ECO220Y1Y, Test \#2, Prof. Murdock

June 19, 2023, 11:10am - 1:00pm

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## Instructions:

- You have 110 minutes. Keep these test papers and the Supplement closed and face up on your desk until the start of the test is announced. You must stay for a minimum of 60 minutes.
- You may use a non-programmable calculator.
- There are 7 questions (most with multiple parts) with varying point values worth a total of 95 points.
- This test includes these 8 pages plus the Supplement. The Supplement contains the aid sheets and readings, figures, tables, and other materials for some test questions. For each question referencing the Supplement, carefully review all materials. The Supplement will NOT be collected: write your answers on these test papers. When we announce the end of the test, hand these test papers to us (you keep the Supplement).
- Write your answers clearly, completely, and concisely in the designated space provided immediately after each question. An answer guide ends each question to let you know what is expected. For example, a quantitative analysis, a fully labelled graph, and/or sentences. Any answer guide asking for a quantitative analysis always automatically means that you must show your work and make your reasoning clear.
- Anything requested by the question and/or the answer guide is required. Similarly, limit yourself to the answer guide. For example, if the answer guide does not request sentences, provide only what is requested (e.g. quantitative analysis).
- Marking TAs are instructed to accept all reasonable rounding.
- Your entire answer must fit in the designated space provided immediately after each question. No extra space/pages are possible. You cannot use blank space for other questions, nor can you write answers on the Supplement. Write in PENCIL and use an ERASER as needed so that you can fit your final answer (including work and reasoning) in the appropriate space. We give more blank space than is needed for each answer (with typical handwriting) worth full marks. Follow the answer guides and avoid excessively long answers.
(1) See Supplement for Question (1): Canada labour force status by educational achievement.
(a) [6 pts] A member of which group is more likely to be unemployed: people with at most a high school degree or people with no degree? Interpret. Answer with a quantitative analysis using formal notation \& 1 sentence.
(b) [3 pts] Suppose a person's labour force status were unrelated with their educational achievement but the fraction of people with each labour force status stayed the same and the fraction of people with each education level stayed the same. What number would replace 0.0036 ? Answer with a quantitative analysis using formal notation.
(2) See Supplement for Question (2): Self-assessed willingness to take risks.
(a) [3 pts] Among all ECO220Y students over the last decade, what percent have a self-assessed willingness to take risks above 6.4? Answer with a quantitative analysis.
(b) [3 pts] Among all ECO220Y students over the last decade, what is the value of the $10^{\text {th }}$ percentile of self-assessed willingness to take risks? Answer with a quantitative analysis.
(c) [7 pts] For LECO301 in Summer 2023, is sampling error a plausible explanation for the sample mean being as high as it is? Answer with a quantitative analysis \& 1 sentence.
(3) [6 pts] See Supplement for Question (3): Creating a new variable. What is the standard deviation of the new variable? Answer with a quantitative analysis.
(4) See Supplement for Question (4): Educational attainment, age shares, and employment rates of Canadian women.
(a) [6 pts] For a randomly selected employed woman, what is the chance that she is 55 years or older? Answer with a quantitative analysis using formal notation.
(b) [6 pts] For a random sample of 120 Canadian women, what is the chance that two or more women in the sample are 25 to 54 years old and have eight or fewer years of education? Answer with a quantitative analysis.
(c) [12 pts] For a random sample of 512 Canadian women with above a bachelor's degree, is sampling error a plausible explanation if more than 80 percent are employed? Illustrate the answer with a fully labelled graph. Answer with a quantitative analysis, a fully labelled graph \& 1 sentence.
(5) See Supplement for Question (5): Fighting Climate Change: International Attitudes Toward Climate Policies.
(a) [8 pts] In Table 7, for the category "limit driving" and for high-income countries, what is the 99\% confidence interval? Next, and importantly, interpret the Cl estimate. Answer with a quantitative analysis \& 2 sentences.
(b) [4 pts] Among the twelve high-income countries in Table 7, only two (i.e. 17\%) are within the $99 \% \mathrm{Cl}$ estimate from the previous part. Why so few? Answer with 2 sentences.
(6) See Supplement for Question (6): Does Price Matter in Charitable Giving? Karlan and List (2007)
(a) [8 pts] The $90 \%$ confidence interval estimate of the difference in the response rate between the low example amount group versus the control group is $L C L=0.000530058$ and $U C L=0.006146188$ (software gives these accurate to the ninth decimal place). Interpret the Cl estimate, addressing causality. Answer with $2-3$ sentences.
(b) [5 pts] Comparing those seeing a high example amount versus a low example amount, what is the $85 \%$ confidence interval estimate of the difference in the fraction choosing to donate? Answer with a quantitative analysis.
(c) [6 pts] Comparing response rate estimates across the eight columns, what is the key message of Table 2B? Answer with 2-3 sentences.
(7) See Supplement for Question (7): 2023 Ontario public disclosure of salaries.
(a) [6 pts] Consider both Summary \#1 and Summary \#2. Answer by filling in the blanks.

