ECO220Y, Term Test #2

December 4, 2015, 9:10 – 11:00 am

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Surname (last name):

Given name (first name):

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Instructions:

- You have 110 minutes. Keep these test papers closed on your desk until the start of the test is announced.
- You may use a non-programmable calculator.
- There are 6 questions (some with multiple parts) with varying point values worth a total of 84 points.
- Write your answers clearly, completely and concisely in the designated space provided immediately after each question. No extra space/pages are possible. You *cannot* use blank space for other questions nor can you write answers on the *Supplement*. Your entire answer must fit in the designated space provided immediately after each question.
  - **Write in pencil and use an eraser as needed.** This way you can make sure to fit your final answer (including work and reasoning) in the appropriate space.
  - **Most questions give more blank space than is needed to answer.** Follow the answer guides and avoid excessively long answers.
- Clearly show your work. Make your reasoning clear.
- Apply your understanding to the specific questions asked. Offer context-specific explanations rather than generic definitions or quotes from class or the book. Show that you can successfully *apply* your understanding to the specific circumstances presented.
- A guide for your response ends each question. The guide lets you know what is expected: e.g. a quantitative analysis, a graph, and/or sentences. If the question and/or guide ask for a fully-labeled graph, it is required.
- For questions with multiple parts (e.g (a) – (c)), attempt each part even if you had trouble with earlier parts.
- This test has 8 pages plus the *Supplement*. The *Supplement* contains the aid sheets (formula sheets and Standard Normal table) as well as graphs, tables, and other information needed to answer the test questions. Anything written on the *Supplement* will *not* be graded. You must write your answers in the designated space provided immediately after each question.

(1) [14 pts] Elevators use substantial electricity and climbing stairs is good exercise. A researcher puts a video screen next to an elevator. As each non-mobility-impaired person approaches, it randomly displays one of two messages "Get fit: use the stairs and exercise" or "Help stop global warming: use the stairs and save electricity." Of the 180 people who saw the "exercise" message, 42 used the stairs. Of the 177 people who saw the "electricity" message, 89 used the stairs. *Compute and interpret* the relevant 95% CI estimate for comparing the effectiveness of these messages. Answer with a quantitative analysis and 1 – 2 sentences.

Page Pts:

(2) [26 pts] Recall "Asiaphoria Meets Regression to the Mean."

Page Pts:

(a) [10 pts] How should you *interpret* the four graphs and OLS results in the *Supplement for Question (2) (a)*? *Specifically* reference the graphs and the OLS results in your answer. Which seemingly obvious conclusions do Pritchett and Summers (the authors of "Asiaphoria Meets Regression to the Mean") say we should *not* make from these graphs and OLS results? Answer with 4 – 6 sentences.

(b) [8 pts] Use the graphs and OLS results in the *Supplement for Question (2) (b)* to strengthen and illustrate your arguments in Part (a) (regarding the conclusions that we should *not* make)? *Specifically* reference the relevant numbers that support your points. Answer with 3 – 5 sentences.

Page Pts:

(c) [8 pts] In the *Supplement for Question (2) (c)*, what do the results in PANEL A mean? For *one* row of results, fully interpret all numbers. Use these results to strengthen your position in Parts (a) and (b). Answer with 3 – 5 sentences.

(3) [10 pts] A farmer raising hens knows that there is natural variation in the size of eggs and that the distribution is Normal. If a farmer finds that 2.9% of the eggs weigh less than 42 grams (the minimum to be labeled "Small") and 4.1% of the eggs weigh more than 70 grams (the minimum to be labelled "Jumbo") then what is the mean and standard deviation of egg weights? Answer with a quantitative analysis *that shows your work and reasoning* and illustrate your answer with a *fully-labelled graph* where the x-axis is egg weight (grams).

Page Pts:

(4) [10 pts] Read the *Supplement for Question (4)*.

Page Pts:

(a) [5 pts] Given the *Supplement for Question (4) (a)*, what is the coefficient of correlation between annual GDP growth in the 90's (i.e. 1990-2000) with annual GDP growth the 00's (i.e. 2000-2010) for OECD countries? Answer with a quantitative analysis.

(b) [5 pts] Given the *Supplement for Question (4) (b)*, what is the mean and s.d. of the *change* in annual GDP growth from the 80's (i.e. 1980-1990) versus the 90's (i.e. 1990-2000) for non-OECD countries? Answer with a quantitative analysis.

