SUGGESTED SOLUTIONS

(1) (a)

$\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} \quad where\ \nu = n - 1$

$3571 \pm 1.677\frac{301}{\sqrt{50}} \quad where\ \nu = 49$

3571 ± 71.39

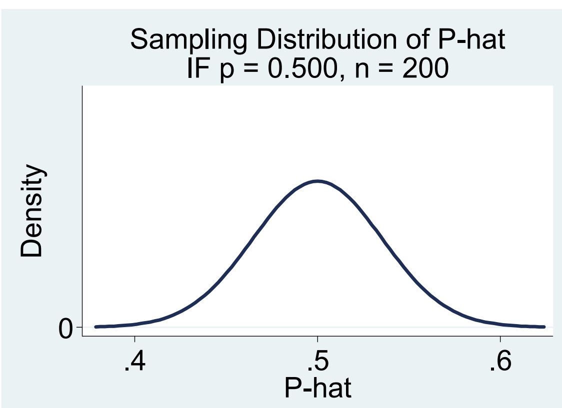
(\$3500, \$3642): LCL=\$3500 and UCL=\$3642 (2012/13: See exercise 15 in Chap. 13 of our textbook.)

(b) We are 90 percent confident that the interval from \$3,500 to \$3,642 contains the true (population) mean increase in the sales tax revenue per retailer. (2012/13: See exercise 15 in Chap. 13 of our textbook.)

(2) (a)

H_0: p = 0.5
H_1: p > 0.5

(b) $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}\sqrt{\frac{0.5(1-0.5)}{200}} = 0.035$



(c) The probability 0.409 is called the power of the hypothesis test: it tells the chance that we will be able to correctly infer the research hypothesis when the population proportion is some specific value, which would make the research hypothesis true. This test has low power: there is only a 40.9% chance that we'll be able to correctly infer the research hypothesis, which means that there is a high chance that we'll make a Type II error (a 0.591 chance).

[Note: Here is how the power is calculated:

Find the unstandardized rejection region.

$P(Z > 1.645) = 0.05$ (1.645 is the standardized critical value)

$$Z = \frac{\hat{P} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

$$1.645 = \frac{\hat{P} - 0.5}{\sqrt{\frac{0.5(1 - 0.5)}{200}}}$$

$\hat{P} = 0.558$ This is the unstandardized critical value.

$$P(\hat{P} > 0.558 \,|\, p = 0.55, n = 200) = P\left(Z > \frac{0.558 - 0.55}{\sqrt{\frac{0.55(1 - 0.55)}{200}}}\right) = P(Z > 0.23) = 0.5 - 0.0910 = 0.409$$

(3) (a) Yes. The positive difference of 1.2 percentage points is statistically significant at any conventional significance level (e.g. α = 0.05, 0.01) as the P-value is very small (and much smaller than any conventional α), which means that it is unlikely that sampling error explains the difference between 40% and 41.2%.

(b) No. It is unlikely that most people would feel that there is a significant difference between a 41.2% and 40.0% fail rate in all-metal hip implants.

(c) In studies with very large sample sizes it is possible that even very small differences (i.e. ones that are not economically or generally significant) are statistically significant. This is because as the sample size grows sampling error decreases (i.e. the standard error of the sample proportion decreases) meaning that we can distinguish even very small differences from chance sampling error.

(4) (a)

H_0: $\mu = 75$
H_1: $\mu \neq 75$

$\bar{X} = 75.6$

$s = 1.3$

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{75.6 - 75}{1.3/\sqrt{22}} = 2.16$$

$$P - value = P(t > 2.16) + P(t < -2.16) = 2 * P(t > 2.16)$$

From table we cannot find the exact P-value for 21 degrees of freedom (ν = n – 1 = 22 – 1). We see that P(t > 2.080) = 0.025 and P(t > 2.518) = 0.01. Hence we know the P-value will be between 0.02 and 0.05.

(b) The CI is the sample proportion plus/minus the margin of error: $\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$. Hence to find $\hat{P}$ we find the mid-point of the interval: $\frac{0.54+0.63}{2} = 0.585$. To find the sample size we use the ME.

$$ME = z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = 2.575\sqrt{\frac{0.585(1-0.585)}{n}}$$

$$2.575\sqrt{\frac{0.585(1-0.585)}{n}} = \frac{0.63-0.54}{2}$$

$$2.575\sqrt{\frac{0.585(1-0.585)}{n}} = 0.045$$

$$\frac{0.585(1-0.585)}{n} = 0.0003054$$

$$n \cong 795$$

(c) We need to find the unstandardized rejection region. Because no significance level is specified we can use a 5% level to find the standardized rejection region.

H_0: p = 0.10
H_1: p < 0.10

$P(Z < -1.645) = 0.05$ (-1.645 is the standardized critical value)

$$Z = \frac{\hat{P} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$-1.645 = \frac{\hat{P} - 0.1}{\sqrt{\frac{0.1(1-0.1)}{400}}}$$

$$\hat{P} = 0.075$$

Hence 0.075 is the unstandardized critical value (edge of the rejection region). For the sample to support the university's claims it would need that 7.5% or less of the sample favors online-only classes, which is 30 or fewer students out of the 400.

(d) The law of small numbers is a not a law but rather the false belief that even small samples will be highly representative of the population in all respects. While there is a law of large numbers that does in fact guarantee that large random samples will be highly representative this result does not extend to small samples, which are highly subject to sampling error. This false belief affects statistical inference by leading people to mentally overestimate the power of their tests and underestimate their P-values (overestimate the strength of their evidence), which is why the authors argue that we must not rely on our instincts and calculate these probabilities as well as consider using confidence interval estimation rather that hypothesis testing because it is easier for people to intuitively grasp.