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UNIVERSITY OF TORONTO
Faculty of Arts & Science

APRIL 2019 EXAMINATIONS

ECO220Y1Y: Quantitative Methods in Economics

Duration: 3 hours

Aids Allowed: A non-programmable calculator

Exam Reminders:

- Fill out your e-mail, name, and UTORid on the top of this page.
- Do not begin writing the actual exam until the announcements have ended and the Exam Facilitator has started the exam.
- As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during an exam, you may be charged with an academic offence.
- Turn off and place all cell phones, smart watches, electronic devices, and unauthorized study materials in your bag under your desk. If it is left in your pocket, it may be an academic offence.
- When you are done your exam, raise your hand for someone to come and collect your exam. Do not collect your bag and jacket before your exam is handed in.
- If you are feeling ill and unable to finish your exam, please bring it to the attention of an Exam Facilitator so it can be recorded before leaving the exam hall.
- In the event of a fire alarm, do not check your cell phone when escorted outside.

Special Instructions:

- Write your answers clearly, completely, and concisely. ***Your entire answer must fit in the designated space provided immediately after each question.*** No extra space/pages are possible. ***Write in PENCIL and use an ERASER as needed.*** Follow the answer guides, which end each question, and avoid excessively long answers.

Exam Format and Grading Scheme:

- This exam includes these ***10 pages*** plus the *Supplement*. There are ***10 questions*** (some with multiple parts) with varying point values worth a total of ***120 points***.
- The *Supplement* is 13 pages and contains graphs, tables, and other materials required for some exam questions and the aid sheets. ***After the exam begins, carefully DETACH the Supplement.*** Remember, you must write your answers on the exam papers in the designated spaces: the Supplement will NOT be graded.

Students must hand in all examination materials at the end

(1) See the ***Supplement for Question (1)****: The TTC's Fare Evasion Problem.*

(a) [12 pts] What is the 90% confidence interval (CI) estimate of the ***difference*** in evasion rates for ***BUSSES*** compared to the ***SUBWAY***, which *includes* both staffed entrances and unstaffed/automatic entrances to the subway? Next, *interpret* the CI estimate. Answer with a quantitative analysis & 1 – 2 sentences.

(b) [12 pts] For ***STREETCARS***, does the sample prove that the evasion rate is ***MORE THAN EIGHT TIMES HIGHER*** than the TTC's claim of 1.8 per cent? Include a calculation of the P-value. What is the conclusion? Answer with hypotheses in formal notation, a quantitative analysis & 1 sentence.

(2) [6 pts] See the ***Supplement for Question (2)****: California and the Death Penalty*. Treating the National Academy of Sciences report as a fact, what is the chance that more than 35 people on California's death row are innocent? Answer with a quantitative analysis.

(3) [10 pts] See the ***Supplement for Question (3)****: Vaccinations*. Given the research plan, what are the hypotheses? *In this context*, what would a Type II error be? *In this context,* is a Type II error very serious (something to worry about)? What is the *best* way to increase the power of the test? Answer with hypotheses in formal notation & 3 sentences.

(4) See the ***Supplement for Question (4)****: Wildfires in the Western United States.*

(a) [4 pts] Which kind of data are described by the figure: cross-sectional, time series, or panel data? How many observations are there? In these data, what are the variables? Answer with 1 – 2 sentences.

(b) [5 pts] Consider the values of the SST (total sum of squares), R^2 (R-squared), and s_e (standard deviation of residuals). None of these values are reported. Below you are asked how they would *change* in two different scenarios. Write "increase," "decrease," or "stay the same" in blanks below.

If area burned were in km^2 (not 1,000s of km^2), the value of the SST would ______________________, the value of the R^2 would ______________________, and the value of the s_e would ______________________.

If we dropped the observation for 2017, the value of the SST would ______________________, the value of the R^2 would ______________________, and the value of the s_e would ______________________.

(c) [5 pts] In the figure, the OLS results are the dashed curve: $\hat{y} = b_0 + b_1 x + b_2 x^2$ where $b_1 = 2.65$ and $b_2 = 1.42$. *Approximately*, what is the value of b_0? Next, *interpret* the value of b_0. Answer with a number & 1 sentence.

(5) [5 pts] Suppose a variable is Normally distributed with a mean of 0.0112 and a standard deviation of 0.0094. What is the value of the 20^{th} percentile? Answer with a quantitative analysis.

(6) See the ***Supplement for Question (6)***: *Museums confront their crowded basements.*

(a) [6 pts] Compare some conditional probabilities. Write "larger," "smaller," or "about the same" in blanks below.

Consider The Indianapolis Museum of Art (IMA). Suppose Museum XX (MXX) has 500,000 objects and 45,000 are on display. Compared to P(Display | IMA), we can say that P(Display | MXX) is **______________________**.

Consider the entire collections (on display and in storage) of the Metropolitan Museum of Art (MET) and Denver Art Museum (DAM) combined. For one randomly selected piece from the combined collections, compared to P(MET | Not on paper), we can say that P(DAM | Not on paper) is **______________________**.

Compared to P(Not on paper | IMA), we can say that P(Not on paper | DAM) is **______________________**.

(b) [5 pts] For the Indianapolis Museum of Art, if the event a piece is ranked D were *independent* of the event that the piece is on display, what is the *approximate* value of P(Display | Ranked D)? Explain why the real probability would be larger, smaller, or about the same as that approximate value. Answer with 2 sentences.

(7) [10 pts] See the ***Supplement for Question (7)****: Mortality Inequality.* Using the Excel regression output, among women in counties in the 10th percentile of poverty, how much did predicted life expectancy *change* from 1990 to 2010? Among women in counties in the 90th percentile of poverty, how much did predicted life expectancy *change* from 1990 to 2010? Comparing the answers across those two questions, what can we conclude about how mortality inequality changed from 1990 to 2010 for women? <u>Answer with a quantitative analysis & 2 – 3 sentences.</u>

(8) See the ***Supplement for Question (8)****: Asiaphoria and PWT 9.0.*

(a) [8 pts] See **Graph #1**. The (x, y) coordinates for the observation for Romania are (-0.0150951, 0.0440858). What are the steps to get that observation from the PWT 9.0 data? Answer with a list of steps (in order).

(b) [6 pts] See **Graph #2**. What is the *interpretation* of 0.002 (the value of the R^2)? What can we conclude given that value? Answer with 2 sentences.

