ECO220Y1Y, Test #2, Prof. Murdock SOLUTIONS

(1) (a) First blank is 14.2 [OK to round to 14]: $P(L2 \mid E) = \frac{P(L2\,\&\,E)}{P(E)} = \frac{0.1204}{0.8473} = 0.142$

Second blank is 64.1 [OK to round to 64]: $P(E \mid L1) = \frac{P(L1\,\&\,E)}{P(L1)} = \frac{0.0371}{0.0579} = 0.641$

(b) $P(L5\ or\ U) = P(L5) + P(U) - P(L5\,\&\,U) = 0.3986 + 0.0384 - 0.0132 = 0.4238$ [Note: "or" can also be written as ∪ and "&" can also be written as "and" or ∩.]

(2) (a) $P(A \mid T) = 0.055$

$P(R \mid A) = 0.100$

(b) Both statements are decidedly false. Column (1) shows that typical applicants, regardless of their race, have a small chance – between 5 and 8 percent – of being admitted to Harvard, but this does NOT mean that they are a small fraction of admitted students. The table is showing chances of admission conditional on various student characteristics – P(A | characteristics) – and NOT the fraction of admitted students with certain characteristics – P(Characteristics | A). Similarly, Column (2) shows the chances of recruited athletes of various races being admitted, and NOT the fraction of admitted recruited athletes of various races. If Statement B were about what this table showed, then the numbers in Column (2) would have to sum to 100%, and they clearly sum to much more.

(3) $P(X > 40{,}000 \mid \mu = 45{,}982, \sigma = 10{,}881) = P\left(Z > \frac{40{,}000-45{,}982}{10{,}881}\right) = P(Z > -0.55) = 0.5 + 0.2088 = 0.7088$

(4) (a) [Because the rule of thumb for the Normal approximation fails, we must use Binomial probability formula.]

$P(X = 0 \mid n = 60, p = 0.10) = \frac{60!}{0!(60-0)!}0.10^0(1-0.10)^{60} = 0.0018$

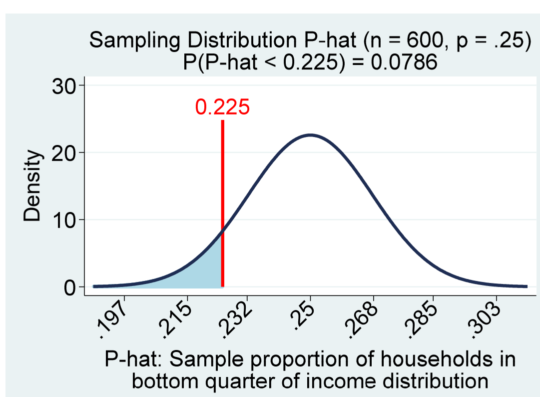
$P(X = 1 \mid n = 60, p = 0.10) = \frac{60!}{1!(60-1)!}0.10^1(1-0.10)^{60-1} = 0.0120$

$P(X = 2 \mid n = 60, p = 0.10) = \frac{60!}{2!(60-2)!}0.10^2(1-0.10)^{60-2} = 0.0393$

$P(X > 2) = 1 - P(X = 0) - P(X = 1) - P(X = 2) = 0.947$

(b) $P(\hat{P} < 0.225 \mid n = 600, p = 0.25) = P\left(Z < \frac{0.225-0.25}{\sqrt{\frac{0.25*0.75}{600}}}\right) =$

$P\left(Z < \frac{-0.025}{\sqrt{0.0003125}}\right) = P(Z < -1.414) \approx 0.5 - 0.42 = 0.08$



(5) (a) $E[\text{p22\_25\_54} - \text{p12\_25\_54}] = E[\text{p22\_25\_54}] - E[\text{p12\_25\_54}] = 81.13087 - 77.9194 = 3.21147$

$\text{V}[\text{p22\_25\_54} - \text{p12\_25\_54}] = \text{V}[\text{p22\_25\_54}] + \text{V}[\text{p12\_25\_54}] + 2\text{SD}[\text{p22\_25\_54}]\text{SD}[\text{p12\_25\_54}]\text{r}$

$\text{V}[\text{p22\_25\_54} - \text{p12\_25\_54}] = (8.951574\text{^}2) + (10.21291\text{^}2) - 2(8.951574)(10.21291)(0.9613)$

$\text{V}[\text{p22\_25\_54} - \text{p12\_25\_54}] = 8.6670$

$\text{SD}[\text{p22\_25\_54} - \text{p12\_25\_54}] = 2.944$

(b) Across 37 member nations of the OECD, older women's labor force participation increased by 11.8 percentage points from 2012 to 2022. For women aged 55 to 64 years, this is a very large increase from 48.8% to 60.6% participating in the labor force, in just one decade. However, the huge standard deviation of 8.2 percentage points means that this increase was *not* uniform across the countries: some had much larger gains for older women – in one case a 26.8 percentage points rise! – and some saw much smaller gains or even losses – in one case a 7.1 percentage point decline! – for older women. For younger women, aged 25 to 54, the gains from 2012 to 2022 were much more modest – 3.2 percentage points – and there was also much less variation in those gains across OECD countries (standard deviation of 2.9 percentage points), compared to older women.

(6) (a) $(0.738 - 0.370) \pm 2.576\sqrt{\frac{0.738(1-0.738)}{13{,}365} + \frac{0.370(1-0.370)}{52{,}465}}$

$(0.368) \pm 2.576\sqrt{0.00001891}$

$0.368 \pm 2.576 * 0.0043486$

0.368 ± 0.011

$LCL = 0.357$ and $UCL = 0.379$

For *all* students in these southern US states who were in grade 9 in during the three academic years from 2000/01 to 2002/03, we are 99% confident that reliance on food stamps as an adult is between 35.7 and 37.9 percentage points higher for students who attended high schools rated "unsatisfactory" compared to students who attended high schools rated "excellent." This is a huge difference that we have very precisely estimated: the margin of error is only 1.1 percentage points.

(b) 7.7 percentage points; 1.44; 116,122; 0.0014

(7) (a) 90; 25; 0.007; 0.7

(b) $E[\bar{X}] = \mu = 121.7868$

$SD[\bar{X}] = \frac{\sigma}{\sqrt{n}} = \frac{38.43549}{\sqrt{10}} = 12.154$

(c) $P(\bar{X} < ?\,|\, \mu = 121.7868, \sigma = 38.43549, n = 30)$

$P(Z < 2.33) = 0.99$

$Z = \frac{? - 121.7868}{\frac{38.43549}{\sqrt{30}}}$

$2.33 = \frac{? - 121.7868}{7.0173}$

$? = 138.137$