ECO220Y1Y, Test #2, Prof. Murdock

November 29, 2024, 9:10 am – 11:00 am

- Keep ALL pages closed and face up on your desk until we announce the start, and only then you may detach the *Supplement*, which has the aid sheets and readings, figures, tables, and other materials for some test questions.
- There are 8 test pages with 7 questions, with varying numbers of parts, worth a total of 95 points.
- You have 110 minutes. You must stay for a minimum of 60 minutes.

Instructions:

- For each question referencing the *Supplement*, carefully review *all* materials. The *Supplement is* <u>NOT</u> *collected*: write your answers on the test papers. At the end, hand in your test papers (you keep the *Supplement*).
- Write your answers clearly, completely, and concisely in the designated space provided immediately after each question. An <u>answer guide</u> ends each question to let you know what is expected. For example, a <u>quantitative</u> <u>analysis</u>, a <u>fully labelled graph</u>, and/or <u>sentences</u>. Any answer guide asking for a <u>quantitative analysis</u> *always* automatically means that you must show your work and make your reasoning clear.
 - o Marking TAs are instructed to accept all reasonable rounding.
- Your entire answer must fit in the designated space provided immediately after each question. No extra space/pages are possible. You *cannot* use blank space for other questions, nor can you write answers on the *Supplement*. Write in <u>PENCIL</u> and use an <u>ERASER</u> as needed so that you can fit your final answer (including work and reasoning) in the appropriate space. We give more blank space than is needed for each answer (with typical handwriting) worth full marks. Follow the <u>answer guides</u> and avoid excessively long answers.

(1) See *Supplement for Question (1)*: Three large Ontario universities and 2023 disclosed salaries.

(a) [2 pts] Locate 0.0521 in Table 1. Write it as a probability statement. <u>Answer with a probability statement in formal notation.</u>

(b) [2 pts] Locate 0.3446 in Table 1. Write it as a probability statement. <u>Answer with a probability statement in formal</u> <u>notation.</u>

(c) [3 pts] What is the chance a University of Toronto employee in the salary disclosure is making \$225,000 or more? <u>Answer with a quantitative analysis using formal notation</u>.

(d) [3 pts] An employee makes \$100,000 to \$125,000 and works at one of the three universities. What is the chance that they work at the University of Waterloo or York University? <u>Answer with a quantitative analysis using formal notation</u>.

(e) [4 pts] Are events S4 and Y *independent*? Use the definition of independent events. <u>Answer with a quantitative</u> <u>analysis using formal notation.</u>

(2) [8 pts] See *Supplement for Question (2)*: *Competitiveness and Happiness.* What is the mean and standard deviation of the new variable abbreviated as X_A that is described in the *Supplement*? <u>Answer with a quantitative analysis.</u>

(3) [6 pts] For a 78.4% confidence level, what is the approximate value of $z_{\alpha/2}$? Illustrate your answer with a graph. Answer with a quantitative analysis using formal notation & a fully labelled graph.



(4) See Supplement for Question (4): Simulation with Ontario Public Disclosure Data from the Universities Sector.

(a) [6 pts] The figure title has two blank lines. Using the figure and given information, determine the two missing values from among these choices [10 / 25 / 250 / 1,000 / 10,000 / 100,000 / 250,000]: n = ______ and m = ______. Next, *explain* each choice. Answer by filling in the two blanks & with 2 sentences that include any quantitative analyses.

(b) [4 pts] If n were doubled, how would the figure change? <u>Answer by filling in the four blanks below.</u>

The number of bars ir	the histogram would	[increase / decrease / stay about the same],
the center would	[increase	: / decrease / stay about the same], the spread would
	[increase / decrease / stay	about the same], and the shape would become more
	[positively skewed / negat	ively skewed / symmetric].

(5) See Supplement for Question (5): Education and Preferences for Desired Traits in Children.

(a) [4 pts] To identify each of the following values in Column (1) of Table 2, select the *best* choice from among these $\left[\mu, p, s, \sigma, \hat{P}, \bar{X}, SE[\hat{P}], SE[\bar{X}]\right]$: 44.298 = _____; 10.893 = ____; 0.246 = ____; 0.431 = ____. Answer by filling in the four blanks.

(b) [5 pts] In Column (1), is the value 4.187 large or small in this context? Explain. Answer with 2 sentences.