In formal notation, 124.9381 is $\qquad$ , and 35.81888 is $\qquad$ , and 119.9348 is $\qquad$ and 31.2389 is $\qquad$ .

Compared to the random sample with $n=30$ in the Supplement, if we drew another random sample with $n=30$, we expect the sample mean to $\qquad$ [be higher / be lower / stay the same], the sample standard deviation to $\qquad$ [be higher / be lower / stay the same], and the sample $75^{\text {th }}$ percentile to $\qquad$ [be higher / be lower / stay the same].
(b) [6 pts] Consider Summary \#3. Answer by filling in the blanks.

For $n=30$, the probability of getting a sample mean below $\$ 107,000$ is about $\qquad$ [\#] and the probability of getting a sample mean above $\$ 140,000$ is somewhere between $\qquad$ [\#] and $\qquad$ [\#].

Next, if $n=10$ instead of $n=30$, then we expect the value in the spot 117.303 to $\qquad$ [increase / decrease / barely change], the value in the spot 187.3988 to $\qquad$ [increase / decrease / barely change], the value in the spot 42.74376 to $\qquad$ [increase / decrease / barely change], and the value in the spot 124.9463 to $\qquad$ [increase / decrease / barely change].

This Supplement will NOT be collected or graded: write your answers on the test papers. Supplement: Page $\mathbf{1}$ of 6
Supplement for Question (1): Consider the joint probability table below, retrieved from the Statistics Canada website.

| Canada, 2022, Both Sexes, Aged 25 to 54 Years |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Labour Force Status |  |  |  |
| Highest level of educational achievement | Employed | Unemployed | Not in labor force | Total |
| No degree, certificate, or diploma | 0.0371 | 0.0036 | 0.0172 | 0.0579 |
| High school graduate | 0.1204 | 0.0075 | 0.0265 | 0.1544 |
| High school graduate, some post-secondary | 0.0272 | 0.0017 | 0.0053 | 0.0341 |
| Postsecondary certificate or diploma | 0.3099 | 0.0124 | 0.0326 | 0.3549 |
| University degree | 0.3527 | 0.0132 | 0.0327 | 0.3986 |
| Total | 0.8473 | 0.0384 | 0.1143 | 1 |

Supplement for Question (2): Recall the survey question: "How do you see yourself: Are you a person who is generally willing to take risks, or do you try to avoid taking risks? Please choose a value on the scale below, where the value 0 means 'completely unwilling' and the value 10 means 'completely willing'."

Suppose that all ECO220Y students over the last decade answered this question but with a slider from 0 to 10 as illustrated below, which means they were not limited to integers. Suppose that the responses for this large population are Normally distributed with a mean of 5.2 and a standard deviation of 1.6.