(9) [6 pts] See the ***Supplement for Question (9)****: Correlations among Economic Preferences.* Is the correlation between ***NEGATIVE RECIPROCITY*** and ***PATIENCE*** statistically significant (different from zero)? If so, at which of these common significance levels: 10%, 5%, 1%, or 0.1%? Answer with a quantitative analysis & 1 sentence.

(10) See the ***Supplement for Question (10)****: California Energy.*

(a) [5 pts] In **Column (1)** of **Table 1**, what does 0.1876 mean? In other words, offer an *interpretation* of 0.1876. Answer with 1 sentence.

(b) [8 pts] In ***COLUMN (3)*** in **Table 1**, what is the *interpretation* of the coefficient for ***CLIMATE ZONE 13***? Also, *why* is the coefficient for Climate zone 13 so different in **Column (4)**? Answer with 2 – 3 sentences.

(c) [7 pts] For which kind of energy use (electricity or natural gas) does Levinson (2016) find a bigger impact of building codes on energy efficiency? *Explain* making specific reference to **Table 1**. Answer with 2 – 3 sentences.

PLEASE CAREFULLY <u>DETACH</u> THIS SUPPLEMENT FROM YOUR EXAM PAPERS NOW

This *Supplement* contains graphs, tables, and other materials required for some exam questions and the aid sheets (formulas and Normal, t and F statistical tables). Review all relevant materials for each question.

Supplement for Question (1): On February 21, 2019 the Auditor General released: "Review of Toronto Transit Commission's Revenue Operations: Phase One – Fare Evasion and Fare Inspection." (https://www.toronto.ca/city-government/accountability-operations-customer-service/accountability-officers/auditor-general/reports/auditor-generals-reports/report-two/)

The excerpts below explain the sampling and data collection and the analysis of these data for the report.

> **Excerpts, p. 10:** To measure fare evasion rates, we conducted six weeks of fare evasion observation in November and early December 2018.
>
> To ensure we obtained a sufficient sample size, we deployed on average four staff members for each observation day, and in total, spent 136 hours in observation over the six-week period. In total, over 24,000 passengers were inspected by TTC Fare Inspectors during our six-week observation period (including on-boarding and off-boarding, and uniformed and plain-clothes Inspectors).
>
> **Excerpts, p. 12:** Based on our sample observations, we estimated that the TTC's overall fare evasion rate is 5.4 per cent for all three modes [busses, streetcars, subway].
>
> **Excerpts, p. 15:** The TTC has been publicly reporting its overall system-wide fare evasion rate as 2% for the last seven years.
>
> Senior TTC staff cited a 1.8 per cent fare evasion rate for streetcars.

Table 1: Fare Evasion Rate by Mode for Sample Observations from Plain-Clothes Inspectors

Mode	Number of Evaders	Total Inspections	Evasion Rate
Bus	88	1,722	5.11%
Streetcar	603	3,957	15.24%
Subway, Staffed Entrances	303	9,342	3.24%
Subway, Unstaffed/Automatic Entrances	218	4,626	4.71%

Notes: An except from Figure 3 titled "Fare Evasion Rate by Mode, with Total Number of Evasions and Total Inspections (based on Plain Clothes inspection results)" p. 13.

Supplement for Question (2): On March 13, 2019, The Editorial Board of *The New York Times* wrote "A Pause on the Nation's Biggest Death Row: The California governor's moratorium on executions in the state should signal the demise of a barbaric practice." (https://www.nytimes.com/2019/03/13/opinion/california-death-penalty-gavin-newsom.html)

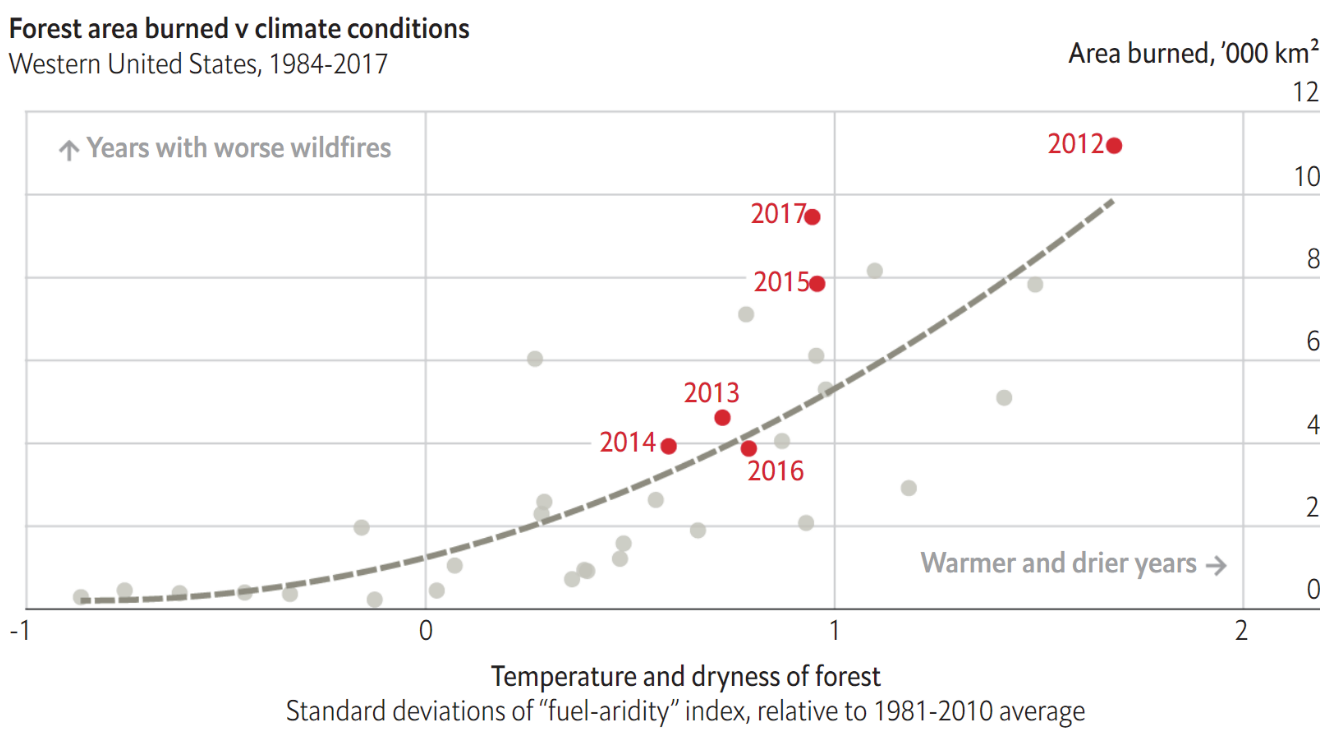
> **Excerpts:** Governor Gavin Newsom of California announced Wednesday that no executions will occur on his watch, granting temporary reprieves to all 737 inmates on the state's death row.
>
> This act of executive mercy recognizes the extreme failures of the death penalty. In announcing his order, Mr. Newsom noted a National Academy of Sciences report estimating that one out of every 25 people on death row is innocent. "If that's the case, that means if we move forward executing 737 people in California, we will have executed roughly 30 people that are innocent," he said.