(5) [12 pts] In June 2014 Starbucks announced the "Starbucks College Achievement Plan." It helps pay for eligible employees to complete a university degree online. Starbucks employs about 191,000 people worldwide (2014 Annual Report). Suppose among all employees, 50 percent are eligible and that an analyst forecast that 20 percent of eligible employees would take advantage of the program.

Page Pts:

(a) [6 pts] If you randomly sampled 12 eligible employees, how surprising would it be if as few as 2 plan to take advantage (only 16.7%) if the claim of 20% were true? Answer with a quantitative analyses and 1 sentence.

(b) [6 pts] If you randomly sampled 1,200 eligible employees, how surprising would it be if as few as 200 plan to take advantage (only 16.7%) if the claim of 20% were true? Answer with a quantitative analyses and 1 sentence.

(6) [12 pts] The *Supplement for Question (6)* describes a population and a Monte Carlo simulation.

Page Pts:

(a) [6 pts] If you randomly selected 30 employees, what is the probability that the sample median is less than $105,000? Would that be surprising or is sampling error a plausible explanation for such a low sample median? Answer with 2 – 3 sentences that show your work/reasoning.

(b) [2 pts] How would you expect the answer to Part (a) to differ if the simulation had used 1,000,000 simulation draws instead of 500,000? Why? Answer with 1 – 2 sentences.

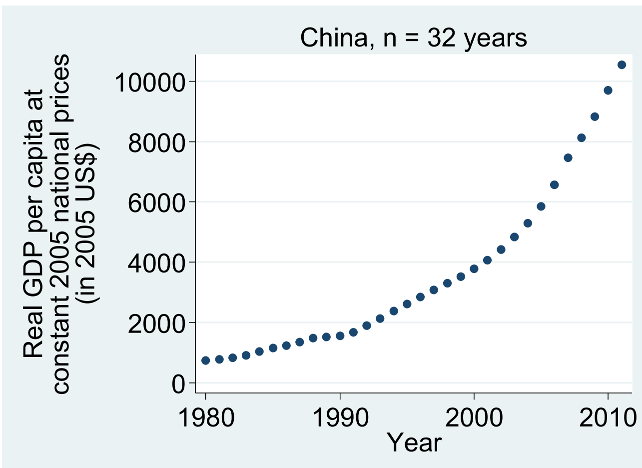
(c) [4 pts] How would you expect the answer to Part (a) to differ if the simulation had used sample sizes of 60 instead of 30? Why? Answer with 2 – 3 sentences.

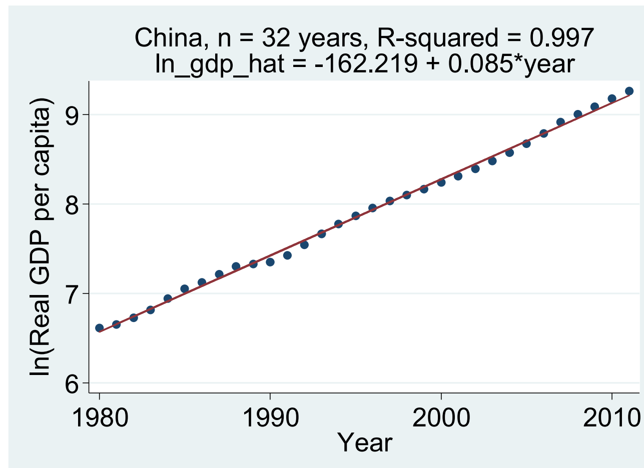
Supplement The pages of this supplement will *not* be graded: write your answers on the test papers.

This *Supplement* contains the aid sheets (formula sheets and Standard Normal table) as well as graphs, tables, and other information needed to answer the test questions. For each question directing you to this *Supplement*, make sure to carefully review all relevant materials. Remember, only your answers written on the test papers (in the designated space immediately after each question) will be graded. Any writing on this *Supplement* will not be graded.

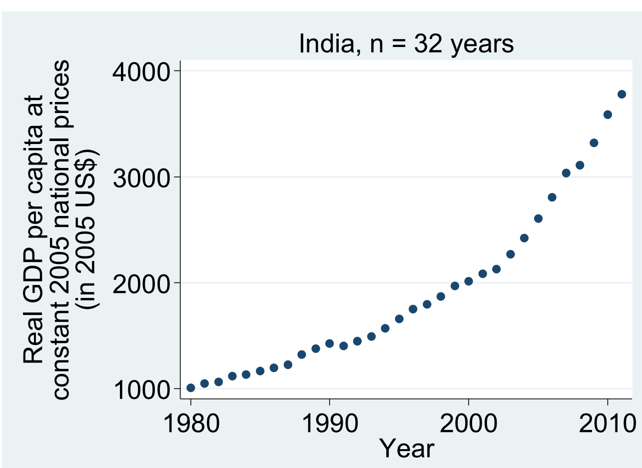
Supplement for Question (2): Recall the readings and study materials assigned prior to this test for "Asiaphoria Meets Regression to the Mean," *NBER Working Paper 20573*, Oct. 2014, by Lant Pritchett and Larry Summers. All results in this *Supplement* use the more recent PWT 8.1 data.[1]

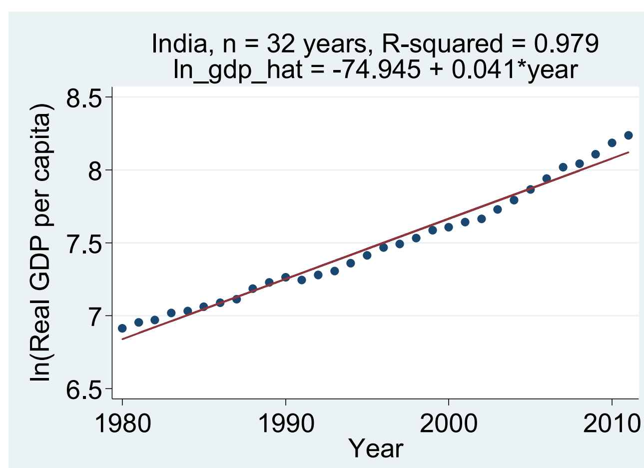
Supplement for Question (2) (a):





OLS results: `ln(gdp)-hat = -162.219 + 0.085*year, R-squared = 0.997, n = 32`





OLS results: `ln(gdp)-hat = -74.945 + 0.041*year, R-squared = 0.979, n = 32`

[1] Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer (2015), "The Next Generation of the Penn World Table" forthcoming *American Economic Review*, available for download at www.ggdc.net/pwt. PWT 8.1 is an updated version of PWT 8.0, covering the same countries and period. Released on: April 13, 2015. (DOI: 10.15141/S5NP4S, Retrieved June 8, 2015.)

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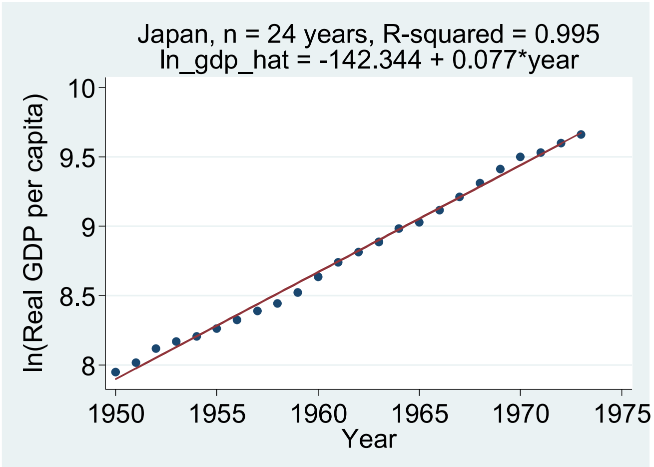
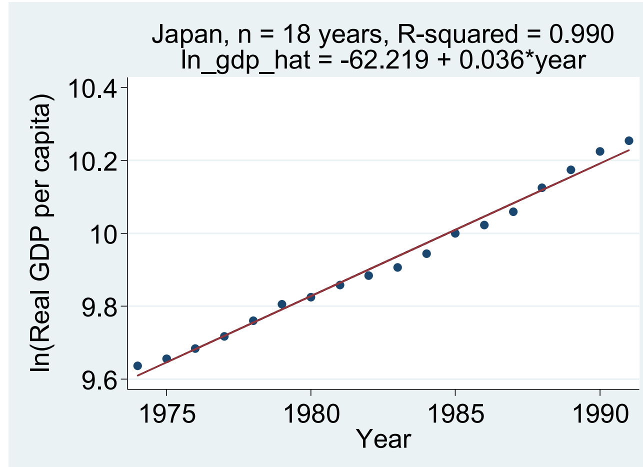
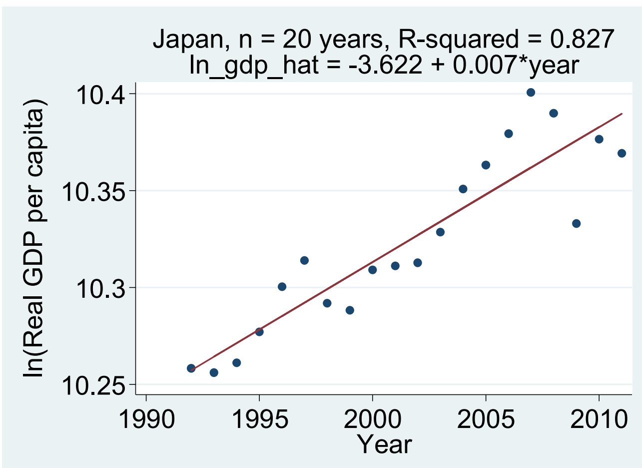
Supplement for Question (2) (b):







