UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL 2016 EXAMINATIONS

ECO220Y1Y

Duration - 3 hours

Examination Aids: A non-programmable calculator

This exam includes Part 1, an *Attachment* (with Part 2), and a separate BUBBLE FORM. ***Once the exam begins, please carefully detach the 16-page Attachment.*** The *Attachment* will not be collected and will not be graded.

Part 1 is open-ended questions. Part 2 is multiple-choice questions. The *Attachment* includes: ● Any graphs, tables, or other information that you need to reference for the Part 1 questions, ● The Part 2 multiple-choice questions, and ● The formula sheets and statistical tables (Standard Normal, Student t and F).

Write in pencil and use an eraser as needed. Write your answers to Part 1 on the exam papers and record your answers to Part 2 on the BUBBLE FORM. Marks for Part 2 are based SOLEY on the BUBBLE FORM: in ALL cases what is (or is not) marked on the BUBBLE FORM is your answer. ***You are responsible for turning in all 8 pages of Part 1 and the BUBBLE FORM. You must complete both, including entering your name and student number, before the end of the exam is announced.***

Part 1: 2 written questions, with multiple parts and varying point values, worth a total of 80 points. Write your answers clearly, concisely, and completely below each question. Make sure to show your work and reasoning. Make sure your graphs are fully labeled. A guide for your response ends each question to let you know what is expected: e.g. a quantitative analysis, a graph, and/or sentences. ***Write in pencil and use an eraser as needed. This way you can make sure to fit your answer in the appropriate space.*** Unless otherwise specified, you choose the significance level. (If there are no special considerations, you may choose a 5% significance level.)

Part 2: 18 multiple-choice questions with point values from 1 to 3 points each for a total of 40 points.

Surname (last name):

Given name (first name):

Student #:

	Q1	Q2	Part 1 Total	Part 2 Total	Raw Total	Percent Mark
Point Value:	42	38	80	40	120	
Points Earned:						

(1) [42pts] See the *Attachment for Question (1)*.

(a) [4pts] For Regression #1 (which is statistically significant overall at $\alpha = 0.05$), *fully interpret* the coefficient on corruption_index. Make sure to be context-specific. Answer with 1 sentence.

(b) [6pts] *Show* that Regression #2 is *not* statistically significant overall at $\alpha = 0.10$. Conclusions? How do the conclusions differ from Regression #1? Answer with formal hypotheses, a quantitative analysis, and 1 – 2 sentences.

(c) [6pts] For Regression #3, *fully interpret* both the intercept and slope. Make sure to be context-specific and to comment on the magnitude (size) of the coefficients. Answer with 2 – 3 sentences.

(d) [6pts] For Regression #3, is there a violation of the equal variance assumption (i.e. homoscedasticity assumption, equal spread condition)? *Explain.* The value of s_e (s.d. of residuals, Root MSE, $\sqrt{MSE}$) is 25: is this a good measure of the amount of scatter around the OLS line? *Explain.* Answer with 2 – 3 sentences.

(e) [8pts] For Regression #5, *fully interpret* the coefficient on ln_gni_pc_2005. What does the reported "P-value" of 0.022 mean? Which hypothesis test does it refer to? Answer with 2 – 3 sentences and formal hypotheses.

(f) [12pts] Compare and contrast the conclusions for Regression #5 versus Regression #4. In which important way are they *similar*? In other words, highlight the key conclusion that both regressions support. *Different*? In other words, highlight the key conclusion that differs between the two regressions. Why? In your discussion use the fact that the coefficient of correlation between corruption_index and ln_gni_pc_2005 is -0.7680. Make sure to address whether these data are observational or experimental and the relative importance of corruption norms versus legal enforcement. Answer with 4 – 6 sentences.

(2) [38pts] See the *Attachment for Question (2)*.

(a) [6pts] Each of the bars in Figures 1 and 2 shows a point estimate: the bands at the top of each bar are $\pm$ one standard error (SE). In Figure 2, show how to compute the SE for the first white bar. In Figure 2, *explain why* the SE band is smaller for the first white bar compared to the first gray bar. Answer with the appropriate formula, a quantitative analysis, and 1 – 2 sentences.

(b) [8pts] What if the figures gave the margin of error (ME) for each bar instead of the SE? Illustrate by computing the ME for the first gray bar and the fourth gray bar in Figure 2. Overall, how may this affect the conclusions suggested by the figures? Is checking whether confidence intervals overlap a correct way of checking if callback rates are statistically different? Answer with a quantitative analysis and 2 – 3 sentences.

(c) [12pts] For Asian applicants and the subsample of jobs ads with pro-diversity language, how strong is evidence that the callback rate is higher for applicants who "whitened first name and experience" compared to "no whitening"? Overall, what should you conclude? Infer causality? Answer with formal hypotheses in standard notation, a quantitative analysis, and 2 – 3 sentences.

(d) [6pts] In this table, fill in the six blanks, which appear as _______. Show your work below.

Table 1: Comparing Callback Rates for Applications with No Whitening versus Any Whitening, Both Races Combined

	No whitening	Any whitening	Difference in Callback Rate	z statistic
Panel A: *All job ads, both races combined*				
Callback rate	_______ (_______)	0.187 (0.011)	_______	3.68
Observations	400	1,200	-	-
Panel B: *Job ads with pro-diversity language, both races combined*				
Callback rate	0.110 (0.022)	_______ (_______)	_______	2.88
Observations	200	600	-	-

Notes: "Any Whitening" includes ads that received an application with a "Whitened first name," "Whitened experience," or "Whitened first name and experience." Standard errors given in parentheses below point estimates.

(e) [6pts] Consider the rationale for Table 1 above. *Why* should combining races lead to stronger evidence about the effects of whitening in contrast to doing the analysis separately for each race? *Why* should combining the three levels of whitening into a single category ("any whitening") lead to weaker evidence about the effects of whitening in contrast to comparing "no whitening" with "whitened first name and experience" like in Part (c). Answer with a 3 – 4 sentences.