On May 24, 2023, a sample of ECO220Y students in LEC0301 in Summer 2023 answered with a 0 to 10 integer scale: see the tabulation below.

| risk \| | Freq. | Percent | Cum. |
| :---: | :---: | :---: | :---: |
| 0 \| | 7 | 11.67 | 11.67 |
| 2 \| | 2 | 3.33 | 15.00 |
| 31 | 5 | 8.33 | 23.33 |
| 4 \| | 4 | 6.67 | 30.00 |
| 5 \| | 8 | 13.33 | 43.33 |
| 6 \| | 10 | 16.67 | 60.00 |
| 7 \| | 14 | 23.33 | 83.33 |
| 8 । | 6 | 10.00 | 93.33 |
| 91 | 2 | 3.33 | 96.67 |
| 10 \| | 2 | 3.33 | 100.00 |
| Total \| | 60 | 100.00 |  |

Supplement for Question (3): For 169 students in ECO220Y in Summer 2023 who answered both the risk question and the competitiveness question, the standard deviation of the risk question (on a zero to 10 scale) is 2.1073 . The standard deviation of the competitiveness question (on a zero to 10 scale) is 2.2093 . The correlation between these two variables is 0.5137 . A new variable averages the reply to the risk question and the reply to the competitiveness question for each student. (For example, for a student who said 8 and 7 , value of the new variable is 7.5.)

Supplement for Question (4): The most recent release (January 2023) from Stats Canada describes the population shares and the employment rates in 2022 by educational attainment and age for women.

Table 14-10-0020-01. Educational attainment and age shares of women aged 15 years and up, Canada, 2022 and Employment rate by educational attainment and age

| Highest educational attainment (share) | Age (share) |  |  |  | Employment rate |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Total, 15 yrs. and up | $\begin{gathered} 15 \text { to } 24 \\ \text { yrs. } \end{gathered}$ | $\begin{gathered} 25 \text { to } 54 \\ \text { yrs. } \end{gathered}$ | 55 yrs. and up | Total, 15 yrs. and up | $\begin{gathered} 15 \text { to } 24 \\ \text { yrs. } \end{gathered}$ | $\begin{gathered} 25 \text { to } 54 \\ \text { yrs. } \end{gathered}$ | 55 yrs. and up |
|  | 1.000 | 0.135 | 0.470 | 0.395 | 0.583 | 0.602 | 0.814 | 0.303 |
| Total, all education levels |  |  |  |  |  |  |  |  |
| 0 to 8 years | 0.038 | 0.004 | 0.005 | 0.030 | 0.141 | 0.301 | 0.390 | 0.079 |
| Some high school | 0.093 | 0.038 | 0.015 | 0.040 | 0.319 | 0.401 | 0.545 | 0.159 |
| High school graduate | 0.187 | 0.032 | 0.062 | 0.093 | 0.481 | 0.649 | 0.704 | 0.274 |
| Some postsecondary | 0.052 | 0.024 | 0.015 | 0.012 | 0.578 | 0.628 | 0.733 | 0.295 |
| Postsecondary certificate or diploma | 0.323 | 0.022 | 0.165 | 0.135 | 0.631 | 0.788 | 0.840 | 0.351 |
| Bachelor's degree | 0.212 | 0.013 | 0.140 | 0.059 | 0.717 | 0.759 | 0.852 | 0.386 |
| Above bachelor's degree | 0.096 | 0.002 | 0.069 | 0.026 | 0.760 | 0.723 | 0.877 | 0.446 |

Supplement for Question (5): Consider an NBER Working Paper (2022) "Fighting Climate Change: International Attitudes Toward Climate Policies." It uses an international survey run between March 2021 and March 2022.

Table 7. Share of people willing to adopt climate-friendly behaviors

|  |  |  |  | $\begin{array}{\|l\|} \frac{\pi}{0} \\ \mathbf{0} \\ \tilde{0} \\ \hline \end{array}$ |  |  |  |  | $$ | $\begin{aligned} & \frac{1}{n} \\ & \stackrel{0}{2} \\ & \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline \mathbf{C} \\ \frac{\pi}{0} \\ \hline 0 \\ \hline \end{array}$ |  |  |  |  | « |  |  | - |  |  |  |  | $\begin{aligned} & \frac{0}{x} \\ & \dot{x} \\ & \end{aligned}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Have a fuel-efficient or electric vehicle | 54 |  | 45 | 52 | 60 | 45 | 4 |  | 8 | 48 | 53 | 57 | 60 | 0 | 51 | 50 |  | 69 | 78 | 65 | 74 | 46 |  | 0 | 60 | 73 | 62 |
| Limit flying | 51 |  | 375 | 53 | 49 | 56 | 6 | 46 | 64 | 37 | 58 | 43 | 62 | 52 | 6 | 39 |  | 55 | 52 | 59 | 66 | 5 |  | 59 | 48 | 44 | 449 |
| Limit beef/meat consumption | 40 |  | 31 | 38 | 33 | 38 | 45 |  | 62 | 24 | 49 | 36 | 4 | 4 | 4 | 36 |  | 44 | 44 | 48 | 62 | 4 |  | 40 | 33 | 35 | 35 |
| Limit driving | 37 |  | 26 | 35 | 33 | 32 | 4 | 15 | 57 | 37 | 41 | 36 | 4 | 7 | 37 | 29 |  | 49 | 41 | 62 | 66 | 5 |  | 47 | 38 | 46 | 625 |
| Limit heating or cooling your home | 34 |  | 25 | 27 | 33 | 39 | 36 | 65 | 55 | 26 | 37 | 29 |  | 6 | 30 | 28 |  | 48 | 46 | 56 | 68 |  |  | 59 | 39 | 34 | 4 |
| Sample size (number of people responding to the survey) | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{n} \end{aligned}$ |  | $\stackrel{\infty}{\underset{\sim}{n}} \underset{\sim}{2}$ | $\begin{gathered} \underset{N}{N} \\ \underset{\sim}{n} \end{gathered}$ | $\underset{\sim}{n} \underset{\sim}{n}$ | $\begin{aligned} & \mathrm{O} \\ & \hline \mathrm{o} \\ & i \end{aligned}$ | i |  | ~ | $\begin{aligned} & \mathrm{O} \\ & \underset{i}{9} \\ & \hline \end{aligned}$ | $\sim$ | $\begin{aligned} & \underset{\sim}{\underset{\sim}{2}} \end{aligned}$ |  | N | j | $\sim$ |  | $\begin{aligned} & -\infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | - | $\cdots$ | N |  |  | $\begin{gathered} n \\ \mathbf{N} \\ \mathbf{N} \end{gathered}$ | $\underset{\sim}{\sim}$ |  | - |

Notes: For "To what extent would you be willing to adopt the following behaviors" respondents answer on a 5-point scale: "Not at all," "A little," "Moderately," "A lot," and "A great deal." The table reports the percent answering either "A lot" or "A great deal" for each climate-friendly behavior listed above.

Supplement for Question (6): Recall Karlan and List (2007) "Does Price Matter in Charitable Giving? Evidence from a Large-Scale Natural Field Experiment."

- For letters asking for a donation mailed to 50,083 people, they randomly vary some aspects of the letter.
- The control group - a random subset of the people - receive an ordinary letter that does not mention a match.
- For the treatment group (everyone not in the control group), the letter includes a match.
- The "match threshold" is the stated total amount of money available to match received donations: it takes values of $\$ 25,000, \$ 50,000, \$ 100,000$ or is left unstated (implying that there may be no limit to the amount available to match donations).
- The "example amount" - a specific dollar amount used to explain how the match works - is either the person's previous highest donation amount ("low"), $25 \%$ more than that ("medium"), or $50 \%$ more than that ("high").
- Various subsets of the treatment group receive these different letters at random.

Below is an excerpt from Table 2B, except that the estimates are reported accurate to the fifth decimal place. (This is poor table design and is not what the author's did: they rounded to the nearest third decimal place. However, this gives you more accurate numbers to work with.)

Table 2B - Mean Responses
(Mean and standard errors)

| Implied price of $\$ 1$ of public good: | Control | Match |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Threshold |  |  |  | Example amount |  |  |
|  |  | \$25,000 | \$50,000 | \$100,000 | Unstated | Low | Medium | High |
|  |  | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Response rate | $\begin{gathered} 0.01786 \\ (0.00103) \end{gathered}$ | $\begin{gathered} 0.02156 \\ (0.00159) \end{gathered}$ | $\begin{gathered} 0.02241 \\ (0.00162) \end{gathered}$ | $\begin{gathered} 0.02204 \\ (0.00161) \end{gathered}$ | $\begin{gathered} 0.02215 \\ (0.00161) \end{gathered}$ | $\begin{gathered} 0.02120 \\ (0.00137) \end{gathered}$ | $\begin{gathered} 0.02219 \\ (0.00140) \end{gathered}$ | $\begin{gathered} 0.02273 \\ (0.00141) \end{gathered}$ |
| Dollars given, unconditional | $\begin{gathered} 0.81327 \\ (0.06330) \end{gathered}$ | $\begin{gathered} 1.06000 \\ (0.10882) \end{gathered}$ | $\begin{gathered} 0.88904 \\ (0.09128) \end{gathered}$ | $\begin{gathered} 0.90347 \\ (0.08352) \end{gathered}$ | $\begin{gathered} 1.01493 \\ (0.10648) \end{gathered}$ | $\begin{gathered} 0.91371 \\ (0.07993) \end{gathered}$ | $\begin{gathered} 1.00368 \\ (0.09057) \end{gathered}$ | $\begin{gathered} 0.98324 \\ (0.08402) \end{gathered}$ |
| Dollars given, conditional on giving | $\begin{aligned} & 45.54027 \\ & (2.39707) \end{aligned}$ | $\begin{aligned} & 49.17222 \\ & (3.52238) \end{aligned}$ | $\begin{aligned} & 39.67380 \\ & (2.89969) \end{aligned}$ | $\begin{aligned} & 41.00000 \\ & (2.33647) \end{aligned}$ | $\begin{aligned} & 45.81460 \\ & (3.47450) \end{aligned}$ | $\begin{aligned} & 43.10678 \\ & (2.55742) \end{aligned}$ | $\begin{aligned} & 45.23887 \\ & (2.93179) \end{aligned}$ | $\begin{aligned} & 43.25099 \\ & (2.54156) \end{aligned}$ |
| Observations | 16,687 | 8,350 | 8,345 | 8,350 | 8,351 | 11,134 | 11,133 | 11,129 |

Supplement for Question (7): Recall the 2023 Ontario public disclosure data with all public sector employees earning $\$ 100,000$ CAN or more in the 2022 calendar year. The variable named salary is in $\$ 1,000$ s of Canadian dollars. Below are three different summaries produced by Stata.

Summary \#1: This summarizes all employees in the 2023 disclosure.

|  | Percentiles | Smallest |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1\% | 100.2922 | 100 |  |  |
| 5\% | 101.3975 | 100 |  |  |
| 10\% | 102.0332 | 100 | Obs | 266,903 |
| 25\% | 103.5281 | 100 | Sum of Wgt. | 266,903 |
| 50\% | 112.4374 |  | Mean | 124.9381 |
|  |  | Largest | Std. Dev. | 35.81888 |
| 75\% | 131.799 | 856.5519 |  |  |
| 90\% | 162.5692 | 968.7103 | Variance | 1282.992 |
| 95\% | 190.2173 | 1690 | Skewness | 4.493073 |
| 99\% | 266.9359 | 1730 | Kurtosis | 59.31905 |

Summary \#2: This summarizes a random sample of 30 employees.