(c) [12 pts] To make an inference about preferences on **imagination** using results in Column (1), compute the 95% confidence interval estimate. *Interpret* the LCL and UCL. <u>Answer with a quantitative analysis & 2 sentences.</u>

(d) [6 pts] In Columns (2) and (3), interpret the difference between 0.175 and 0.195. Answer with 2 sentences.

(e) [6 pts] For a 90% confidence level, what is the margin of error for the difference in the preferences on **independence** between the treatment and control groups? <u>Answer with a quantitative analysis & fill in the two blanks.</u>

The margin of error is ______[#] ______ [percent (%) / percentage points (p.p.)].

(6) [15 pts] Suppose Premier Doug Ford claims that 60% of Torontonians support his proposals regarding bike lanes. If his claim were true, is sampling error a *plausible* explanation for a sample proportion as small as 0.58 in a survey of 300 Torontonians? *Explain* and support your arguments with quantitative evidence. Also, illustrate your answer graphically. Answer with a quantitative analysis, a fully labelled graph & 2 - 3 sentences.



(7) See Supplement for Question (7): Full-Time Working Arrangements in the US as of 2023, Percentage Distributions.

(a) [3 pts] Indicate whether each statement is TRUE or FALSE. Answer by writing TRUE or FALSE on each line.

P(C) + P(G) = 0.033: _____

Events F1 and F2 are independent: _____

Events R and F3 are disjoint (mutually exclusive): _____

Events E and S are disjoint (mutually exclusive): _____

(b) [2 pts] Locate 17.8 in Column (3) of Table 1. Write it as a probability statement. <u>Answer with a probability statement</u> in formal notation.

(c) [4 pts] How would Table 1 change if "how" people work (e.g., onsite or hybrid) and "for whom" people work (e.g., for an employer or as a gig worker) were *independent*? You must assume that the distribution of "how" is unchanged and the distribution of "for whom" is unchanged. <u>Answer by filling in the 23 blanks below, skipping the cells shaded grey.</u>

Table 1. Full-Time Working Arrangements in the United States as of 2023, Percentage Distributions						
	Fully	Hybrid	Fully	Percent of		
	onsite	arrangement	remote	all workers		
	(1)	(2)	(3)	(4)		
All workers				100.0		
Self-employed, excluding contractors and gig workers						
Contractors and gig workers						
All employees						
In firms with 1 to 9 employees						
In firms with 10 to 49 employees						
In firms with 50 to 99 employees						
In firms with 100 to 499 employees						
In firms with 500 to 4,999 employees						
In firms with 5,000 or more employees						

This Supplement will NOT be collected or graded: write your answers on the test papers. Supplement: Page 1 of 5

Supplement for Question (1): Consider the 2023 salaries of employees of the University of Toronto, the University of Waterloo, and York University in the Ontario public sector salary disclosure data. See the joint probability table below. Seven events are shown in boldface. *Note:* You must use events as defined in Table 1 in your answers.

Table 1. Three Large Ontario Universities and Distribution of Disclosed 2023 Salaries							
	University of Toronto	University of Waterloo	York University	Total			
		vv	ř				
\$100K ≤ salary < \$125K S1	0.1774	0.0645	0.0637	0.3056			
\$125K ≤ salary < \$175K S2	0.1898	0.0661	0.0886	0.3446			
\$175K ≤ salary < \$225K S3	0.1068	0.0521	0.0496	0.2085			
\$225K ≤ salary S4	0.0976	0.0229	0.0209	0.1413			
Total	0.5716	0.2056	0.2228	1.0000			

Supplement for Question (2): The class answered survey questions in Workshop 6 via an anonymous MS Form. The responses to the happiness (Cantril ladder) question and the competitiveness question are summarized below.

. summarize happ	comp;				
Variable	Obs	Mean	Std. Dev.	Min	Max
happ comp	291 291	6.395189 6.790378	1.835994 2.159353	0 0	10 10
. correlate happ (obs=291)	comp;				
	happ 	comp			
happ comp	1.0000 0.2071	1.0000			

A new variable named "ave_hap_com" is the mean of happiness and competitiveness for each student. For example, if a student answered 8 for happiness and 5 for competitiveness then "ave_hap_com" has a value of 6.5 for this student. Use these abbreviated variable names: X_H for "happ", X_C for "comp", and X_A for "ave_hap_com."

Supplement for Question (3): N/A (all information given with the question)

Supplement for Question (4): In the Ontario public sector salary data, the 2023 salaries of employees in the universities sector making at least \$100,000 are disclosed. The mean salary is \$160,906 with a standard deviation of \$52,843.