Please carefully detach this 16-page *Attachment*. This *Attachment* will not be collected. Anything you write on these pages will not be marked: you must write your answers to Part 1 on the Part 1 exam papers (in the designated space immediately after each question) and you must record your answers to Part 2 on the BUBBLE FORM.

This *Attachment* includes:

- Any graphs, tables, or other information that you need to reference for the Part 1 questions
- The Part 2 multiple-choice questions
- The formula sheets and statistical tables (Standard Normal, Student t and F)

Attachment for Question (1): Consider the 2007 academic article "Corruption, Norms, and Legal Enforcement: Evidence from Diplomatic Parking Tickets" published in the *Journal of Political Economy*: Fisman and Miguel (2007). (http://www.jstor.org/stable/10.1086/527495)

ABSTRACT We study cultural norms and legal enforcement in controlling corruption by analyzing the parking behavior of United Nations officials in Manhattan. Until 2002, diplomatic immunity protected UN diplomats from parking enforcement actions, so diplomats' actions were constrained by cultural norms alone. We find a strong effect of corruption norms: diplomats from high-corruption countries (on the basis of existing survey-based indices) accumulated significantly more unpaid parking violations. In 2002, enforcement authorities acquired the right to confiscate diplomatic license plates of violators. Unpaid violations dropped sharply in response. Cultural norms and (particularly in this context) legal enforcement are both important determinants of corruption.

An excerpt of Table 1, which is very long (spread over 6 pages), is below. A footnote to this table explains "The corruption index is from Kaufmann et al (2005). A higher score in the corruption index denotes more corruption."

TABLE 1
AVERAGE UNPAID ANNUAL NEW YORK CITY PARKING VIOLATIONS PER DIPLOMAT, NOVEMBER 1997 TO NOVEMBER 2005

Parking Violations Rank	Country Name	Violations per Diplomat, Pre-enforcement (11/1997–11/2002)	Violations per Diplomat, Postenforcement (11/2002–11/2005)	UN Mission Diplomats in 1998	Corruption Index, 1998
1	Kuwait	249.4	.15	9	−1.07
2	Egypt	141.4	.33	24	.25
3	Chad	125.9	.00	2	.84
4	Sudan	120.6	.37	7	.75
5	Bulgaria	119.0	1.64	6	.50
6	Mozambique	112.1	.07	5	.77
7	Albania	85.5	1.85	3	.92
8	Angola	82.7	1.71	9	1.05
9	Senegal	80.2	.21	11	.45
10	Pakistan	70.3	1.21	13	.76
11	Ivory Coast	68.0	.46	10	.35
12	Zambia	61.2	.15	9	.56
13	Morocco	60.8	.40	17	.10
14	Ethiopia	60.4	.62	10	.25
15	Nigeria	59.4	.44	25	1.01
16	Syria	53.3	1.36	12	.58
17	Benin	50.4	6.50	8	.76
18	Zimbabwe	46.2	.86	14	.13
19	Cameroon	44.1	2.86	8	1.11
20	Montenegro and Serbia	38.5	.05	6	.97
21	Bahrain	38.2	.65	7	−.41

Attachment for Question (1), cont'd:

Figure 1 shows the change after legal enforcement began (notice the vertical scale).

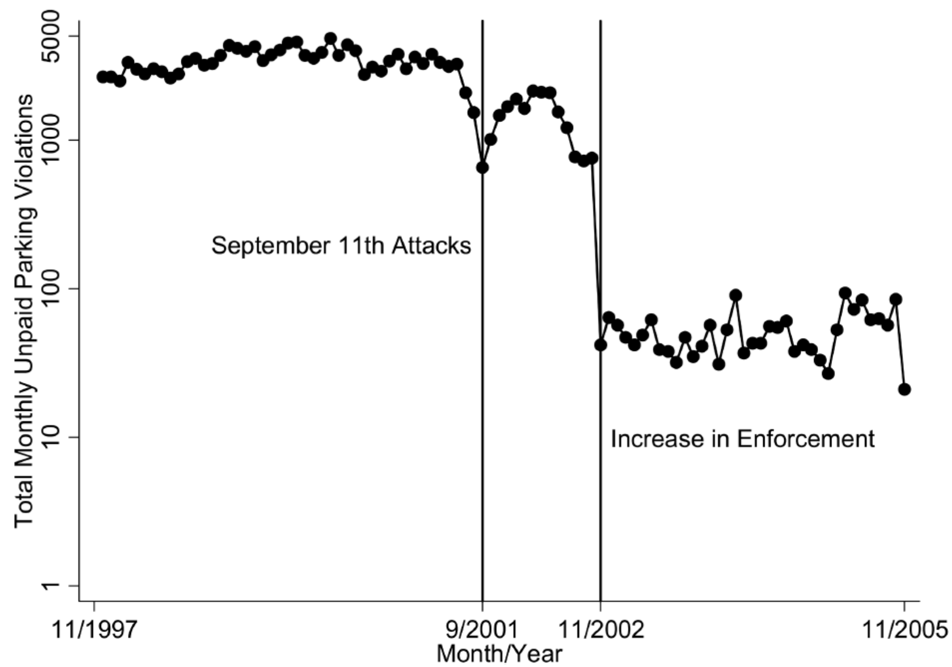
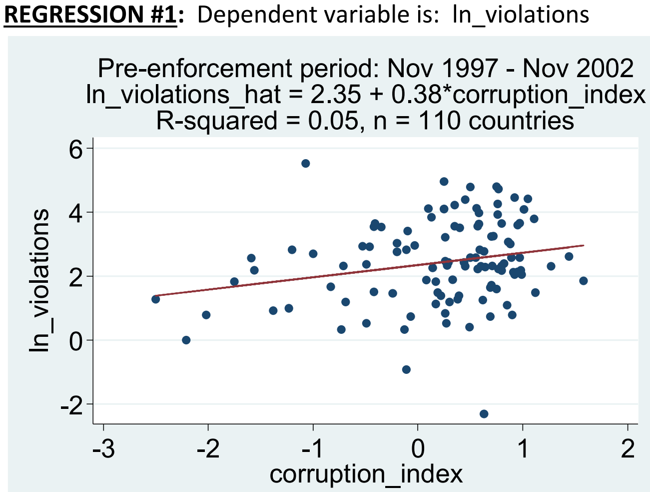


FIG. 1.—Total monthly New York City parking violations by diplomats, 1997–2005 (vertical axis on log scale).