|  | Percentiles | Smallest |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1\% | 100.6484 | 100.6484 |  |  |
| 5\% | 101.4398 | 101.4398 |  |  |
| 10\% | 101.523 | 101.497 | Obs | 30 |
| 25\% | 101.8269 | 101.549 | Sum of Wgt. | 30 |
| 50\% | 103.3656 |  | Mean | 119.9348 |
|  |  | Largest | Std. Dev. | 31.2389 |
| 75\% | 119.2247 | 163.9943 |  |  |
| 90\% | 164.3571 | 164.7199 | Variance | 975.8688 |
| 95\% | 189.4578 | 189.4578 | Skewness | 2.202837 |
| 99\% | 233.6431 | 233.6431 | Kurtosis | 7.516149 |

Summary \#3: This summarizes the results of a simulation to obtain the simulated sampling distribution of the sample mean for a sample size of $30(n=30)$ using 200,000 simulation draws ( $m=200,000$ ).

|  | Percentiles | Smallest |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1\% | 112.9676 | 106.1555 |  |  |
| 5\% | 115.6573 | 106.4117 |  |  |
| 10\% | 117.303 | 106.4634 | Obs | 200,000 |
| 25\% | 120.3363 | 107.2116 | Sum of Wgt. | 200,000 |
| 50\% | 124.2093 |  | Mean | 124.9463 |
|  |  | Largest | Std. Dev. | 6.537871 |
| 75\% | 128.7583 | 187.3988 |  |  |
| 90\% | 133.4708 | 188.9387 | Variance | 42.74376 |
| 95\% | 136.6278 | 190.3812 | Skewness | . 83021 |
| 99\% | 143.5549 | 205.1489 | Kurtosis | 5.001855 |

Sample mean: $\bar{X}=\frac{\sum_{i=1}^{n} x_{i}}{n} \quad$ Sample variance: $s^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{X}\right)^{2}}{n-1}=\frac{\sum_{i=1}^{n} x_{i}^{2}}{n-1}-\frac{\left(\sum_{i=1}^{n} x_{i}\right)^{2}}{n(n-1)} \quad$ Sample s.d.: $s=\sqrt{s^{2}}$
Sample coefficient of variation: $C V=\frac{s}{\bar{X}} \quad$ Sample covariance: $s_{x y}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{X}\right)\left(y_{i}-\bar{Y}\right)}{n-1}=\frac{\sum_{i=1}^{n} x_{i} y_{i}}{n-1}-\frac{\left(\sum_{i=1}^{n} x_{i}\right)\left(\sum_{i=1}^{n} y_{i}\right)}{n(n-1)}$
Sample interquartile range: $I Q R=Q 3-Q 1 \quad$ Sample coefficient of correlation: $r=\frac{s_{x y}}{s_{x} s_{y}}=\frac{\sum_{i=1}^{n} z_{x_{i}} z_{y_{i}}}{n-1}$
Addition rule: $P(A$ or $B)=P(A)+P(B)-P(A$ and $B) \quad$ Conditional probability: $P(A \mid B)=\frac{P(A \text { and } B)}{P(B)}$
Complement rules: $P\left(A^{C}\right)=P\left(A^{\prime}\right)=1-P(A) \quad P\left(A^{C} \mid B\right)=P\left(A^{\prime} \mid B\right)=1-P(A \mid B)$
Multiplication rule: $P(A$ and $B)=P(A \mid B) P(B)=P(B \mid A) P(A)$

Expected value: $E[X]=\mu=\sum_{\text {all } x} x p(x) \quad$ Variance: $V[X]=E\left[(X-\mu)^{2}\right]=\sigma^{2}=\sum_{\text {all } x}(x-\mu)^{2} p(x)$
Covariance: $\operatorname{COV}[X, Y]=E\left[\left(X-\mu_{X}\right)\left(Y-\mu_{Y}\right)\right]=\sigma_{X Y}=\sum_{\text {all } x} \sum_{\text {all } y}\left(x-\mu_{X}\right)\left(y-\mu_{Y}\right) p(x, y)$

