(1) (a) We are 95% confident that between 10.6 and 13.4 percent of *all Canadians* in the first half of October 2013 believed that science is not yet conclusive that global warming is happening.

(b) $CI = \hat{P} \pm ME = \hat{P} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = 0.12 \pm 1.96\sqrt{\frac{0.12*0.88}{2{,}003}} = 0.12 \pm 0.014$. The summary is giving a margin of error for various proportions and hence uses the worst case scenario of a sample proportion of 0.5, which results in the highest possible margin of error: $ME = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = 1.96\sqrt{\frac{0.5*0.5}{2{,}003}} = 0.022$. For a sample proportion of only 12% the margin of error will be considerably smaller (i.e. 1.4% as correctly calculated above).

(c)

$H_0: p = 0.5$

$H_1: p < 0.5$ (Note students may write H_1 or H_A.)

$\hat{P} = 0.47$

We can approximate the relevant sample size as 229 $\left(= 2{,}003 * \frac{4025.1}{35{,}158.3}\right)$.

Compute the P-value, which is a quantitative measure of the strength of our evidence in favor of the research hypothesis:

$$P - value = P\left(\hat{P} < 0.47 \middle| p = 0.5, n = 229\right) = P\left(Z < \frac{0.47 - 0.5}{\sqrt{\frac{0.5 * 0.5}{229}}}\right) = P(Z < -0.91) = 0.18$$

We have only very weak evidence in favor of the research hypothesis. The sample proportion is less than half, but even if half of all Alberta residents did believe this there is an 18% chance we could see a sample proportion as low as 47%. This result is not statistically significant at any conventional significance level.

(2) (a) The numbers in parentheses are the <u>standard errors of the reported sample proportions</u> (response rates), which indicate how much sampling error could affect the reported statistics. For example, the results for the 3:1 match in Blue States are $\hat{P} = 0.021$ and $n = 6{,}574$. To find the reported s.e.: $se[\hat{P}] = \sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = \sqrt{\frac{0.021(1-0.021)}{6{,}574}} = 0.0018$. Rounding to the nearest thousandth we obtain 0.002 just as reported in the table.

[Note to markers: It is a serious error if students use the following: $SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$ because we do not know the population proportion (a parameter) and hence cannot plug into this formula.] (Further explanation: our estimate of the s.d. of the sample proportion is 0.2 percentage points. Hence $\hat{P}$ is a fairly precise estimate of the population proportion: the point estimate is 2.1 percentage points versus as standard error of 0.2 percentage points.)

(b) Define p_1 as the proportion responding (response rate) for a \$1:\$1 match and p_2 as the proportion responding (response rate) for a \$3:\$1 match.

$H_0: (p_1 - p_2) = 0$

$H_1: (p_1 - p_2) \neq 0$

$\hat{P}_1 = \frac{X_1}{n_1} = 0.021$; $\hat{P}_2 = \frac{X_2}{n_2} = 0.026$

Pooled proportion: $\bar{P} = \frac{X_1+X_2}{n_1+n_2} \approx \frac{0.021*4{,}490+0.026*4{,}547}{4{,}490+4{,}547} = 0.0235$

Test statistic: $z = \frac{\hat{P}_1-\hat{P}_2}{\sqrt{\frac{\bar{P}(1-\bar{P})}{n_1}+\frac{\bar{P}(1-\bar{P})}{n_2}}} = \frac{0.021-0.026}{\sqrt{0.0235(1-0.0235)\left(\frac{1}{4{,}490}+\frac{1}{4{,}547}\right)}} = \frac{-0.005}{0.00319} = -1.57$

P-value: $P-value = P(Z < -1.57) + P(Z > 1.57) = 2*(0.5-0.4418) = 0.1164$

Because the P-value is greater than the selected significance level (0.10) we conclude that there is NOT a statistically significant difference in the (population) response rates between a $1:$1 match with a $3:$1 match in Red States.

[Note to markers: This question can be correctly approached either by using the P-value approach and comparing it to the significance level (as shown above), by using the standardized rejection region approach, or by using the unstandardized rejection region approach. If a student uses the standardized rejection region, the rejection region has two parts: $Z \leq -1.645$ and $Z \geq 1.645$. If the student uses the unstandardized rejection region, the rejection region has two parts: $\left(\hat{P}_1 - \hat{P}_2\right) \leq -0.0052$ and $\left(\hat{P}_1 - \hat{P}_2\right) \geq 0.0052$.]

(c) To answer we find a Confidence Interval estimate. Since no significance level is specified and there is nothing special here to suggest departing from the usual convention in economics of a 5% significance level, we use that.

$$(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}+\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$$

$$(0.026 - 0.015) \pm 1.96\sqrt{\frac{0.026(1-0.026)}{4{,}547}+\frac{0.015(1-0.015)}{6{,}648}}$$

$$0.011 \pm 1.96 * 0.0028$$

$$0.011 \pm 0.0055$$

Hence the estimated effect of moving from no match to a $3:$1 match in Red States on the response rate is a 1.1 percentage point increase in the response rate with a margin of error of plus or minus 0.6 percentage points to achieve 95% confidence.

(3) Define the parameter p to be the proportion of boys born in the relevant population. Write down the null and research hypotheses:

$H_0: p = 0.512$

$H_1: p > 0.512$ (Note students may write H_1 or H_A.)

To answer the question requires selecting a significance level and finding the rejection region. Given the sensitive nature of this question I would choose a fairly high burden of proof (higher than the conventional significance level of 5%). Suppose we choose 1%. [Note to markers: The important fact is that the student demonstrates understanding of this issue. They could choose a different significance level – so long as s/he justifies its use in this situation – because this is a judgment call.]

Find the rejection region:

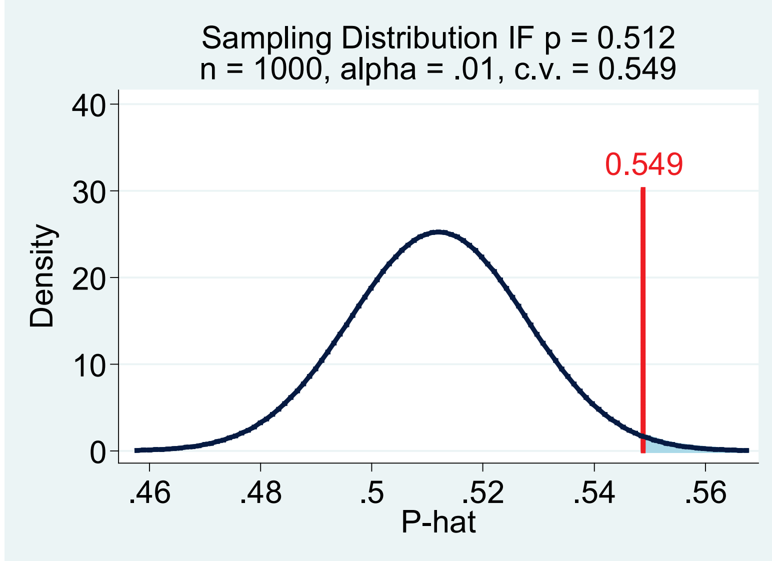
$P(\hat{P} > ?\,|p_0 = 0.512, n = 1{,}000) = \alpha = 0.01$

$P(Z > 2.33) = 0.01$

$$Z = \frac{\hat{P} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$2.33 = \frac{\hat{P} - 0.512}{\sqrt{\frac{0.512(1-0.512)}{1000}}}$$

$$2.33 = \frac{\hat{P} - 0.512}{0.0158}$$



$0.0368 = \hat{P} - 0.512$. Hence the critical value (c.v.) – edge of the rejection region – is 0.549 (= 0.512 + 0.0368). Given that our sample size is 1,000, this means I would need to see *at least* 549 boys (i.e. 549 *or more* boys) to be convinced that there is interference to select for male babies.

(4) (a) The Empirical Rule, which states that in a sample drawn from a Normal population about 68.3% of observations should lie within 1 standard deviation of the mean, about 95.4% should lie within 2 standard deviations of the mean, and about 99.7% should lie within 3 standard deviations of the mean does NOT apply here because distribution of Fortune 500 companies profits is clearly positively skewed and not Normal. According to the STATA summary the mean is $1.64 billion and the s.d. is $4.09 billion. If the Empirical Rule were to hold then about 68.3% of Fortune 500 companies should have profits between -$2.45 billion and $5.73 billion. However, the Stata summary shows that the 5^{th} percentile is $-0.65 billion and the 90^{th} percentile is $3.89 billion, which means that more than 85% of companies fall within 1 s.d. of the mean: this is way off from what the Empirical Rule would say.

(b) In this example the Central Limit theorem, which states that the sampling distribution of the sample mean $(\bar{X})$ is Normally distributed no matter what the shape of the population so long as the sample size is sufficiently large, does NOT apply in the Fortune 500 example for either n = 10 or n = 40. In both cases we are closer to Normal than the original population shown in Histogram #1 – which is strongly positively skewed – but the simulated sampling distributions of the sample mean clearly still shows some positive skew for both a sample size of n = 10 and n = 40, and hence are not Normal. Histogram #3 is closer to Normal than Histogram #2 because the sample size is bigger but it is still clearly skewed. We'd need an even bigger sample size to meet the "sufficiently large" requirement.

(c) Yes the analyst is correct to be surprised that the mean profit of a random sample of 40 Fortune 500 companies is a low as $0.5 billion. Histogram #3 shows the sampling distribution of the sample mean for a sample size of 40 (as obtained from a Monte Carlo simulation). From that sampling distribution we see that it is unlikely to obtain a random sample of 40 companies with a mean profits as low as $0.5 billion: it looks like the chance of that happening is clearly less than 0.05. The fact that many companies have such a low level of profits refers to the original population distribution and NOT the distribution of the sample mean.

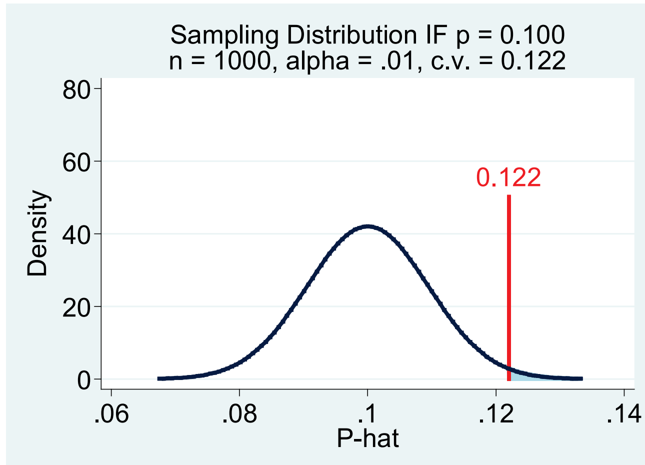
(5)

$H_0: p = 0.10$

$H_1: p > 0.10$

Find the unstandardized rejection region for $n = 1000$ and $\alpha = 0.01$:

$P(Z > 2.33) = 0.01$



$$Z = \frac{\hat{P} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$2.33 = \frac{\hat{P} - 0.10}{\sqrt{\frac{0.10(1-0.10)}{1000}}}$$

$$2.33 = \frac{\hat{P} - 0.10}{0.0095}$$

Solving for $\hat{P}$ gives the unstandardized critical value (c.v.) = 0.122. Hence the rejection region is (0.122, 1): i.e. we need a sample proportion of at least 12.2 percent making purchases to reject the null and infer the true proportion is over 10%.

$$Power = P(\hat{P} > 0.122 | p = 0.14, n = 1000) = P\left(Z > \frac{0.122 - 0.14}{\sqrt{\frac{0.14(1-0.14)}{1000}}}\right) = P\left(Z > \frac{-0.018}{0.0110}\right) = P(Z > -1.64) = 0.9496$$