.677	2.010	2.405	2.680
13	1.350	1.771	2.160	2.650	3.012	50	1.299	1.676	2.009	2.403	2.678
14	1.345	1.761	2.145	2.624	2.977	51	1.298	1.675	2.008	2.402	2.676
15	1.341	1.753	2.131	2.602	2.947	52	1.298	1.675	2.007	2.400	2.674
16	1.337	1.746	2.120	2.583	2.921	53	1.298	1.674	2.006	2.399	2.672
17	1.333	1.740	2.110	2.567	2.898	54	1.297	1.674	2.005	2.397	2.670
18	1.330	1.734	2.101	2.552	2.878	55	1.297	1.673	2.004	2.396	2.668
19	1.328	1.729	2.093	2.539	2.861	60	1.296	1.671	2.000	2.390	2.660
20	1.325	1.725	2.086	2.528	2.845	65	1.295	1.669	1.997	2.385	2.654
21	1.323	1.721	2.080	2.518	2.831	70	1.294	1.667	1.994	2.381	2.648
22	1.321	1.717	2.074	2.508	2.819	75	1.293	1.665	1.992	2.377	2.643
23	1.319	1.714	2.069	2.500	2.807	80	1.292	1.664	1.990	2.374	2.639
24	1.318	1.711	2.064	2.492	2.797	90	1.291	1.662	1.987	2.368	2.632
25	1.316	1.708	2.060	2.485	2.787	100	1.290	1.660	1.984	2.364	2.626
26	1.315	1.706	2.056	2.479	2.779	120	1.289	1.658	1.980	2.358	2.617
27	1.314	1.703	2.052	2.473	2.771	140	1.288	1.656	1.977	2.353	2.611
28	1.313	1.701	2.048	2.467	2.763	160	1.287	1.654	1.975	2.350	2.607
29	1.311	1.699	2.045	2.462	2.756	180	1.286	1.653	1.973	2.347	2.603
30	1.310	1.697	2.042	2.457	2.750	200	1.286	1.653	1.972	2.345	2.601
31	1.309	1.696	2.040	2.453	2.744	250	1.285	1.651	1.969	2.341	2.596
32	1.309	1.694	2.037	2.449	2.738	300	1.284	1.650	1.968	2.339	2.592
33	1.308	1.692	2.035	2.445	2.733	400	1.284	1.649	1.966	2.336	2.588
34	1.307	1.691	2.032	2.441	2.728	500	1.283	1.648	1.965	2.334	2.586
35	1.306	1.690	2.030	2.438	2.724	750	1.283	1.647	1.963	2.331	2.582
36	1.306	1.688	2.028	2.434	2.719	1000	1.282	1.646	1.962	2.330	2.581
37	1.305	1.687	2.026	2.431	2.715	∞	1.282	1.645	1.960	2.326	2.576

Degrees of freedom: ν