| Laws of expected value: | Laws of variance: | Laws of covariance: |
| :---: | :---: | :---: |
| $E[c]=c$ | $V[c]=0$ | $\operatorname{COV}[X, c]=0$ |
| $E[X+c]=E[X]+c$ | $V[X+c]=V[X]$ | $\operatorname{COV}[a+b X, c+d Y]=b d * \operatorname{COV}[X, Y]$ |
| $E[c X]=c E[X]$ | $V[c X]=c^{2} V[X]$ |  |
| $E[a+b X+c Y]=a+b E[X]+c E[Y]$ | $\begin{aligned} & V[a+b X+c Y]= \\ & V[a+b X+c Y]= \\ & \text { where } \rho=\text { CORRELA } \end{aligned}$ | $\begin{aligned} & X]+c^{2} V[Y]+2 b c * \operatorname{COV}[X, Y] \\ & X]+c^{2} V[Y]+2 b c * S D(X) * S D(Y) * \rho \\ & {[X, Y]} \end{aligned}$ |

Combinatorial formula: $C_{x}^{n}=\frac{n!}{x!(n-x)!} \quad$ Binomial probability: $p(x)=\frac{n!}{x!(n-x)!} p^{x}(1-p)^{n-x} \quad$ for $x=0,1,2, \ldots, n$
If $X$ is Binomial $(X \sim B(n, p))$ then $E[X]=n p$ and $V[X]=n p(1-p)$
If $X$ is Uniform $(X \sim U[a, b])$ then $f(x)=\frac{1}{b-a}$ and $E[X]=\frac{a+b}{2}$ and $V[X]=\frac{(b-a)^{2}}{12}$

Sampling distribution of $\bar{X}: \quad$ Sampling distribution of $\widehat{\boldsymbol{P}}: \quad$ Sampling distribution of ( $\left(\widehat{\boldsymbol{P}}_{2}-\widehat{\boldsymbol{P}}_{1}\right)$ :
$\mu_{\bar{X}}=E[\bar{X}]=\mu$
$\mu_{\hat{P}}=E[\hat{P}]=p$
$\mu_{\hat{P}_{2}-\hat{P}_{1}}=E\left[\hat{P}_{2}-\hat{P}_{1}\right]=p_{2}-p_{1}$
$\sigma_{\bar{X}}^{2}=V[\bar{X}]=\frac{\sigma^{2}}{n} \quad \sigma_{\hat{P}}^{2}=V[\hat{P}]=\frac{p(1-p)}{n}$
$\sigma_{\hat{P}_{2}-\hat{P}_{1}}^{2}=V\left[\hat{P}_{2}-\hat{P}_{1}\right]=\frac{p_{2}\left(1-p_{2}\right)}{n_{2}}+\frac{p_{1}\left(1-p_{1}\right)}{n_{1}}$
$\sigma_{\bar{X}}=S D[\bar{X}]=\frac{\sigma}{\sqrt{n}}$
$\sigma_{\hat{P}}=S D[\hat{P}]=\sqrt{\frac{p(1-p)}{n}}$

$$
\sigma_{\hat{P}_{2}-\hat{P}_{1}}=S D\left[\hat{P}_{2}-\hat{P}_{1}\right]=\sqrt{\frac{p_{2}\left(1-p_{2}\right)}{n_{2}}+\frac{p_{1}\left(1-p_{1}\right)}{n_{1}}}
$$

Inference about a population proportion:
Cl estimator: $\hat{P} \pm z_{\alpha / 2} \sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

Inference about comparing two population proportions:
Cl estimator: $\left(\hat{P}_{2}-\hat{P}_{1}\right) \pm z_{\alpha / 2} \sqrt{\frac{\hat{P}_{2}\left(1-\hat{P}_{2}\right)}{n_{2}}+\frac{\hat{P}_{1}\left(1-\hat{P}_{1}\right)}{n_{1}}}$


The Standard Normal Distribution:

| $z$ | Second decimal place in z |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | . 753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | . 4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |

