ECO220Y1Y, Test #2, Prof. Murdock SOLUTIONS

(1) (a) $P(S3 \,\&\, W) = 0.0521$ [Note the order of events S3 and W does not matter for a joint probability.]

(b) $P(S2) = 0.3446$

(c) $P(S4 \mid T) = \frac{P(S4 \,\&\, T)}{P(T)} = \frac{0.0976}{0.5716} = 0.171$

(d) $P(W \mid S1) + P(Y \mid S1) = \frac{P(W \,\&\, S1)}{P(S1)} + \frac{P(Y \,\&\, S1)}{P(S1)} = \frac{0.0645}{0.3056} + \frac{0.0637}{0.3056} = 0.420$ [Also, $1 - P(T \mid S1) = 0.420$]

(e) Events S4 and Y are independent if and only if P(S4 | Y) = P(S4) [or you could equivalently check that P(Y | S4) = P(Y)]. P(S4 | Y) = 0.0209/0.2228 = 0.094 and P(S4) = 0.1413. These are unequal so Events S4 and Y are *not* independent.

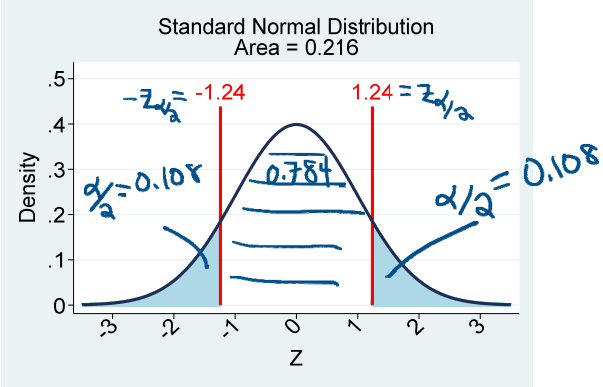
(2) It is a linear combination of the original variables: $X_A = \frac{X_H + X_C}{2}$. The Laws of Expectation for linear combinations work for sample statistics so $\bar{X}_A = \frac{\bar{X}_H + \bar{X}_C}{2} = \frac{6.395189 + 6.790378}{2} = 6.59$.

$$V[X_A] = V\left[\frac{X_H + X_C}{2}\right] = \frac{1}{4}V[X_H] + \frac{1}{4}V[X_C] + 2 * \frac{1}{2} * \frac{1}{2} * SD[X_H] * SD[X_C] * CORR[X_H, X_C]$$
$$= 0.25(1.835994)^2 + 0.25(2.159353)^2 + 0.5 * 1.835994 * 2.159353 * 0.2071 = 2.419$$
$$SD[X_A] = \sqrt{2.419} = 1.555$$

(3) $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 0.784$

$P(0 < Z < z_{\alpha/2}) = 0.392$

We get a reasonable approximate value using the Standard Normal table: $P(0 < Z < 1.24) \cong 0.392$ so $z_{\alpha/2} \cong 1.24$



(4) (a) n = 250 and m = 10,000. The sampling distribution of the sample mean is *approximately* Normal, and using the Empirical Rule the only case where $SD[\bar{X}] = \frac{\sigma}{\sqrt{n}} = \frac{52.843}{\sqrt{n}}$ can possible match the figure is for $SD[\bar{X}] = \frac{52.843}{\sqrt{250}} = 3.34$. For the number of simulation draws, we add the heights of the bars. Of the choices, only 10,000 is possible.

(b) stay about the same / stay about the same, / decrease / symmetric

(5) (a) 44.298 = $\bar{X}$; 10.893 = s; 0.246 = $\hat{P}$; 0.431 = s.

(b) Given that the average years of schooling is about 12 years, a standard deviation of 4.2 is huge! It means there is a lot of variation across respondents: some are very poorly educated while others are very well educated.

(c) $\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

$0.246 \pm 1.960\sqrt{\frac{0.246(1-0.246)}{25{,}160}}$

$0.246 \pm 1.960 * 0.0027$

0.246 ± 0.005

From 1981 through 2022 and across 19 European countries, we are 95% confident that between 24.1 percent and 25.1 percent believe imagination is an especially important quality for children to be encouraged to learn at home. We can make a very precise inference – very narrow width – because of the very large sample size (over 25,000 respondents).

(d) From 1981 through 2022 and across 19 European countries, 19.5 percent of those in the control group and 17.5 percent of those in the treatment group (who were exposed to education reforms) think that religious faith is an especially important quality for children to be encouraged to learn at home. It is 2 percentage points higher among those not exposed to the education reforms.

(e) Use the part after the plus/minus in: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

$ME = 1.645\sqrt{\frac{0.538(1-0.538)}{12{,}921} + \frac{0.501(1-0.501)}{12{,}239}}$

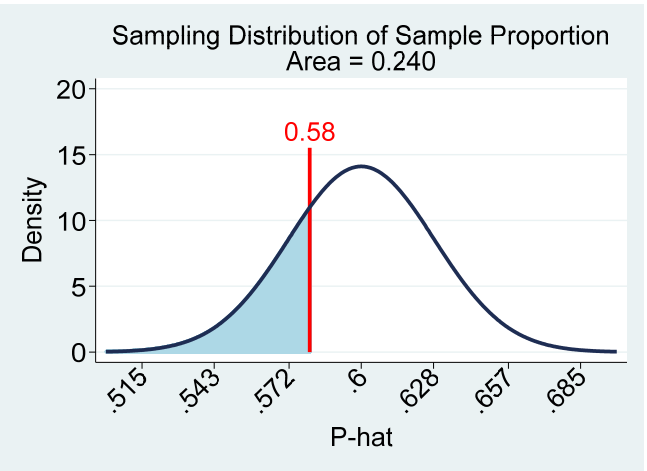
$= 1.645 * 0.0063 = 0.010$ Hence, the margin of error is <u>1.0 percentage points</u>.

(6) $E[\hat{P}] = p = 0.60$ and $SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6(1-0.6)}{300}} = 0.02828$

We expect at least 10 successes and at least 10 failures so we can use the Normal approximation, so $\hat{P} \sim N(0.6, 0.0008)$.

$P(\hat{P} < 0.58 \mid n = 300, p = 0.60) = P\left(Z < \frac{0.58-0.6}{0.02828}\right) =$
$P(Z < -0.7071) \cong P(Z < -0.71) = 0.5 - 0.2611 = 0.24$

If Doug Ford's claim were true, there is a 24% chance we could see a survey result as low as 58% – two full percentage points below his claim of 60% support – just because of sampling error. 24% is a high chance so sampling error IS a plausible explanation for "such a low" level of support in the survey.



(7) (a) TRUE; FALSE; FALSE; TRUE

(b) P(R | F6) = 0.178

(c)

Table 1. Full-Time Working Arrangements in the United States as of 2023, Percentage Distributions

	Fully onsite (1)	Hybrid arrangement (2)	Fully remote (3)	Percent of all workers (4)
All workers	55.9	28.6	15.5	100.0
Self-employed, excluding contractors and gig workers	55.9	28.6	15.5	7.4
Contractors and gig workers	55.9	28.6	15.5	3.3
All employees	55.9	28.6	15.5	89.3
In firms with 1 to 9 employees	55.9	28.6	15.5	7.1
In firms with 10 to 49 employees	55.9	28.6	15.5	14.2
In firms with 50 to 99 employees	55.9	28.6	15.5	13.3
In firms with 100 to 499 employees	55.9	28.6	15.5	19.7
In firms with 500 to 4,999 employees	55.9	28.6	15.5	19.6
In firms with 5,000 or more employees	55.9	28.6	15.5	15.3