Note: Be sure to review the OLS results given in the title of each of these graphs.

Supplement for Question (2) (c):

Table 1: Little persistence in cross-national growth rates across decades				
Period 1	Period 2	Regression Coefficient	R-squared	N
PANEL A: *Adjacent decades*				
1950 – 60	1960 – 70	0.3375783	0.1236	66
1960 – 70	1970 – 80	0.4084345	0.1234	108
1970 – 80	1980 – 90	0.3225473	0.1138	142
1980 – 90	1990 – 00	0.2884994	0.1304	142
1990 – 00	2000 – 10	0.2051206	0.0562	142
Source: Calculations based on PWT 8.1.				

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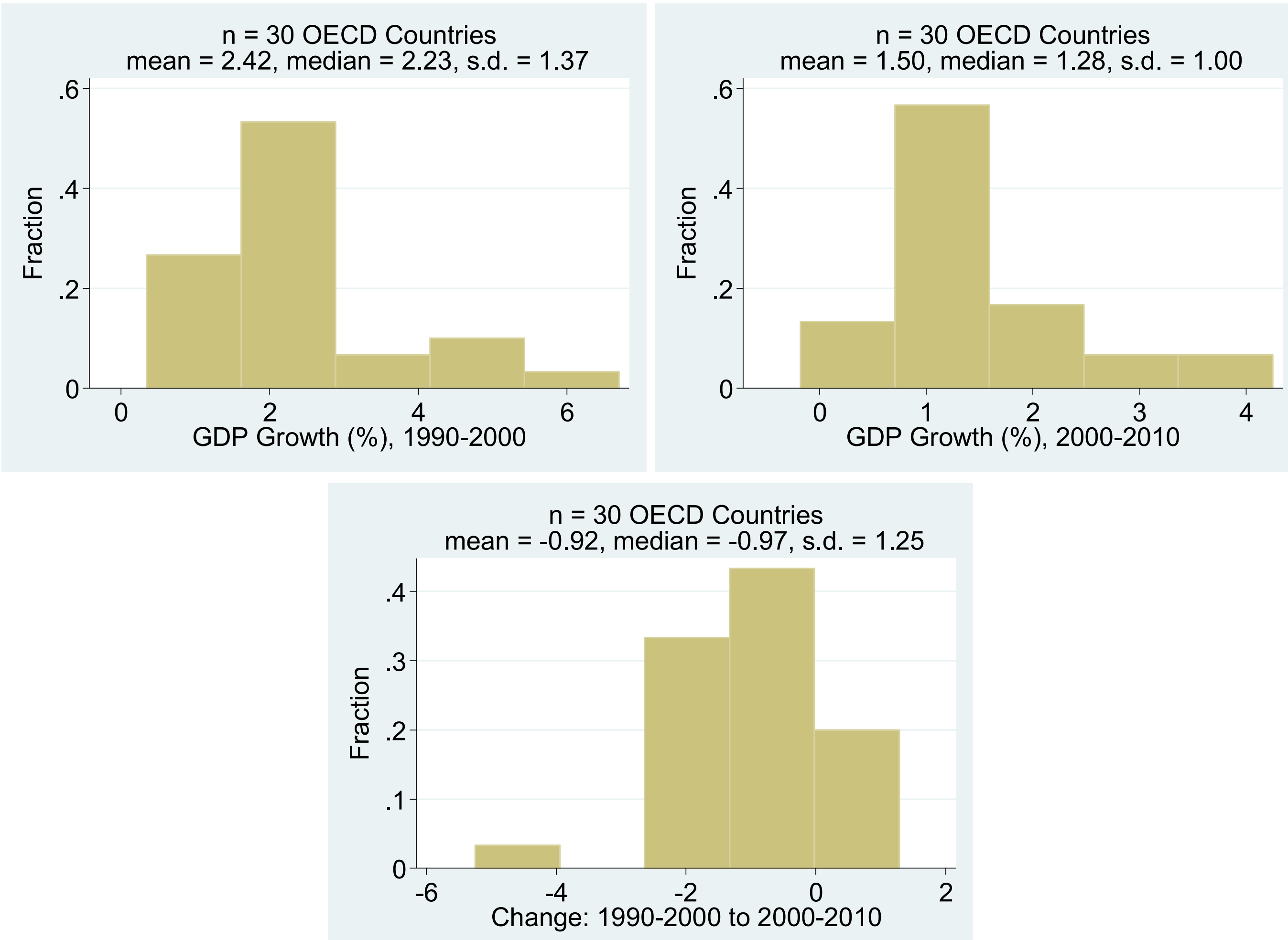
Supplement for Question (4): Recall the PWT 8.1 data discussed in the *Supplement for Question (2)*.