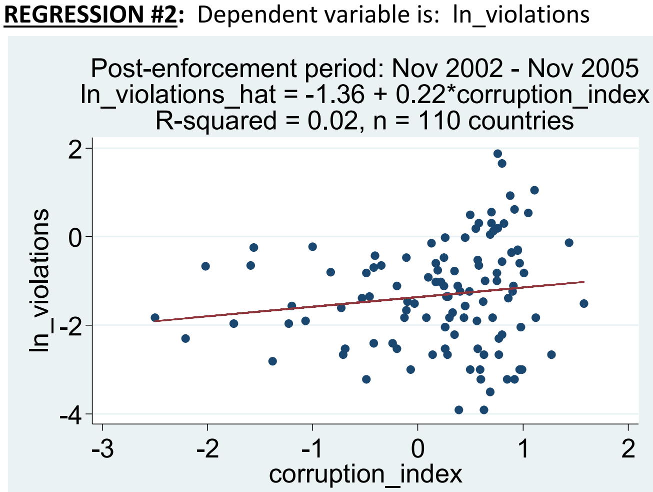
Using 110 countries with non-zero violations, violations per diplomat are observed for two time periods – the pre-enforcement period and the post-enforcement period – for each country.

Variable	Variable description	Obs.	Mean	Median	S.D.	Min.	Max.
violations	Number of violations per diplomat	220	12.449	1.7	27.646	0.02	249.4
ln_violations	Natural logarithm of number of violations per diplomat	220	0.556	0.531	2.259	-3.912	5.519
corruption_index	Corruption Index, 1998	220	0.219	0.420	0.788	-2.500	1.580
post_enforcement	= 1 if post-enforcement period (11/2002 – 11/2005) and 0 otherwise	220	0.500	0.500	-	0	1
ln_gni_pc_2005	Natural logarithm of Gross National Income (GNI) per capita in 2005 ($1,000s of USD)	220	0.644	0.429	1.483	-2.120	3.651

<u>REGRESSION #1</u>: Dependent variable is: ln_violations

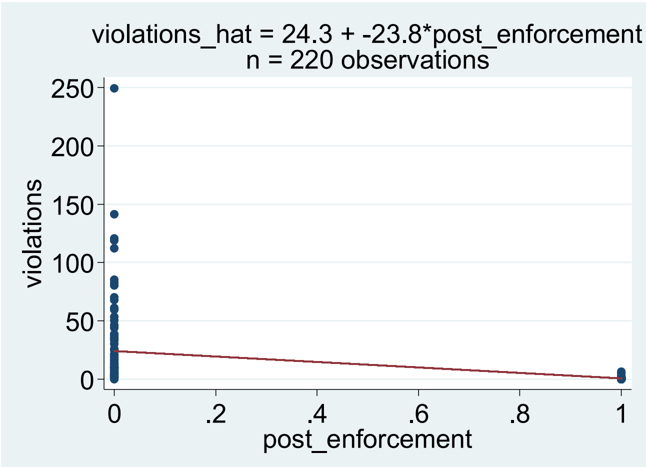


<u>REGRESSION #2</u>: Dependent variable is: ln_violations



Attachment for Question (1), cont'd:

<u>REGRESSION #3</u>: Dependent variable is: violations



<u>REGRESSION #4</u>: Dependent variable is: ln_violations

R-squared = 70.22% R-squared (adjusted) = 69.95%; s_e = 1.2383 with 220 – 3 = 217 degrees of freedom

Source	Sum of Squares	DF	Mean Square	F-ratio
Regression	784.738002	2	392.369001	255.88
Residual	332.749603	217	1.53340831	

Variable	Coefficient	SE(Coeff)	*t*-ratio	P-value
Intercept	2.364769	0.1203283	19.65	0.000
post_enforcement	-3.747768	0.1669735	-22.45	0.000
corruption_index	0.2997261	0.106169	2.82	0.005

<u>REGRESSION #5</u>: Dependent variable is: ln_violations

R-squared = 70.94% R-squared (adjusted) = 70.54%; s_e = 1.2261 with 220 – 4 = 216 degrees of freedom

Source	Sum of Squares	DF	Mean Square	F-ratio
Regression	792.750393	3	264.250131	175.77
Residual	324.737212	216	1.50341302	

Variable	Coefficient	SE(Coeff)	*t*-ratio	P-value
Intercept	2.558218	0.1456623	17.56	0.000
post_enforcement	-3.747768	0.1653323	-22.67	0.000
corruption_index	0.0086781	0.1641518	0.05	0.958
ln_gni_pc_2005	-0.2014249	0.0872512	-2.31	0.022

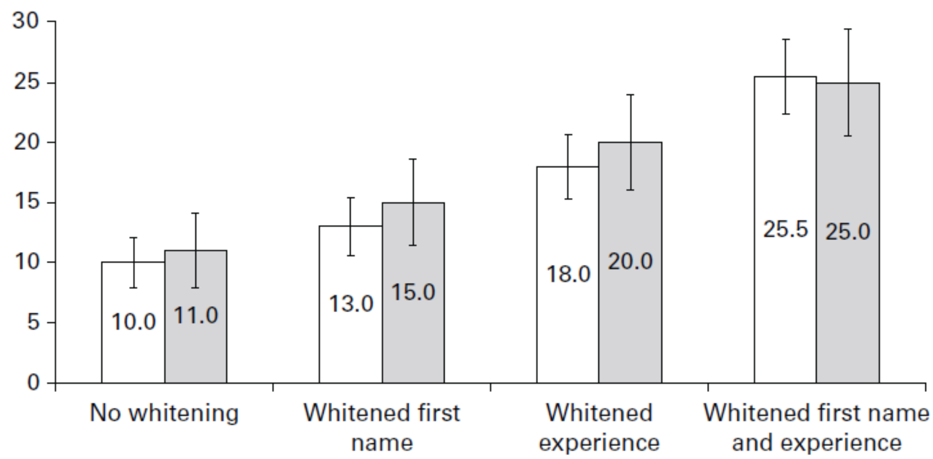
Attachment for Question (2): A March 17, 2016 article in *The Star* – "Jobseekers resort to 'resumé whitening' to get a foot in the door, study shows" (http://www.thestar.com/news/immigration/2016/03/17/jobseekers-resort-to-resum-whitening-to-get-a-foot-in-the-door-study-shows.html) – cites a 2016 journal article.