Supplement for Question (3): On April 4, 2019, *The Toronto Star* published "Ontario's vaccination coverage is not what you think." (https://www.thestar.com/news/investigations/2019/04/04/torontos-vaccination-coverage-is-not-what-you-think-nearly-30-percent-of-7-year-olds-are-not-fully-immunized.html) Consider the short excerpt below. After that, there is a description of a research plan.

> **Excerpt:** In the city of Toronto, only 76 per cent of students who were 7 years old during the 2017-2018 school year (born in 2010) are immunized for measles, the most highly contagious disease that can be prevented through two doses of vaccines that are typically given before age of 6. Toronto Public Health confirmed this figure on March 27, 2019.
>
> **<u>Research Plan:</u>** Given concerns about data reliability raised elsewhere in the article, a researcher does not believe the currently reported value of 76 per cent. She wishes to prove the actual immunization rate is below 70 per cent. She collects random sample of 500 students born in 2010 in the city of Toronto. The research team carefully investigates the actual immunization record of each child.

Supplement for Question (4): On November 17, 2018, *The Economist* published "Burning out: Despite California's inferno, global wildfires are fizzling out: Climate change makes fires worse, but agricultural development limits them." (https://www.economist.com/graphic-detail/2018/11/17/despite-californias-inferno-global-wildfires-are-fizzling-out)

> **Excerpts:** Measured by area burned, nine of California's ten worst fires have occurred since 2000.
>
> John Abatzoglou and Park Williams, two academics, have shown that temperature and dryness exacerbate [which means make worse] wildfires in the western United States. Without global warming, they reckon, only half as much woodland would have burned between 1984 and 2015.



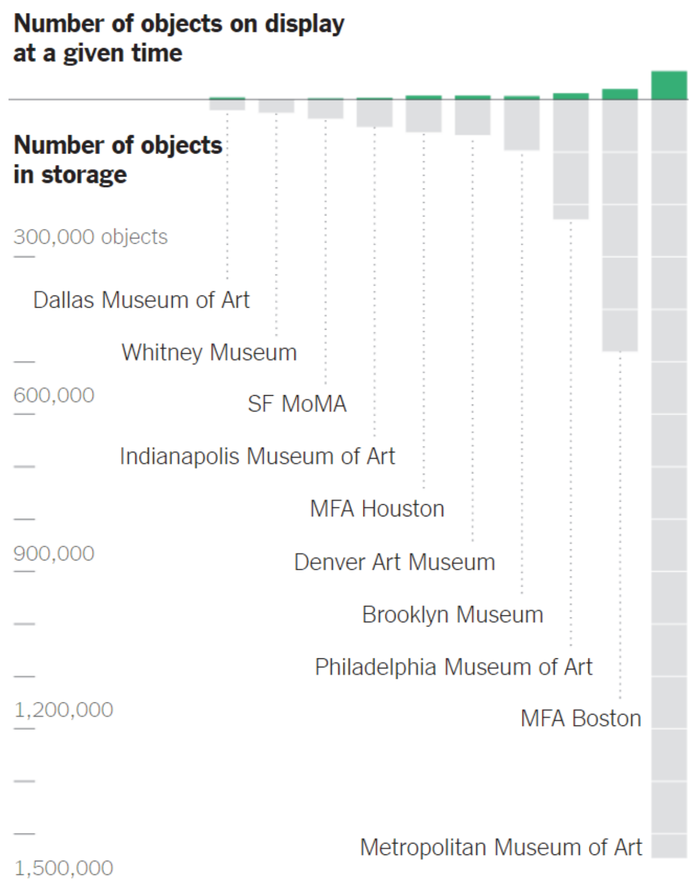
> ***Note:*** "Fuel-aridity" measures how dry the fuel for forest fires is. In other words, are the grasses, trees, and other things that fuel wildfires very dried out and hence highly flammable? The higher the index, the drier the fuel.

Supplement for Question (6): On March 10, 2019, *The New York Times* published "Clean House to Survive? Museums Confront Their Crowded Basements: With storage spaces filled with works that may never be shown, some museums are rethinking the way they collect art, and at least one is ranking what it owns." (https://www.nytimes.com/interactive/2019/03/10/arts/museum-art-quiz.html)

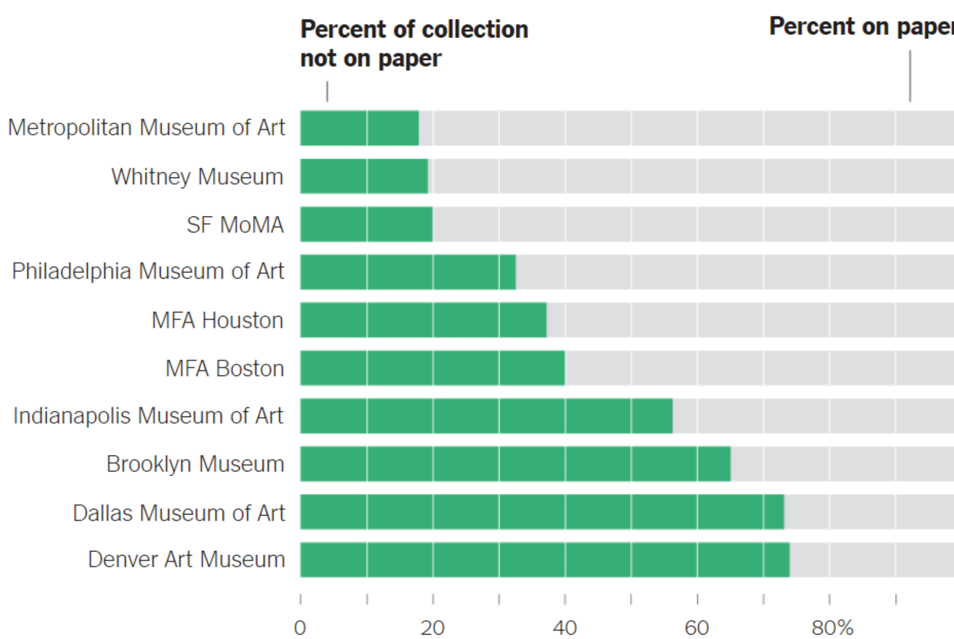
Excerpts: [The Indianapolis Museum of Art] embarked on an ambitious effort to rank each of the 54,000 items in its collection with letter grades [of A, B, C, or D]. Twenty percent of the items received a D, making them ripe to be sold or given to another institution.