Supplement for Question (4) (a): Below are three graphs for the 30 OECD countries in these data.



Supplement for Question (4) (b): Below are summary statistics for the 112 non-OECD countries in these data.

```
. summarize pct_2000_10 pct_1990_00 pct_1980_90 pct_1970_80 pct_1960_70 if oecd~=1;

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 pct_2000_10 |        112    2.915649    2.162696  -2.086586   9.735052
 pct_1990_00 |        112    1.583804    2.555775  -9.341296   9.092055
 pct_1980_90 |        112     .660269    3.169517  -6.393413   8.251417
 pct_1970_80 |        112    2.060368    3.408507  -8.434716   10.38726

. correlate pct_2000_10 pct_1990_00 pct_1980_90 pct_1970_80 if oecd~=1;
(obs=112)

             | pct_2000_10 pct_1990_00 pct_1980_90 pct_1970_80
-------------+--------------------------------------------
 pct_2000_10 |      1.0000
 pct_1990_00 |      0.2806      1.0000
 pct_1980_90 |      0.2979      0.3422      1.0000
 pct_1970_80 |     -0.0124      0.2047      0.3237      1.0000
```

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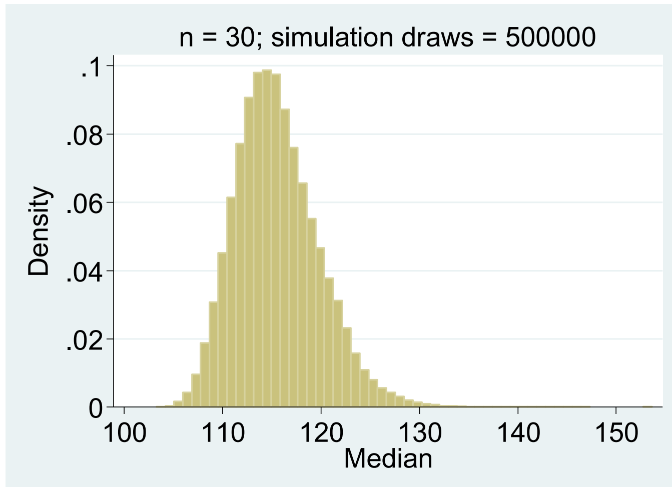
Supplement for Question (6): Recall the salary data for ON public sector employees with salaries of $100,000 or more (http://www.fin.gov.on.ca/en/publications/salarydisclosure/pssd/). Consider the 98,942 employees in the 2014 disclosure that make $300,000 or less. A STATA summary shows the distribution of salaries (measured in $1,000s).

```
                              Salary
-------------------------------------------------------------
      Percentiles      Smallest
 1%     100.2091            100
 5%     100.9725            100
10%     102.0857            100       Obs               98942
25%     105.7196            100       Sum of Wgt.       98942

50%     115.1083                      Mean           125.3419
                        Largest       Std. Dev.      29.96436
75%     132.4765       299.9739
90%     162.9707            300       Variance        897.863
95%     187.7392            300       Skewness       2.382879
99%      254.231            300       Kurtosis       10.12159
```

Consider a Monte Carlo simulation. In each simulation draw, a random sample of 30 employees is drawn from the population of 98,942 employees. For each random sample, the sample median is computed. 500,000 simulation draws are used. A histogram and STATA summary show the simulation results.




```
                           Sample median
-------------------------------------------------------------
      Percentiles      Smallest
 1%     107.3953       103.2731
 5%     109.3153       103.4182
10%     110.4721       103.4636       Obs              500000
25%      112.562       103.4783       Sum of Wgt.      500000

50%     115.1357                      Mean           115.5542
                        Largest       Std. Dev.      4.250755
75%     118.1434       144.8759
90%     121.2057       146.0857       Variance       18.06892
95%     123.0212       147.2199       Skewness       .6106821
99%     127.3034       153.6206       Kurtosis        3.77098
```

Sample mean: $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ **Sample variance:** $s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{X})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$ **Sample s.d.:** $s = \sqrt{s^2}$

Sample coefficient of variation: $CV = \frac{s}{\bar{X}}$ **Sample covariance:** $s_{xy} = \frac{\sum_{i=1}^{n}(x_i-\bar{X})(y_i-\bar{Y})}{n-1} = \frac{\sum_{i=1}^{n} x_i y_i}{n-1} - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(n-1)}$

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Sample interquartile range: $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{s_{xy}}{s_x s_y} = \frac{\sum_{i=1}^{n} z_{x_i} z_{y_i}}{n-1}$

<u>SIMPLE REGRESSION</u>: OLS line: $\hat{y}_i = b_0 + b_1 x_i$ $\quad b_1 = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x}$ $\quad b_0 = \bar{Y} - b_1 \bar{X}$

Residuals: $e_i = y_i - \hat{y}_i$ **Standard deviation of residuals:** $s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i - 0)^2}{n-2}}$

$SST = \sum_{i=1}^{n}(y_i - \bar{Y})^2 = SSR + SSE \quad SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{Y})^2 \quad SSE = \sum_{i=1}^{n} e_i^{\,2} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

$s_y^2 = \frac{SST}{n-1}$ **Coefficient of determination:** $R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} = (r)^2$

Addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **Conditional probability:** $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Complement rules: $P(A^C) = P(A') = 1 - P(A) \quad P(A^C|B) = P(A'|B) = 1 - P(A|B)$

Multiplication rule: $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$

Expected value: $E[X] = \mu = \sum_{all\ x} x p(x)$ **Variance:** $V[X] = E[(X-\mu)^2] = \sigma^2 = \sum_{all\ x}(x-\mu)^2 p(x)$

Covariance: $COV[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = \sigma_{XY} = \sum_{all\ x}\sum_{all\ y}(x-\mu_X)(y-\mu_Y)p(x,y)$

<u>Laws of expected value</u>:

$E[c] = c$

$E[X+c] = E[X] + c$

$E[cX] = cE[X]$

$E[a+bX+cY] = a + bE[X] + cE[Y]$

<u>Laws of variance</u>:

$V[c] = 0$

$V[X+c] = V[X]$

$V[cX] = c^2 V[X]$

$V[a+bX+cY] = b^2 V[X] + c^2 V[Y] + 2bc * COV[X,Y]$

$V[a+bX+cY] = b^2 V[X] + c^2 V[Y] + 2bc * SD(X) * SD(Y) * \rho$

where $\rho = CORRELATION[X,Y]$

<u>Laws of covariance</u>:

$COV[X,c] = 0$

$COV[a+bX, c+dY] = bd * COV[X,Y]$

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$ for $x = 0,1,2,\ldots,n$

If X is Binomial $(X \sim B(n,p))$ **then** $E[X] = np$ **and** $V[X] = np(1-p)$

If X is Uniform $(X \sim U[a,b])$ **then** $f(x) = \frac{1}{b-a}$ **and** $E[X] = \frac{a+b}{2}$ **and** $V[X] = \frac{(b-a)^2}{12}$

<u>Sampling distribution of $\bar{X}$</u>:

$\mu_{\bar{X}} = E[\bar{X}] = \mu$

$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

<u>Sampling distribution of $\hat{P}$</u>:

$\mu_{\hat{P}} = E[\hat{P}] = p$

$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$

$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

<u>Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$</u>:

$\mu_{\hat{P}_2-\hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$

$\sigma_{\hat{P}_2-\hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$

$\sigma_{\hat{P}_2-\hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

<u>Inference about a population proportion</u>: CI estimator: $\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

<u>Inference about comparing two population proportions</u>: CI estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

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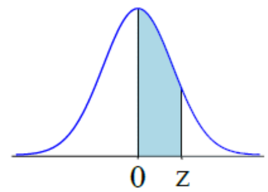

Normal Probabilities:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999