Consider using the on_sal_2023.xlsx dataset (filtered to include only the universities sector) to do a simulation.

The figure to the right summarizes the results of the simulation. Notice that the figure title has two blank lines and recall that m denotes the number of simulation draws.



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Supplement for Question (5): Consider a 2024 NBER working paper "Education and Preferences for Desired Traits in Children." The researchers use data from the Integrated Values Survey (IVS) from surveys run from 1981 through 2022 across 118 countries. They use a subset of those data for 19 European countries that implemented an educational reform to raise the mandatory years of schooling. Hence, some respondents were exposed to the extra years of required schooling when they were still school aged, and others were not.

Recall this IVS question: "Here is a list of qualities that children can be encouraged to learn at home. Which, if any, do you consider to be especially important? Please choose up to five!" The 11 options are: Good manners; Independence; Hard work; Feeling of responsibility; Imagination; Tolerance and respect for other people; Thrift, saving money and things; Determination, perseverance; Religious faith; Not being selfish (unselfishness); Obedience. The authors state: "If the respondent chose a particular child attribute as important, it was coded as one, and zero otherwise."

Below is Table 2 from the paper. The next two sentences are the researchers' interpretation of the first row of results in Table 2. "The overall average education is about 12 years. Those in the treatment group (who were exposed to the education reforms) have on average 0.6 more years of schooling compared to those who are in the control group (who were not exposed to the education reforms)."

Table 2. Descriptive statistics							
	All	All Treatment					
	(1)	(2)	(3)				
Respondent attributes:							
Veers of schooling	12.045	12.313	11.758				
rears of schooling	(4.187)	(4.083)	(4.278)				
A.g.o	44.298	42.742	45.958				
Age	(10.893)	(10.719)	(10.833)				
Malo	0.497	0.495	0.500				
	(0.500)	(0.500)	(0.500)				
Respondent preferences on child attributes:							
Independence	0.520	0.538	0.501				
Independence	(0.500)	(0.499)	(0.500)				
Imagination	0.246	0.262	0.228				
inagination	(0.431)	(0.440)	(0.420)				
Determination persoverance	0.349	0.361	0.335				
e pondent preferences on child attributes: ependence gination ermination, perseverance ing of responsibility erance and respect for others	(0.477)	(0.480)	(0.472)				
Feeling of responsibility	0.749	0.749	0.748				
espondent preferences on child attributes: dependence nagination etermination, perseverance eeling of responsibility olerance and respect for others	(0.434)	(0.433)	(0.434)				
Tolerance and respect for others	0.763	0.766	0.759				
Tolerance and respect for others	(0.425)	(0.424)	(0.428)				
Obedience	0.284	0.276	0.293				
Obedience	(0.451)	(0.447)	(0.455)				
Hardwork	0.377	0.379	0.376				
	(0.485)	(0.485)	(0.484)				
Religious faith	0.184	0.175	0.195				
	(0.388)	(0.380)	(0.396)				
Unselfishness	0.296	0.302	0.290				
0136113111633	(0.457)	(0.459)	(0.454)				
N	25,160	12,921	12,239				

Note: Standard deviations are in parentheses.

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Supplement for Question (6): N/A (all information given with the question)

Supplement for Question (7): From Test #1, recall the 2023 paper "The Evolution of Work from Home" published in the *Journal of Economic Perspectives* and Table 1. For a randomly selected survey respondent, define events as:

- Event O: The respondent works fully onsite
- Event H: The respondent has a hybrid arrangement
- Event R: The respondent works fully remote
- Event S: The respondent is self-employed (but is not a contractor or a gig worker)
- Event C: The respondent is contractor
- Event G: The respondent is gig worker
- Event E: The respondent is an employee
- Event F1: The respondent is an employee in a firm with 1 to 9 employees
- Event F2: The respondent is an employee in a firm with 10 to 49 employees
- Event F3: The respondent is an employee in a firm with 50 to 99 employees
- Event F4: The respondent is an employee in a firm with 100 to 499 employees
- Event F5: The respondent is an employee in a firm with 500 to 4,999 employees
- Event F6: The respondent is an employee in a firm with 5,000 or more employees

Table 1. Full-Time Working Arrangements in the United States as of 2023, Percentage Distributions