[The Indianapolis Museum of Art] shows 8 to 10 percent of its collection at any one time.

Major museums are only able to display a small portion of their collection.



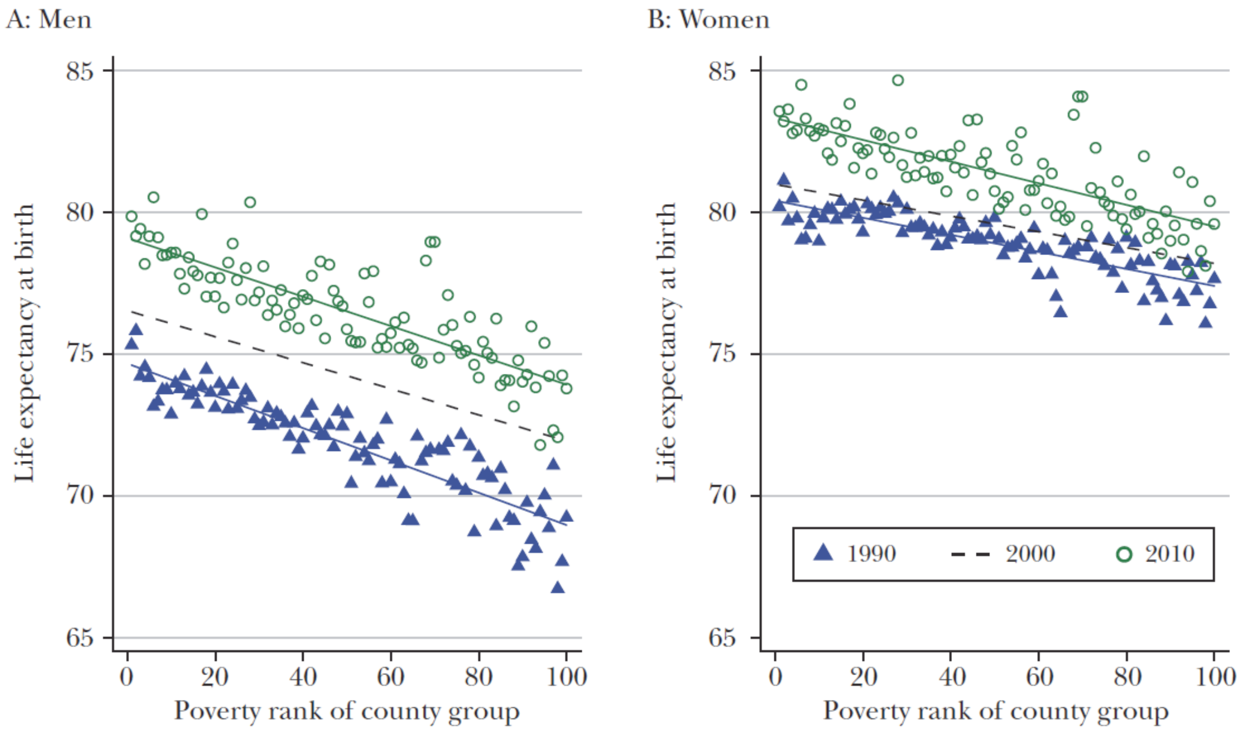
The percentage on display is affected by space constraints, but also by how much of a collection is devoted to works on paper, which cannot be shown for long due to light sensitivity. The Met collection is particularly weighted toward works on paper, but its percentage on display, about 4 percent, is in rough parity with other museums on the list.



Supplement for Question (7): Recall Currie and Schwandt (2016) "Mortality Inequality: The Good News from a County-Level Approach" (https://www.aeaweb.org/articles?id=10.1257/jep.30.2.29) and Figure 2 below. The 199 observations for women in 2010 and 1990 yield the Excel regression output below. The variable quantile is the poverty rank of county group. The variable yr2010 is a dummy equal to 1 for 2010 and 0 otherwise. The variable yr2010xquantile is the product of the variables yr2010 and quantile.

Figure 2

Life Expectancy at Birth across Poverty Percentiles



Source: Authors using data from the Vital Statistics, the US Census, and the American Community Survey.
Note: Counties are ranked by their poverty rate in 1990, 2000, and 2010, and divided into groups each representing about 1 percent of the overall population. Each marker represents the life expectancy at birth in a given county group. Lines are fitted using OLS regression. For 2000, markers are omitted and only the regression line is shown.

Excel regression output: (The dependent variable is female life expectancy at birth measured in years.)

Regression Statistics	
R Squared	0.791256384
Standard Error	0.828469253
Observations	199

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	507.3310304	169.1103435	246.3867687	4.57771E-66
Residual	195	133.840454	0.686361302		
Total	198	641.1714844			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	80.40514533	0.166951284	481.6084281	7.4816E-302	80.07588333	80.73440733
quantile	-0.030036757	0.002876683	-10.44145534	1.47931E-20	-0.035710162	-0.024363351
yr2010	2.920343635	0.236099894	12.36910186	2.55325E-26	2.454706468	3.385980801
yr2010xquantile	-0.008254237	0.004063553	-2.031285555	0.043583212	-0.016268393	-0.000240081

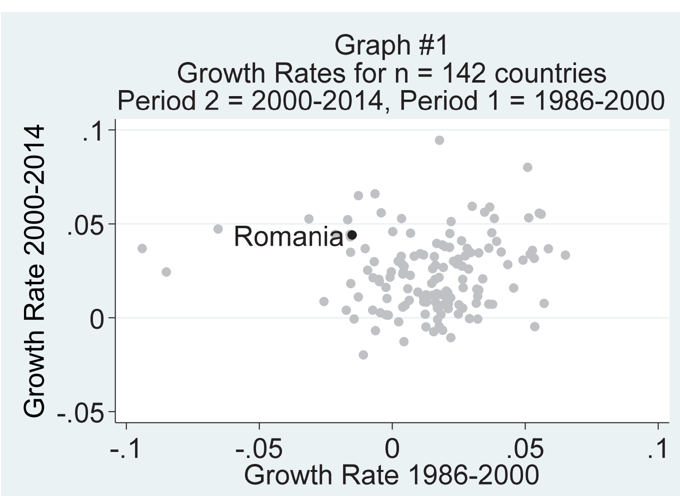
Supplement for Question (8): Recall Pritchett and Summers (2014) "Asiaphoria Meets Regression to the Mean" and the Penn World Tables (PWT) version 9.0. The data file asiap_pwt_90_all.xlsx from DACM gives you the relevant PWT 9.0 data. It includes 7 variables, described below, and 9,439 observations.

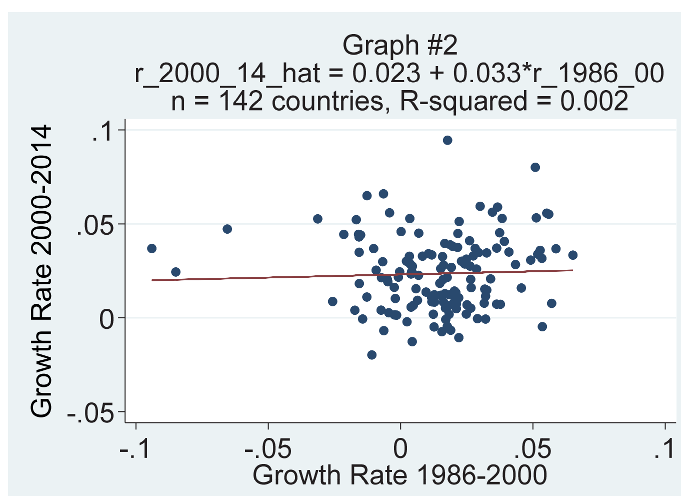
Variable name	Variable description
country	Country name
countrycode	3-letter ISO country code
oecd	=1 if OECD member (Organization for Economic Cooperation and Development)
continent	Continent
year	Year
rgdpna	Real GDP at constant 2011 national prices (in mil. 2011US$)
pop	Population (in millions)

Below are the first four observations in these data to remind you of the data structure and contents.

country	countrycode	oecd	continent	year	rgdpna	pop
Albania	ALB	0	Europe	1970	8099.765	2.150602
Albania	ALB	0	Europe	1971	8423.821	2.20204
Albania	ALB	0	Europe	1972	8761.946	2.253842
Albania	ALB	0	Europe	1973	9113.993	2.305999
…						

Pritchett and Summers (2014) consider time periods of various lengths – one decade (10 years) or two decades (20 years) – to assess how well past annual growth rates of GDP per capita predict future annual growth rates of GDP per capita across countries. The graphs below use the same approach as Pritchett and Summers (2014) except that they consider 1.5 decade-long periods (15 years) and use the most recent PWT 9.0 data. Graph #1 highlights the country of Romania. Graph #2 includes some OLS estimation results in the title and displays the regression line.





Supplement for Question (9): Recall Falk et al. (2018) "Global Evidence on Economic Preferences" (https://doi.org/10.1093/qje/qjy013).

> **Abstract (excerpt):** This article studies the global variation in economic preferences. For this purpose, we present the Global Preference Survey (GPS), an experimentally validated survey data set of time preference [patience], risk preference, positive and negative reciprocity, altruism, and trust from 80,000 people in 76 countries.

For the 76 countries, Table IV reports the correlation matrix for the various preference measures (e.g. time (aka patience), risk, positive reciprocity, etc.) where the value of each preference measure is the mean for each country.

Table IV. Pairwise Correlations between Preferences at the Country Level

	Patience	Risk taking	Pos. reciprocity	Neg. reciprocity	Altruism	Trust
Patience	1					
Risk taking	0.230	1				
Pos. reciprocity	0.016	−0.256	1			
Neg. reciprocity	0.258	0.193	−0.154	1		
Altruism	−0.010	−0.015	0.711	−0.132	1	
Trust	0.190	−0.062	0.363	0.160	0.273	1

Supplement for Question (10): Recall Levinson (2016) "How Much Energy Do Building Energy Codes Save? Evidence from California Houses" (https://www.aeaweb.org/articles?id=10.1257/aer.20150102).

> **Abstract:** Regulations governing the energy efficiency of new buildings have become a cornerstone of US environmental policy. California enacted the first such codes in 1978 and has tightened them every few years since. I evaluate the resulting energy savings three ways: comparing energy used by houses constructed under different standards, controlling for building and occupant characteristics; examining how energy use varies with outdoor temperatures; and comparing energy used by houses of different vintages in California to that same difference in other states. All three approaches yield estimated energy savings significantly short of those projected when the regulations were enacted.

Levinson (2016) uses the 2003 and 2009 Residential Appliance Saturation Study (RASS) surveys of households. These data include many variables describing each house, its owners, the local climate, and the appliances in the house. The key dependent variables are annual household electricity use in MMBTUs and annual household natural gas use in MMBTUs (in either 2003 or 2009).

Table 1, on the next page, shows some regression results using these data. For the explanatory variables (x variables), there are many dummy variables, which have obvious definitions. Some illustrative examples: Climate zone 7 is a dummy variable that equals 1 if the house is located in Climate zone 7 (which is a geographic area) and 0 otherwise, remodeled is a dummy that equals 1 if the house has ever been remodeled in any way and equals 0 otherwise, and Built 1975 – 1977 is a dummy that equals 1 if the house was constructed from 1975-1977 and equals 0 otherwise. Other variables, such as bedrooms and refrigerators are obviously not dummies and instead record the number of each.

Supplement for Question (10), continues on the next page >>>>>

Supplement for Question (10), cont'd:

Table 1. Annual Energy Use by California Households in the RASS Survey

	Dependent variable:	*ln(Annual electricity in MMBTUs)*		*ln(Annual natural gas in MMBTUs)*	
Explanatory variables:		***(1)***	***(2)***	***(3)***	***(4)***
Controls for climate (Climate zone 1 is reference/ omitted category)	Cooling degree-days (100s)	-	0.0209 (0.0032)	-	-0.0158 (0.0060)
	Heating degree-days (100s)	-	-0.0006 (0.0037)	-	0.0287 (0.0068)
	Climate zone 2	-	-0.0074 (0.0270)	-0.2271 (0.0473)	0.0812 (0.0692)
	Climate zone 3	-	0.0522 (0.0234)	-0.2510 (0.0381)	0.1237 (0.0754)
	Climate zone 4	-	-0.0710 (0.0286)	-0.1361 (0.0391)	-0.0143 (0.0690)
	Climate zone 5	-	-0.1203 (0.0362)	-0.0814 (0.0536)	-0.0130 (0.0683)
	Climate zone 7	-	0.0184 (0.0463)	-0.2836 (0.0389)	0.1191 (0.0836)
	Climate zone 8	-	-0.1553 (0.0448)	-0.3172 (0.0484)	0.0142 (0.1049)
	Climate zone 9	-	-0.1427 (0.0359)	-0.2848 (0.0310)	0.0826 (0.0897)
	Climate zone 10	-	-0.0935 (0.0284)	-0.4470 (0.0885)	0.0280 (0.0876)
	Climate zone 11	-	-0.2141 (0.0485)	-0.4733 (0.0310)	-0.0972 (0.0990)
	Climate zone 12	-	-0.0591 (0.0401)	-0.3548 (0.0310)	-0.0502 (0.0892)
	Climate zone 13	-	-0.1563 (0.0476)	-0.4850 (0.0477)	-0.1151 (0.1110)
Controls for building characteristics	ln(square feet)	-	0.2658 (0.0161)	-	0.3768 (0.0206)
	Bedrooms	-	0.0252 (0.0067)	-	0.0156 (0.0080)
	Remodeled	-	0.0272 (0.0102)	-	-0.0100 (0.0125)
Controls for occupant characteristics (HH = Household)	ln(years at address)	-	0.0265 (0.0046)	-	0.0115 (0.0044)
	ln(number of residents)	-	0.2434 (0.0095)	-	0.1306 (0.0128)
	ln(HH income in $1,000s)	0.1876 (0.0150)	0.0951 (0.0084)	-	0.0643 (0.0076)
	HH head graduated college	-	-0.0509 (0.0072)	-	-0.0339 (0.0112)
	Disabled resident	-	0.1431 (0.0086)	-	0.0801 (0.0172)
	Number of residents 0 – 5	-	-0.0494 (0.0050)	-	0.0137 (0.0055)
	Number of residents 65+	-	-0.0307 (0.0066)	-	0.0584 (0.0081)
	HH head black	-	0.0490 (0.0206)	-	0.1156 (0.0232)
	HH head Latino	-	-0.0848 (0.0129)	-	-0.0293 (0.0143)
	Own home	-	-0.0546 (0.0171)	-	-0.0779 (0.0252)
Controls for appliances	Electric cooking	-	0.0361 (0.0068)	-	-0.0374 (0.0070)
	Refrigerators	-	0.1908 (0.0073)	-	-
	Room AC	-	0.0583 (0.0114)	-	-
	Central AC	-	0.1410 (0.0291)	-	-
	Central AC × sq. feet (1,000s)	-	0.0296 (0.0089)	-	-
Survey year	RASS 2009	-	0.0521 (0.0081)	-	-0.1605 (0.0166)
Year house constructed (Pre-1940 is reference/ omitted category)	Built 1940s	-	0.0120 (0.0167)	-	-0.0553 (0.0234)
	Built 1950s	-	0.0331 (0.0261)	-	-0.0370 (0.0199)
	Built 1960s	-	0.0625 (0.0222)	-	-0.0507 (0.0252)
	Built 1970 – 1974	-	0.0567 (0.0255)	-	-0.0944 (0.0266)
	Built 1975 – 1977	-	0.0700 (0.0269)	-	-0.1127 (0.0265)
	Built 1978 – 1982	-	0.0647 (0.0274)	-	-0.1609 (0.0292)
	Built 1983 – 1992	-	0.0382 (0.0238)	-	-0.1957 (0.0253)
	Built 1993 – 1997	-	-0.0048 (0.0255)	-	-0.2551 (0.0300)
	Built 1998 – 2000	-	-0.0163 (0.0216)	-	-0.2780 (0.0343)
	Built 2001 – 2004	-	-0.0254 (0.0296)	-	-0.3125 (0.0361)
	Built 2005 – 2008	-	-0.0871 (0.0285)	-	-0.3498 (0.0408)
	Constant	2.2929 (0.0806)	1.8160 (0.1131)	4.0361 (0.0310)	3.1027 (0.1761)
	Observations	14,045	14,045	12,358	12,358
	R^2	0.0827	0.4370	0.0734	0.2661

Notes: Each column shows a separate regression. The dependent variable is either the natural logarithm of annual electricity use (in MMBTUs) or the natural logarithm of annual natural gas use (in MMBTUs). Robust standard errors in parentheses.

Sample mean: $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ **Sample variance:** $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{X})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$ **Sample s.d.:** $s = \sqrt{s^2}$

Sample coefficient of variation: $CV = \frac{s}{\bar{X}}$ **Sample covariance:** $s_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{X})(y_i - \bar{Y})}{n-1} = \frac{\sum_{i=1}^{n} x_i y_i}{n-1} - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(n-1)}$

Sample interquartile range: $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{s_{xy}}{s_x s_y} = \frac{\sum_{i=1}^{n} z_{x_i} z_{y_i}}{n-1}$

Addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **Conditional probability:** $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Complement rules: $P(A^C) = P(A') = 1 - P(A)$ $P(A^C|B) = P(A'|B) = 1 - P(A|B)$

Multiplication rule: $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$

Expected value: $E[X] = \mu = \sum_{all\ x} x p(x)$ **Variance:** $V[X] = E[(X-\mu)^2] = \sigma^2 = \sum_{all\ x}(x-\mu)^2 p(x)$

Covariance: $COV[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = \sigma_{XY} = \sum_{all\ x}\sum_{all\ y}(x-\mu_X)(y-\mu_Y)p(x,y)$

Laws of expected value:

$E[c] = c$

$E[X + c] = E[X] + c$

$E[cX] = cE[X]$

$E[a + bX + cY] = a + bE[X] + cE[Y]$

Laws of variance:

$V[c] = 0$

$V[X + c] = V[X]$

$V[cX] = c^2 V[X]$

$V[a + bX + cY] = b^2 V[X] + c^2 V[Y] + 2bc * COV[X,Y]$

$V[a + bX + cY] = b^2 V[X] + c^2 V[Y] + 2bc * SD(X) * SD(Y) * \rho$

where $\rho = CORRELATION[X,Y]$

Laws of covariance:

$COV[X,c] = 0$

$COV[a + bX, c + dY] = bd * COV[X,Y]$

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$ for $x = 0,1,2,\ldots,n$

If X is Binomial $(X \sim B(n,p))$ **then** $E[X] = np$ **and** $V[X] = np(1-p)$

If X is Uniform $(X \sim U[a,b])$ **then** $f(x) = \frac{1}{b-a}$ **and** $E[X] = \frac{a+b}{2}$ **and** $V[X] = \frac{(b-a)^2}{12}$

Sampling distribution of $\bar{X}$:

$\mu_{\bar{X}} = E[\bar{X}] = \mu$

$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

Sampling distribution of $\hat{P}$:

$\mu_{\hat{P}} = E[\hat{P}] = p$

$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$

$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$:

$\mu_{\hat{P}_2 - \hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$

$\sigma_{\hat{P}_2 - \hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$

$\sigma_{\hat{P}_2 - \hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

Sampling distribution of $(\bar{X}_1 - \bar{X}_2)$, independent samples:

$\mu_{\bar{X}_1 - \bar{X}_2} = E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_1 - \bar{X}_2}^2 = V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$\sigma_{\bar{X}_1 - \bar{X}_2} = SD[\bar{X}_1 - \bar{X}_2] = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Sampling distribution of $(\bar{X}_d)$, paired ($d = X_1 - X_2$):

$\mu_{\bar{X}_d} = E[\bar{X}_d] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_d}^2 = V[\bar{X}_d] = \frac{\sigma_d^2}{n} = \frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}$

$\sigma_{\bar{X}_d} = SD[\bar{X}_d] = \frac{\sigma_d}{\sqrt{n}} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}}$

Inference about a population proportion:

***z* test statistic:** $z = \frac{\hat{P}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ **CI estimator:** $\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

Inference about comparing two population proportions:

***z* test statistic under Null hypothesis of no difference:** $z = \frac{\hat{P}_2-\hat{P}_1}{\sqrt{\frac{\bar{P}(1-\bar{P})}{n_1}+\frac{\bar{P}(1-\bar{P})}{n_2}}}$ **Pooled proportion:** $\bar{P} = \frac{X_1+X_2}{n_1+n_2}$

CI estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}+\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

Inference about the population mean:

***t* test statistic:** $t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$ **CI estimator:** $\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

Inference about a comparing two population means, independent samples, unequal variances:

***t* test statistic:** $t = \frac{(\bar{X}_1-\bar{X}_2)-\Delta_0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$

Degrees of freedom: $\nu = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2+\frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$

Inference about a comparing two population means, independent samples, assuming equal variances:

***t* test statistic:** $t = \frac{(\bar{X}_1-\bar{X}_2)-\Delta_0}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}$ **Degrees of freedom:** $\nu = n_1 + n_2 - 2$

Pooled variance: $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

Inference about a comparing two population means, paired data: (n is number of pairs and $d = X_1 - X_2$)

***t* test statistic:** $t = \frac{\bar{d}-\Delta_0}{s_d/\sqrt{n}}$ **CI estimator:** $\bar{X}_d \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

SIMPLE REGRESSION:

Model: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ **OLS line:** $\hat{y}_i = b_0 + b_1 x_i$ $b_1 = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x}$ $b_0 = \bar{Y} - b_1\bar{X}$

Coefficient of determination: $R^2 = (r)^2$ **Residuals:** $e_i = y_i - \hat{y}_i$

Standard deviation of residuals: $s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i-0)^2}{n-2}}$ **Standard error of slope:** $s.e.(b_1) = s_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$

Inference about the population slope:

***t* test statistic:** $t = \frac{b_1 - \beta_{10}}{s.e.(b_1)}$ **CI estimator:** $b_1 \pm t_{\alpha/2} s.e.(b_1)$ **Degrees of freedom:** $\nu = n - 2$

Standard error of slope: $s.e.(b_1) = s_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$

Prediction interval for y at given value of x (x_g):

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \textbf{or} \quad \hat{y}_{x_g} \pm t_{\alpha/2} \sqrt{\left(s.e.(b_1)\right)^2 (x_g - \bar{X})^2 + \frac{s_e^2}{n} + s_e^2}$$

Degrees of freedom: $\nu = n - 2$

Confidence interval for predicted mean at given value of x (x_g):

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{\frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \textbf{or} \quad \hat{y}_{x_g} \pm t_{\alpha/2} \sqrt{\left(s.e.(b_1)\right)^2 (x_g - \bar{X})^2 + \frac{s_e^2}{n}}$$

Degrees of freedom: $\nu = n - 2$

SIMPLE & MULTIPLE REGRESSION:

Model: $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + \varepsilon_i$

$$SST = \sum_{i=1}^{n} (y_i - \bar{Y})^2 = SSR + SSE \quad SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{Y})^2 \quad SSE = \sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$s_y^2 = \frac{SST}{n-1} \quad MSE = \frac{SSE}{n-k-1} \quad Root\ MSE = \sqrt{\frac{SSE}{n-k-1}} \quad MSR = \frac{SSR}{k}$$

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \quad Adj. R^2 = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)} = \left(R^2 - \frac{k}{n-1}\right)\left(\frac{n-1}{n-k-1}\right)$$

Residuals: $e_i = y_i - \hat{y}_i$ **Standard deviation of residuals:** $s_e = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i - 0)^2}{n-k-1}}$

Inference about the overall statistical significance of the regression model:

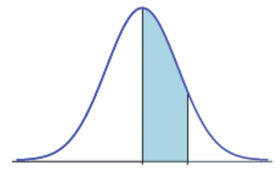
$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)} = \frac{(SST - SSE)/k}{SSE/(n-k-1)} = \frac{SSR/k}{SSE/(n-k-1)} = \frac{MSR}{MSE}$$

Numerator degrees of freedom: $\nu_1 = k$ **Denominator degrees of freedom:** $\nu_2 = n - k - 1$

Inference about the population slope for explanatory variable j:

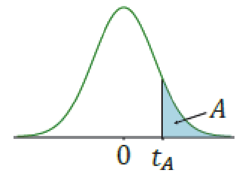
***t* test statistic:** $t = \frac{b_j - \beta_{j0}}{s_{b_j}}$ **CI estimator:** $b_j \pm t_{\alpha/2} s_{b_j}$ **Degrees of freedom:** $\nu = n - k - 1$

Standard error of slope: $s.e.(b_j) = s_{b_j}$ (for multiple regression, must be obtained from technology)



The Standard Normal Distribution:

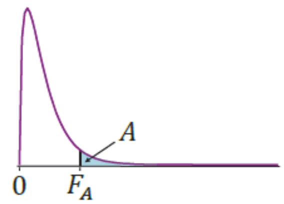
	Second decimal place in z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999



Critical Values of Student t Distribution:

ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	$t_{0.001}$	$t_{0.0005}$	ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	$t_{0.001}$	$t_{0.0005}$
1	3.078	6.314	12.71	31.82	63.66	318.3	636.6	38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
2	1.886	2.920	4.303	6.965	9.925	22.33	31.60	39	1.304	1.685	2.023	2.426	2.708	3.313	3.558
3	1.638	2.353	3.182	4.541	5.841	10.21	12.92	40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610	41	1.303	1.683	2.020	2.421	2.701	3.301	3.544
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869	42	1.302	1.682	2.018	2.418	2.698	3.296	3.538
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959	43	1.302	1.681	2.017	2.416	2.695	3.291	3.532
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408	44	1.301	1.680	2.015	2.414	2.692	3.286	3.526
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041	45	1.301	1.679	2.014	2.412	2.690	3.281	3.520
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781	46	1.300	1.679	2.013	2.410	2.687	3.277	3.515
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587	47	1.300	1.678	2.012	2.408	2.685	3.273	3.510
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437	48	1.299	1.677	2.011	2.407	2.682	3.269	3.505
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318	49	1.299	1.677	2.010	2.405	2.680	3.265	3.500
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221	50	1.299	1.676	2.009	2.403	2.678	3.261	3.496
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140	51	1.298	1.675	2.008	2.402	2.676	3.258	3.492
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073	52	1.298	1.675	2.007	2.400	2.674	3.255	3.488
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015	53	1.298	1.674	2.006	2.399	2.672	3.251	3.484
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965	54	1.297	1.674	2.005	2.397	2.670	3.248	3.480
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922	55	1.297	1.673	2.004	2.396	2.668	3.245	3.476
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883	60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850	65	1.295	1.669	1.997	2.385	2.654	3.220	3.447
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819	70	1.294	1.667	1.994	2.381	2.648	3.211	3.435
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792	75	1.293	1.665	1.992	2.377	2.643	3.202	3.425
23	1.319	1.714	2.069	2.500	2.807	3.485	3.768	80	1.292	1.664	1.990	2.374	2.639	3.195	3.416
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745	90	1.291	1.662	1.987	2.368	2.632	3.183	3.402
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725	100	1.290	1.660	1.984	2.364	2.626	3.174	3.390
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707	120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690	140	1.288	1.656	1.977	2.353	2.611	3.149	3.361
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674	160	1.287	1.654	1.975	2.350	2.607	3.142	3.352
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659	180	1.286	1.653	1.973	2.347	2.603	3.136	3.345
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646	200	1.286	1.653	1.972	2.345	2.601	3.131	3.340
31	1.309	1.696	2.040	2.453	2.744	3.375	3.633	250	1.285	1.651	1.969	2.341	2.596	3.123	3.330
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622	300	1.284	1.650	1.968	2.339	2.592	3.118	3.323
33	1.308	1.692	2.035	2.445	2.733	3.356	3.611	400	1.284	1.649	1.966	2.336	2.588	3.111	3.315
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601	500	1.283	1.648	1.965	2.334	2.586	3.107	3.310
35	1.306	1.690	2.030	2.438	2.724	3.340	3.591	750	1.283	1.647	1.963	2.331	2.582	3.101	3.304
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582	1000	1.282	1.646	1.962	2.330	2.581	3.098	3.300
37	1.305	1.687	2.026	2.431	2.715	3.326	3.574	∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Degrees of freedom: ν



The F Distribution:

ν_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	30	∞
ν_2	**Critical Values of F Distribution for $A = 0.10$:**															
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27	3.24	3.21	3.17	3.10
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28	2.24	2.20	2.16	2.06
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02	1.97	1.92	1.87	1.76
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.91	1.89	1.84	1.79	1.74	1.61
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77	1.72	1.67	1.61	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.74	1.71	1.66	1.61	1.54	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66	1.60	1.54	1.48	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.63	1.60	1.55	1.48	1.41	1.19
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55	1.49	1.42	1.34	1.00
ν_2	**Critical Values of F Distribution for $A = 0.05$:**															
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.62	4.56	4.50	4.36
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.85	2.77	2.70	2.54
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.40	2.33	2.25	2.07
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.20	2.12	2.04	1.84
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.01	1.93	1.84	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.92	1.84	1.74	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.84	1.75	1.65	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83	1.75	1.66	1.55	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75	1.67	1.57	1.46	1.00
ν_2	**Critical Values of F Distribution for $A = 0.01$:**															
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89	9.72	9.55	9.38	9.02
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71	4.56	4.41	4.25	3.91
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.52	3.37	3.21	2.87
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23	3.09	2.94	2.78	2.42
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84	2.70	2.55	2.39	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	2.52	2.37	2.20	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.35	2.20	2.03	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34	2.19	2.03	1.86	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18	2.04	1.88	1.70	1.00
ν_2	**Critical Values of F Distribution for $A = 0.001$:**															
5	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2	26.9	26.6	26.4	25.9	25.4	24.9	23.8
10	21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96	8.75	8.59	8.45	8.13	7.80	7.47	6.76
15	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.94	5.81	5.54	5.25	4.95	4.31
20	14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.94	4.82	4.56	4.29	4.00	3.38
30	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.11	4.00	3.75	3.49	3.22	2.59
40	12.6	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.75	3.64	3.40	3.14	2.87	2.23
60	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	3.54	3.42	3.32	3.08	2.83	2.55	1.89
120	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.12	3.02	2.78	2.53	2.26	1.54
∞	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	2.96	2.84	2.74	2.51	2.27	1.99	1.00

Numerator degrees of freedom: ν_1; Denominator degrees of freedom: ν_2