> **EXCERPT, *The Star*:** It's a disturbing practice called "resumé whitening" and involves deleting telltale signs of race or ethnicity from a CV in the hopes of landing a job. According to a two-year study led by University of Toronto researchers, as many as 40 percent of minority jobseekers "whiten" their resumés by adopting Anglicized names and downplaying experience with racial groups to bypass biased screeners and just get their foot in the door. It's when "Lamar J. Smith" becomes "L. James Smith" or "Lei Zhang" morphs to "Luke Zhang" – and the callback rates soar. "It's really a wake-up call for organizations to do something. Discrimination is still a reality," said Sonia Kang, lead author of [the 2016 journal article] "Whitened Resumés, Race and Self-Presentation in the Labour Market."

Attachment for Question (2), cont'd:

EXCERPT, 2016 journal article: To explore the consequences of resumé whitening in the labor market, we conducted a randomized resumé audit study. This field experimental method involves sending applications from fictitious but realistic job seekers in response to actual job postings. Researchers then examine how randomly assigned resumé content, such as the name or an experience associated with a racial minority group [such as "Vice President, Aspiring African American Business Leaders, Stanford University, 2013–2015"] affects the probability that an applicant is contacted for a job interview. … Our overall sampling frame included entry-level job ads (one per employer) for college graduates, posted in the past 30 days, in 16 geographically dispersed U.S. metropolitan areas. … In total, we responded to 1,600 job postings, of which 800 contained explicit pro-diversity language. … We randomly assigned [each job posting] to one of the eight experimental conditions [shown in Figures 1 and 2]. … In total, 267 (or 16.7 percent) of the 1,600 applications led to a job interview request. Callback rates by condition across all job ads are depicted via the white bars in Figure 1 (for black applicants) and Figure 2 (for Asian applicants). … We [also] examined the subsample of job postings that contained explicit pro-diversity language (see gray bars in Figures 1 and 2).
(doi: 10.1177/0001839216639577)

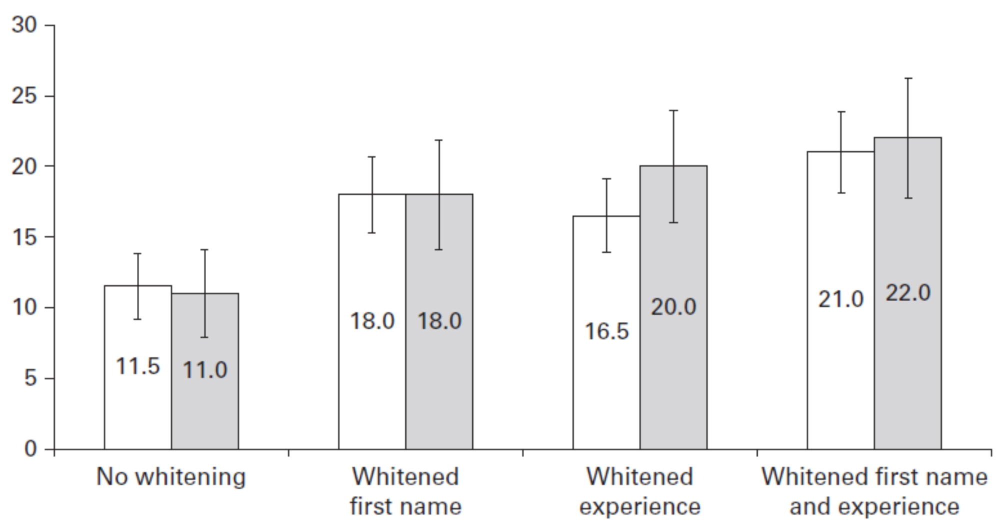
Figure 1. Callback rates for black applicants.



☐ All job ads (N = 200 in each condition)

☐ Subsample of job ads with pro-diversity language (N = 100 in each condition)

Figure 2. Callback rates for Asian applicants.



☐ All job ads (N = 200 in each condition)

☐ Subsample of job ads with pro-diversity language (N = 100 in each condition)

Part 2: 18 multiple-choice questions with point values from 1 to 3 points each for a total of 40 points. Point value for each question shown by [1pt], [2pts] or [3pts]. Most questions have choices **(A)** – **(E)**. For questions with fewer choices, the correct answer is ALWAYS one of those offered (e.g. if the choices are **(A)** – **(D)**, then **(E)** is NOT a possible correct answer.)

- On the FRONT of the BUBBLE FORM: Print your 9 (or 10) digit student number in the boxes AND darken each number in the corresponding circles; Print your last name and initial in the boxes AND darken each letter in the corresponding circles; Fill in the upper left region of the form. (You may leave the FORM CODE blank.)
- On the BACK of the BUBBLE FORM: Write in your name, sign, and record your answers.
- Use a pencil and make dark solid marks that fill the bubble completely.
- Erase completely any marks you want to change; Crossing out a marked box is incorrect.
- Choose the best answer for each question. If more than one answer is selected that question earns 0 points.
- For questions with numeric answers that require rounding, round your final answer to be consistent with the choices offered. Use standard rounding rules.

REMEMBER, you must record your answers to these 18 multiple-choice questions on the BUBBLE FORM.

► **Questions (1) – (2):** Consider the "Daily booster wheel" – see image below – from the online game "Candy Crush." Define fair to mean that each spin yields a random outcome that is independent of all other spins and that the probability of landing on each of the eight wedges is equal.

(1) [2pts] If the wheel is fair, what is the chance that in 32 spins a player never wins the "Jackpot"?

(A) less than 0.001
(B) 0.004
(C) 0.014
(D) 0.031
(E) 0.112

(2) [3pts] Notice that one prize appears twice on the wheel: the dark circle with dots, which is called a "color bomb." If the wheel is fair, what is the chance that in 20 spins a player wins a color bomb 2 times?

(A) less than 0.001
(B) 0.067
(C) 0.071
(D) 0.084
(E) 0.093

(3) [2pts] For a Standard Normal random variable what is the value of $P(-2 < Z < 2)$?

(A) 0.475
(B) 0.477
(C) 0.950
(D) 0.954
(E) 0.997

▶ **Questions (4) – (7):** Consider these joint probability tables downloaded from the Statistics Canada website on March 18, 2016 for females in Canada: "Labour force characteristics by sex and age group" (CANSIM, table 282-0002).

2015, Females:

	15 to 19 years	20 to 24 years	25 to 29 years	30 to 34 years	35 to 39 years	40 to 44 years	45 to 49 years	50 to 54 years	55 to 59 years	60 to 64 years	65 to 69 years	70 years and over
In labor force	0.034	0.060	0.065	0.067	0.065	0.065	0.067	0.075	0.059	0.036	0.013	0.006
Not in labor force	0.032	0.019	0.015	0.016	0.014	0.013	0.013	0.017	0.027	0.039	0.052	0.129

1995, Females:

	15 to 19 years	20 to 24 years	25 to 29 years	30 to 34 years	35 to 39 years	40 to 44 years	45 to 49 years	50 to 54 years	55 to 59 years	60 to 64 years	65 to 69 years	70 years and over
In labor force	0.040	0.061	0.070	0.084	0.083	0.079	0.067	0.045	0.027	0.012	0.004	0.002
Not in labor force	0.041	0.022	0.022	0.026	0.024	0.021	0.021	0.023	0.029	0.040	0.046	0.109

(4) [1pt] If you randomly selected a female 15 years old or older in 2015, she is most likely to be in which age range?

(A) 20 to 24 years old
(B) 45 to 49 years old
(C) 50 to 54 years old
(D) 70 years and over

(5) [2pts] If you randomly selected a woman who is 60 years old or older in 1995, what is the chance that she is in the labor force?

(A) 0.018
(B) 0.031
(C) 0.049
(D) 0.064
(E) 0.085

(6) [2pts] For 2015, if the age distribution and the overall female participation rate were the same, but female labor force participation had nothing to do with age, then what value would be in the cell for "Not in labor force" and "15 to 19 years"?

(A) 0.025
(B) 0.029
(C) 0.033
(D) 0.036
(E) 0.042

(7) [2pts] Compared to women in their 30's in 1995, what can we say about women in their 30's in 2015?

(A) In 2015, women in their 30's are less likely to be in the labor force
(B) In 2015, women in their 30's are more likely to be in the labor force
(C) In 2015, women in their 30's are equally likely to be in the labor force

(8) [2pts] Consider a lottery with an 84% chance of winning nothing, a 10% chance of $10, a 5% chance of $100 and a 1% chance of $10,000. If the random variable X records the outcome of the lottery, what is the standard deviation of X?

(A) $887.32
(B) $994.62
(C) $1,002.33
(D) $1,186.69
(E) $1,254.97

(9) [2pts] Let X be a random variable where the probability model is: $P(X=-1)=P(X=1)=\frac{1}{2}$. Let Y be a random variable where the probability model is: $P(Y=-10)=P(Y=10)=\frac{1}{2}$. Which of these statements is true?

(A) $E[X]>E[Y]$
(B) $E[X]<E[Y]$
(C) $V[X]=V[Y]$
(D) $V[X]>V[Y]$
(E) $V[X]<V[Y]$

(10) [2pts] The weight of a certain species of fish at 5 years of age follows the Normal model. The population mean is 1.7 kg and the population s.d. is 0.2 kg. What is the chance of catching a fish that weighs over 1.8 kg?

(A) 0.1915
(B) 0.2229
(C) 0.2500
(D) 0.2771
(E) 0.3085

(11) [3pts] A bus arrives every 10 minutes to a bus stop. Consider five independent trips. Assume that for each trip the bus is equally likely to arrive any time between 0 to 10 minutes. The expected *total wait time* for the five trips is 25 minutes. What is the standard deviation of *total wait time* for the five trips?

(A) 1.3 minutes
(B) 2.9 minutes
(C) 5.0 minutes
(D) 6.5 minutes
(E) 14.4 minutes

(12) [3pts] Consider an investment portfolio with 10 shares of Stock A and 20 shares of Stock B. An investor is interested in assessing the riskiness of this portfolio. Because future stock performance is unknown, this assessment requires assumptions. Assume that the:

- expected value of Stock A is $40 per share with a standard deviation of $4,
- expected value of Stock B is $60 per share with a standard deviation of $7, and
- Stock A and Stock B share prices have a positive correlation of 0.40.

The expected total value of the investment portfolio is $1,600. What is the standard deviation of the total value of the investment portfolio?

(A) $120
(B) $140
(C) $160
(D) $180
(E) $200

(13) [3pts] Consider the hypothesis test $H_0: p = 0.5$ versus $H_1: p > 0.5$. If $\alpha = 0.05$, $n = 1{,}000$ and the true proportion is $p = 0.52$ then the power $(1 - \beta)$ of the test is 0.35. Which of these would make the test more powerful?

(A) using a larger sample size such as $n = 2{,}000$
(B) using a higher burden of proof such as $\alpha = 0.01$
(C) changing the hypotheses to $H_0: p = 0.51$ versus $H_1: p > 0.51$
(D) using the P-value approach instead of the rejection region approach
(E) All of the above

(14) [2pts] For $H_0: \mu = 0$ versus $H_1: \mu \neq 0$ with a 10% significance level and 9 degrees of freedom, how big must the test statistic be (in absolute value) to reject the null and infer the research (alternative) hypothesis is true?

(A) At least 1.28 (in absolute value)
(B) At least 1.383 (in absolute value)
(C) At least 1.645 (in absolute value)
(D) At least 1.833 (in absolute value)
(E) At least 1.96 (in absolute value)

(15) [3pts] For a random sample of 115 houses, the mean selling price is $531,042 with a s.d. of $159,189. A multiple regression model predicts selling price measured in $1,000's. It includes living area, age, and number of bathrooms as x-variables. The value of s_e = Root MSE = $\sqrt{MSE}$ = 101.081. To test the overall statistical significance of the model, what is the value of the test statistic?

(A) 57
(B) 59
(C) 62
(D) 64
(E) 68

▶ **Questions (16) – (18):** While the association between skipping breakfast and obesity is well-documented, advice on which food is best for breakfast is mixed. A 2010 study[1] emphasizes the benefits of ready-to-eat breakfast cereals, such as Corn Flakes. A 2013 study[2] emphasizes the benefits of a high protein breakfast. Both studies show that skipping breakfast is associated with a higher body mass index (BMI), where BMI = kg/m^2 where a person's weight is measured in kg and height in meters. However, the studies disagree about the optimal breakfast food for controlling BMI.

The 2010 study, which was funded by *Kellogg's* (a cereal manufacturer), uses cross-sectional data from the *National Health and Nutrition Examination Survey*, which is a survey conducted throughout the U.S., that includes 4,320 children (aged 9-13) and 5,339 teenagers (aged 14-18).

The 2013 study, which was funded by *The Beef Checkoff* and the *American Egg Board*, uses data from a randomized controlled trial with 20 participants aged 15–20 living in Missouri (a state in the U.S.). The participants are overweight girls who normally skip breakfast, but are randomized into one of three groups: skip breakfast, eat a high protein breakfast, or eat a normal protein breakfast. Each group is required to follow their assignments for a period of 7 consecutive days.

(16) [2pts] Which of these statements are true?

(A) Both the 2010 and the 2013 study use experimental data
(B) Both the 2010 and the 2013 study rely on observational data
(C) The 2010 study uses observational data and the 2013 study uses experimental data
(D) The 2010 study uses experimental data and the 2013 study uses observational data

(17) [2pts] Which of these is NOT a plausible explanation for any differences in the conclusions across these studies?

(A) The sampling frame differs across the two studies
(B) The 2010 study uses a cluster sample whereas the 2013 study uses a stratified random sample
(C) The 2013 study has a small sample size, which means a lot of sampling error (sampling variability)
(D) The results from the 2010 study are affected by lurking (unobserved/omitted/confounding) variables

(18) [2pts] Consider a multiple regression model to assess how eating breakfast (regbreak = 1 if regularly eat breakfast and 0 otherwise) and its protein intensity (high_pro = 1 if relatively high in protein and 0 otherwise) are associated with the body mass index (BMI). $\widehat{BMI}_i = \beta_0 + \beta_1 regbreak_i + \beta_2 high\_pro_i * regbreak_i + \varepsilon_i$ Which of these results would directly support the claim that high protein breakfasts are better for controlling BMI?

(A) Fail to reject $H_0: \beta_1 = 0$ vs. $H_1: \beta_1 < 0$ with a P-value of 0.300
(B) Fail to reject $H_0: \beta_2 = 0$ vs. $H_1: \beta_2 \neq 0$ with a P-value of 0.300
(C) Reject $H_0: \beta_1 = 0$ in favor of $H_1: \beta_1 > 0$ with a P-value of 0.008
(D) Reject $H_0: \beta_2 = 0$ in favor of $H_1: \beta_2 < 0$ with a P-value of 0.008
(E) Reject $H_0: \beta_2 = 0$ in favor of $H_1: \beta_2 > 0$ with a P-value of 0.008

This is the end of Part 2. Make sure you have recorded your answers on the BUBBLE FORM.

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[1] "The Relationship of Breakfast Skipping and Type of Breakfast Consumption with Nutrient Intake and Weight Status in Children and Adolescents: The National Health and Nutrition Examination Survey 1999-2006" in the *Journal of the American Dietetic Association* (doi: 10.1016/j.jada.2010.03.023).

[2] "Beneficial effects of a higher-protein breakfast on the appetitive, hormonal, and neural signals controlling energy intake regulation in overweight/obese, 'breakfast-skipping,' late-adolescent girls" in the *American Journal of Clinical Nutrition* (doi: 10.3945/ajcn.112.053116).

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Sample mean: $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ **Sample variance:** $s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{X})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$ **Sample s.d.:** $s = \sqrt{s^2}$

Sample coefficient of variation: $CV = \frac{s}{\bar{X}}$ **Sample covariance:** $s_{xy} = \frac{\sum_{i=1}^{n}(x_i-\bar{X})(y_i-\bar{Y})}{n-1} = \frac{\sum_{i=1}^{n} x_i y_i}{n-1} - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(n-1)}$

Sample interquartile range: $IQR = Q3 - Q1$ **Sample coefficient of correlation:** $r = \frac{s_{xy}}{s_x s_y} = \frac{\sum_{i=1}^{n} z_{x_i} z_{y_i}}{n-1}$

Addition rule: $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ **Conditional probability:** $P(A|B) = \frac{P(A\ and\ B)}{P(B)}$

Complement rules: $P(A^C) = P(A') = 1 - P(A)$ $P(A^C|B) = P(A'|B) = 1 - P(A|B)$

Multiplication rule: $P(A\ and\ B) = P(A|B)P(B) = P(B|A)P(A)$

Expected value: $E[X] = \mu = \sum_{all\ x} xp(x)$ **Variance:** $V[X] = E[(X-\mu)^2] = \sigma^2 = \sum_{all\ x}(x-\mu)^2 p(x)$

Covariance: $COV[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = \sigma_{XY} = \sum_{all\ x}\sum_{all\ y}(x-\mu_X)(y-\mu_Y)p(x,y)$

<u>Laws of expected value</u>:

$E[c] = c$

$E[X + c] = E[X] + c$

$E[cX] = cE[X]$

$E[a + bX + cY] = a + bE[X] + cE[Y]$

<u>Laws of variance</u>:

$V[c] = 0$

$V[X + c] = V[X]$

$V[cX] = c^2 V[X]$

$V[a + bX + cY] = b^2 V[X] + c^2 V[Y] + 2bc * COV[X,Y]$

$V[a + bX + cY] = b^2 V[X] + c^2 V[Y] + 2bc * SD(X) * SD(Y) * \rho$

where $\rho = CORRELATION[X,Y]$

<u>Laws of covariance</u>:

$COV[X,c] = 0$

$COV[a + bX, c + dY] = bd * COV[X,Y]$

Combinatorial formula: $C_x^n = \frac{n!}{x!(n-x)!}$ **Binomial probability:** $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$ for $x = 0,1,2,\dots,n$

If X is Binomial $(X \sim B(n,p))$ **then** $E[X] = np$ **and** $V[X] = np(1-p)$

If X is Uniform $(X \sim U[a,b])$ **then** $f(x) = \frac{1}{b-a}$ **and** $E[X] = \frac{a+b}{2}$ **and** $V[X] = \frac{(b-a)^2}{12}$

<u>Sampling distribution of $\bar{X}$</u>:

$\mu_{\bar{X}} = E[\bar{X}] = \mu$

$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma^2}{n}$

$\sigma_{\bar{X}} = SD[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

<u>Sampling distribution of $\hat{P}$</u>:

$\mu_{\hat{P}} = E[\hat{P}] = p$

$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$

$\sigma_{\hat{P}} = SD[\hat{P}] = \sqrt{\frac{p(1-p)}{n}}$

<u>Sampling distribution of $(\hat{P}_2 - \hat{P}_1)$</u>:

$\mu_{\hat{P}_2-\hat{P}_1} = E[\hat{P}_2 - \hat{P}_1] = p_2 - p_1$

$\sigma_{\hat{P}_2-\hat{P}_1}^2 = V[\hat{P}_2 - \hat{P}_1] = \frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}$

$\sigma_{\hat{P}_2-\hat{P}_1} = SD[\hat{P}_2 - \hat{P}_1] = \sqrt{\frac{p_2(1-p_2)}{n_2} + \frac{p_1(1-p_1)}{n_1}}$

<u>Sampling distribution of $(\bar{X}_1 - \bar{X}_2)$, independent samples</u>:

$\mu_{\bar{X}_1-\bar{X}_2} = E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_1-\bar{X}_2}^2 = V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$\sigma_{\bar{X}_1-\bar{X}_2} = SD[\bar{X}_1 - \bar{X}_2] = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

<u>Sampling distribution of $(\bar{X}_d)$, paired $(d = X_1 - X_2)$</u>:

$\mu_{\bar{X}_d} = E[\bar{X}_d] = \mu_1 - \mu_2$

$\sigma_{\bar{X}_d}^2 = V[\bar{X}_d] = \frac{\sigma_d^2}{n} = \frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}$

$\sigma_{\bar{X}_d} = SD[\bar{X}_d] = \frac{\sigma_d}{\sqrt{n}} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 - 2*\rho*\sigma_1*\sigma_2}{n}}$

Inference about a population proportion:

***z* test statistic:** $z = \frac{\hat{P}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ **CI estimator:** $\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

Inference about comparing two population proportions:

***z* test statistic under Null hypothesis of no difference:** $z = \frac{\hat{P}_2-\hat{P}_1}{\sqrt{\frac{\bar{P}(1-\bar{P})}{n_1}+\frac{\bar{P}(1-\bar{P})}{n_2}}}$ **Pooled proportion:** $\bar{P} = \frac{X_1+X_2}{n_1+n_2}$

CI estimator: $(\hat{P}_2 - \hat{P}_1) \pm z_{\alpha/2}\sqrt{\frac{\hat{P}_2(1-\hat{P}_2)}{n_2} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_1}}$

Inference about the population mean:

***t* test statistic:** $t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$ **CI estimator:** $\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

Inference about a comparing two population means, independent samples, unequal variances:

***t* test statistic:** $t = \frac{(\bar{X}_1-\bar{X}_2)-\Delta_0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Degrees of freedom: $\nu = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2+\frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$

Inference about a comparing two population means, independent samples, assuming equal variances:

***t* test statistic:** $t = \frac{(\bar{X}_1-\bar{X}_2)-\Delta_0}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}$ **CI estimator:** $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$ **Degrees of freedom:** $\nu = n_1 + n_2 - 2$

Pooled variance: $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

Inference about a comparing two population means, paired data: (n is number of pairs and $d = X_1 - X_2$)

***t* test statistic:** $t = \frac{\bar{d}-\Delta_0}{s_d/\sqrt{n}}$ **CI estimator:** $\bar{X}_d \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$ **Degrees of freedom:** $\nu = n - 1$

SIMPLE REGRESSION:

Model: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ **OLS line:** $\hat{y}_i = b_0 + b_1 x_i$ $b_1 = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x}$ $b_0 = \bar{Y} - b_1\bar{X}$

Coefficient of determination: $R^2 = (r)^2$ **Residuals:** $e_i = y_i - \hat{y}_i$

Standard deviation of residuals: $s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i-0)^2}{n-2}}$ **Standard error of slope:** $s.e.(b_1) = s_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$

Inference about the population slope:

***t* test statistic:** $t = \frac{b_1 - \beta_{10}}{s.e.(b_1)}$ **CI estimator:** $b_1 \pm t_{\alpha/2} s.e.(b_1)$ **Degrees of freedom:** $\nu = n - 2$

Standard error of slope: $s.e.(b_1) = s_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$

Prediction interval for y at given value of x (x_g):

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \textbf{or} \quad \hat{y}_{x_g} \pm t_{\alpha/2} \sqrt{\left(s.e.(b_1)\right)^2 (x_g - \bar{X})^2 + \frac{s_e^2}{n} + s_e^2}$$

Degrees of freedom: $\nu = n - 2$

Confidence interval for predicted mean at given value of x (x_g):

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{\frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \textbf{or} \quad \hat{y}_{x_g} \pm t_{\alpha/2} \sqrt{\left(s.e.(b_1)\right)^2 (x_g - \bar{X})^2 + \frac{s_e^2}{n}}$$

Degrees of freedom: $\nu = n - 2$

SIMPLE & MULTIPLE REGRESSION:

Model: $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + \varepsilon_i$

$$SST = \sum_{i=1}^{n} (y_i - \bar{Y})^2 = SSR + SSE \quad SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{Y})^2 \quad SSE = \sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$s_y^2 = \frac{SST}{n-1} \quad MSE = \frac{SSE}{n-k-1} \quad Root\ MSE = \sqrt{\frac{SSE}{n-k-1}} \quad MSR = \frac{SSR}{k}$$

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \quad Adj.R^2 = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)} = \left(R^2 - \frac{k}{n-1}\right)\left(\frac{n-1}{n-k-1}\right)$$

Residuals: $e_i = y_i - \hat{y}_i$ **Standard deviation of residuals:** $s_e = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i - 0)^2}{n-k-1}}$

Inference about the overall statistical significance of the regression model:

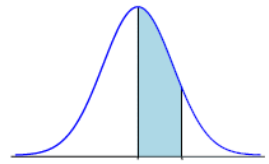
$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)} = \frac{(SST-SSE)/k}{SSE/(n-k-1)} = \frac{SSR/k}{SSE/(n-k-1)} = \frac{MSR}{MSE}$$

Numerator degrees of freedom: $\nu_1 = k$ **Denominator degrees of freedom:** $\nu_2 = n - k - 1$

Inference about the population slope for explanatory variable j:

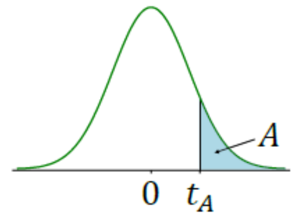
***t* test statistic:** $t = \frac{b_j - \beta_{j0}}{s_{b_j}}$ **CI estimator:** $b_j \pm t_{\alpha/2} s_{b_j}$ **Degrees of freedom:** $\nu = n - k - 1$

Standard error of slope: $s.e.(b_j) = s_{b_j}$ (for multiple regression, must be obtained from technology)



Normal Probabilities:

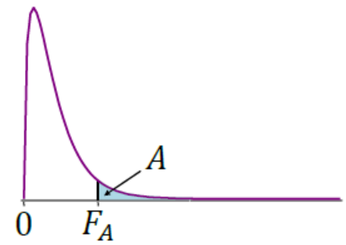
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999



Critical Values of t:

ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	ν	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
1	3.078	6.314	12.706	31.821	63.657	33	1.308	1.692	2.035	2.445	2.733
2	1.886	2.920	4.303	6.965	9.925	34	1.307	1.691	2.032	2.441	2.728
3	1.638	2.353	3.182	4.541	5.841	35	1.306	1.690	2.030	2.438	2.724
4	1.533	2.132	2.776	3.747	4.604	36	1.306	1.688	2.028	2.434	2.719
5	1.476	2.015	2.571	3.365	4.032	37	1.305	1.687	2.026	2.431	2.715
6	1.440	1.943	2.447	3.143	3.707	38	1.304	1.686	2.024	2.429	2.712
7	1.415	1.895	2.365	2.998	3.499	39	1.304	1.685	2.023	2.426	2.708
8	1.397	1.860	2.306	2.896	3.355	40	1.303	1.684	2.021	2.423	2.704
9	1.383	1.833	2.262	2.821	3.250	41	1.303	1.683	2.020	2.421	2.701
10	1.372	1.812	2.228	2.764	3.169	42	1.302	1.682	2.018	2.418	2.698
11	1.363	1.796	2.201	2.718	3.106	43	1.302	1.681	2.017	2.416	2.695
12	1.356	1.782	2.179	2.681	3.055	44	1.301	1.680	2.015	2.414	2.692
13	1.350	1.771	2.160	2.650	3.012	45	1.301	1.679	2.014	2.412	2.690
14	1.345	1.761	2.145	2.624	2.977	46	1.300	1.679	2.013	2.410	2.687
15	1.341	1.753	2.131	2.602	2.947	47	1.300	1.678	2.012	2.408	2.685
16	1.337	1.746	2.120	2.583	2.921	48	1.299	1.677	2.011	2.407	2.682
17	1.333	1.740	2.110	2.567	2.898	49	1.299	1.677	2.010	2.405	2.680
18	1.330	1.734	2.101	2.552	2.878	50	1.299	1.676	2.009	2.403	2.678
19	1.328	1.729	2.093	2.539	2.861	55	1.297	1.673	2.004	2.396	2.668
20	1.325	1.725	2.086	2.528	2.845	60	1.296	1.671	2.000	2.390	2.660
21	1.323	1.721	2.080	2.518	2.831	70	1.294	1.667	1.994	2.381	2.648
22	1.321	1.717	2.074	2.508	2.819	80	1.292	1.664	1.990	2.374	2.639
23	1.319	1.714	2.069	2.500	2.807	90	1.291	1.662	1.987	2.368	2.632
24	1.318	1.711	2.064	2.492	2.797	100	1.290	1.660	1.984	2.364	2.626
25	1.316	1.708	2.060	2.485	2.787	120	1.289	1.658	1.980	2.358	2.617
26	1.315	1.706	2.056	2.479	2.779	140	1.288	1.656	1.977	2.353	2.611
27	1.314	1.703	2.052	2.473	2.771	160	1.287	1.654	1.975	2.350	2.607
28	1.313	1.701	2.048	2.467	2.763	180	1.286	1.653	1.973	2.347	2.603
29	1.311	1.699	2.045	2.462	2.756	200	1.286	1.653	1.972	2.345	2.601
30	1.310	1.697	2.042	2.457	2.750	250	1.285	1.651	1.969	2.341	2.596
31	1.309	1.696	2.040	2.453	2.744	400	1.284	1.649	1.966	2.336	2.588
32	1.309	1.694	2.037	2.449	2.738	∞	1.282	1.645	1.960	2.326	2.576

Degrees of freedom: ν



Critical Values of F: $A = 0.10$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	11	12
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.91	1.89
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.74	1.71
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.63	1.60
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55

Critical Values of F: $A = 0.05$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	11	12
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75

Critical Values of F: $A = 0.01$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	11	12
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.96	9.89
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

Numerator degrees of freedom: ν_1; Denominator degrees of freedom: ν_2