	Fully	Hybrid	Fully	Percent of
	onsite	arrangement	remote	all workers
	(1)	(2)	(3)	(4)
All workers	55.9	28.6	15.5	100.0
Self-employed, excluding contractors and gig workers	24.9	26.8	48.3	7.4
Contractors and gig workers	32.9	22.7	44.4	3.3
All employees	59.3	29.0	11.8	89.3
In firms with 1 to 9 employees	67.5	17.6	14.9	7.1
In firms with 10 to 49 employees	68.3	24.1	7.6	14.2
In firms with 50 to 99 employees	57.2	34.1	8.7	13.3
In firms with 100 to 499 employees	56.5	32.4	11.2	19.7
In firms with 500 to 4,999 employees	50.7	37.7	11.6	19.6
In firms with 5,000 or more employees	63.5	18.8	17.8	15.3

Note: This table considers full-time American workers who are 20–64 years old as of the survey, where "full-time" means working for pay five or more days in the survey reference week. "Fully Onsite" refers to those who worked at their employer's worksite (or a client's location) each workday in the reference week. "Fully Remote" refers to those who worked from home on all workdays in the reference week. "Hybrid Arrangement" refers to those who split the workweek between home and their employer's worksite (or client locations). Column (4) reports the sample percentage of persons in the indicated row.

Sample mean: $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ Sample variance: $s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{X})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{(\sum_{i=1}^{n} x_i)^2}{n(n-1)}$ Sample s.d.: $s = \sqrt{s^2}$ Sample coefficient of variation: $CV = \frac{s}{\bar{x}}$ Sample covariance: $s_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{X})(y_i - \bar{Y})}{n-1} = \frac{\sum_{i=1}^{n} x_i y_i}{n-1} - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(n-1)}$ Sample interquartile range: IQR = Q3 - Q1 Sample coefficient of correlation: $r = \frac{s_{xy}}{s_x s_y} = \frac{\sum_{i=1}^{n} z_{x_i} z_{y_i}}{n-1}$ Addition rule: P(A or B) = P(A) + P(B) - P(A and B) Conditional probability: $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Complement rules: $P(A^C) = P(A') = 1 - P(A)$ $P(A^C|B) = P(A'|B) = 1 - P(A|B)$ **Multiplication rule:** P(A and B) = P(A|B)P(B) = P(B|A)P(A)

Expected value: $E[X] = \mu = \sum_{all \ x} xp(x)$ Variance: $V[X] = E[(X - \mu)^2] = \sigma^2 = \sum_{all \ x} (x - \mu)^2 p(x)$ Covariance: $COV[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] = \sigma_{XY} = \sum_{all \ x} \sum_{all \ y} (x - \mu_X)(y - \mu_Y)p(x, y)$

Laws of expected value:	Laws of variance:	Laws of covariance:
E[c] = c	V[c] = 0	COV[X, c] = 0
E[X+c] = E[X] + c	V[X+c] = V[X]	COV[a + bX, c + dY] = bd * COV[X, Y]
E[cX] = cE[X]	$V[cX] = c^2 V[X]$	
E[a + bX + cY] = a + bE[X] + cE[Y]	$V[a + bX + cY] = b^2 V[$	$[X] + c^2 V[Y] + 2bc * COV[X, Y]$
	$V[a + bX + cY] = b^2 V[$	$[X] + c^2 V[Y] + 2bc * SD(X) * SD(Y) * \rho$
	where $\rho = CORRELATION$	N[X,Y]

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ Binomial probability: $p(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$ for x = 0,1,2,...,nIf X is Binomial $(X \sim B(n,p))$ then E[X] = np and V[X] = np(1-p)

If X is Uniform $(X \sim U[a, b])$ then $f(x) = \frac{1}{b-a}$ and $E[X] = \frac{a+b}{2}$ and $V[X] = \frac{(b-a)^2}{12}$

Sampling distribution of \overline{X} :	Sampling distribution of \widehat{P} :	Sampling distribution of $(\widehat{P}_2 - \widehat{P}_1)$:
$\mu_{\bar{X}} = E[\bar{X}] = \mu$	$\mu_{\hat{P}} = E[\hat{P}] = p$	$\mu_{\hat{P}_2 - \hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$
$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$	$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$	$\sigma_{\hat{P}_2 - \hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1 - p_2)}{n_2} + \frac{p_1(1 - p_1)}{n_1}$
$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$	$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$	$\sigma_{\hat{P}_2 - \hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1 - p_2)}{n_2} + \frac{p_1(1 - p_1)}{n_1}}$

Inference about a population proportion:

Cl estimator: $\hat{P} \pm z_{\alpha/2} \sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

Inference about comparing two population proportions:

Cl estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2} \sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

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	Second decimal place in z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999

The Standard Normal Distribution: