Endowments, Skill-Biased Technology, and Factor Prices: A Unified Approach to Trade *

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ABSTRACT:

We develop a multi-factor, multi-sector Eaton-Kortum model in order to examine the impact of trade costs, factor endowments, and technology (both Ricardian and factor-augmenting) on factor prices, trade in goods, and trade in the services of primary factors (value-added trade). This framework nests the Heckscher-Ohlin-Vanek (HOV) model and the Vanek factor content of trade prediction. We take the model to the data using skilled and unskilled data for 38 countries. We have two findings. First, the key determinants of international variation in the factor content of trade are endowments and international variation in factor inputs used per dollar of output. Input-usage variation in turn is driven by (1) factor-augmenting international technology differences and (2) international factor price differences. Second, our estimates of factor-augmenting international technology differences — which imply cross-country variation in skill-biased technologies — are empirically similar to those used to rationalize cross-country evidence on income differences and directed technical change.

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This paper develops a multi-factor, multi-sector Eaton and Kortum (2002) model featuring the interindustry linkages of Caliendo and Parro (2015) to provide a unified framework for thinking about how technology, endowments, and trade costs impact factor prices, trade in goods, and the factor content of trade. The factor content of trade remains a critical object of interest because of its intimate connection to value-added trade. Value-added trade is the *value* of primary factor services embodied in trade; relatedly, the factor content of trade is the *quantity* of primary factor services embodied in trade i.e., value added trade is the factor content of trade evaluated at domestic factor prices. Therefore the current interest in value-added trade (e.g. Johnson and Noguera, 2012) and global value chains (e.g. Alfaro, Antràs, Chor and Conconi, 2015) should translate into an interest in the factor content of trade and factor prices. This paper is about these two and their determinants.

Our unified framework nests the Heckscher-Ohlin-Vanek (HOV) factor content prediction into a more general prediction that allows for Ricardian and factor-augmenting international technology differences, trade costs, and departures from factor price equalization. Taking the model to empirics using the World Input-Output Database, we make two empirical contributions. *First*, we show that the failure of the Vanek factor-content prediction is largely explained (*i*) by departures from factor price equalization, (*ii*) by factor augmenting international technology differences and, to a lesser extent, (*iii*) by trade costs for government services. *Second*, the factor-augmenting international technology differences that make the Vanek prediction work display the same pattern of skill bias needed to explain cross-country income differences (e.g., Caselli and Coleman, 2006, Caselli, 2016) and cross-country evidence on directed technical change (Acemoglu, 1998). This provides a bridge between the development accounting and directed technical change literatures on the one hand and HOV empirics on the other.

Our starting point is the observation that existing HOV empirics are not embedded in a unified theory of the impacts of productivity, endowments, and trade costs on factor prices and trade. This makes it impossible to reconcile the diverse findings in the literature, let alone assess the relative importance of various determinants within a single framework. Depending on the study, the most important determinant is endowments (e.g., Davis, Weinstein, Bradford and Shimpo, 1997), or Hicks-neutral productivity differences (e.g., Trefler, 1995, Debaere, 2003), or Ricardian productivity differences (e.g., Marshall, 2012, but not Nishioka, 2012), or factor-augmenting productivity differences (e.g., Trefler, 1993a, but not Gabaix, 1997a), or factor prices (e.g., Fadinger, 2011). Trade costs are rarely considered (e.g., Staiger, Deardorff and Stern, 1987). Davis and Weinstein (2001) are unique in ambitiously considering all of these determinants. However, they do not provide an integrated theory and their empirical modelling is in some places based on informal reasoning that creates problems. For example, they use reduced-form estimation to conclude that the failure of the HOV prediction is due in part to the failure of factor price equalization. Using our unified framework we show that their reduced-form estimating equation is very different from that implied by our theory and that the parameters needed to support their conclusion are not identified by their data. This points to the need for a unified theory.

We show that the failure of the Vanek prediction is primarily due to international differences in the use of skilled and unskilled labor that result from (1) substitution effects associated with international factor price differences and (2) factor-augmenting international technology differences. Notice that separately identifying the two is a non-trivial inference problem. For example, China's heavy reliance on unskilled labor across industries is due to a combination of low wages and low productivity for unskilled labor: A model is needed to separately identify these wage and productivity effects. Yet this issue has never been raised in the HOV literature.

Our estimated model successfully fits the entire supply-side content of the model. Specifically, it fits the following: (*a*) A *Techniques* equation relating international differences in factor demands ('techniques') to international differences in wages and technology; (*b*) A *Wage* or factor market clearing equation relating international differences in factor prices to international differences in endowments and technology; and, (*c*) A *Vanek* equation relating the factor content of trade (itself a function of factor prices and technology) to endowments. These three equations describe the *entire* supply side of the model so there are no other supply-side equations we can exploit.

In addition to identifying the key roles of factor prices and factor-augmenting technology, we find that the Vanek equation displays 'missing trade' (Trefler, 1995) unless we treat Government Services as nontradable. This highlights the benefits of developing a model with explicit trade costs.[1]

Turning to the our second contribution, we relate our analysis to the literatures on development accounting and directed technical change. The development accounting literature has shown that richer countries are more productive and that this productivity advantage is more pronounced for skilled than unskilled labor i.e., technology differences are factor-augmenting and skill-biased (Caselli, 2005, Caselli and Coleman, 2006, Caselli and Feyrer, 2007, Caselli, 2016). Using our multi-sector, multi-country method of estimating factor-augmenting technology parameters we find that richer countries are more productive in their use of both skilled and unskilled labor. (In contrast, Caselli and Coleman (2006) find a *negative* relationship between income and unskilled-augmenting technology.) Further, the ratio of our skilled-to-unskilled technology parameters almost exactly equals those that come off of a CES aggregate production function. It is interesting that estimates from a detailed micro model match those from an aggregate macro model.[2]

The directed technical change literature has shown that skill bias is systematically related to factor endowments: Relatively skill-abundant countries should have relatively low skilled wages, but nevertheless direct innovation towards improving the productivity of skilled labor. (See Acemoglu, 1998, Caselli and Coleman, 2006, and Acemoglu, 2009. For open-economy empirics see Acemoglu and Zilibotti, 2001 and Blum, 2010). Our estimates also support a core prediction of the directed technical change literature: On the balanced growth path with an elasticity of

[1] In this paper we focus primarily on the supply side. There is an important literature on the interaction between demand- and supply-side determinants. Notably, Caron, Fally and Markusen (2014) offer important insights into missing trade by introducing demand-side non-homotheticities.

[2] We treat factor-augmenting parameters as technology parameters. Burstein and Vogel (2017) provide a theoretical way of endogenizing these and relating them to firm-size heterogeneity. Malmberg (2017) treats the technology parameters not as technologies but as factor-quality parameters. The two are isomorphic. He then uses the factor-quality parameters in a fascinating decomposition of the sources of cross-country income differences and finds that they are hugely important. We do not attempt such a decomposition. Nor do we estimate parameters using the same type of data. Malmberg uses trade data whereas in our primary specification we use production data. Since trade data likely depend on preferences (e.g., Armington home bias), technology/factor-quality estimates based on trade data likely also depend on preferences.

substitution between skilled and unskilled labor that exceeds unity ($\sigma > 1$), skill-abundant countries are relatively more productive in their use of skilled labor i.e., a country's technical change is directed towards its abundant factor. To our knowledge, Blum (2010) is the only other empirical international trade paper to investigate this issue and he finds the opposite result. Although our multi-sector open-economy model does not nest Acemoglu's (2009, chapter 15) more aggregate closed-economy model, Acemoglu's core induced technical change equation suggests a way of backing out the elasticity of substitution between skilled and unskilled labor from our estimated factor-augmenting technology parameters. The resulting elasticity of 1.67 is squarely within the range of existing estimates from the labor literature.

Literature Review

This paper is most closely related to Davis and Weinstein (2001). There are three notable differences between our work and that of Davis and Weinstein. First, being an older study, they had more limited access to data. (*a*) They only had data for 10 OECD countries, which means that they did not have much variation in the development status of the sample countries. (*b*) They did not have data separately for skilled and unskilled labor, which means that they could not investigate skill bias or directed technical change. (*c*) They did not have factor price data, which means they could not directly investigate their core claim about the importance of failures of factor price equalization for understanding the factor content of trade. We use the World Input-Output Database (Timmer, Dietzenbacher, Los, Stehrer and Vries, 2015), which has data for 38 countries, for skilled and unskilled labor, and for factor prices. Second, Davis and Weinstein modelled substitution effects and the failure of factor price equalization in a reduced-form way; in contrast, we micro-found our model and, in the process, show that Davis and Weinstein were basing their conclusions on parameters that are not identified by their data.[3] Third, they considered Hicks-neutral technology differences whereas we show that factor-augmenting technology differences between skilled and unskilled labor are a key feature of the data for HOV, development accounting, and directed technical change.

Our model is an extension of Caliendo and Parro (2015) to allow for trade in final goods, multiple primary factors and factor-augmenting international technology differences. Other multi-sector and/or multi-factor extensions of Eaton and Kortum (2002) appear in Burstein, Cravino and Vogel (2013), Caron et al. (2014), Caliendo, Dvorkin and Parro (2017b) and Burstein and Vogel (2017). Our model also generalizes the HOV model to allow for heterogeneous firms and trade costs [as in the Melitz-based (2003) models of Bernard, Redding and Schott (2007b) and Burstein and Vogel (2011)]. Finally, our paper is related to the literature on global value chains (e.g. Caliendo, Parro and Tsyvinski, 2017a, and Antràs and de Gortari, 2017).[4]

Sections 1–3 present our general equilibrium model, describe our three estimating equations,

[3]As noted by Diamond, McFadden and Rodriguez (1978), one cannot separately identify substitution effects from factor-augmenting technology differences unless one has data on factor prices and the elasticity of substitution between factors, neither of which Davis and Weinstein used.

[4]To see this, note that value-added trade depends both on the world input-output table of intermediates used per unit of output (**B**) and on a matrix of inputs per unit of output for each factor (**D**). Caliendo et al. (2017a) and Antràs and de Gortari (2017) parameterize and estimate **B**, we parameterize and estimate **D**.

and discuss identification. Section 4 describes the data. Section 5 presents our baseline results and evaluates the performance of the model. Section 6 draws out the implications for factor price equalization, substitution effects, factor-augmenting technology differences, skill bias, development accounting, and directed technical change. Section 7 links our results to HOV 'folklore', and section 8 concludes.

1. Theory

We slightly modify the Caliendo and Parro (2015) multi-sector Eaton and Kortum (2002) model. Let $i,j = 1,...,N$ index countries, $g,h = 1,...,G$ index goods or industries, and let $\omega_g \in [0,1]$ index varieties of good g. A variety is potentially produced by many firms producing a homogeneous product and selling it in perfectly competitive international markets. Unit costs of producing ω_g in country i are given by $c_{gi}/z_{gi}(\omega_g)$ where $z_{gi}(\omega_g)$ is Fréchet-distributed efficiency and c_{gi} is described below. There are also iceberg trade costs: $\tau_{gi,j}$ is the cost of shipping any variety of g from country i to country j or, more succinctly, the cost of shipping (g,i) to j. The $\tau_{gi,j}$ satisfy the triangle inequality. The price of ω_g in country j is therefore

$$p_{gj}(\omega_g) = \min_i \frac{c_{gi}\tau_{gi,j}}{z_{gi}(\omega_g)} \,. \tag{1}$$

1.1. *Households*

Preferences in country i are given by

$$U = \prod_{g=1}^{G} \left\{ \left(\int_0^1 q'_{gi}(\omega_g)^{\frac{\rho_g - 1}{\rho_g}} d\omega_g \right)^{\frac{\rho_g}{\rho_g - 1}} \right\}^{\gamma_{gi}^U}$$

where $q'_{gi}(\omega_g)$ is the amount of ω_g consumed in country i, $\rho_g > 1$ is the elasticity of substitution for consumption, and the non-negative Cobb-Douglas share parameters satisfy $\sum_g \gamma_{gi}^U = 1$.

1.2. *Goods Producers*

Each country i is endowed with an inelastic supply of primary factors V_{fi} where f indexes factors e.g., unskilled labor. Output $q_{gi}(\omega_g)$ of variety ω_g in country i is produced using the Caliendo et al. (2017b) extension of the one-factor Caliendo and Parro (2015) technology:

$$q_{gi}(\omega_g) = z_{gi}(\omega_g) \left\{ \sum_f \alpha_{fg}^{1/\sigma} \left[\lambda_{gi} \pi_{fi} V_{fgi}(\omega_g) \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}\gamma_{gi}} \prod_{h=1}^{G} \left[Y_{h,gi}(\omega_g) \right]^{\gamma_{h,gi}} \tag{2}$$

where $Y_{h,gi}(\omega_g)$ is a good-h CES input bundle with elasticity of substitution ρ_h.[5] $V_{fgi}(\omega_g)$ is the input of primary factor f. σ is the elasticity of substitution between primary factors. π_{fi} is the efficiency of factor f in country i and captures factor-augmenting international technology

[5]As in Caliendo and Parro (2015), this input bundle has the same form as the good-h consumption bundle that appears in braces in the utility function.

differences. It is central to this paper. $z_{gi}(\omega_g)$ are Fréchet with location parameter 1 and shape parameter θ_g. λ_{gi} is the efficiency of industry g in country i and captures Ricardian technology differences.[6] The Cobb-Douglas parameters satisfy $\gamma_{gi} + \sum_{h=1}^{G} \gamma_{h,gi} = 1$ and are non-negative. $\gamma_{h,gi}$ is the share of the good-h intermediate bundle in the cost of producing varieties of (g,i).[7]

The unit cost function for $q_{gi}(\omega_g)$ is $c_{gi}/z_{gi}(\omega_g)$. We next develop an expression for c_{gi}. The unit cost function in i for a good-g input bundle is $P_{gi} \equiv \left[\int \left[p_{gi}(\omega_g)\right]^{1-\rho_g} d\omega_g\right]^{1/(1-\rho_g)}$. A deep insight from Caliendo and Parro (2015) is that in this setting of multi-sector 'roundabout' production, P_{gi} and c_{gi} are jointly determined as solutions to the following two equations:

$$P_{gi} = \kappa_g \left[\sum_{j=1}^{N} \left(c_{gj}\tau_{gj,i}\right)^{-\theta_g}\right]^{-1/\theta g} \tag{3}$$

$$c_{gi} = \kappa_{gi} \left\{ \left[\sum_f \alpha_{fg} \left(\frac{w_{fi}}{\lambda_{gi}\pi_{fi}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \right\}^{\gamma_{gi}} \prod_{h=1}^{G} \left(P_{hi}\right)^{\gamma_{h,gi}} \tag{4}$$

where w_{fi} is the price of factor f in country i and the term in braces is the unit cost function for primary factors.[8]

1.3. *Expenditure Shares*

We next derive the trade shares which Eaton and Kortum denote by π and which we denote by $\widetilde{\pi}$. Specifically, let $\widetilde{\pi}_{gi,j}$ be the share of g that j sources from i. As suggested by equation (3),

$$\widetilde{\pi}_{gi,j} = \frac{\left(c_{gi}\tau_{gi,j}\right)^{-\theta_g}}{\sum_{i'=1}^{N} \left(c_{gi'}\tau_{gi',j}\right)^{-\theta_g}} . \tag{5}$$

The proof is similar to that in Caliendo and Parro (2015).[9]

1.4. *Equilibrium*

We now depart from Caliendo and Parro (2015) in allowing for trade in final goods, an important feature of our empirical analysis.

Income and expenditure: Let Q_{gi} be the value of (g,i) output summed across varieties: $Q_{gi} \equiv \int \left[c_{gi}/z_{gi}(\omega_g)\right] q_{gi}(\omega_g)\, d\omega_g$. The condition that sales of (g,i) equal expenditures on (g,i) is

$$Q_{gi} = \sum_{j=1}^{N} \left[\widetilde{\pi}_{gi,j} \sum_{h=1}^{G} \left(\gamma_{g,hj} + \gamma_{gj}^{U} \gamma_{hj} \right) Q_{hj} \right] . \tag{6}$$

An explanation and proof of this equation appear in the appendix. For what follows, the reader only needs to know that the term in brackets is country j's expenditures on (g,i).

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[6] $(\lambda_{gi})^{\gamma_{gi}}$ replaces Eaton and Kortum's (2002) T_{gi}. For our empirical purposes it makes no difference whether we take λ_{gi} out of the production function and put it into the Fréchet location parameter.

[7] The $\alpha_{h,gi} \geq 0$ control input intensities. The $\alpha_{fg} \geq 0$ control factor intensities and $\sum_f \alpha_{fg} = 1$.

[8] In equations (3)–(4), $\kappa_g = \Gamma((1+\theta_g-\sigma_g)/\theta_g)^{1/(1-\sigma_g)}$ and $\kappa_{gi} = (\gamma_{gi})^{-\gamma_{gi}} \prod_h (\gamma_{h,gi}/\alpha_{h,gi})^{-\gamma_{h,gi}}$. Equation (3) follows from standard Fréchet properties e.g., Caliendo and Parro (2015, eqn. 4). Equation (4) is the dual of equation (2).

[9] λ_{gi} does not appear directly in equation (5) because λ_{gi} is subsumed within c_{gi}. See equation (4).

Goods market clearing: Setting supply equal to demand for a variety of (g,i) yields

$$q_{gi}(\omega_g) = \sum_{j\in J(i)} \tau_{gi,j} \frac{p_{gj}(\omega_g)^{-\rho_g}}{P_{gj}^{1-\rho_g}} \left[\widetilde{\pi}_{gi,j} \sum_{h=1}^{G} \left(\gamma_{g,hj} + \gamma_{gj}^{U} \gamma_{hj} \right) Q_{hj} \right] \tag{7}$$

where $J(i)$ is the set of importers that source ω_g from country i.[10]

Factor market clearing: The exposition of factor market clearing usually builds on an expression for factor demand per unit of a firm's output. However, the input-output data that we will use only records factor demand per *dollar* of an industry's output i.e., $d_{fgi} \equiv V_{fgi}/Q_{gi}$ where $V_{fgi} \equiv \int V_{fgi}(\omega_g)d\omega_g$ is industry (g,i) employment of factor f. We therefore work with d_{fgi}. Using Shephard's lemma and aggregating to the industry level,

$$d_{fgi} = \frac{\gamma_{gi}}{\pi_{fi}} \frac{\alpha_{fg} \left(w_{fi}/\pi_{fi} \right)^{-\sigma}}{\sum_{f'} \alpha_{f'g} \left(w_{f'i}/\pi_{f'i} \right)^{1-\sigma}} \,. \tag{8}$$

The proof appears in the appendix. Factor market clearing is $\sum_g V_{fgi} = V_{fi}$ or

$$\textstyle\sum_{g=1}^{G} d_{fgi} Q_{gi} = V_{fi} \,. \tag{9}$$

Equilibrium: Equilibrium is a set of prices w_{fi} and $p_{gi}(\omega_g)$ which clear factor markets domestically (equation 9) and clear product markets internationally (equation 7) subject to producers minimizing costs (equation 8) and consumers maximizing utility. In equations (7)–(9), the variables $(P_{gi}, c_{gi}, \widetilde{\pi}_{gi,j}, Q_{gi})$ satisfy equations (3)–(6) and the definition of Q_{gi}.

2. Empirical Counterparts, Value-Added Trade and HOV

Input-output tables report data that are aggregated up from varieties to goods (industries) and that are in values. Recall that Q_{gi} is the value of (g,i). Let $C_{gi,j}$ be the value of country j's consumption of (g,i), let $M_{gi,j}$ be the value of country j's imports of (g,i), let X_{gi} be the value of country i's exports of g, and let $b_{gi,hj}$ be the value of intermediate purchases of (g,i) per dollar of (h,j) output. The following lemma relates the theoretical parameters to the data. If we were interested in calibrating trade in final and intermediate goods to underlying technology parameters, as in Caliendo et al. (2017a) and Antràs and de Gortari (2017), then this lemma would be of importance. However, we are not headed in this direction so the lemma is stated primarily for completeness. The proof is in the appendix.

Lemma 1 *(1)* $C_{gi,j} = \widetilde{\pi}_{gi,j} \gamma_{gj}^{U} \sum_{h=1}^{G} \gamma_{hj} Q_{hj}$ *. (2)* $M_{gi,j} = \widetilde{\pi}_{gi,j} \sum_{h=1}^{G} (\gamma_{g,hj} + \gamma_{gj}^{U} \gamma_{hj}) Q_{hj}$ *for* $j \neq i$ *. (3)* $X_{gi} = \sum_{j\neq i} M_{gi,j}$ *. (4)* $b_{gi,hj} = \widetilde{\pi}_{gi,j} \gamma_{g,hj}$ *.*

[10] $\tau_{gi,j}$ appears just after the summation in order to convert demand for delivered goods (q/τ) into demand for supplied goods (q).

Let $\mathbf{Q}_i$, $\mathbf{C}_{ij}$, $\mathbf{M}_{ij}$, and $\mathbf{X}_i$ be $G \times 1$ vectors with gth elements of Q_{gi}, $C_{gj,i}$, $M_{gj,i}$, and X_{gi}, respectively. Let $\mathbf{B}_{ij}$ be a $G \times G$ matrix whose (g,h)-th element is $b_{gi,hj}$. Define the matrices

$$\mathbf{Q} \equiv \begin{bmatrix} \mathbf{Q}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{Q}_N \end{bmatrix}, \qquad \mathbf{C} \equiv \begin{bmatrix} \mathbf{C}_{11} & \cdots & \mathbf{C}_{N1} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_{1N} & \cdots & \mathbf{C}_{NN} \end{bmatrix},$$

$$\mathbf{T} \equiv \begin{bmatrix} \mathbf{X}_1 & -\mathbf{M}_{21} & \cdots & -\mathbf{M}_{N1} \\ -\mathbf{M}_{12} & \mathbf{X}_2 & \cdots & -\mathbf{M}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{M}_{1N} & -\mathbf{M}_{2N} & \cdots & \mathbf{X}_N \end{bmatrix} \text{ and } \mathbf{B} \equiv \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \mathbf{B}_{1N} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \mathbf{B}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{N1} & \mathbf{B}_{N2} & \cdots & \mathbf{B}_{NN} \end{bmatrix}$$

where $\mathbf{Q}$, $\mathbf{C}$, and $\mathbf{T}$ are $NG \times N$ and $\mathbf{B}$ is $NG \times NG$. Output is used for intermediates ($\mathbf{BQ}$), for consumption final demand ($\mathbf{C}$) and for trade $\mathbf{T}$ so that $\mathbf{Q} = \mathbf{BQ} + \mathbf{C} + \mathbf{T}$ or

$$\mathbf{T} = (\mathbf{I}_{NG} - \mathbf{B})\mathbf{Q} - \mathbf{C} \tag{10}$$

where $\mathbf{I}_{NG}$ is the $NG \times NG$ identity matrix. Equation (10) is the goods-market clearing equation and always holds both in equilibrium and in the data.[11]

Let $\mathbf{D}_{fi}$ be a $1 \times G$ vector with gth column element d_{fgi}. Define the $1 \times NG$ vectors $\mathbf{D}_f \equiv [\mathbf{D}_{f1} \cdots \mathbf{D}_{fN}]$ and $\mathbf{A}_f \equiv \mathbf{D}_f(\mathbf{I}_{NG} - \mathbf{B})^{-1}$. Let $\mathbf{T}_i$ be the ith column of $\mathbf{T}$. Define

$$F_{fi} \equiv \mathbf{A}_f \mathbf{T}_i = \mathbf{D}_f(\mathbf{I}_{NG} - \mathbf{B})^{-1}\mathbf{T}_i\,. \tag{11}$$

F_{fi} is the amount of factor f employed worldwide to produce country i's net trade vector $\mathbf{T}_i$. Following Trefler and Zhu (2010), we explain this as follows. Consider tracking the global supply chain needed to produce $\mathbf{T}_i$. Production directly requires a vector $\mathbf{BT}_i$ of intermediate inputs. This is the first link in the global value chain. In the second link, producing $\mathbf{BT}_i$ requires $\mathbf{B}(\mathbf{BT}_i) = \mathbf{B}^2\mathbf{T}_i$ of intermediate inputs and this too can be divided into domestic and foreign intermediates. Adding an infinite number of links to the global value chain, the intermediate inputs required to deliver $\mathbf{T}_i$ is $\sum_{n=1}^{\infty} \mathbf{B}^n\mathbf{T}_i$. Further, the gross output (intermediates plus final output) required is $\mathbf{T}_i + \sum_{n=1}^{\infty} \mathbf{B}^n\mathbf{T}_i = \sum_{n=0}^{\infty} \mathbf{B}^n\mathbf{T}_i = (\mathbf{I}_{NG} - \mathbf{B})^{-1}\mathbf{T}_i$. This explains how our equation (11) definition of the factor content of trade tracks global value chains.

In light of this, it is not surprising that this definition is intimately related to what Johnson and Noguera (2012, forthcoming) as well as Hummels, Ishii and Yi (2001) and Yi (2003, 2010) refer to as trade in value added. To see this without getting bogged down in notation, suppose that the unit in which each primary factor is measured is the annual value of its service flow i.e., its value added. For labor this would be payroll. Then the Johnson-Noguera trade in value added for country i equals $\sum_f w_{fi}F_{fi}$. Extensions to bilateral trade are straightforward.

The Vanek factor content of trade prediction is $F_{fi} = V_{fi} - s_i \sum_j V_{fj}$. Consider a generalization of it. Let $\mathbf{C}_j$ be the jth column of $\mathbf{C}$ and let $\mathbf{C}_w = \sum_j \mathbf{C}_j$. A typical element of $\mathbf{C}_w$ is $C_{gi,w} = \sum_j C_{gi,j}$ i.e., world consumption of (g,i). Let s_i be country i's share of world consumption ($\sum_i s_i = 1$).

[11] Though not obvious, it can be derived by aggregating up equation (7) from varieties to goods. See Trefler and Zhu (2010) for a detailed derivation.

Theorem 1 *(Trefler and Zhu, 2010): Define* $\nu_{fi} \equiv \mathbf{A}_f(\mathbf{C}_i - s_i\mathbf{C}_w)$*. Goods market clearing (equation 10) alone implies*

$$F_{fi} = V_{fi} - s_i \sum_{j=1}^{N} V_{fj} + \nu_{fi}\,. \tag{12}$$

The proof appears in the appendix. There are many reasons to expect $\nu_{fi} \neq 0$ including trade costs ($\tau_{gi,j} > 1$ for $i \neq j$), nontraded sectors ($\tau_{gi,j} = \infty$ for $i \neq j$), international differences in preferences (γ^U_{gi} depends on i), and non-homotheticies (as in Caron et al. 2014). The significance of the theorem flows from the empirical observation, established below, that after correcting for the nontradability of government services, the ν_{fi} are small.

It will be useful later to have a shorthand notation for $\mathbf{C}_j - s_j\mathbf{C}_w$ and its typical element $C_{gi,j} - s_jC_{gi,w}$. We say that *consumption similarity* holds if $C_{gi,j} = s_jC_{gi,w}$ i.e., if country j's share of world consumption of each (g,i) equals s_j. This is a common expression; for instance, if there are no intermediate inputs then $C_{gi,j} = M_{gi,j}$ and $C_{gi,w} = Q_{gi}$, then consumption similarity is just a restatement of gravity without trade costs ($M_{gi,j} = s_jQ_{gi}$). It is straightforward to show in our model that if preferences are internationally identical and trade costs are zero then $\nu_{fi} = 0$.[12]

Finally, it may seem surprising that equation (12) holds so generally: All that is required is goods market clearing. To make the point that much (but not all) of the empirics of this paper hold more generally, in the online appendix we establish all the core equations needed for our empirics using a Krugman (1980) love-of-variety model. There we continue to assume that the production function is separable in primary factors and intermediates and that the primary-factors aggregator is CES as these are both important for our empirics.

3. Empirical Specification

Using equation (8) to calculate $d_{fgi}/d_{fg,\text{us}}$ and rearranging yields

$$d_{fgi} = \beta_{fi} d_{fg,\text{us}}/\delta_{gi} \tag{13}$$

where

$$\beta_{fi} \equiv \left(\frac{w_{fi}/\pi_{fi}}{w_{f,\text{us}}/\pi_{f,\text{us}}}\right)^{-\sigma} \left(\frac{\pi_{fi}}{\pi_{f,\text{us}}}\right)^{-1} \tag{14}$$

and

$$\delta_{gi} \equiv \frac{\gamma_{g,\text{us}}}{\gamma_{gi}} \; \frac{\sum_{f'} \alpha_{f'g}\left(w_{f'i}/\pi_{f'i}\right)^{1-\sigma}}{\sum_{f'} \alpha_{f'g}\left(w_{f',\text{us}}/\pi_{f',\text{us}}\right)^{1-\sigma}}\,. \tag{15}$$

These β_{fi} are central to what follows. The terms in the first and second pairs of parentheses of (14) crystallize the substitution and productivity effects at the heart of Davis and Weinstein (2001) and

[12] To see this, normalize world expenditures to unity so that s_j is j's total expenditures. γ^U_{gj} is the fraction of j's final consumption expenditure allocated to g. $\gamma^U_{gj}\, s_j$ is what j spends on final consumption of g. $\widetilde{\pi}_{gi,j}\, \gamma^U_{gj}\, s_j$ is what j spends on final consumption of (g,i). Hence $C_{gi,j} = \widetilde{\pi}_{gi,j}\, \gamma^U_{gj}\, s_j$. Internationally identical preferences means that $\gamma^U_{gj} = \gamma^U_g$ for all g and j. Zero trade costs means $\tau_{gi,j} = 1$ for all (g,i) and j and so implies that $\widetilde{\pi}_{gi,j}$ is constant across all j for a given (g,i) (see equation 5). Call this $\widetilde{\pi}_{gi}$. Hence, $C_{gi,j} = \widetilde{\pi}_{gi}\, \gamma^U_g\, s_j$. Summing this over j and using $\sum_j s_j = 1$ yields $C_{gi,w} = \widetilde{\pi}_{gi}\, \gamma^U_g$ so that $s_jC_{gi,w} = \widetilde{\pi}_{gi}\, \gamma^U_g\, s_j$. This establishes $C_{gi,j} = s_jC_{gi,w}$ or, in matrix notation, $\mathbf{C}_j = s_j\mathbf{C}_w$. Hence $\nu_{fj} = 0$.

Trefler (1993a), respectively. The (direct) amount of a factor used to produce a unit of output can be high relative to the United States either because of low productivity adjusted wages (substitution effects) or because of low productivity. To avoid confusion, we refer to the $\pi_f \equiv (\pi_{fi}, \ldots, \pi_{fN})$ as 'productivity' parameters and the $\beta_f \equiv (\beta_{fi}, \ldots, \beta_{fN})$ as 'reduced-form' parameters since they capture international differences in both productivity and factor prices.[13]

3.1. *The Three Estimating Equations*

With the additional structure that flows from equations (13)-(15), we can now develop our three estimating equations. These equations completely describe the supply side of the model — there are no other supply-side equations of interest. Consider first the Vanek equation. Recall that $F_{fi} = \mathbf{D}_f(\mathbf{I}_{NG} - \mathbf{B})^{-1}\mathbf{T}_i$ is the factor content of trade using observed factor usage $\mathbf{D}_f$. Let $\mathbf{D}_f(\beta_f)$ be a $1 \times GN$ matrix with typical element $\beta_{fi} d_{fg,\text{us}}/\delta_{gi}$ (the right-hand side of equation 13) and define

$$F_{fi}(\beta_f) \equiv \mathbf{D}_f\left(\beta_f\right)\left[\mathbf{I}_{NG} - \mathbf{B}\right]^{-1}\mathbf{T}_i\ . \tag{16}$$

(We suppress the δ_{gi} as arguments.) Under the assumptions of our model, $\mathbf{D}_f(\beta_f)$ equals the data $\mathbf{D}_f$ and hence $F_{fi}(\beta_f)$ equals the data F_{fi}. It follows that our generalized Vanek equation becomes

$$F_{fi}(\beta_f) = V_{fi} - s_i \textstyle\sum_{j=1}^{N} V_{fj} + \nu_{fi}\ . \tag{V}$$

The equation label (V) is for Vanek. Since $F_{fi}(\beta_f)$ is linear in $\mathbf{D}_f(\beta_f)$ and $\mathbf{D}_f(\beta_f)$ is linear in β_f, $F_{fi}(\beta_f)$ is linear in β_f. Hence, equation (V) can be written as a system of linear equations that uniquely solve for the vector β_f. Notice that the unknown parameters (β_{fi}) show up on the left-hand side of the Vanek equation as in Davis and Weinstein (2001).

Turning to the factor-market clearing equation, or Wage equation for short, substitute factor demands (equation 13) into the factor-market clearing condition (equation 9) and solve for productivity adjusted wages to obtain

$$\frac{w_{fi}/\pi_{fi}}{w_{f,\text{us}}/\pi_{f,\text{us}}} = \left[\frac{\pi_{f,\text{us}}V_{f,\text{us}}}{\pi_{fi}V_{fi}}\right]^{1/\sigma}\left(\sum_{g=1}^{G}\frac{d_{fg,\text{us}}Q_{gi}}{\delta_{gi}V_{f\text{us}}}\right)^{1/\sigma}\ . \tag{17}$$

See the appendix for a proof. The first term (in square brackets) shows that productivity adjusted factor prices are decreasing in productivity adjusted factor supplies, *ceteris paribus*. The second term shows that the price of factor f is bid up if output Q_{gi} is large in sectors with high per-unit demands for factor f, *ceteris paribus*. These demands are high when the sector is intensive in factor f ($d_{fg,\text{us}}$ is large). Rearranging this equation yields our second estimating equation:

$$W_{fi}(\mathbf{D}_f, \mathbf{Q}, V_{fi}, \delta) \equiv \left[\sum_{g=1}^{G}\frac{d_{fg,\text{us}}Q_{gi}}{\delta_{gi}V_{fi}}\right]^{-1} = \beta_{fi} + \varepsilon_{fi}^{W} \tag{W}$$

where $\delta \equiv \{\delta_{gi}\}_{\forall g,i}$, $W_{fi}()$ is a function, and the error term ε_{fi}^{W} is discussed below. The equation label (W) is for Wage, a short form for 'labor-market clearing'.

[13] On a trivial identification issue, we only identify the $\pi_{fi}/\pi_{f,\text{us}}$ and not the π_{fi} and $\pi_{f,\text{us}}$ separately. This is a standard feature of international productivity comparisons.

Turning to the third and last equation, the Techniques equation, we aggregate equation (13) up to the same level as the Vanek and Wage equations, namely, to the factor-country level. Specifically, taking the employment-weighted average of equation (13) yields $\sum_g \theta_{fgi} d_{fgi}/d_{fg,\text{us}} = \sum_g \theta_{fgi}\beta_{fi}/\delta_{gi}$ where $\theta_{fgi} \equiv V_{fgi}/V_{fi}$ is the share of f that is employed in industry g. The θ_{fgi} are data and satisfy $\sum_g \theta_{fgi} = 1$. Rearranging to isolate β_{fi} yields

$$T_{fi}(\mathbf{D}_f, \delta) \equiv \frac{\sum_{g=1}^{G} \theta_{fgi}\left(d_{fgi}/d_{fg,\text{us}}\right)}{\sum_{g=1}^{G} \theta_{fgi}/\delta_{gi}} = \beta_{fi} + \varepsilon_{fi}^{T} \tag{T}$$

where $T_{fi}()$ is a function and the error term ε_{fi}^{T} is discussed below. (The dependence of T_{fi} on the θ_{fgi} is suppressed.) Equation (T) also shows that average factor f usage is hig when θ_{fgi} is large and δ_{gi} small so that $\sum_g \theta_{fgi}/\delta_{gi}$ is large. The equation label (T) is for Techniques. 'Techniques' refers to factor demand choices whereas technology refers to parameters of the cost function.

Turning to the error terms, suppose that the production function (equation 2) is mis-specified. Then equation (13) is mis-specified and requires an error term: $d_{fgi} = \beta_{fi} d_{fg,\text{us}}/\delta_{gi} + \varepsilon_{fgi}$. Starting with this equation and re-deriving equations (W) and (T) generates the errors ε_{fi}^{W} and ε_{fi}^{T}.[14]

3.2. *Calibrating the* δ_{gi}

The only things that are not data in equations (T), (V) and (W) are the δ_{gi} and β_{fi}. Before explaining how we calibrate the δ_{gi} we make the following observations.

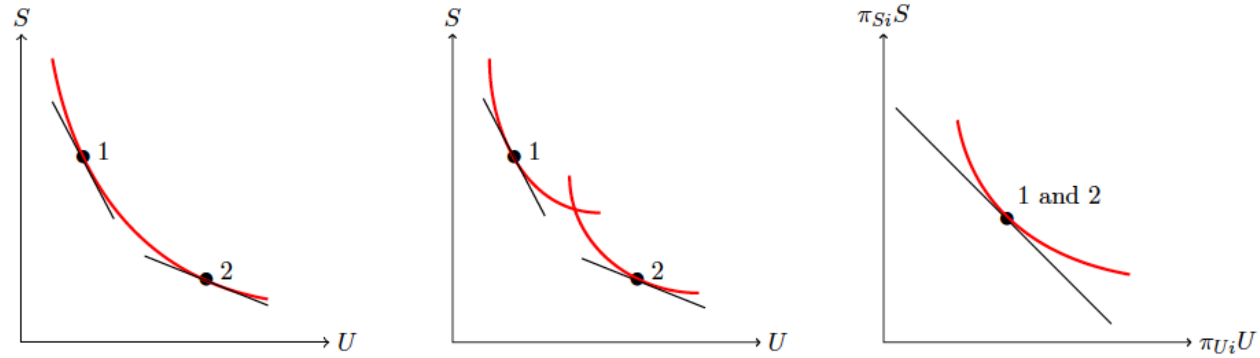
First, equations (13)–(15) imply something very surprising: Conditional on factor prices, factor demands are independent of λ_{gi}. That is, the key data that might be expected to inform Ricardian comparative advantage — the d_{fgi} — are *not* informative of the Ricardian parameters λ_{gi}. To be clear, factor demands *per unit of output* do depend on the λ_{gi}, but these demands are not observable. We only observe factor demands *per dollar of output*. It had been our expectation that in our core equation $d_{fgi} = \beta_{fi} d_{fg,\text{us}}/\delta_{gi}$, the β_{fi} would capture factor-augmenting technology differences (π_{fi}) and the δ_{gi} would capture Ricardian technology differences (λ_{gi}). The latter is incorrect and, as a result, the δ_{gi} are not terribly interesting.

Second, the λ_{gi} and π_{fi} only enter into the model via the term $\lambda_{gi}\pi_{fi}$ in the production function (equation 2). We can therefore always rescale them by a country-specific scalar i.e., $\lambda_{gi}\pi_{fi} = (\lambda_{gi}/\phi_i)(\pi_{fi}\phi_i)$ for any positive constant ϕ_i. ϕ_i is a free parameter that corresponds to a normalization of the productivity terms. This normalization logic extends to the terms β_{fi} and δ_{gi} since they only enter the model via the term $d_{fgi} = d_{fg,\text{us}}\left(\beta_{fi}/\delta_{gi}\right)$ in equation (13). We have already argued that the δ_{gi} are not terribly interesting so it is convenient to normalize them using $\sum_g \theta_{Lgi}\delta_{gi} = 1$ where θ_{Lgi} is the share of country i's labor employed in good g. This loads aggregate productivity onto the π_{fi} allowing us to compare them with previous work (e.g. Caselli and Coleman (2006)). Since d_{fgi} is invariant to this scaling, so is $F_{fi}\left(\beta_f\right)$ (see equation (16)). It follows that our results in section 5 are invariant to our choice of normalization.[15]

[14] $\varepsilon_{fi}^{W} \equiv \sum_g \varepsilon_{fgi} Q_{gi} / \sum_g (d_{fg,\text{us}}/\delta_{gi}) Q_{gi}$ and $\varepsilon_{fi}^{T} \equiv \sum_g (\varepsilon_{fgi}\theta_{gi}/d_{fg,\text{us}}) / \sum_g (\theta_{gi}/\delta_{gi})$.

[15] Specifically, what matters for the section 5 plots is the position of the data relative to the 45° line. Those positions are invariant to the choice of scaling; scaling just relabels each axis.

Figure 1: Identification



(*a*): Identical technologies
Choose curvature

(*b*): Identical curvature
Choose tech. diff.

(*c*): Technology differences
with PFPE

Third and intuitively, $\delta_{gi}/\delta_{g,\text{us}}$ is the average difference in input requirements $d_{fg,\text{us}}/d_{fgi}$ after purging the latter of their β_{fi} components. This suggests three different ways of calibrating the δ_{gi}.[16] (*i*) Regress $\ln d_{fgi}/d_{fg,\text{us}}$ on (g,i) fixed effects and treat the fixed effects as the calibrated $\widehat{\delta}_{gi}$. (*ii*) Regress $\ln d_{fgi}/d_{fg,\text{us}}$ on (g,i) and (f,i) fixed effects and treat the (g,i) fixed effects as the calibrated $\widehat{\delta}_{gi}$. (*iii*) From equation (13) and our normalization, $\delta_{gi} = \delta_{gi}/\sum_{g'}\theta_{Lg'i}\delta_{g'i} = (d_{fg,\text{us}}/d_{fgi})/\sum_{g'}\theta_{Lg'i}(d_{fg',\text{us}}/d_{fg'i})$. δ_{gi} computed in this way depends on the choice of factor f. Taking the geometric mean over two factors (skilled labor S and unskilled labor U) yields our third calibration method:

$$\widehat{\delta}_{gi} \equiv \frac{(d_{Ug,\text{us}}/d_{Ugi})^{1/2}(d_{Sg,\text{us}}/d_{Sgi})^{1/2}}{\sum_{g'=1}^{G}\theta_{Lg'i}(d_{Ug',\text{us}}/d_{Ug'i})^{1/2}(d_{Sg',\text{us}}/d_{Sg'i})^{1/2}}\,. \tag{18}$$

Fortunately, the three calibration methods are equivalent and we thus use the method in equation (18) for the remainder of the paper.[17,18]

To conclude, equations (T), (V) and (W) with the δ_{gi} set equal to the $\widehat{\delta}_{gi}$ are our three estimating equations.

3.3. *Identification*

We say that productivity-adjusted factor price equalization holds if

$$\frac{w_{fi}}{\pi_{fi}} = \frac{w_{f,\text{us}}}{\pi_{f,\text{us}}}\,. \tag{19}$$

[16] By equation (15), $\delta_{g,\text{us}} = 1\,\forall g$.

[17] More precisely, they are equivalent up to a country-specific normalization which we take to be $\sum_g \theta_{Lgi}\delta_{gi} = 1$.

[18] The above three methods weight each factor equally. In the regression methods of (*i*) and (*ii*), the optimal GMM weights depend on the variance of $\ln d_{Ugi}/d_{Ug,\text{us}}$ relative to the variance of $\ln d_{Sgi}/d_{Sg,\text{us}}$. Since these two covariances are very close to being equal, equal weights are close to optimal. Nevertheless, we have experimented by doubling the weight on one factor and halving the weight on the other. In terms of equation (18), this involves changing the exponents from (1/2,1/2) to either (1/4,3/4) or (3/4,1/4). The results reported below are not at all sensitive to this re-weighting.

A surprising conclusion emerges from examination of equations (T), (V) and (W): By themselves they cannot identify the factor-augmenting technology parameters π_{fi} nor the substitution effects associated with failure of PFPE i.e., they cannot be used to answer our major question. In these equations the only unknown parameters are the β_{fi}. Further, the only place where w_{fi}, π_{fi} and σ appear are in the β_{fi}. Let $\widehat{\beta}_{fi}$, $\widehat{\pi}_{fi}$ and $\widehat{\sigma}$ be estimates of β_{fi}, π_{fi} and σ, respectively, so that

$$\widehat{\beta}_{fi} = \left(\frac{w_{fi}/\widehat{\pi}_{fi}}{w_{f,\text{us}}/\widehat{\pi}_{f,\text{us}}} \right)^{-\widehat{\sigma}} \left(\frac{\widehat{\pi}_{fi}}{\widehat{\pi}_{f,\text{us}}} \right)^{-1} . \tag{20}$$

Hence, given data on factor prices and given estimates $\widehat{\beta}_{fi}$, we cannot uniquely identify $(\widehat{\pi}_{fi} , \widehat{\sigma})$: We can only identify combinations of the $\widehat{\pi}_{fi}$ and $\widehat{\sigma}$. This lack of identification is well-known (Diamond et al., 1978) and intimately connected to the main concerns of this paper. To see this, suppose that we observe data on factor prices and the amounts of U and S per unit of output used in two different countries i.e., suppose we observe (w_{Ui},w_{Si}) and (d_{Ugi},d_{Sgi}) for countries $i = 1,2$. Figure 1(a) plots an isoquant in (U,S) space. Points correspond to (d_{Ugi},d_{Sgi}) and slopes to $-w_{Ui}/w_{Si}$. Now consider the problem of estimating cost or demand functions that are consistent with these data. One approach is to make the identifying assumption that technologies are internationally identical and then fit the data by adjusting the curvature of the isoquant. See panel (a). In our context where primary factors enter the production function via a CES aggregator, this means adjusting σ. Another approach is to assume that there are international technology differences so that isoquants differ across countries. See panel (b). In our CES context this means adjusting the π_{fi}. In between there are countless other possibilities involving mixtures of curvature and international technology differences. That is, σ and the π_{fi} are not identified.

Trefler (1993a) is the special case where PFPE is imposed so that $\widehat{\beta}_{fi} = (\widehat{\pi}_{fi}/\widehat{\pi}_{f,\text{us}})^{-1}$ i.e., the π_{fi} are identified, but not σ. It follows that Trefler could not address the Davis and Weinstein (2001) question about the importance of substitution effects when PFPE fails. The point is further illustrated in panel (c) where the axes are productivity adjusted factor inputs so that international differences in technology and factor prices disappear. Since all data for an industry are on a single point, substitution effects along an isoquant cannot be examined and σ cannot be estimated.

Davis and Weinstein (2001) is the special case in which PFPE fails and there are only Hicks-neutral productivity differences ($\pi_{Si} = \pi_{Ui} = \pi_i$). From equation (20), this implies $\widehat{\beta}_{fi} = (w_{fi}/w_{f,\text{us}})^{-\widehat{\sigma}}(\pi_i/\pi_{us})^{\widehat{\sigma}-1}$ which means that one cannot use the reduced-form $\widehat{\beta}_{fi}$ to infer the w_{fi}, π_{fi} and σ separately i.e., to make claims about whether it is international differences in factor prices or technology that are needed to 'fix' the Vanek equation. Since Davis and Weinstein do not use data on factor prices w_{fi} or external estimates of the elasticity of substitution σ, their reduced-form estimates cannot support their claims. And, given Diamond et al.'s (1978) more general non-identification result, this identification problem holds for any cost function. Surprisingly, then, Davis and Weinstein (2001) ask a great question but do not answer it.[19]

Given these identification issues our strategy is as follows. We estimate the β_{fi} from equations (T) and (W), choose a value of $\widehat{\sigma}$ that comes out of a directed technical change equation and that is also consistent with the labor literature, and use wage data together with the $\widehat{\beta}_{fi}$ and equation (20) to back out the $\widehat{\pi}_{fi}$.[20] We can then examine whether the $\widehat{\pi}_{fi}$ are consistent with PFPE (i.e. $w_{fi}/\widehat{\pi}_{fi} = w_{f,\text{us}}/\widehat{\pi}_{f,\text{us}}$) and answer the Davis and Weinstein question. This in turn is linked to the development accounting exercise of Caselli and Coleman (2006) in which wage and endowment data are used to calculate and characterize the skill bias of technology and the directed technical change of Acemoglu (1998) in which the skill bias is partially explained by endowments.

4. The Data

Unless otherwise noted, all data come from the World Input-Output Database (WIOD) as assembled by Timmer et al. (2015). This data set has the advantage of providing information on the full world input-output matrix **B** and satisfying all data identities. Our data cover 38 developed and developing countries and 22 industries in the year 2006.[21] Countries and industries are listed in the appendix. WIOD includes trade data (**T**), input-output data (**B**), output data (Q_{gi}), and data on labor by industry, type, and country (V_{fgi}). Direct input requirements are $d_{fgi} = V_{fgi}/Q_{gi}$. Skilled workers (S) are those possessing some tertiary education. Unskilled workers (U) are the remainder of the labor force. The wage (w_{fi}) for each factor in a given country is given by aggregate compensation to the factor divided by the aggregate number of workers possessing that level of education.[22]

5. Results

5.1. *The Wage and Techniques Equations*

We begin by estimating the β_{fi} from the (W) and (T) equations. We do this separately by factor. To this end, we stack the T_{Ui} and W_{Ui} and regress the stacked vector on a set of country dummies to estimate the β_{Ui}. We then repeat this for skilled labor to estimate the β_{Si}. Denote these OLS estimates by $\widehat{\beta}_{fi}$. (Throughout this paper a β_{fi} with a 'hat' always refers to these estimates.) To deepen our understanding of these estimates, note that for each (f,i) pair, equation (W) defines a

[19]What makes lack of identification surprising in their context is that their approach is very intuitive. They assume a reduced-form relationship between wages and endowments that is reminiscent of that in Katz and Murphy (1992). In our setting this is $\ln(w_{Si}/w_{Ui}) = -\xi \ln(V_{Si}/V_{Ui})$ for some constant ξ. It would thus seem that endowments can be used in place of wages and estimates of $1/\xi$ can be used in place of σ. We have tried without success to write down a cost function to support this intuition. Part of the problem is that the Katz and Murphy logic is based on the demand for labor in a single sector model. Adding in market clearing (demand *and* supply) and multiple sectors undermines this logic i.e., $\ln(w_{Si}/w_{Ui}) = -\xi \ln(V_{Si}/V_{Ui})$ is incompatible with labor-market clearing (equation 9).

[20]We discuss our choice of $\widehat{\sigma}$ in detail below. It will be 1.67 in our baseline specification below.

[21]We use 2006 because it is the most recent year before the Great Recession and the subsequent trade collapse.

[22] The Techniques and Wage equations (T) and (W) are defined so as to be unit free and thus naturally scaled. In contrast, the Vanek equation (V) is not unit free. Therefore, throughout this paper, we scale the Vanek equation by σ_f where $\sigma_f^2 \equiv \Sigma_i(V_{fi} - s_i V_{fw})^2/N$. (This is a variance because $V_{fi} - s_i V_{fw}$ has a zero mean.) All the important results in this paper are invariant to the choice of scaling: Scaling simply eases visual exposition.

β^W_{fi} that makes the Wage equation fit perfectly. Likewise, equation (T) defines a β^T_{fi} that makes the Techniques equation fit perfectly. Our OLS estimator satisfies $\widehat{\beta}_{fi} = (\beta^W_{fi} + \beta^T_{fi})/2$.[23]

Figure 2 presents the results. The left- and right-hand plots display results for unskilled and skilled labor, respectively. The top row plots the Wage equation (W), meaning, it plots $W_{fi}(\mathbf{D}_f, \mathbf{Q}, V_{fi}, \widehat{\delta})$ against $\widehat{\beta}_{fi}$. The middle row plots the Techniques equation (T), meaning, it plots $T_{fi}(\mathbf{D}_f, \widehat{\delta})$ against $\widehat{\beta}_{fi}$. It is clear that the fit is very good.

The Techniques and Wage equations are not collinear. However, there is only one source of error that prevents them from fitting perfectly, namely, that equation (13) does not fit perfectly.[24] If equation (13) were to fit perfectly, then the (W) and (T) equations would fit perfectly. To investigate the fit of equation (13), the bottom panels of figure 2 plot $\ln d_{fgi}/d_{fg,\text{us}}$ against $\ln \widehat{\beta}_{fi}/\widehat{\delta}_{gi}$. As is apparent, the fit is very good. The R^2 for unskilled and skilled labor are 0.89 and 0.84, respectively. This explains why the (W) and (T) equations *both* fit so well.

The $\widehat{\beta}_{fi}$ are reported in appendix table A1. Note that we can easily reject the hypothesis that $\widehat{\beta}_{Ui} = \widehat{\beta}_{Si}$ for all i at way less than the 1% level.[25]

5.2. *The Vanek Equation*

We begin by plotting the factor content of trade against its prediction. Since we predict techniques so well, actual factor demands $\mathbf{D}_f$ are very close to estimated factor demands $\mathbf{D}_f(\widehat{\beta}_f)$ and, consequently, actual factor contents F_{fi} are very close to estimated factor contents $F_{fi}(\widehat{\beta}_f)$. We therefore only report the results for $F_{fi}(\widehat{\beta}_f)$.[26] The top row of figure 3 plots $F_{fi}(\widehat{\beta}_f)$ against $V_{fi} - s_i V_{fw}$. The left- and right-hand panels are for unskilled and skilled labor, respectively. The good news for the Vanek equation is that the correlation is very high: the spearman rank correlation is 0.97 for unskilled labor and 0.98 for skilled labor. There are two outliers, China to the right and the United States to the left. The correlations without these outliers are also very high, 0.96 and 0.98 for unskilled and skilled labor, respectively. Also note that the share of observations for which $F_{fi}(\widehat{\beta}_f)$ and $V_{fi} - s_i V_{fw}$ have the same sign (the 'sign test') is 0.95 for both unskilled and skilled labor. This good fit of the Vanek equation for skilled and unskilled labor is a new result in the literature. [27]

[23] Alternatively, we could have done feasible GLS. That is, for factor f, let h^W_f and h^T_f be estimates of the inverse of the variances of the (W) and (T) equations. Then the feasible GLS estimator is $\widehat{\beta}^{FGLS}_{fi} = \beta^W_{fi} h^W_f + \beta^T_{fi} h^T_f$. $\widehat{\beta}^{FGLS}_{fi}$ is virtually identical to the OLS estimate $\widehat{\beta}_{fi}$. Note that feasible GLS is GMM. Also note that GMM with optimal weighting is both biased in small samples (Altonji and Segal, 1996) and unworkable here because the optimal-weight GMM estimator will fit one equation perfectly and set the weight on the other equation to 0. That is, it will either choose β^W_{fi} and $h^T_f = 0$ or choose β^T_{fi} and $h^W_f = 0$. Finally, since the correlation between β^W_{fi} and β^T_{fi} is 0.99, all these estimators yield very similar estimates.

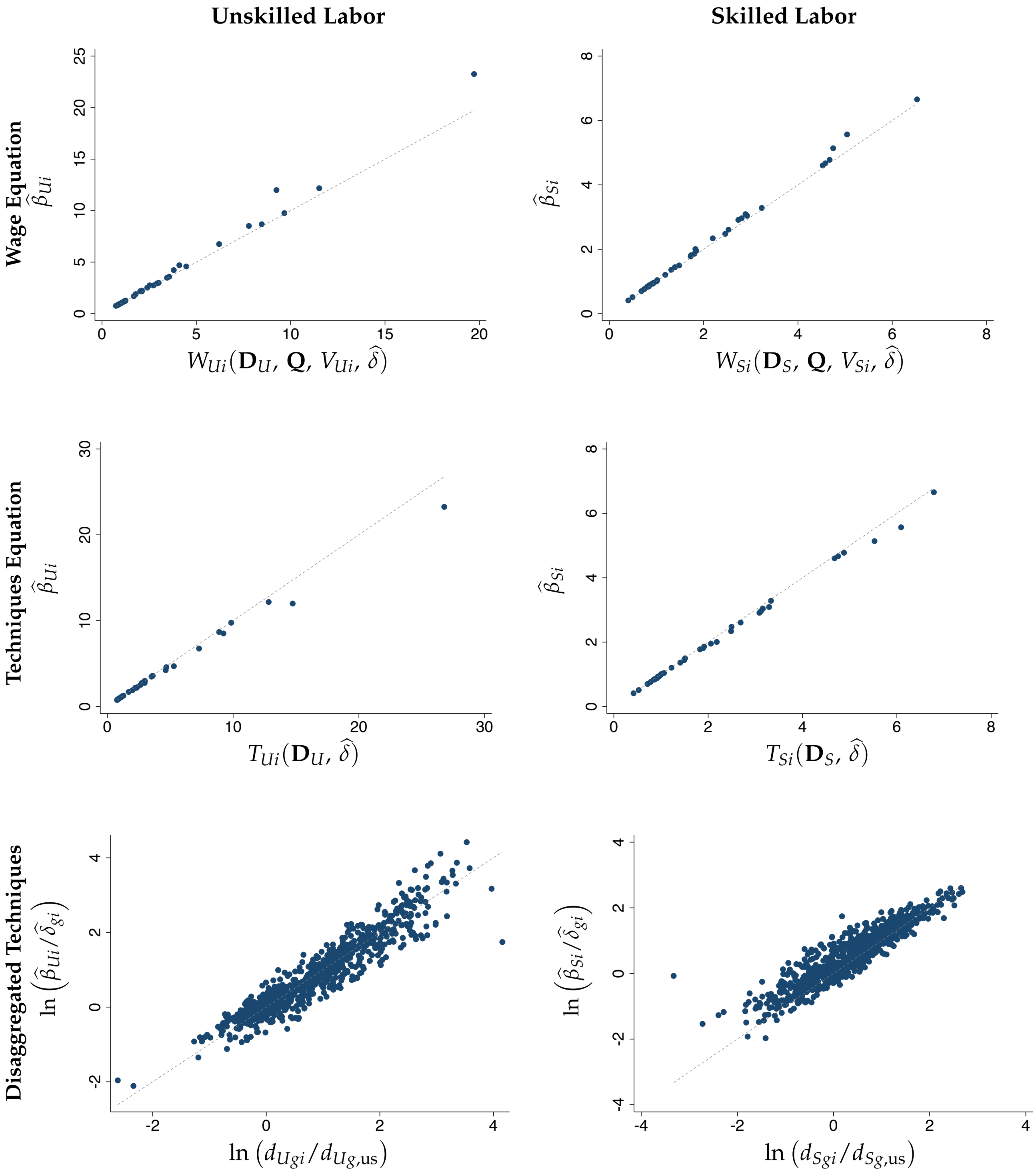
[24] See foonote 14 above.

[25] $F^{38}_{76} = 27.56$ where the 1% critical value is $F^{38}_{76} = 1.88$.

[26] The reason for the lack of a perfect fit between F_{fi} and $F_{fi}(\widehat{\beta}_f)$ is the error term ε_{fgi} described in the last paragraph of section 3.1.

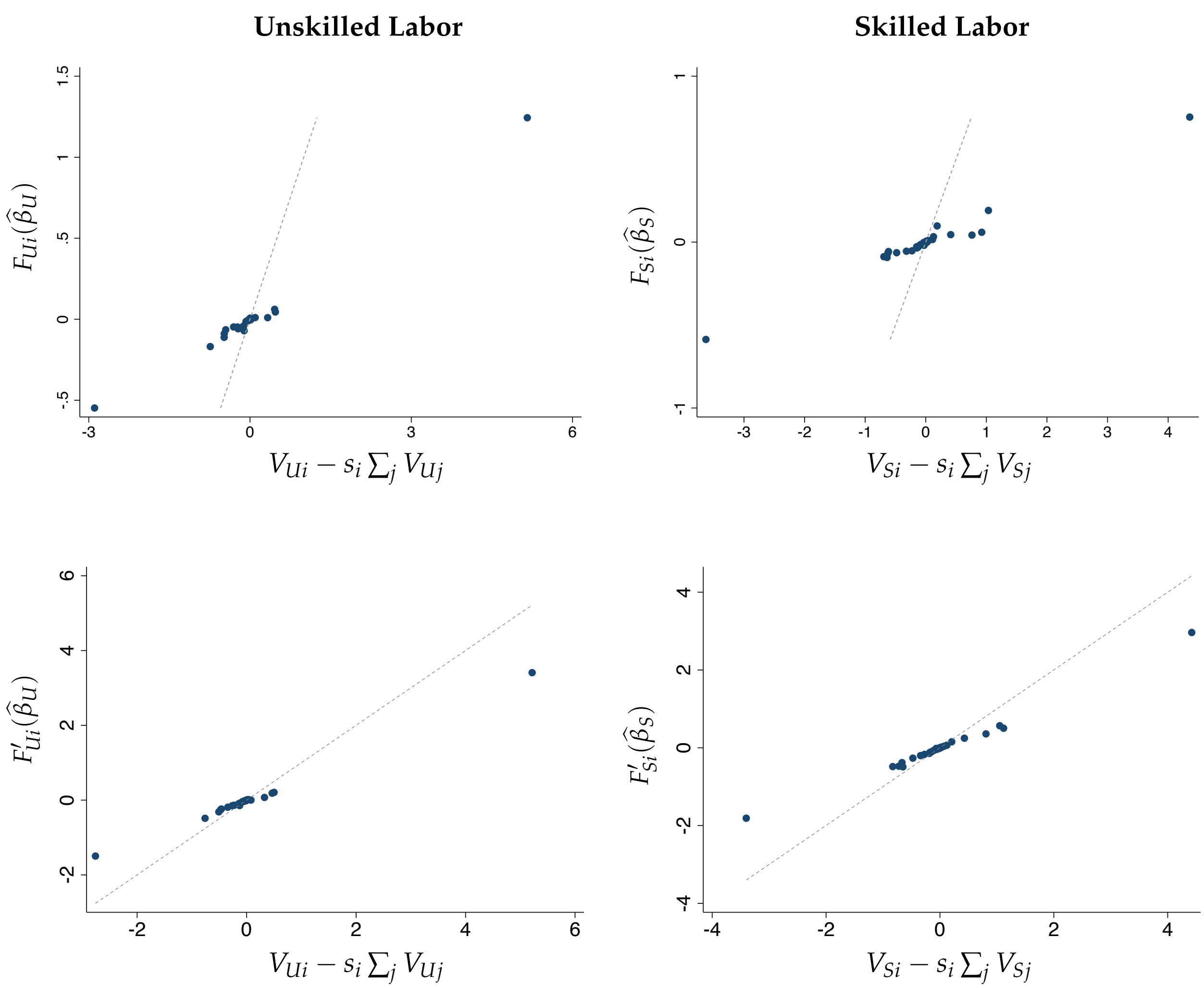
[27] The good fit has been documented by Trefler and Zhu (2010) for the case of aggregate labor. We cannot directly compare our results to Davis and Weinstein (2001) because they use aggregate labor and Hicks-neutral technology, but the results are much better than those associated with their most similar specification (T3).

Figure 2: Performance of the Wage and Techniques Equations: Two-Equation Approach



Notes: The left-hand side plots are for unskilled labor, the right-hand side plots are for skilled labor. The top panels are the Wage equation (W). The middle panels are the Techniques equations (T). The bottom panels are relative factor demands from equation (13), namely, $\ln(d_{fgi}/d_{fg,\mathrm{us}}) = \ln(\beta_{fi}/\delta_{gi})$. All equations are evaluated at the estimated values $\widehat{\beta}_{fi}$ and calibrated values $\widehat{\delta}_{gi}$. In the top and middle panels, each observation is a factor and country (f,i) while in the bottom panels each observation is a factor, industry and country (f,g,i). The 45° line is displayed in each panel.

Figure 3: Performance of the Vanek Equation



Notes: All panels plot the Vanek equation (V). The left- and right-hand columns are for unskilled and skilled labor, respectively. The top row uses the factor content of trade i.e., $F_{fi}(\widehat{\beta}_f)$. The bottom row uses the Government Services adjusted factor content of trade i.e., $F'_{fi}(\widehat{\beta}_f)$. Each observation is a factor and country (f,i) and the most extreme observations in each panel are the United States (to the left) and China (to the right). All lines are 45° lines.

5.2.1. Missing Trade

There is only one problem with the performance of the Vanek equation, namely, Trefler's (1995) 'missing trade.' This is apparent from the displayed 45° line, which is steeper than a line of best fit.[28] It is obvious that if one wants to explain missing trade then one must deal with trade costs and especially nontraded services e.g., Trefler (1995) and Davis and Weinstein (2001). Further, an immediate implication of theorem 1 and equation (12) is that if nontradables lead to departures from the Vanek equation then they do so via departures from consumption similarity.

Since this paper is primarily concerned with the role of factor prices and factor-augmenting technology, and since the handling of missing trade is orthogonal to these issues (i.e., it has no impact on how we estimate the β_{fi}), we focus solely on the most obvious source of nontradables, namely, Government Services, which *by definition* should be nontraded.[29]

We follow Trefler and Zhu (2010) in our treatment of Government Services. For expositional simplicity, assume for the moment that there are no intermediate inputs so that consumption similarity is the same as gravity without trade costs. Also, let $g = G$ denote Government Services. Had consumption similarity held in Government Services, country i would have consumed a share s_i of the Government Services produced by country j (Q_{Gj}). Hence, i's imports of Government Services from j would have been $s_i Q_{Gj}$ and country j's exports to the world would have been $(1 - s_j)Q_{Gj}$ (production less consumption). Let $\mathbf{T}'_i$ be the vector $\mathbf{T}_i$ in equation (11), but with the elements corresponding to $M_{Gi,j}$ and X_{Gj} replaced by $s_j Q_{Gi}$ and $(1 - s_j)Q_{Gj}$, respectively. The easy generalization of $\mathbf{T}'_i$ to include intermediate inputs, which is what we use for the remainder of the paper, is described in the appendix.

With $\mathbf{T}'_i$ in hand we can compute the adjusted factor content of trade as $F'_{fi} \equiv \mathbf{D}_f \left[\mathbf{I}_{NG} - \mathbf{B}\right]^{-1} \mathbf{T}'_i$ or, using estimated techniques,

$$F'_{fi}(\widehat{\beta}_f) \equiv \mathbf{D}_f \left(\widehat{\beta}_f\right) \left[\mathbf{I}_{NG} - \mathbf{B}\right]^{-1} \mathbf{T}'_i \,. \tag{21}$$

The bottom row of figure 3 plots $F'_{fi}(\widehat{\beta}_f)$ against $V_{fi} - s_i V_{fw}$. As is apparent, missing trade is much less: The OLS slope is 0.65 for unskilled labor and 0.66 for skilled labor. Online appendix figure A1 drops the outliers China and the United States and shows that the fit inside the 'pack' is very good. Further, we could raise the slope even more if we treated other sectors such as Construction as nontradable.

Our approach is surprisingly similar to that of Davis and Weinstein (2001), who deal with nontradables by netting out the endowments used to produce nontradables i.e., by netting out the factor content of nontradables. The following lemma establishes this. It appeals to the fact that if preferences are internationally identical and trade costs are zero then $C_{gj,i} = s_i C_{gj,w}$.[30]

[28] The slope from an OLS regression of $F_{fi}(\widehat{\beta}_f)$ on $V_{fi} - s_i V_{fw}$ is 0.23 for unskilled labor and 0.16 for skilled labor.

[29] That is, System of National Accounts manuals instruct national statistical agencies to exclude from this sector all government services that are sold via market transactions. By way of example, Canadian postal services are sold to the public but police services are not so that Government Services excludes the post office but not the police. Since by definition Government Services are not sold on markets, they are nontraded — we do not see California state troopers patrolling the streets of São Paolo. We therefore need to treat Government Services differently.

[30] That is, i's consumption of (g,j) is proportional to world consumption of (g,j) for all g where the constant of proportionality is i's expenditure share s_i. See footnote 12.

Lemma 2 *Let G' be the set of nontradable goods. Define $V'_{fi} \equiv \sum_{g \in G'} A_{fgi} C_{gii}$ as the factor content of nontradable consumption or, equivalently, the endowments used to produce nontradable consumption. Let $V'_{fw} \equiv \sum_j V'_{fj}$. Assume that $C_{gj,i} = s_i C_{gj,w}$ for all i and j and all $g \notin G'$ (i.e., for tradable goods). Recall from theorem 1 that $F_{fi} = V_{fi} - s_i V_{fw} - \mathbf{A}_f\left(\mathbf{C}_i - s_i \mathbf{C}_w\right)$. Then*

1. $\mathbf{A}_f\left(\mathbf{C}_i - s_i \mathbf{C}_w\right) = V'_{fi} - s_i V'_{fw}$ *and* $F_{fi} = V_{fi} - s_i V_{fw} - \left[V'_{fi} - s_i V'_{fw}\right]$
2. $\mathbf{A}_f\left(\mathbf{C}_i - s_i \mathbf{C}_w\right) = F'_{fi} - F_{fi}$ *and* $F'_{fi} = \left(V_{fi} - s_i V_{fw}\right)$

Part 1 states that under the conditions of the lemma, one can derive the type of estimating equation first examined by Davis and Weinstein. The second part derives our estimating equation. Together, the two parts imply that the approaches are equivalent. The term $\mathbf{A}_f\left(\mathbf{C}_i - s_i \mathbf{C}_w\right)$ appears on the right side in Davis and Weinstein and on the left side in our work.

The advantage of putting $\mathbf{A}_f\left(\mathbf{C}_i - s_i \mathbf{C}_w\right) = V'_{fi} - s_i V'_{fw}$ on the right side is that netting out the factor content of nontradable endowments is intuitive. The disadvantage is that this term is endogenous and so belongs on the left side. Empirically, if we put it on the right side there is not much improvement in missing trade (the slope is only 0.28 for unskilled labor and 0.29 for skilled labor). The reason is simple: $V'_{fi} - s_i V'_{fw}$ is small relative to $V_{fi} - s_i V_{fw}$ so that the latter term dominates and the slope does not rise much.

5.2.2. *The Role of the $\widehat{\beta}_f$ for the Vanek Equation*

To investigate the role of the $\widehat{\beta}_f$ for the fit of the Vanek equation, in figure 4 we set $\beta_f = (\beta_{f1}, \ldots, \beta_{fN}) = \iota$ where ι is an N-vector of ones. $F'_{fi}(\iota)$ is the factor content of trade if all countries use US techniques save for differences in δ_{gi}. We plot $F'_{fi}(\iota)$ against $V_{fi} - s_i V_{fw}$. The fit is horrible in two dimensions. First, there is an increase in missing trade, which is related to previous findings (Trefler, 1995, Davis and Weinstein, 2001, Trefler and Zhu, 2010) that missing trade is exacerbated when all countries are forced to have the same choice of techniques.[31] Second, the rank correlation deteriorates: 0.44 for unskilled labor and 0.26 for skilled labor. We conclude from this that the β_f play a central role for understanding the Vanek prediction.

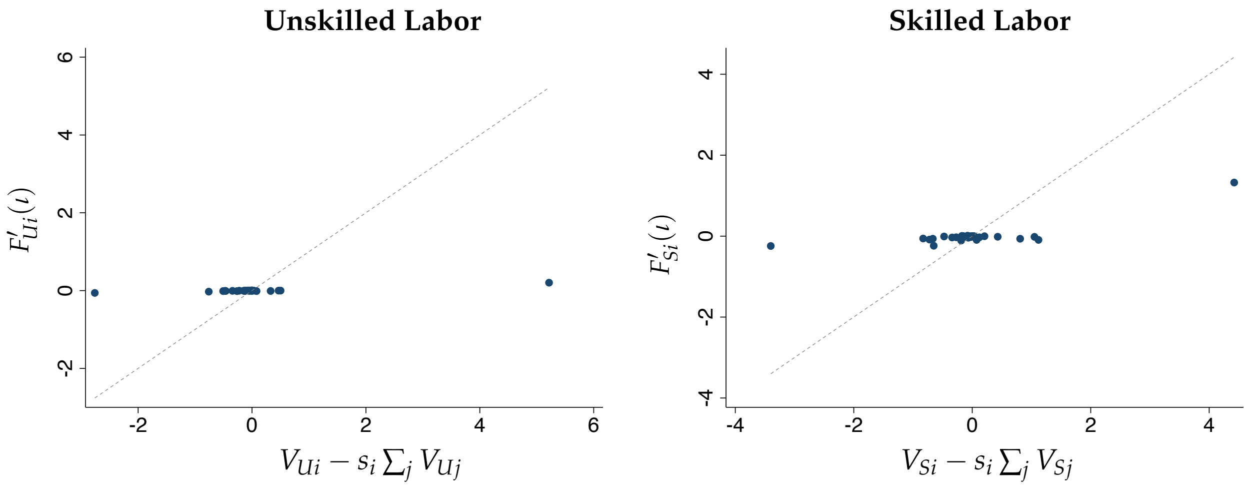
5.3. *Spirit of HO*

When we teach the Heckscher-Ohlin model to our students we focus on the role of *relative* abundance and *relative* factor intensities. In contrast, since Vanek (1968) and Leamer (1980), empirical research has examined the Vanek equation one factor at a time. We return to the earlier tradition by examining skilled relative to unskilled labour. This also serves as a 'stress test' of our results.

Figure 5 reports the results in relative terms. The Wage and Techniques terms are in ratios. We put the Vanek equation in differences rather than ratios because — since F'_{fi} can be positive, negative or zero — F'_{Si}/F'_{Ui} is both hard to interpret and becomes extreme when F'_{Ui} is close to 0. Turning to figure 5, each equation does extremely well when examined in terms of skilled relative to unskilled labour! No previous paper has subjected its results to this stress test. Further, these

[31] Slopes are 0.03 and 0.19 for unskilled and skilled labor, respectively.

Figure 4: Performance of the Vanek Equations ($\beta_{fi} = 1$)



Notes: Both panels plot the Vanek equation (V). The left- and right-hand columns are for unskilled and skilled labor, respectively. The plots use the Government Services adjusted factor content of trade, but sets the $\beta_{fi} = 1$ i.e., $F'_{fi}(\iota)$. The 45° line is displayed. Each observation is a factor and country (f,i).

results will be of great interest to those who teach the pre-Vanek/Leamer characterization of the Heckscher-Ohlin model.

5.4. *The Full-Information, Three-Equation Approach*

We want to ensure that we have exploited *all* of the supply-side information about productivity in estimating the β_{fi}. The only source that we have not yet exploited is the comparative advantage information contained in trade flows, which suggests that we should also use the Vanek equation for estimation of the β_{fi}. A reason for being cautious is that trade flows combine information from both the supply and demand sides and so are 'contaminated' by demand. We therefore begin by understanding how to quarantine this contamination. The intuition is simple: If consumption similarity holds then demand patterns are proportional across countries and the Vanek equation is uncontaminated by demand. We formalize this.

Lemma 3 *Suppose $C_{gij} = s_i C_{gwj}$ $\forall g,i,j$. Then equation (V) is equivalent to*

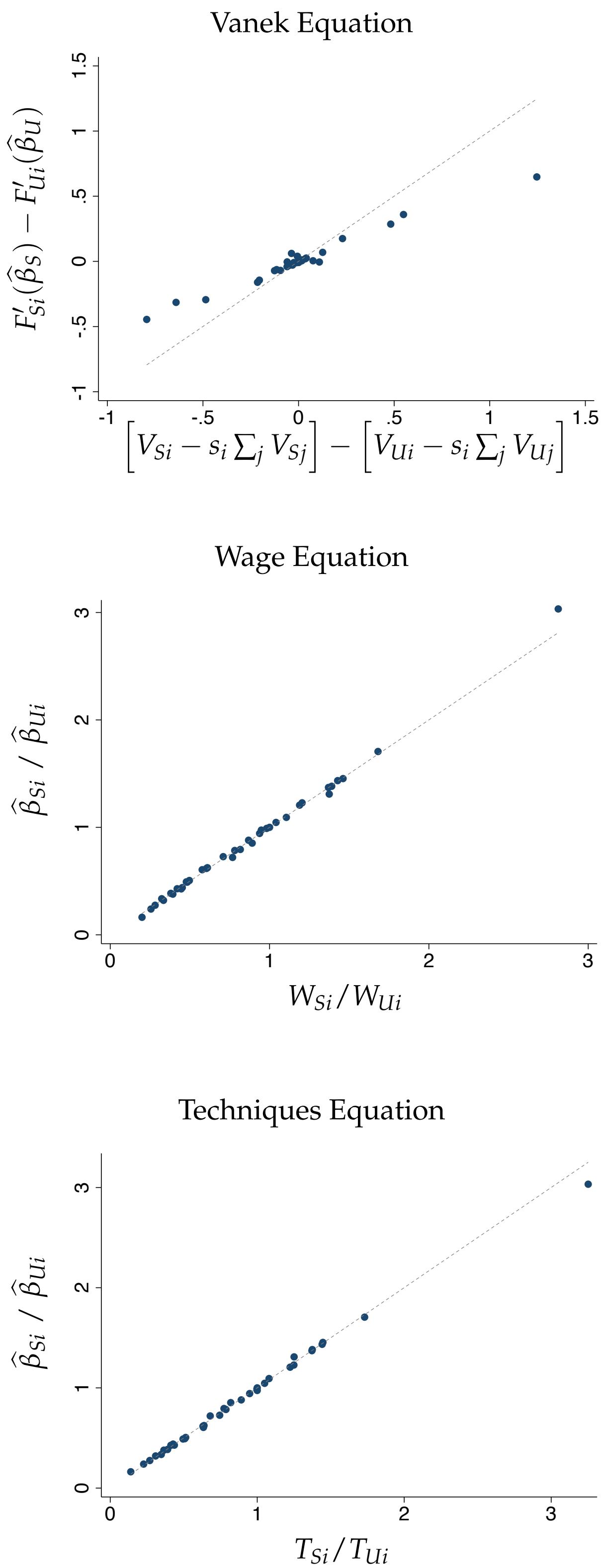
$$F_{fi}(\iota) = \beta_{fi}^{-1} V_{fi} - s_i \sum\nolimits_j \beta_{fj}^{-1} V_{fj} \, . \tag{VT}$$

Further, the β_{fi} that make this equation fit perfectly are given by

$$(\beta_{fi}^{VT})^{-1} \equiv \frac{s_i / V_{fi}}{s_{us} / V_{f,us}} + \frac{s_i}{V_{fi}} \left(\frac{F_{fi}(\iota)}{s_i} - \frac{F_{f,us}(\iota)}{s_{us}} \right) . \tag{22}$$

Equation (VT) states that under consumption similarity we can generalize Trefler's (1993) Vanek equation to the case where there are Ricardian productivity differences and where productivity

Figure 5: Differencing Across Factors: Two Equation Approach (T and W)



Notes: The top row plots the difference in the Government Services adjusted factor content of trade $F'_{Si}(\widehat{\beta}_S) - F'_{Ui}(\widehat{\beta}_U)$ against the difference in the predicted factor content of trade $\left[V_{Si} - s_i \sum_j V_{Sj}\right] - \left[V_{Ui} - s_i \sum_j V_{Uj}\right]$. The middle row plots W_{Si}/W_{Ui} against $\widehat{\beta}_{Si}/\widehat{\beta}_{Ui}$. The bottom row plots relative unit input requirements (T_{Si}/T_{Ui}) against $\widehat{\beta}_{Si}/\widehat{\beta}_{Ui}$. All lines are 45∘ lines.

adjusted factor price equalization fails.[32] Equation (22) states that if we estimated the β_{fi} solely from the modified Vanek equation (VT) then we would end up with $\beta_{fi} = \beta_{fi}^{VT}$.

We next show that the β_{fi}^{VT} capture important aspects of productivity from the development accounting literature. To make sense of the first term on the right-hand side of equation (22), consider a single-good economy with an aggregate production function

$$Y_i = \left[\alpha_U (\pi_{Ui} V_{Ui})^{\frac{\sigma-1}{\sigma}} + \alpha_S (\pi_{Si} V_{Si})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{23}$$

where Y_i is both output and income. Equating the marginal product of factor f with its factor price yields

$$w_{fi} = \alpha_f (\pi_{fi})^{\frac{\sigma-1}{\sigma}} \left(Y_i / V_{fi}\right)^{\frac{1}{\sigma}} \ . \tag{24}$$

Dividing by the corresponding equation for the United States and substituting out Y_i using $s_i = Y_i / \sum_j Y_j$ yields

$$\beta_{fi}^{-1} = \frac{s_i / V_{fi}}{s_{\text{us}} / V_{f,\text{us}}}$$

That is, the first term in equation (22) comes straight out of the most basic Development Accounting exercise.[33]

The second term in equation (22) is the information about productivity contained in trade flows. Specifically, after dividing through by s_i to control for country size, if factor f in country i has a larger factor content of trade than in the United States then factor f is revealed by trade to be more productive in i than in the United States.

This discussion demonstrates that if consumption similarity holds — or empirically by equation (12), if the Vanek equation fits well — then the Vanek equation is a third (and final) source of information about productivity. Moving to estimation using the Vanek equation, we note that $F'_{fi}(\beta_f)$ is linear in $\beta_f \equiv (\beta_{f1}, \ldots, \beta_{fN})$ so that equation (V) can be rewritten as

$$H_{fi}(V_{f1}, \ldots, V_{fN}, F'_{f1}(\iota), \ldots, F'_{fN}(\iota), \widehat{\delta}) = \beta_{fi} \tag{V'}$$

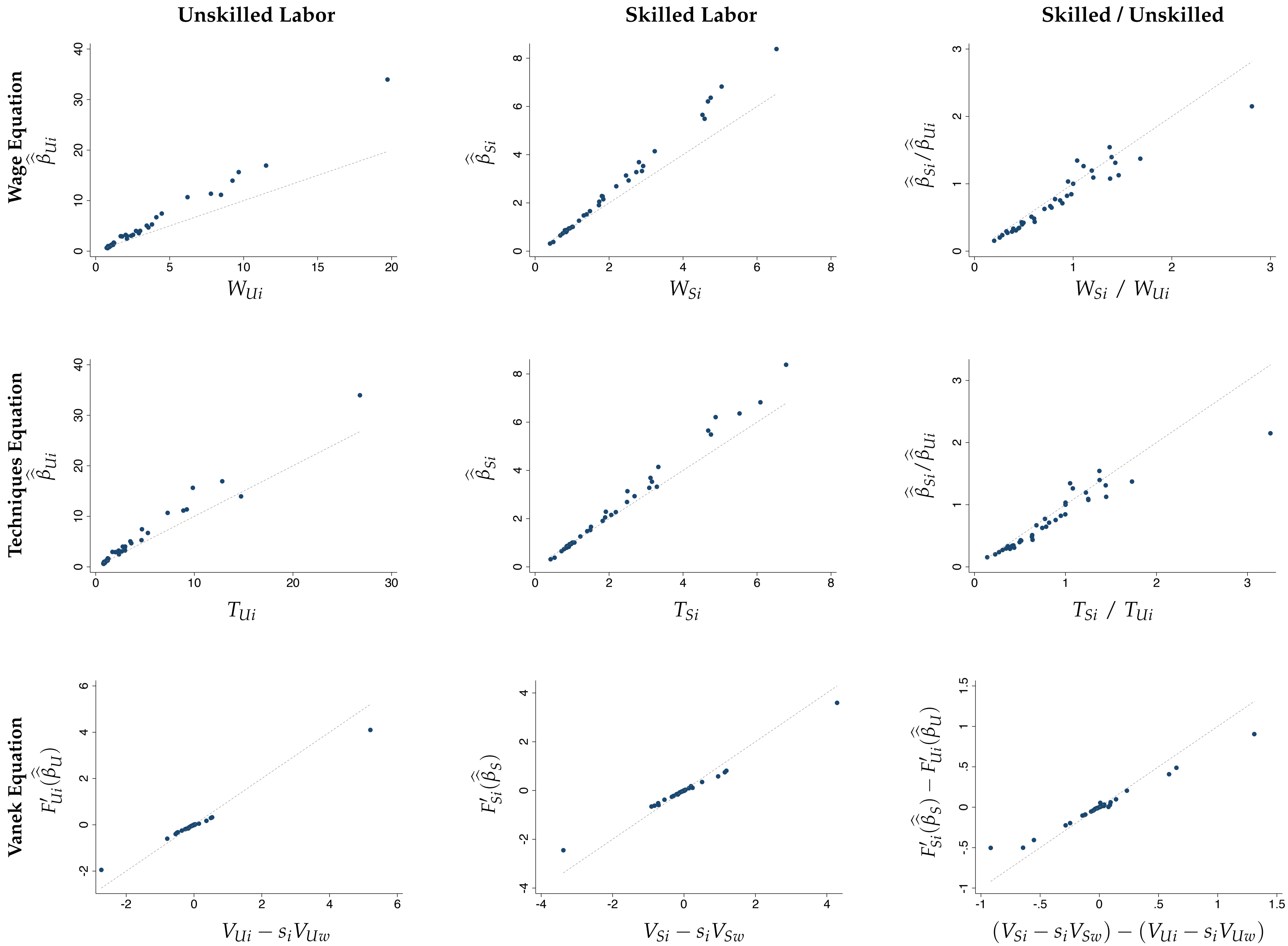
for some function H_{fi} which depends only on data. Doing this allows us to simply stack equations (W), (T) and (V') and estimate the β_{fi} using OLS and country-factor dummy variables. Let $\widehat{\widehat{\beta}}_f$ denote the vector of estimates.

The first thing to note about the estimates is that $\widehat{\widehat{\beta}}_f$ is very close to the two-equation estimate $\widehat{\beta}_f$: the correlation for each factor is 0.99. The second thing to note is that the full-information, three-equation approach produces good results. These are on display in figure 6. The three columns

[32] To see that it is a generalization of Trefler note the following. Under productivity adjusted factor price equalization, $(\beta_{fi})^{-1} = \pi_{fi}/\pi_{f,us}$, $\delta_{gi}/\delta_{g,us} = 1$, and $F_{fi}(\iota)$ is the factor content of trade using US techniques (denoted F_{fi}^{us}). Hence equation (VT) becomes $F_{fi}^{\text{us}} = \pi_{fi} V_{fi} - s_i \sum_j \pi_{fj} V_{fj}$, which is Trefler's equation. Also note that without factor price equalization, it is very unlikely that $C_{gij} = s_i C_{gwj}\ \forall g,i,j$ holds.

[33] Here are the details. $w_{fi}/w_{f,\text{us}} = (\pi_{fi}/\pi_{f,\text{us}})^{\frac{\sigma-1}{\sigma}} \left[(s_i/V_{fi})/(s_{\text{us}}/V_{f,\text{us}})\right]^{\frac{1}{\sigma}}$. Rearranging yields $(w_{fi}/w_{f,\text{us}})^{\sigma} (\pi_{fi}/\pi_{f,\text{us}})^{1-\sigma} = (s_i/V_{fi})/(s_{\text{us}}/V_{f,\text{us}})$. From equation (20), the left-hand side is β_{fi}^{-1}. The π_{fi} that satisfies this equation is exactly the same measure of productivity as in Caselli and Coleman (2006) for the case without capital. See their footnote 7.

Figure 6: Full-Information, Three-Equation Approach



Notes: The left-hand column plots are for unskilled labor, the middle column plots are for skilled labor, and the right-hand column plots are for skilled relative to unskilled labor. The top row is the Wage equation (W), the middle row is the Techniques equation (T), and the bottom row is the Vanek equation (V). All equations are evaluated at the full-information, three-equation estimates of the β_{fi} and calibrated values of the δ_{gi}. Each observation is a factor and country. The 45° line is displayed in each panel.

Table 1: Test Statistics for the Fit of the Vanek Equation

Factor Content of Trade	Unskilled Labor: Rank Corr.	Variance Ratio	Sign Test	Slope Test	Skilled Labor: Rank Corr.	Variance Ratio	Sign Test	Slope Test
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1. F_{fi}	0.953	0.038	0.868	0.193	0.974	0.016	0.947	0.127
2. $F_{fi}(\widehat{\beta}_f)$	0.965	0.052	0.947	0.226	0.982	0.027	0.947	0.163
3. $F'_{fi}(\widehat{\beta}_f)$	0.978	0.424	0.947	0.648	0.990	0.452	0.921	0.664
4. $F_{fi}(\widehat{\beta}_f)$	0.962	0.078	0.895	0.276	0.976	0.089	0.921	0.29
5. $F'_{fi}[\iota]$	0.441	0.001	0.763	0.034	0.256	0.052	0.474	0.185
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.995	0.588	0.947	0.765	0.995	0.607	0.974	0.776

Notes: This table presents test statistics for the fit of the Vanek equation (V) for different specifications of the factor content of trade. In row 1, the actual factor content of trade is used. In row 2, the factor content of trade is calculated using $\widehat{\beta}_f$ (the two-equation estimate of β_f) and equation (16). In row 3, the factor content of trade is adjusted for nontraded Government Services using equation (21). In row 4, the nontraded Government Services adjustment is put on the right-hand side of the Vanek equation as in part 1 of lemma 2 and as in Davis and Weinstein (2001). In row 5, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but all elements of the vector $\widehat{\beta}_f$ are set to 1. In row 6, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but the estimate of β_f is from the three-equation approach. 'Rank Corr.' is the rank or Spearman correlation between the factor content of trade and $V_{fi} - s_i V_{fw}$. 'Variance Ratio' is the variance of the factor content of trade divided by the variance of $V_{fi} - s_i V_{fw}$. 'Sign Test' is the proportion of observations for which the factor content of trade and $V_{fi} - s_i V_{fw}$ have the same sign. 'Slope Test' is the OLS slope estimate from a regression of the factor content of trade on $V_{fi} - s_i V_{fw}$.

from left to right are for unskilled labor, skilled labor, and the ratio of skilled to unskilled labor. The three rows from top to bottom are for the Wage equation, the Techniques equation, and the Vanek equation. In all nine panels, the fit is very good. Turning specifically to the Vanek equation, notice that there is less missing trade than before. The slope coefficient from a regression of $F'_{fi}(\widehat{\widehat{\beta}}_f)$ on $V_{fi} - s_i V_{fw}$ is now 0.77 for unskilled labor and 0.78 for unskilled labor. The rank correlation statistics for each are high at 0.99. Online appendix figure A1 redoes the figure without the US and Chinese outliers.

It is conventional in HOV papers to report a large number of test statistics. While we feel that the plots tell the full story, we give a nod to convention in Table 1. The table notes explain the familiar tests. It is worth noting that WIOD reports V_{fgi} across high, medium, and low skilled labor. The online appendix presents analogous results to those in figures 2 and 3 and table 1 when we look at these three types of labor separately.[34]

[34] The online appendix also reports separately results for capital and labor where labor is defined as the sum of the the three labor types. We report these two sets of results separately because our exercise requires a single elasticity of substitution across factors for results that are comparable to those in the text (e.g. the elasticity of substitution between skilled and unskilled labor is the same as between labor and capital). Constraining the elasticity of substitution to be the same between two types of labor as between labor and capital delivers similar results. We choose not to report results for capital in the main text for two reasons. First, internationally comparable and reliable data on the cost of capital is difficult to come by and, second, capital can easily be thought of as a traded good as in Fitzgerald and Hallak (2004).

6. Factor Prices, Substitution Effects, Factor Bias, and Directed Technical Change

Up to this point, much of the paper has been concerned with issues surrounding the specification and fit of the Vanek, Wage and Techniques equations, issues which depend only on the reduced-form β_{fi}. We now turn to a host of questions which depend critically on the underlying structural parameters i.e., on the elasticity of substitution between skilled and unskilled labor (σ) and the factor-augmenting technology parameters (π_{fi}).

The intuition for how we can extract π_{fi} from the β_f comes from Caselli and Coleman (2006) and our discussion of identification. Simplify notation with the normalization $\pi_{f,\text{us}} = 1$ and by normalizing wages using $w_{f,\text{us}} = 1$ for $f = S,U$. Then from equation (20)

$$\pi_{fi} = (\beta_{fi}/w_{fi}^{-\sigma})^{1/(\sigma-1)} \,. \tag{25}$$

What this says is that for a given value of σ, international factor price differences generate international differences in factor demands according to $w_{fi}^{-\sigma}$, which in turn generates differences in the estimates β_{fi}. Any variation in the β_{fi} not explained by variation in factor prices must be due to international technology differences (the π_{fi}). How so depends on whether or not σ is greater or less than unity. For example, suppose that a country has greater unskilled-intensive demand (relative to the United States) than is predicted based on its wages and the elasticity of substitution. If skilled and unskilled labor are substitutes ($\sigma > 1$), then unskilled-augmenting productivity must be higher in this country. If skilled and unskilled labor are complements ($\sigma < 1$), then unskilled-augmenting productivity must be lower. Thus, we need an estimate of σ.

Typical values for the elasticity of substitution between skilled and unskilled labor range between 1.4 and 2. Although we discuss and justify our choice of σ in detail in section 6.5, we start near the midpoint with σ=1.67.[35] Since our results are completely insensitive to how we estimate the β_{fi}, we use our two-equation estimates ($\widehat{\beta}_{fi}$). Plugging these into equation (25) generates our productivity estimates. These are also reported in appendix table A1. We turn now to using those estimates to answer substantive questions.

6.1. Are Productivity Adjusted Factor Prices Equalized?

Leamer and Levinsohn's (1995, p. 1360) factor price insensitivity theorem states that in the FPE set, factor prices are insensitive to endowments.[36] In the context of the factor-market clearing condition (equation 17), this means that the impact on wages of differences in productivity adjusted factor endowments

$$S_{fi} \equiv \left[\frac{\pi_{f,\text{us}}V_{f,\text{us}}}{\hat{\pi}_{fi}V_{fi}}\right]^{1/\sigma}$$

[35] The seminal citation is Katz and Murphy (1992) which provides $\sigma = 1.4$. Caselli and Coleman (2006) use values of σ between 1.1 and 2. Card and Lemieux (2001) find $\sigma = 2.5$. See Ciccone and Peri (2005) for a review of estimate of the elasticity of substitution between skilled and unskilled labor. See Antràs (2004) for estimates of the elasticity of substitution between capital and aggregate labor.

[36] In our context, the theorem is productivity adjusted factor price insensitivity.

Table 2: Decomposition of the Wage Equation

Panel A: Unskilled Labor

	(1)	(2)	(3)
	$\ln(S_{Ui})$	$\ln(R_{Ui})$	$\ln(\epsilon^W_{Ui})$
$\ln\left(\frac{w_{Ui}}{\pi_{Ui}} / \frac{w_{U\text{us}}}{\pi_{U\text{us}}}\right)$	1.38***	-0.34	-0.040***
	(0.44)	(0.45)	(0.01)
R-squared	0.21	0.02	0.23

Panel B: Skilled Labor

	$\ln(S_{Si})$	$\ln(R_{Si})$	$\ln(\epsilon^W_{Si})$
$\ln\left(\frac{w_{Si}}{\pi_{Si}} / \frac{w_{S\text{us}}}{\pi_{S\text{us}}}\right)$	0.89***	0.12	-0.0085***
	(0.22)	(0.22)	(0.00)
R-squared	0.31	0.01	0.22

Panel C: Skilled Relative to Unskilled Labor

	$\ln(S_{SUi})$	$\ln(R_{SUi})$	$\ln(\epsilon^W_{SUi})$
$\ln\left(\frac{w_{Si}\pi_{Ui}}{w_{Ui}\pi_{Si}} / \frac{w_{S\text{us}}\pi_{U\text{us}}}{w_{U\text{us}}\pi_{S\text{us}}}\right)$	0.96***	0.027*	0.014**
	(0.01)	(0.01)	(0.01)
R-squared	0.99	0.10	0.15

Notes: Within each panel, each column represents a separate regression. The dependent variable is identified by the column header and the independent variable is identified by the column on the left. All regressions are OLS with 38 observations. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

are exactly offset by differences in industrial composition

$$R_{fi} \equiv \left(\sum_{g=1}^{G} \frac{d_{fg,\text{us}} Q_{gi}}{\widehat{\delta}_{gi} V_{f\text{us}}} \right)^{1/\sigma} .$$

Restated, Rybczynski effects (R_{fi}) exactly offset supply effects (S_{fi}). Then from equation (17) in logs,

$$\ln\left(\frac{w_{fi}/\widehat{\pi}_{fi}}{w_{f,\text{us}}/\pi_{f,\text{us}}}\right) = \ln S_{fi} + \ln R_{fi} + \ln \epsilon^W_{fi} \tag{26}$$

where we have added an error term so that equation (26) is an identity.[37]

Given that equation (26) is an identity we can use a variance decomposition to assess the relative importance of each of the three right-hand side components of (26). We implement this following Bernard, Jensen, Redding and Schott (2007a). Consider column 1 of the upper panel of table 2, which deals with unskilled labor. It is a regression of $\ln S_{fi}$ on the left-hand side term of equation (26). The coefficient is large, indicating that the supply component explains most of the variation in wages. Repeating this for $\ln R_{fi}$ in column 2 and $\ln \varepsilon^W_{fi}$ in column 3 shows that the supply component is by far the most important. Notice that the coefficients in columns 1–3 must sum to

[37] The error term is intimately related to ε^W_{fi} described in footnote 14.

unity by construction. In this sense, the three coefficients provide a variance decomposition of the left-hand side of equation (26) into its components.[38]

We repeat the exercise for skilled labor in panel B of table 2 and skilled relative to unskilled labor in panel C of table 2. As is apparent, the same conclusions emerge, namely, productivity adjusted factor prices are highly sensitive to productivity adjusted factor supplies. We conclude from this that Davis and Weinstein (2001) were correct to emphasize failure of factor price equalization and the role of factor supplies.[39]

6.2. *International Differences in Unit Factor Demands: Substitution Effects or Productivity?*

We can now answer the fundamental question posed by Davis and Weinstein, namely, are international differences in average choice of techniques due more to substitution effects (failure of PFPE) or to international technology differences? We start with the Techniques equation (T), take logs and rewrite it to isolate the object of interest, $\sum_{g=1}^{G} \theta_{fgi} \left(\frac{d_{fgi}}{d_{fg,\text{us}}}\right)$:

$$\ln\left[\sum_{g=1}^{G} \theta_{fgi} \left(\frac{d_{fgi}}{d_{fg,\text{us}}}\right)\right] = \ln\left(\frac{w_{fi}/\widehat{\pi}_{fi}}{w_{f,us}/\pi_{f,us}}\right)^{-\sigma} + \ln\left(\frac{\widehat{\pi}_{fi}}{\pi_{f,us}}\right)^{-1} + \ln\left(\sum_{g=1}^{G} \theta_{fgi}/\widehat{\delta}_{gi}\right) + \ln \epsilon_{fi}^{T} \tag{27}$$

where we have added an error term so that equation (27) is an identity.[40] Looking to the right of the equal sign, differences in average techniques are associated with (1) substitution effects due to differences in productivity adjusted factor prices, (2) productivity effects due to differences in factor-augmenting technology, (3) the δ_{gi} terms, and (4) an error term. Similar to our decomposition of the Wage equation, we assess the contribution of each of these four components by separately regressing each on the left-hand side term of equation (27). The coefficients from the four separate regressions again sum to unity.

Table 3 reports the results. Panels A and B present results for unskilled and skilled labor, respectively. They show that most of the variance in techniques relative to the United States is due to international differences in factor-augmenting technology. Factor prices, the δ_{gi} terms, and errors are less important. In contrast, panel C presents results for the ratio of skilled to unskilled labour. (Roughly, panels A and B are about $\ln d_{Ugi}$ and $\ln d_{Sgi}$, respectively, whereas panel C

[38]It is tempting to interpret the coefficients in table 2 in terms of first moments (slopes); however, the coefficients speak to second moments e.g., a negative coefficient means that the component is associated with a smaller *variance* of factor prices.

[39]Our finding of a lack of 'Rybczynski effects' is consistent with the results of Gandal, Hanson and Slaughter (2004), Lewis (2004) and Blum (2010), though less consistent with the results of Hanson and Slaughter (2002).

[40]The error term is intimately related to ε_{fi}^{T} described in footnote 14.

Table 3: Decomposition of the Techniques Equation

Panel A: Unskilled Labor

	(1)	(2)	(3)	(4)
	$\ln\left(\frac{w_{Ui}/\widehat{\pi}_{Ui}}{w_{U,us}/\pi_{U,us}}\right)^{-\sigma}$	$\ln\left(\frac{\widehat{\pi}_{Ui}}{\pi_{U,us}}\right)^{-1}$	$\ln(\sum_g \theta_{Ugi}/\hat{\delta}_{gi})$	$\ln\left(\epsilon^T_{Ui}\right)$
$\ln\left[\sum_{g=1}^{G}\theta_{Ugi}\left(\frac{d_{Ugi}}{d_{Ug,\text{us}}}\right)\right]$	-0.32***	1.20***	0.087***	0.032***
	(0.09)	(0.09)	(0.02)	(0.01)
R-squared	0.26	0.85	0.36	0.51

Panel B: Skilled Labor

	$\ln\left(\frac{w_{Si}/\widehat{\pi}_{Si}}{w_{S,us}/\pi_{S,us}}\right)^{-\sigma}$	$\ln\left(\frac{\widehat{\pi}_{Si}}{\pi_{S,us}}\right)^{-1}$	$\ln(\sum_g \theta_{Sgi}/\hat{\delta}_{gi})$	$\ln\left(\epsilon^T_{Si}\right)$
$\ln\left[\sum_{g=1}^{G}\theta_{Sgi}\left(\frac{d_{Sgi}}{d_{Sg,\text{us}}}\right)\right]$	-0.45*	1.29***	0.15***	0.015***
	(0.27)	(0.22)	(0.05)	(0.00)
R-squared	0.07	0.49	0.19	0.29

Panel C: Skilled Relative to Unskilled Labor

	$\ln\left(\frac{w_{Si}\widehat{\pi}_{Ui}}{\widehat{\pi}_{Si}w_{Ui}}\Big/\frac{w_{S,us}\pi_{U,us}}{\pi_{S,us}w_{U,us}}\right)^{-\sigma}$	$\ln\left(\frac{\widehat{\pi}_{Si}}{\widehat{\pi}_{Ui}}\Big/\frac{\pi_{S,us}}{\pi_{U,us}}\right)^{-1}$	$\ln\left(\frac{\sum_g \theta_{Sgi}/\hat{\delta}_{gi}}{\sum_g \theta_{Ugi}/\hat{\delta}_{gi}}\right)$	$\ln\left(\epsilon^T_{SUi}\right)$
$\ln\left[\frac{\sum_{g=1}^{G}\theta_{Sgi}\left(d_{Sgi}/d_{Sg,\text{us}}\right)}{\sum_{g=1}^{G}\theta_{Ugi}\left(d_{Ugi}/d_{Ug,\text{us}}\right)}\right]$	1.85***	-0.73***	0.16***	-0.29***
	(0.22)	(0.20)	(0.04)	(0.08)
R-squared	0.66	0.28	0.30	0.25

Notes: Within each panel, each column represents a separate regression. The dependent variable is identified by the column header and the independent variable is identified by the column on the left. All regressions are OLS with 38 observations. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

is about $\ln d_{Sgi}/d_{Ugi}$). For variation in relative factor demands it is factor prices that are most important. Recapping, differences in factor-augmenting productivity are most important when analyzing per unit input requirements, but substitution effects caused by departures from PFPE are most important when looking at input requirements of skilled relative to unskilled labor.

6.3. *The Vanek Equation: Technology Differences or Failure of Factor Price Equalization?*

Davis and Weinstein (2001) and Trefler (1993a) place substitution effects and factor-augmenting productivity at the forefront of their respective analyses. We are the first to integrate the two within a unified empirical framework and can now examine the relative importance of each. The Vanek equation is not log-linear in the $\widehat{\beta}_{fi}$ so no simple variance decomposition is possible. The most obvious thing to do is shut down the wage and productivity terms one at a time. Recall from equation (20) together with the normalizations $\pi_{f,\text{us}} = 1$ and $w_{f,\text{us}} = 1$ that

$$\widehat{\beta}_{fi} = (w_{fi}/\widehat{\pi}_{fi})^{-\widehat{\sigma}}\ \widehat{\pi}_{fi}^{-1}$$

can be decomposed into a productivity adjusted factor price term $\left(w_{fi}/\widehat{\pi}_{fi}\right)^{-\widehat{\sigma}}$ and a factor-augmenting technology term $\widehat{\pi}_{fi}^{-1}$.

We begin by shutting down the factor price term and computing what the factor content of trade would look like. In our notation, this is $F'_{fi}(\widehat{\pi}_f^{-1})$ where $\widehat{\pi}_f^{-1} = (\widehat{\pi}_{f1}^{-1}, \ldots, \widehat{\pi}_{fN}^{-1})$. Column 1 of figure 7 plots $F'_{fi}(\widehat{\pi}_f^{-1})$ against $V_{fi} - s_i V_{fw}$ for unskilled labor (top row) and skilled labor (bottom) row. The rank correlations are 0.80 and 0.50, respectively. Thus for unskilled labor and to a lesser extent for skilled labor, factor augmentation is important. We next shut down the factor-augmenting technology term and compute what the factor content of trade would look like. In our notation this is $F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}})$ where $[w_f/\widehat{\pi}_f]^{-\widehat{\sigma}} = ([w_{f1}/\widehat{\pi}_{f1}]^{-\widehat{\sigma}}, \ldots, [w_{fN}/\widehat{\pi}_{fN}]^{-\widehat{\sigma}})$. Column 2 of figure 7 plots $F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}})$ against $V_{fi} - s_i V_{fw}$. The fit is horrible and the rank correlations are negative, which means that factor-augmenting technology is exceedingly important.

These results taken together imply that there are important interactions between factor prices and productivity. To examine these we start by defining the 'marginal contribution of productivity'

$$F'_{fi}(\widehat{\beta}_f) - F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}}) \; .$$

The idea is that if β_f were linear in $[w_f/\widehat{\pi}_f]^{-\widehat{\sigma}}$ and $\widehat{\pi}_f^{-1}$ then this expression would equal $F'_{fi}(\widehat{\pi}_f^{-1})$. So the above term is the marginal contribution of (nonlinearly) adding in factor-augmenting technology. We likewise defined the 'marginal contribution of factor prices' as

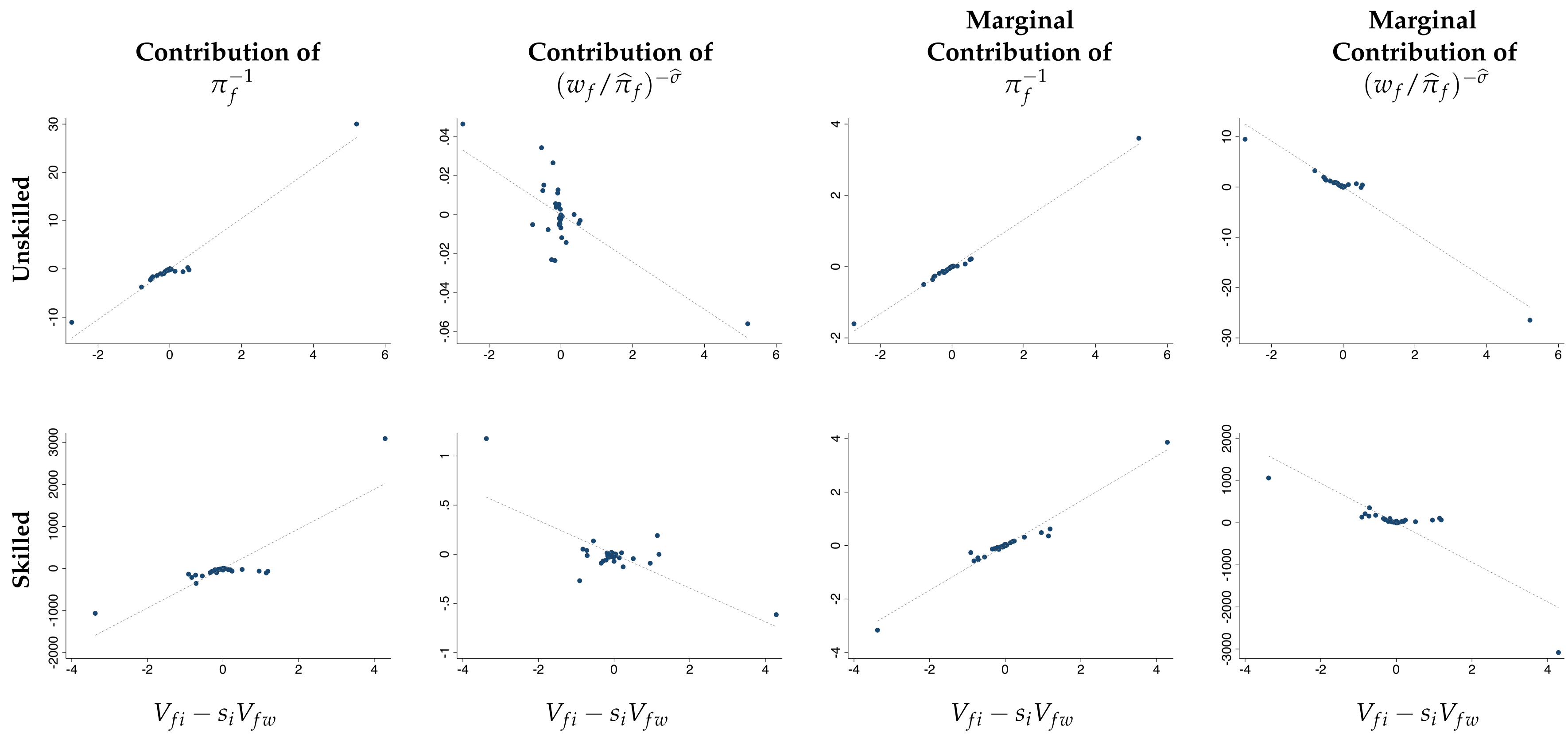
$$F'_{fi}(\widehat{\beta}_f) - F'_{fi}(\widehat{\pi}_f^{-1}) \; .$$

These marginal contributions appear in columns 3 and 4 of figure 7, respectively. Two things stand out. First, from column 3, *the marginal contribution of productivity is very important and performs extremely well both in terms of correlations and missing trade.* Second, from column 4, the marginal contribution of factor prices is unimportant. However, because neither productivity nor factor prices performs very well by itself, one must ultimately conclude that both productivity and factor prices are important for understanding the Vanek equation.

6.4. Development Accounting Revisited: Is There Skill Bias in Cross-Country Technology Differences?

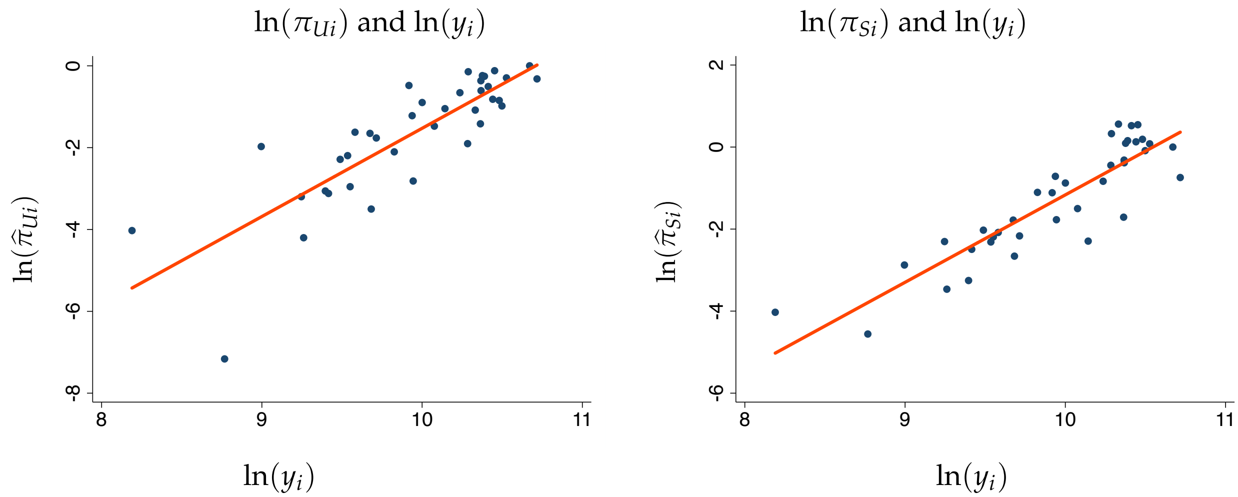
In this subsection we address two questions. First, how similar are our estimates of the π_{fi} to those in the development accounting literature? Second, do our estimates display the skill bias which is

Figure 7: Vanek Equation: Relative Importance of Substitution Effects vs. Factor Augmentation



Notes: The top row deals with unskilled labor. The bottom row deals with skilled labor. The horizontal axis is always $V_{fi} - s_i V_{fw}$. In the first column, the vertical axis is $F'_{fi}(\widehat{\pi}_f^{-1})$. In the second column the vertical axis is $F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}})$. In the third column, the vertical axis is $F'_{fi}(\widehat{\beta}_f) - F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}})$ i.e., the marginal contribution of $\widehat{\pi}_f^{-1}$ after controlling for $(w_f/\widehat{\pi}_f)^{-\widehat{\sigma}}$. In the fourth column the vertical axis is $F'_{fi}(\widehat{\beta}_f) - F'_{fi}(\widehat{\pi}_f^{-1})$ i.e., the marginal contribution of $(w_f/\widehat{\pi}_f)^{-\widehat{\sigma}}$ after controlling for $\widehat{\pi}_f^{-1}$. All lines are OLS best fits.

Figure 8: Factor-augmenting International Technology Differences



Notes: The left column plots $\ln(\widehat{\pi}_{Ui})$ against (log) real income per worker from the Penn World Tables. Each dot is a country. right column plots $\ln(\widehat{\pi}_{Si})$ against (log) real income per worker. All lines are OLS best fits.

so central to Caselli and Coleman (2006)? As discussed in sections 3.3 and 5.4, there are significant methodological similarities between our approach and theirs. Nevertheless, these questions are not trivial because there are also many differences between the two approaches.[41]

Are our estimates of the π_{fi} close to those of Caselli and Coleman (2006)? The answer is: not quite. Caselli and Coleman (2006, figures 1 and 2) show that while π_{Si} has the expected positive correlation with log real income per worker, π_{Ui} does not. Figure 8 shows that our $\widehat{\pi}_{Si}$ and $\widehat{\pi}_{Ui}$ are *both* positively correlated with log real income per worker. We think that this is a sensible result: As countries grow rich *both* their skilled and unskilled workers become more productive.

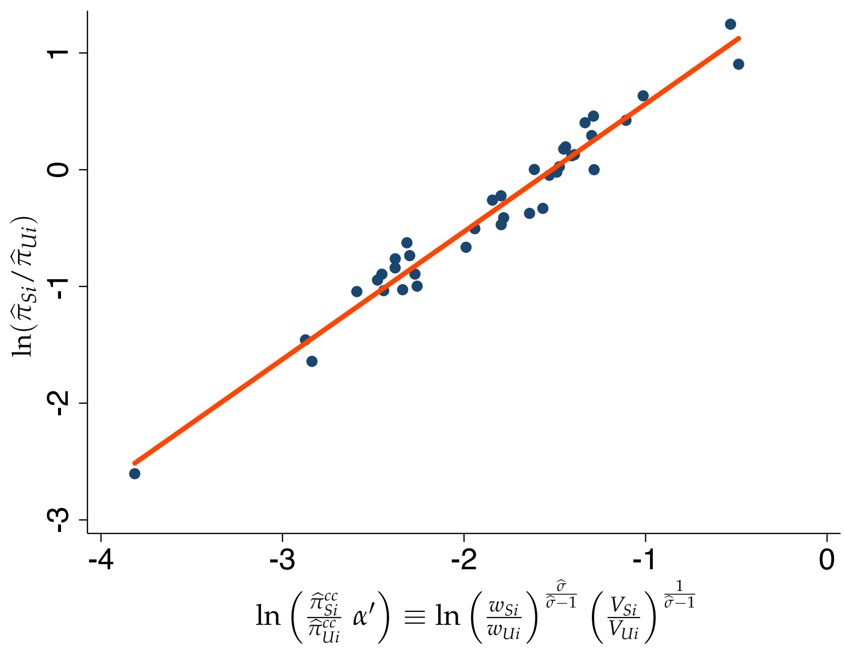
More importantly, we confirm Caselli and Coleman's measures of skill bias in cross-country technology differences. To see this, start with an aggregate production function, use equation (24) to equate the ratio of marginal productivities to the ratio of factor prices, and then invert to solve for the Caselli and Coleman technology ratios:

$$\frac{\pi^{cc}_{Si}}{\pi^{cc}_{Ui}} = \alpha' \left(\frac{w_{Si}}{w_{Ui}}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{V_{Si}}{V_{Ui}}\right)^{\frac{1}{\sigma-1}} . \tag{28}$$

where $\alpha' \equiv (\alpha_S/\alpha_U)^{\frac{\sigma}{1-\sigma}}$. Figure 9 plots the log of these (up to a constant α') against the log of our $\widehat{\pi}_{Si}/\widehat{\pi}_{Ui}$. *The fit is remarkable* ($R^2 = 0.95$)*, which establishes a strong link between the HOV and*

[41] By way of just a few examples, Caselli and Coleman (2006) includes capital and precisely measures endowments of human capital whereas we use highly disaggregated sectoral data and the factor market clearing equation (W).

Figure 9: Factor Bias — HOV versus Development Accounting



development accounting literatures. It is also interesting that the multi-sector HOV model reproduces results from an aggregate production function.

6.5. *Directed Technical Change: Are the π_{fi} Biased Towards a Country's Abundant Factor?*

The previous result about skill bias is static i.e., it is derived from a time-invariant production function. In a series of papers summarized in Acemoglu (2009, chapter 15), the author argues that whether innovation is directed towards skilled or unskilled labor will depend on offsetting effects: Innovation will be directed towards the expensive factor (the price effect) and towards the more abundant factor (the market-size effect). Under fairly general assumptions, the market-size effect dominates so that technical change is directed towards a country's abundant factor. The key piece in the proof of Acemoglu's argument deals with the innovation process. Letting η_i be the efficiency of R&D in the skill-intensive relative to the unskilled-intensive sectors, Acemoglu (2009, eq. 15.27) derives the following equation, which contains most of the economics of directed technical change:

$$\ln\frac{\pi_{Si}}{\pi_{Ui}} = \gamma_0 + \gamma_1 \ln \eta_i + (\sigma - 1)\ln\left(\frac{V_{Si}}{V_{Ui}}\right) \tag{29}$$

where γ_0 and γ_1 are exogenous parameters of Acemoglu's model and σ is the (derived) elasticity of substitution.[42] It is important to note that this is a macro elasticity whereas in our model σ is a micro elasticity. Nevertheless, we equate the two.[43]

We investigate this equation and also use it to estimate σ. Recall that in order to define the $\widehat{\pi}_{fi}$ we needed to choose a value for σ and combine it with $\widehat{\beta}_{fi}$ and w_{fi} where here we treat $\widehat{\beta}_{fi}$ as data. (See equation 25.) We therefore write $\widehat{\pi}_{fi} = \pi_{fi}(\sigma;\ \widehat{\beta}_{fi}, w_{fi})$ and rewrite equation (29) as

$$\ln \frac{\pi_{Si}(\sigma;\ \widehat{\beta}_{Si}, w_{Si})}{\pi_{Ui}(\sigma;\ \widehat{\beta}_{Ui}, w_{Ui})} = \gamma_0 + \gamma_1 \ln \eta_i + (\sigma - 1) \ln \left(\frac{V_{Si}}{V_{Ui}} \right) . \tag{30}$$

This is a nonlinear equation in σ, which we estimate as follows. Collapse $\gamma_0 + \gamma_1 \ln \eta_i$ into an intercept γ (we relax this below), pick an initial value of σ — call it σ_0 — and run the regression

$$\ln \frac{\pi_{Si}(\sigma_0;\ \widehat{\beta}_{Si}, w_{Si})}{\pi_{Ui}(\sigma_0;\ \widehat{\beta}_{Ui}, w_{Ui}))} = \gamma + (\sigma_1 - 1) \ln \left(\frac{V_{Si}}{V_{Ui}} \right) + \epsilon_i \tag{31}$$

to recover an estimate of σ_1. Iterate until $\sigma_0 = \sigma_1$. For all starting values of σ_0 between 0.5 and 50 we quickly converge to a final value $\widehat{\sigma} = 1.67$, which is the value used throughout this paper. The t-statistic is 2.93 and the $R^2 = 0.28$. The plot of the data and fitted line appear in the left panel of figure 10.

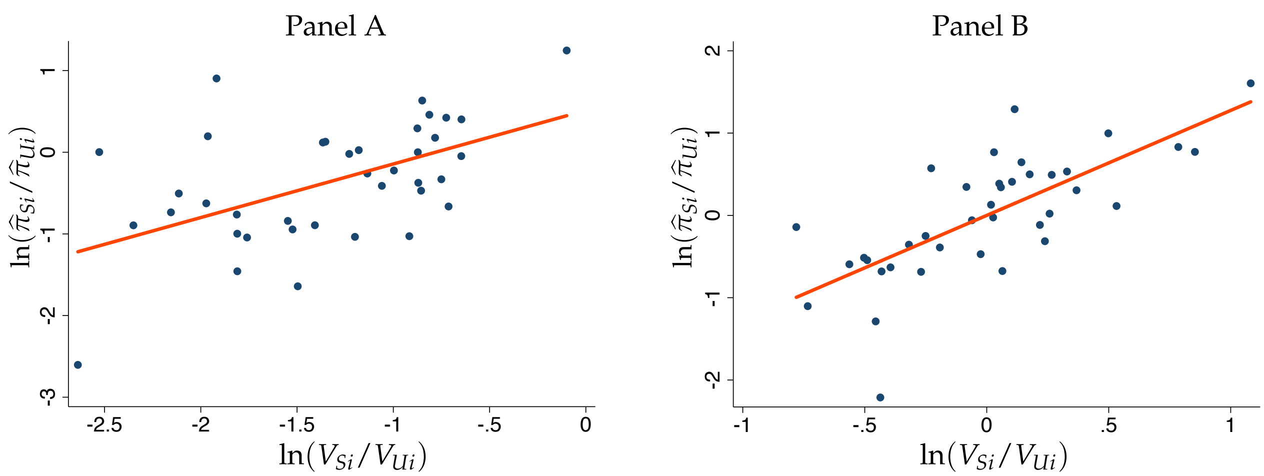
This establishes three things. First, we have now generated a $\widehat{\sigma}$ which is consistent with a (closed-economy) model of directed technical change. Second, this $\widehat{\sigma}$ is in the middle of the range of existing estimates of σ. See footnote 35. Third, $\widehat{\sigma} > 1$, which implies that countries indeed direct their innovation towards improving the productivity of their abundant factor.

We can refine the analysis somewhat by recognizing that the intercept γ in equation (31) depends on $\ln \eta_i$ and so is not a constant. We proxy it by income per worker (y_i) and, since we do not know what this function looks like we experiment with polynomials of order 1 through 4 and with semi-parametric estimators. All approaches yield virtually identical results so we only report

[42] To understand why Acemoglu (2009) refers to this as a 'derived' demand, we must review the setup of his model. There are two sectors. Sector S (U) is produced with machines and factor S (U). Aggregate output is CES in the two sectoral outputs. It follows that aggregate output is a function of S and U, which in turn implies a 'derived' elasticity of substitution σ. One way of seeing that this derived elasticity behaves like one is that it equals the inverse of the elasticity of relative wages with respect to endowments. See the discussion following Acemoglu's equation 15.19.

[43] Two additional caveats about the link between our model and Acemoglu's model flow from the fact that the Acemoglu (2009) model is for a closed economy. First, Rybczynski effects are limited in scope in a closed economy. Second, innovation is driven by market-size effects, including export markets. (See Lileeva and Trefler (2010) for some empirics on this point.) With trade costs, presumably the relevant market size will look like a market potential function in which weight is given to each trading partner's market in proportion to trade costs. These caveats point to the need for developing a dynamic, open-economy model to pursue the analysis rigorously.

Figure 10: Linking HOV with Directed Technical Change



Notes: Panel A plots $\ln(V_{Si}/V_{Ui})$ against $\ln(\widehat{\pi}_{Si}/\widehat{\pi}_{Ui})$ where π_{fi} is evaluated at σ=1.67. Panel B presents a partial regression plot of $\ln(V_{Si}/V_{Ui})$ against $\ln(\widehat{\pi}_{Si}/\widehat{\pi}_{Ui})$ after controlling for a fourth-order polynominal in real income per worker (see equation 32).

the 4th-order result. Specifically, we estimate

$$\ln \frac{\pi_{Si}(\sigma;\ \widehat{\beta}_{Si}, w_{Si})}{\pi_{Ui}(\sigma;\ \widehat{\beta}_{Ui}, w_{Ui})} = \sum_{k=0}^{4} \gamma_k (\ln y_i)^k + (\sigma - 1) \ln \left(\frac{V_{Si}}{V_{Ui}} \right) + \epsilon_i \tag{32}$$

using the same iterative procedure as before. This yields $\widehat{\sigma} = 1.89$ ($t = 4.92$, $R^2 = 0.54$). 1.89 is sufficiently close to 1.67 that it has no perceptible impact on our calculated $\pi_{fi}(\sigma;\ \widehat{\beta}_{fi}, w_{fi})$. The right panel of figure 10 displays the partial regression plot. *Once again, the fit is very good and supports the conclusion that countries direct their innovation towards improving the productivity of their abundant factors.*

7. An Observation on Some HOV Folklore

Since Gabaix (1997b), there is a sense that something was terribly wrong with the approach in Trefler (1993a). Because lemma 3 establishes a close connection between Trefler's approach and our current approach, some comment may be of help. There are two elements of the folklore.

First, depending on how the β_f are estimated, one ends up with very different conclusions about the performance of the Vanek equation. Specifically, return to Trefler's (1993) specification, which is nested by equation (VT) of lemma 3 and recall that β_{fi}^{VT} makes equation (VT) fit perfectly. Using Trefler's data, if one compares $(\beta_{fi}^{VT})^{-1}$ with $w_{fi}/w_{f,\text{us}}$ then one arrives at a positive view of the HOV model with PFPE. On the other hand, if one sets $(\beta_{fi})^{-1} = w_{fi}/w_{f\text{us}}$ and plugs this into equation (VT) one arrives at a negative view. It is important to understand that *this problem does not carry over to our current approach.* In case this is not obvious, recall from the start of section 5.1 that β_{fi}^{W} and β_{fi}^{T} are the values of β_{fi} that respectively make the Wage and Techniques equations fit perfectly. As shown in online appendix figure A2, plugging either β_{fi}^{W} or β_{fi}^{T} into the Vanek equation results in a very good fit. Indeed, the fit is virtually identical to what we saw in the bottom row of figure 3 because empirically, $\widehat{\beta}_{fi} = (\beta_{fi}^{W} + \beta_{fi}^{T})/2 \approx \beta_{fi}^{W} \approx \beta_{fi}^{T}$. Thus the first element of the folklore does not hold in the current setting.

The second element of the folklore is that the estimates of the β_{fi} do not change when trade is set to 0. There are two points of note. (*a*) We do not use the Vanek equation or *any* trade data in estimating $\widehat{\beta}_{fi}$ so this observation is not germane to this paper. (*b*) Setting trade to zero has big impacts elsewhere in the model. In particular, when trade is set to 0 we have $F_{fi}(\widehat{\beta}_f) = 0$. In that case we get a horrible fit of the Vanek equation: A plot of 0 $(= F_{fi}(\widehat{\beta}_f))$ against $V_{fi} - s_i V_{fw}$ is a horizontal line with slope 0 and correlation 0. It obviously matters if trade is set to 0. In summary, the folklore concerns about Trefler's (1993a) approach are of considerable interest, but not applicable here.[44]

8. Conclusion

This paper develops a multi-factor, multi-sector model that integrates the insights and machinery of Eaton and Kortum (2002) and Caliendo and Parro (2015) to provide a unified framework for thinking about how technology, endowments, and trade costs impact factor prices, trade in goods and the factor content of trade. Our motivation for developing this unified framework is that existing HOV empirics are not embedded in a unified theory of the impacts of productivity, endowments and trade costs on factor prices and trade. This makes it impossible to reconcile the diverse findings in the literature, let alone assess the relative importance of various determinants within a single framework. While there are many results in this paper, we believe that we have made two primary empirical contributions of note.

First, because our framework nests the Heckscher-Ohlin-Vanek (HOV) factor content prediction into a more general prediction, we show that the failure of the HOV ("Vanek") prediction is largely explained (*i*) by departures from factor price equalization, (*ii*) by factor augmenting international

[44] A deeper explanation of the second element of the folklore appears in online Appendix C.

technology differences and, to a lesser extent, (*iii*) by trade costs for government services. We note that distinguishing the first two is a non-trivial inference problem that has never before been raised in the HOV literature. *Second*, the factor-augmenting international technology differences needed to rationalize the Vanek prediction display the same pattern of skill bias needed to rationalize cross-country income differences (e.g. Caselli and Coleman (2006)) and cross-country evidence on directed technical change (e.g. (Acemoglu, 1998, 2009)). This provides a bridge between these literatures and HOV empirics.

Along the way, we have reconciled the diverse findings in the HOV literature which have previously been different to reconcile due to the lack of a unified framework. Most prominently, we have reconciled the findings of Trefler (1993b) and Davis and Weinstein (2001) who focus factor augmenting productivity and a failure of productivity-adjusted factor price equalization, respectively. We also show how an equation that nests the Vanek prediction can be derived even when trade costs and differences in preferences are admitted to the model.

Appendix

Proof of Equation (6) From equation (2), a fraction γ_{hj} of the sales of (h,j) go to primary factors so that $\gamma_{hj}Q_{hj}$ is the income generated by sales of (h,j). Hence $\sum_h \gamma_{hj}Q_{hj}$ is the income of country j.

1. Final goods: The utility function implies that $\gamma^U_{gj}\sum_{h=1}^{G}\gamma_{hj}Q_{hj}$ (i.e., γ^U_{gj} times income) is j's expenditures on g for use as final goods. A fraction $\widetilde{\pi}_{gi,j}$ is sourced from i. Hence $\gamma^U_{gj}\widetilde{\pi}_{gi,j}\sum_h \gamma_{hj}Q_{hj}$ is j's expenditures on (g,i) for use as final goods.
2. Intermediates: From equation (2) with (g,i) and (h,j) swapped, $\gamma_{g,hj}Q_{hj}$ is the expenditures by (h,j) on g for use as intermediates. A fraction $\widetilde{\pi}_{gi,j}$ is sourced from i. Hence $\sum_{h=1}^{G}\widetilde{\pi}_{gi,j}\gamma_{g,hj}Q_{hj}$ is j's expenditures on (g,i) for use as intermediates.
3. From points 1 and 2, $\widetilde{\pi}_{gi,j}\sum_{h=1}^{G}(\gamma_{g,hj}+\gamma^U_{gj}\gamma_{hj})Q_{hj}$ is j's total expenditures on (g,i).

Hence, $\sum_j \widetilde{\pi}_{gi,j}\sum_{h=1}^{G}(\gamma_{g,hj}+\gamma^U_{gj}\gamma_{hj})Q_{hj}$ is world expenditures on (g,i). But this must equal Q_{gi}. ■

Proof of Equation (8) By Shephard's lemma, firm ω_g's per unit demand for primary factor f is just the derivative of the unit cost function i.e., the derivative of $c_{gi}/z_{gi}(\omega)$. Hence the firm's total demand is $V_{fgi}(\omega) = \{\partial[c_{gi}/z_{gi}(\omega)]/\partial w_{fi}\}q_{gi}(\omega)$. Rearranging, $V_{fgi}(\omega) = [\partial c_{gi}/\partial w_{fi}](c_{gi})^{-1}\{[c_{gi}/z_{gi}(\omega)]q_{gi}(\omega)\}$. Integrating over ω_g and recalling the definitions of V_{fgi} (above equation 8) and Q_{gi} (above equation 6) yields $V_{fgi} = [\partial c_{gi}/\partial w_{fi}](c_{gi})^{-1}Q_{gi}$. Hence $d_{fgi} \equiv V_{fgi}/Q_{gi} = [\partial c_{gi}/\partial w_{fi}]/c_{gi}$. From the unit cost function (equation 4), $[\partial c_{gi}/\partial w_{fi}]/c_{gi}$ is just the right-hand side of equation (8). ■

Proof of Lemma 1 (1) $C_{gi,j}$ is country j's consumption of (g,i). The result follows from point 1 of the above proof of equation (6). (2) Follows immediately from point 3 of the above proof of equation (6). (3) $M_{gi,j}$ is country i's exports of good g to country j. Summing over j yields country i's exports of g, which is X_{gi}. (4) From the production function (equation 2), the production of (g,i) uses $\gamma_{h,gi}$ dollars of intermediate inputs per dollar of output. A fraction $\widetilde{\pi}_{hi,j}$ of these intermediate purchases are sourced from j. Hence, purchases of intermediates (h,j) per dollar of (g,i) output is $\widetilde{\pi}_{hi,j}\gamma_{h,gi}$. ■

Proof of Theorem 1 Pre-multiplying equation (10) by $\mathbf{A}_f$ yields $\mathbf{A}_f\mathbf{T} = \mathbf{A}_f(\mathbf{I}_{NG}-\mathbf{B})\mathbf{Q} - \mathbf{A}_f\mathbf{C} = \mathbf{D}_f\mathbf{Q} - \mathbf{A}_f\mathbf{C} = \begin{bmatrix} V_{f1} & \cdots & V_{fN} \end{bmatrix} - \mathbf{A}_f\mathbf{C}$. Consider column i of this equation, namely,

$$\mathbf{A}_f\mathbf{T}_i = V_{fi} - \mathbf{A}_f\mathbf{C}_i \tag{33}$$

where $\mathbf{T}_i$ and $\mathbf{C}_i$ are the ith columns of T and C, respectively. Hence

$$\mathbf{A}_f\Sigma_j\mathbf{T}_j = \Sigma_j V_{fj} - \mathbf{A}_f\Sigma_j\mathbf{C}_j. \tag{34}$$

Consider each of the three terms in this equation. $V_{fw} \equiv \Sigma_j V_{fj}$ is the world endowment of f. Recall that $\mathbf{T}_j$ is composed of blocks of $G\times 1$ matrices. Let $\mathbf{T}_{ij}$ be the ith block of $\mathbf{T}_j$. Then by inspection of the definition of $\mathbf{T}$ together with balanced trade, $\Sigma_j\mathbf{T}_{ij} = \mathbf{X}_i - \Sigma_j\mathbf{M}_{ji} = \mathbf{0}_G$ where $\mathbf{0}_G$ is the $G\times 1$ vector of zeros. Hence $\Sigma_j\mathbf{T}_j = \mathbf{0}_{NG}$ where $\mathbf{0}_{NG}$ is the $NG\times 1$ vector of zeros. Recall that $\mathbf{C}_w \equiv \sum_j \mathbf{C}_j$. Thus, equation (34) can be written as $0 = V_{fw} - \mathbf{A}_f\mathbf{C}_w$ or $0 = s_iV_{fw} - \mathbf{A}_f(s_i\mathbf{C}_w)$. Subtracting this from equation (33) yields $F_{fi} = V_{fi} - s_iV_{fw} - \mathbf{A}_f(\mathbf{C}_i - s_i\mathbf{C}_w)$. ■

Proof of Equations (17) ***and*** (W)***:*** Factor-market clearing (equation 9) yields $V_{fi} = \Sigma_g d_{fgi}Q_{gi}$. Substituting in $d_{fgi} = d_{fg,\mathrm{us}}\beta_{fi}/\delta_{gi}$ (equation 13) delivers $V_{fi} = \beta_{fi}\Sigma_g d_{fg,\mathrm{us}}Q_{gi}/\delta_{gi}$. Rearranging this yields equations (17) and (W). ■

Proof of Lemma 2 Part 1. By definition $V'_{fi} \equiv \sum_{g \in G'} A_{fgi} C_{gi,i}$ where G' is the set of nontradable goods. Since $C_{gj,i} = 0$ for nontradables (i does not buy from j), $V'_{fi} \equiv \sum_j \sum_{g \in G'} A_{fgj} C_{gj,i}$. Hence $V'_{fw} \equiv \sum_i V'_{fi} = \sum_i \sum_j \sum_{g \in G'} A_{fgj} C_{gj,i} = \sum_j \sum_{g \in G'} A_{fgj} C_{gj,w}$. Hence $V'_{fi} - s_i V'_{fw} = \sum_j \sum_{g \in G'} A_{fgj}(C_{gj,i} - s_i C_{gj,w})$. But consumption similarity holds for tradables i.e., $(C_{gj,i} - s_i C_{gj,w}) = 0$ for $g \notin G'$. Hence $V'_{fi} - s_i V'_{fw} = \sum_j \sum_g A_{fgj}(C_{gj,i} - s_i C_{gj,w}) = \mathbf{A}_f(\mathbf{C}_i - s_i \mathbf{C}_w)$.

Part 2. Let $\mathbf{B}_{gj,i}$ be a row of the $G \times G$ matrix $\mathbf{B}_{ji}$. Then $\mathbf{B}_{gj,i} Q_i$ is country i's intermediate demand for good g produced in country j. i's imports of g produced in j are

$$M_{gj,i} = \mathbf{B}_{gj,i} Q_i + C_{gj,i} \ .$$

Adjusted imports impose consumption similarity (meaning $C_{gj,i} = s_i C_{gj,w}$) for $g \in G'$ and are thus

$$M'_{gj,i} = \mathbf{B}_{gj,i} Q_i + s_i C_{gj,w} \ \text{ for } \ g \in G'.$$

Likewise, exports are

$$X_{gi} = Q_{gi} - B_{gi,i} Q_i - C_{gi,i} \ .$$

Adjusted exports impose consumption similarity ($C_{gj,i} = s_i C_{gj,w}$) for $g \in G'$ and are thus

$$X'_{gi} = Q_{gi} - B_{gi,i} Q_i - s_i C_{gi,w} \ \text{ for } \ g \in G'.$$

Consider the section 2 expression for the $GN \times 1$ vector $\mathbf{T}_i$. Let $\mathbf{T}'_i$ be a $GN \times 1$ vector whose elements are as follows. When g is tradable ($g \notin G'$), the gi element is X_{gi} and the gj element ($j \neq i$) is $-M_{gj,i}$. When g is nontradable ($g \in G'$), the gi element is X'_{gi} and the gj element ($j \neq i$) is $-M'_{gj,i}$. It follows that the nonzero elements of $\mathbf{T}'_i - \mathbf{T}_i$ are either $X'_{gi} - X_{gi} = C_{gi,i} - s_i C_{gi,w}$ or $(-M'_{gj,i}) - (-M_{gj,i}) = C_{gj,i} - s_i C_{gj,w}$. That is, the nonzero elements are always of the form $C_{gj,i} - s_i C_{gj,w}$. Since consumption similarity holds for tradables, the zero elements can also be expressed as $C_{gj,i} - s_i C_{gj,w} (= 0)$. Hence $F'_{fi} - F_{fi} = \mathbf{A}_f(\mathbf{T}'_i - \mathbf{T}_i) = \mathbf{A}_f(\mathbf{C}_i - s_i \mathbf{C}_w)$. ■

Consumption Similarity Calculation: From part 2 of the proof of lemma 2, we have already defined $M'_{gj,i}$, X'_{gi} and $\mathbf{T}'_i$ in terms of the data $\mathbf{Q}_i$, $\mathbf{B}_{ij}$, and s_i as well as $\mathbf{C}_{jw}$ where $\mathbf{C}_{jw}$ is defined implicitly using the world goods market clearing condition:

$$\mathbf{Q}_i = \mathbf{C}_{iw} + \sum_{j=1}^{N} \mathbf{B}_{ij} \mathbf{Q}_j. \tag{35}$$

Note that imposing consumption similarity on Government Services affects consumption patterns, but not production patterns $\mathbf{Q}$ or intermediate input usage ratios $\mathbf{B}$.

List of Countries: Australia , Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Great Britain, Greece, Hungary, Indonesia, Ireland, Italy, Japan, Korea, Latvia, Lithuania, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sweden, Taiwan, Turkey, USA.

List of Industries and ISIC codes: Agriculture (110); Mining (200); Food, Beverages, Tobacco (311); Textiles and Textile Products (321); Leather and Footwear (323); Wood and Products of Wood (331); Pulp, Paper, Printing, and Publishing (341); Chemicals (351); Rubber and Plastics (355); Non-Metallic Minerals (369); Basic and Fabricated Metals (371); Machinery, nec. (381); Transport Equipment (384); Electrical and Optical Equipment (385); Manufacturing, nec.(390); Electricity, Gas, Water Supply (400); Construction (500); Wholesale and Retail Trade (600); Hotels and Restaurants (630); Transport (700); Finance, Insurance, Real Estate (800); Government Services (900);

Table A1: Values of β_{fi} and π_{fi}

	Unskilled Labor				Skilled Labor			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Country	$\widehat{\beta}_{Ui}$	w_{Ui}	$\widehat{\pi}_{Ui}$	$\frac{w_{Ui}}{\widehat{\pi}_{Ui}}$	$\widehat{\beta}_{Si}$	w_{Si}	$\widehat{\pi}_{S,i}$	$\frac{w_{Si}}{\widehat{\pi}_{Si}}$
Australia	1.13	0.98	1.14	0.86	0.70	0.89	0.44	2.04
Austria	1.22	0.86	0.92	0.93	0.76	0.79	0.37	2.13
Belgium	0.81	1.42	1.76	0.81	0.41	1.10	0.33	3.29
Brazil	12.17	0.07	0.06	1.26	4.60	0.18	0.14	1.31
Bulgaria	8.68	0.07	0.03	2.20	2.91	0.10	0.01	6.63
Canada	1.19	0.97	1.21	0.80	0.93	0.74	0.43	1.74
China	11.99	0.04	0.01	3.50	1.95	0.04	0.00	50.46
Cyprus	1.89	0.44	0.33	1.33	2.48	0.48	0.62	0.77
Czech Republic	2.93	0.26	0.17	1.52	1.44	0.26	0.06	4.39
Denmark	0.79	1.43	1.73	0.83	0.96	0.98	0.89	1.10
Estonia	2.74	0.23	0.11	2.01	4.67	0.20	0.17	1.14
Finland	0.84	0.98	0.72	1.35	1.21	0.77	0.69	1.11
France	0.81	1.30	1.39	0.93	0.84	1.04	0.86	1.21
Germany	1.06	1.03	1.17	0.88	0.93	0.94	0.77	1.22
Great Britain	0.93	1.08	1.10	0.99	1.02	0.90	0.79	1.14
Greece	2.18	0.34	0.22	1.55	1.86	0.38	0.23	1.68
Hunary	3.59	0.23	0.17	1.35	2.61	0.29	0.19	1.52
Indonesia	23.25	0.03	0.02	1.70	5.57	0.07	0.02	4.03
Ireland	0.76	0.87	0.47	1.85	1.04	0.86	0.72	1.18
Italy	1.19	0.75	0.64	1.18	0.51	0.69	0.15	4.73
Japan	1.27	0.74	0.68	1.09	1.36	0.65	0.54	1.20
Latvia	3.48	0.21	0.13	1.60	3.28	0.20	0.10	1.94
Lithuania	4.58	0.17	0.12	1.40	6.66	0.17	0.20	0.85
Malta	2.16	0.40	0.33	1.22	0.83	0.48	0.12	3.97
Mexico	6.75	0.12	0.08	1.42	2.96	0.15	0.04	3.41
Netherlands	0.85	1.14	1.08	1.05	0.84	0.98	0.74	1.33
Poland	4.22	0.17	0.10	1.69	3.04	0.21	0.11	1.92
Portugal	2.75	0.41	0.49	0.84	0.88	0.66	0.29	2.25
Romanis	8.51	0.11	0.10	1.10	2.34	0.17	0.04	4.13
Russia	9.76	0.11	0.11	0.95	4.78	0.12	0.05	2.32
Slovakia	3.00	0.18	0.07	2.57	1.81	0.17	0.03	5.78
Slovenia	2.24	0.43	0.42	1.04	1.78	0.49	0.41	1.22
South Korea	1.69	0.29	0.10	2.90	5.14	0.25	0.35	0.70
Spain	1.08	0.68	0.43	1.58	1.50	0.60	0.52	1.17
Sweden	0.91	1.31	1.69	0.77	0.88	0.88	0.60	1.46
Taiwan	2.52	0.29	0.18	1.61	3.09	0.29	0.24	1.19
Turkey	4.70	0.11	0.04	2.80	2.00	0.19	0.05	4.17
United States	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Notes: Columns 1 and 5 present $\widehat{\beta}_{fi}$. Columns 2 and 6 are data described in section 4. Columns 3, 4, 7, and 8 are calculated using the data in columns 1, 2, 5, and 6, the definition of $\widehat{\beta}_{fi}$, and σ=1.67.

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Online Appendix to

"Endowments, Factor Prices, and Skill-Biased Technology: Importing Development Accounting into HOV"

Appendix A. A Helpman and Krugman (1985) Model

This section shows that our estimating equations can also be generated using a model of monopolistic competition, increasing returns to scale, and costless trade.

Appendix A.1. Preferences, Endowments and Technology

Let $i,j = 1,\ldots,N$ index countries, let $g = 1,\ldots,G$ index goods (or industries), and let $\omega \in \Omega_{gi}$ index varieties of a horizontally differentiated good g produced in country i as in Krugman (1980). Preferences are internationally identical and homothetic with the nested structure:

$$U = \prod_{g=1}^{G} (U_g)^{\eta_g} \quad \text{and} \quad U_g = \left(\sum_{i=1}^{N} \int_{\omega \in \Omega_{gi}} x_{gi}(\omega)^{\frac{\rho_g - 1}{\rho_g}} d\omega \right)^{\frac{\rho_g}{\rho_g - 1}},$$

where $\rho_g > 1$ is the elasticitiy of substitution, $\eta_g > 0$, $\Sigma_g \eta_g = 1$, and $x_{gi}(\omega)$ is a quantity consumed. Let $p_{gi}(\omega)$ be the corresponding price. We assume that trade is costless so that all consumers worldwide face the same price $p_{gi}(\omega)$. Then the price index associated with U_g is $P_g = \left(\sum_{i=1}^{N} \int_{\omega \in \Omega_{gi}} p_{gi}(\omega)^{1-\rho_g} d\omega \right)^{\frac{1}{1-\rho_g}}$.

Let $f = 1,\ldots,K$ index primary factors such as labor. V_{fi} is country i's exogenous endowment of factor f and w_{fi} is its price. Let $\mathbf{w}_i = (w_{1i},\ldots,w_{Ki})$. We assume that factors are mobile across industries within a country and immobile across countries.

Turning to technology, a firm uses both primary factors and intermediate inputs of goods $h = 1,\ldots,G$. The production function is Cobb-Douglas in (*a*) an index of primary factors and (*b*) CES indexes of each of the G intermediate goods. This results in a unit cost function for $\omega \in \Omega_{gi}$ of the form

$$\phi_{gi}(\mathbf{w}_i,\mathbf{p}) = \left[c_{gi}(\mathbf{w}_i) \right]^{\gamma_{g0}} \prod_{h=1}^{G} P_{gh}^{\gamma_{gh}} \tag{36}$$

where

$$P_{gh} = \left(\sum_{j=1}^{N} \int_{\nu \in \Omega_{hj}} \alpha_{gh}\ p_{hj}(\nu)^{1-\rho_h}\ d\nu \right)^{\frac{1}{1-\rho_h}},$$

$\mathbf{p} = \{p_{hj}(\nu) : \nu \in \Omega_{hj}, \forall h,j\}$ is the set of all product prices, $\nu \in \Omega_{hj}$ indexes varieties when used as inputs, and the γ_{gh} are positive constants with $\Sigma_{h=0}^{G} \gamma_{gh} = 1$. $c_{gi}(\mathbf{w}_i)$ is a constant-returns-to-scale unit cost function associated with primary factors. P_{gh} is the unit cost function associated with the CES index of intermediate good h in the production of good g. The α_{gh} are constants that allow for empirically relevant factor intensity asymmetries.[45]

Marginal costs are $\phi_{gi}(\mathbf{w}_i,\mathbf{p})$. Per variety variable costs are $\phi_{gi}(\mathbf{w}_i,\mathbf{p})q_{gi}(\omega)$. We assume that fixed costs are proportional to marginal costs and given by $\phi_{gi}(\mathbf{w}_i,\mathbf{p})\overline{\phi}_g$ for some constant $\overline{\phi}_g > 0$.

Appendix A.2. Firm Behavior

Profits for any variety $\omega \in \Omega_{gi}$ are

$$\left[p_{gi} - \phi_{gi}(\mathbf{w}_i,\mathbf{p}) \right] q_{gi} - \phi_{gi}(\mathbf{w}_i,\mathbf{p})\overline{\phi}_g.$$

[45] We assume that a firm does not buy from itself. Since anything it bought from itself would have zero measure, we do not have to keep track of this in the expression for P_{gh}; however, we will have to keep track of this in the discussion of profit maximization below.

There are two sources of demand for $\omega \in \Omega_{gi}$: (1) Consumers in country j demand $p_{gi}{}^{-\rho_g} P_g^{\rho_g - 1} \eta_g Y_j$ where Y_j is national income. (2) Downstream producers of variety $\nu \in \Omega_{hj}$ each demand $b_{ij}(g,h) \left[q_{hj} + \overline{\phi}_h\right]$ where, by Shephard's Lemma,

$$b_{ij}(g,h;\mathbf{w}_j,\mathbf{p}) = \frac{\partial \phi_{hj}(\mathbf{w}_j,\mathbf{p})}{\partial p_{gi}}.$$

$b_{ij}(g,h;\mathbf{w}_j,\mathbf{p})$ is necessarily complicated notation because we need to track the entire global supply chain. Aggregating over both final and intermediate-input demands for a typical variety $\omega \in \Omega_{gi}$ yields the following result that will be useful later:

Lemma 1 *$q_{gi} = p_{gi}^{-\rho_g} \kappa_g$ for some $\kappa_g > 0$ and all i.*

The proof appears in the appendix to this section. The last line of the proof is an expression for κ_g, from which it is apparent that κ_g is a constant from the firm's perspective.

Using lemma 1, profit maximization for $\omega \in \Omega_{gi}$ yields the standard optimal price:

$$p_{gi} = \frac{\rho_g}{\rho_g - 1} \phi_{gi}(\mathbf{w}_i,\mathbf{p}). \tag{37}$$

Zero profits for $\omega \in \Omega_{gi}$, together with this pricing rule, yield:

$$q_{gi} = (\rho_g - 1)\overline{\phi}_g \,. \tag{38}$$

Turning to factor markets, consider the demand for factor f by firm $\omega \in \Omega_{gi}$. By Shephard's Lemma this (direct) demand per unit of output is

$$d_{fgi}(\mathbf{w}_i,\mathbf{p}) = \frac{\partial \phi_{gi}(\mathbf{w}_i,\mathbf{p})}{\partial w_{fi}}.$$

Factor market clearing in country i is thus

$$V_{fi} = \sum_{g=1}^{G} n_{gi} d_{fgi}(\mathbf{w}_i,\mathbf{p}) \left[q_{gi} + \overline{\phi}_g\right] \tag{39}$$

where $n_{gi} = \int_{\omega \in \Omega_{gi}} d\omega$ is the measure of identical firms producing varieties of g in country i.

Appendix A.3. Equilibrium

Define $\mathbf{n}^* = \left\{n_{gi}^*\right\}_{\forall g,i}$, $\mathbf{w}^* = \left\{w_{fi}^*\right\}_{\forall f,i}$, and $\mathbf{p}^* = \{p_{gi}^*(\omega) \colon \omega \in \Omega_{gi}, \forall g,i\}$. An equilibrium is a triplet $(\mathbf{w}^*,\mathbf{p}^*,\mathbf{n}^*)$ such that when consumers maximize utility and firms maximize profits, product markets clear internationally for each variety, factor markets clear nationally for each factor, and profits are zero. Market clearing and zero profits imply that all income is factor income ($Y_i = \Sigma_f w_{fi} V_{fi}$) and that trade is balanced. It follows from this definition of equilibrium that $(\mathbf{w}^*,\mathbf{p}^*,\mathbf{n}^*)$ is an equilibrium if it satisfies equations (37)–(39). [46]

[46] From equation (38), firm output $q_{gi} = q_g$ is independent of i. Since $q_g = p_{gi}^{-\rho_g} \kappa_g$, it follows that price $p_{gi} = p_g$ is also independent of i. As discussed in Remark 1 of the appendix to this section, this plays no role and is for expositional simplicity.

Appendix A.4. Empirical Specification

This subsection shows how the alternate model of the previous section delivers the empirical specifications in the main text. Assume that the cost function for primary factors is

$$c_{gi}(\mathbf{w}_i) = \left[\sum_f \frac{\alpha_{fg}}{\delta_{gi}} \left(\frac{w_{fi}}{\pi_{fi}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{40}$$

where the α_{fg} control (exogenous) factor intensities, the π_{fi} are factor-augmenting productivity (technology) parameters, σ is the elasticity of substitution, and the δ_{gi} are Ricardian technology parameters. We will assume that cost functions satisfy equation (40) for the remainder of this appendix. When the $\delta_{gi} = 1$ for all g and i, equation (40) is a special case of Trefler's (1993) factor-augmenting technology and PFPE is straightforwardly defined as

$$\frac{w_{fi}}{\pi_{fi}} = \frac{w_{f,\text{us}}}{\pi_{f,\text{us}}} \,. \tag{41}$$

We consider a diversified equilibrium in which the Ricardian technology differences δ_{gi} lead to failure of PFPE.[47] It is straightforward to show that there are δ_{gi} which support a diversified equilibrium, but we will need to ensure that our empirical counterparts of δ_{gi}, $\widehat{\delta}_{gi}$, are consistent with such an equilibrium. Here we review several minor points about diversification. First, the δ_{gi} can be interpreted as differences in quality, in which case our diversification has the flavour of Schott (2004).[48] Schott provides abundant evidence of diversification in his analysis of 'product overlap' at the 10-digit HS level. Second, we can treat observed diversification as trade in varieties rather than as a function of aggregation bias. Third, country-level productivity can be loaded onto either the π_{fi} or the δ_{gi} so a normalization is needed. As in th main text, we normalize the δ_{gi} using $\delta_{g,\text{us}} = 1$ for all g and $\Sigma_g \theta_{Lgi} \delta_{gi} = 1 \; \forall i$ where θ_{Lgi} is the share of country i's total labor endowment employed in industry g.

If varieties of good g are produced both by country i and by the United States, then Shephard's lemma implies

$$d_{fgi} = \beta_{fi} d_{fg,\text{us}} / \delta_{gi} \tag{42}$$

where

$$\beta_{fi} \equiv \left(\frac{w_{fi}/\pi_{fi}}{w_{f,\text{us}}/\pi_{f,\text{us}}} \right)^{-\sigma} \left(\frac{\pi_{fi}}{\pi_{f,\text{us}}} \right)^{-1} \,. \tag{43}$$

Appendix A.5. The Three Estimating Equations

We now show that the above model delivers three estimating equations that are identical to those derived in the main text. Consider first the Vanek equation. Recall that $F_{fi} = \mathbf{D}_f(\mathbf{I}_{NG} - \mathbf{B})^{-1}\mathbf{T}_i$ is the factor content of trade using observed factor usage $\mathbf{D}_f$. Let $\mathbf{D}_f(\beta_f)$ be a $1 \times GN$ matrix with typical element $\beta_{fi} d_{fg,\text{us}} / \delta_{gi}$ (the right-hand side of equation 42) and define

$$F_{fi}(\beta_f) \equiv \mathbf{D}_f\left(\beta_f\right) \left[\mathbf{I}_{NG} - \mathbf{B}\right]^{-1} \mathbf{T}_i \,. \tag{44}$$

(We suppress the δ_{gi} as arguments.) Under our cost function assumption (equation 40), $\mathbf{D}_f(\beta_f)$ equals the data $\mathbf{D}_f$ and $F_{fi}(\beta_f)$ equals the data F_{fi}. It follows that the Vanek equation becomes

$$F_{fi}(\beta_f) = V_{fi} - s_i \sum\nolimits_{j=1}^{N} V_{fj} \,. \tag{V}$$

[47] δ_{gi} prevents international goods price equalization from leading to international productivity adjusted factor price equalization.

[48] This is similar to calibrating 'wedges' e.g., Hottman, Redding and Weinstein (2014) infer quality as the wedge that rationalizes demand for a given set of prices and quantities.

As in the main text, because $F_{fi}(\beta_f)$ is linear in $\mathbf{D}_f(\beta_f)$, $\mathbf{D}_f(\beta_f)$ is linear in β_f, (therefore) $F_{fi}(\beta_f)$ is linear in β_f, and equation (V) can be written as a system of linear equations that uniquely solve for the vector β_f.

Turning to the Wage equation, substitute factor demands (equation 42) into the factor-market clearing condition (equation 39) and solve for productivity adjusted wages to obtain

$$\frac{w_{fi}/\pi_{fi}}{w_{f,\text{us}}/\pi_{f,\text{us}}} = \left[\frac{\pi_{f,\text{us}} V_{f,\text{us}}}{\pi_{fi} V_{fi}}\right]^{1/\sigma} \left(\sum_{g=1}^{G} \frac{d_{fg,\text{us}} Q_{gi}}{\delta_{gi} V_{f\text{us}}}\right)^{1/\sigma} . \tag{45}$$

See the appendix for a proof. Rearranging this equation yields the “Wage Equation":

$$W_{fi}(\mathbf{D}_f, \mathbf{Q}, V_{fi}, \delta) \equiv \left[\sum_{g=1}^{G} \frac{d_{fg,\text{us}} Q_{gi}}{\delta_{gi} V_{fi}}\right]^{-1} = \beta_{fi} \tag{W}$$

where $\delta \equiv \{\delta_{gi}\}_{\forall g,i}$ and $W_{fi}()$ is a function.

Turning to the third and last equation, the Techniques equation, we aggregate equation (42) up to the same level as the Vanek and Wage equations, namely, at the factor-country level. Specifically, taking the employment-weighted average of equation (42) yields $\sum_g \theta_{fgi} d_{fgi}/d_{fg,\text{us}} = \sum_g \theta_{fgi} \beta_{fi}/\delta_{gi}$ where θ_{fgi} is the share of V_{fi} that is employed in industry g. The θ_{fgi} are data and satisfy $\sum_g \theta_{fgi} = 1$. Rearranging to isolate β_{fi} yields

$$T_{fi}(\mathbf{D}_f, \delta) \equiv \frac{\sum_{g=1}^{G} \theta_{fgi} \left(d_{fgi}/d_{fg,\text{us}}\right)}{\sum_{g=1}^{G} \theta_{fgi}/\delta_{gi}} = \beta_{fi} \tag{T}$$

where $T_{fi}()$ is a function.

While the interpretation of δ_{gi} is different than in the main text, our strategy to calibrate it remains the same. A in the main text, we use the normalizations $\delta_{g,\text{us}} = 1$ and $\sum_g \theta_{Lgi} \delta_{gi} = 1$. From equation (42), $\delta_{gi} = (d_{fg,\text{us}}/d_{fgi})\beta_{fi}$. Hence, $\delta_{gi} = \delta_{gi}/\sum_g \theta_{Lgi}\delta_{gi} = (d_{fg,\text{us}}/d_{fgi})/\sum_g \theta_{Lgi}(d_{fg,\text{us}}/d_{fgi})$. This establishes that we can calibrate the δ_{gi} using data on factor usages d_{fgi}.[49] Note that since the calibrated δ_{gi} satisfy equation (13), they are consistent with a diversified equilibrium. As in the main text, this calibration of δ_{gi} depends on f and so is not unique. As in the main text, we work with the geometric mean of the two: $\delta_{gi} = [(d_{Ug,\text{us}}/d_{Ugi})\beta_{Ui}]^{1/2}[(d_{Sg,\text{us}}/d_{Sgi})\beta_{Si}]^{1/2}$. This yields

$$\widehat{\delta}_{gi} \equiv \frac{(d_{Ug,\text{us}}/d_{Ugi})^{1/2}(d_{Sg,\text{us}}/d_{Sgi})^{1/2}}{\sum_{g'=1}^{G} \theta_{Lg'i}(d_{Ug',\text{us}}/d_{Ug'i})^{1/2}(d_{Sg',\text{us}}/d_{Sg'i})^{1/2}} . \tag{46}$$

[49]Intuitively, a Ricardian technology difference $\delta_{gi}/\delta_{g,\text{us}}$ is the average difference in input requirements $d_{fg,\text{us}}/d_{fgi}$ after purging them of their factor-augmenting productivity and wage components β_{fi}.

Appendix to Appendix A

Proof of Lemma 1 We start with a preliminary result involving change of indexes. From equation (36), $\phi_{hj}(\mathbf{w}_j,\mathbf{p}) = \left[c_{hj}(\mathbf{w}_j)\right]^{\gamma_{h0}} \prod_{g=1}^{G} P_{hg}^{\gamma_{hg}}$ where $P_{hg} = \left(\sum_{i=1}^{N} \int_{\omega\in\Omega_{gi}} \alpha_{hg}\ p_{gi}(\omega)^{1-\rho_g}\ d\omega\right)^{\frac{1}{1-\rho_g}}$. Also, note that $\partial P_{hg'}^{\gamma_{hg'}}/\partial p_{gi}(\omega) = 0$ for $g' \neq g$ and $\partial P_{hg}^{\gamma_{hg}}/\partial p_{gi}(\omega) = \gamma_{hg} P_{hg}^{\gamma_{hg}-1+\rho_g}\ \alpha_{hg}\ p_{gi}(\omega)^{-\rho_g}$. Hence

$$\begin{aligned} b_{ij}(g,h,\mathbf{w}_j,\mathbf{p}) &= \partial\phi_{hj}(\mathbf{w}_j,\mathbf{p})/\partial p_{gi}(\omega) \\ &= c_{hj}^{\gamma_{g0}} \prod_{g'\neq g} P_{hg'}^{\gamma_{hg'}} \left[\gamma_{hg} P_{hg}^{\gamma_{hg}-1+\rho_g} \alpha_{hg} p_{gi}(\omega)^{-\rho_g}\right] \\ &= \phi_{hj}(\mathbf{w}_j,\mathbf{p})\gamma_{hg} P_{hg}^{-1+\rho_g} \alpha_{hg} p_{gi}(\omega)^{-\rho_g}\ . \end{aligned} \tag{47}$$

As explained in section (Appendix A.2), demand for variety $\omega \in \Omega_{gi}$ is the sum of demands for final goods and intermediate inputs: $q_{gi}(\omega) = p_{gi}(\omega)^{-\rho_g} P_g^{\rho_g-1} \eta_g \Sigma_{j=1}^{N} Y_j + \Sigma_{h=1}^{G}\Sigma_{j=1}^{N} \int_{\nu\in\Omega_{hj}} b_{ij}(g,h,\mathbf{w}_j,\mathbf{p}) \left[q_{hj}(\nu) + \overline{\phi}_h\right] d\nu$. Substituting in equation (47), the lemma follows with $\kappa_g \equiv P_g^{\rho_g-1} \eta_g \Sigma_{j=1}^{N} Y_j + \Sigma_{h=1}^{G}\Sigma_{j=1}^{N} \int_{\nu\in\Omega_{hj}} \phi_{hj}(\mathbf{w}_j,\mathbf{p})\gamma_{hg} P_{hg}^{-1+\rho_g} \alpha_{hg} \left[q_{hj}(\nu) + \overline{\phi}_h\right] d\nu$. ■

Proof of Equation** (42)**: By Shephard's lemma, $d_{fgi} = \partial\phi_{gi}/\partial w_{fi}$. Hence, $d_{fgi} = \left[\partial\phi_{gi}/\partial c_{gi}\right]\left[\partial c_{gi}/\partial w_{fi}\right] = \left[\gamma_{g0} c_{gi}^{-1}\phi_{gi}\right]\left[\alpha_{fg}\left(w_{fi}\right)^{-\sigma}\left(\pi_{fi}\right)^{\sigma-1}\left(\delta_{gi}\right)^{-1}\left(c_{gi}\right)^{\sigma}\right]$. Recall that at the end of section Appendix A.3 we established that $p_{gi} = p_g$. Hence from equation (37), $1 = p_{gi}/p_{g,\mathrm{us}} = \phi_{gi}(\mathbf{w}_i,\mathbf{p})/\phi_{g,\mathrm{us}}(\mathbf{w}_{\mathrm{us}},\mathbf{p}) = \left(c_{gi}^{\gamma_{g0}} \prod_{h=1}^{G} P_{gh}^{\gamma_{gh}}\right) / \left(c_{g,\mathrm{us}}^{\gamma_{g0}} \prod_{h=1}^{G} P_{gh}^{\gamma_{gh}}\right) = \left(c_{gi}/c_{g,\mathrm{us}}\right)^{\gamma_{g0}}$ or $c_{gi} = c_{g,\mathrm{us}}$ and $\phi_{gi}(\mathbf{w}_i,\mathbf{p}) = \phi_{g,\mathrm{us}}(\mathbf{w}_{\mathrm{us}},\mathbf{p})$. Hence, $d_{fgi}/d_{fg,\mathrm{us}} = (w_{fi}/w_{f,\mathrm{us}})^{-\sigma}(\pi_{fi}/\pi_{f,\mathrm{us}})^{\sigma-1}(\delta_{gi}/\delta_{g,\mathrm{us}})^{-1}$. Equation (42) follows with the normalization $\delta_{g,\mathrm{us}} = 1$. ■

Remark 1 *If $p_{gi} \neq p_{g,us}$ then we get $d_{fgi} = d_{fg,us}(w_{fi}/w_{f,us})^{-\sigma}(\pi_{fi}/\pi_{f,us})^{\sigma-1}(\delta'_{gi}/\delta'_{g,us})^{-1}$ where $\delta'_{gi} = \delta_{gi}/p_{gi}$. That is, this leads to a reinterpretation of the δ_{gi}, but does not otherwise affect anything in the paper.*

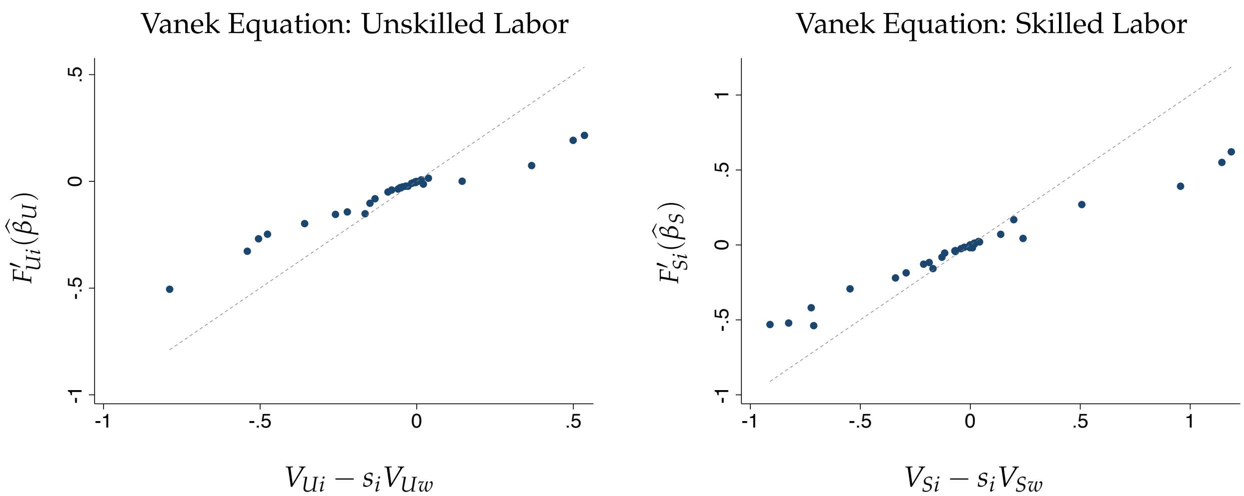
Proof of Equation** (W)**: Recall that $Q_{gi} = n_{gi}(q_g + \overline{\phi}_g)$. Plugging this into the factor-market clearing equation (39) yields $V_{fi} = \Sigma_g d_{fgi} Q_{gi}$. Substituting in $d_{fgi} = d_{fg,\mathrm{us}}\beta_{fi}/\delta_{gi}$ (equation 13) into this expression delivers $V_{fi} = \beta_{fi}\Sigma_g d_{fg,\mathrm{us}} Q_{gi}/\delta_{gi}$. Equation ($W$) follows from a simple re-arrangement. ■

Appendix B. Additional Empirical Results

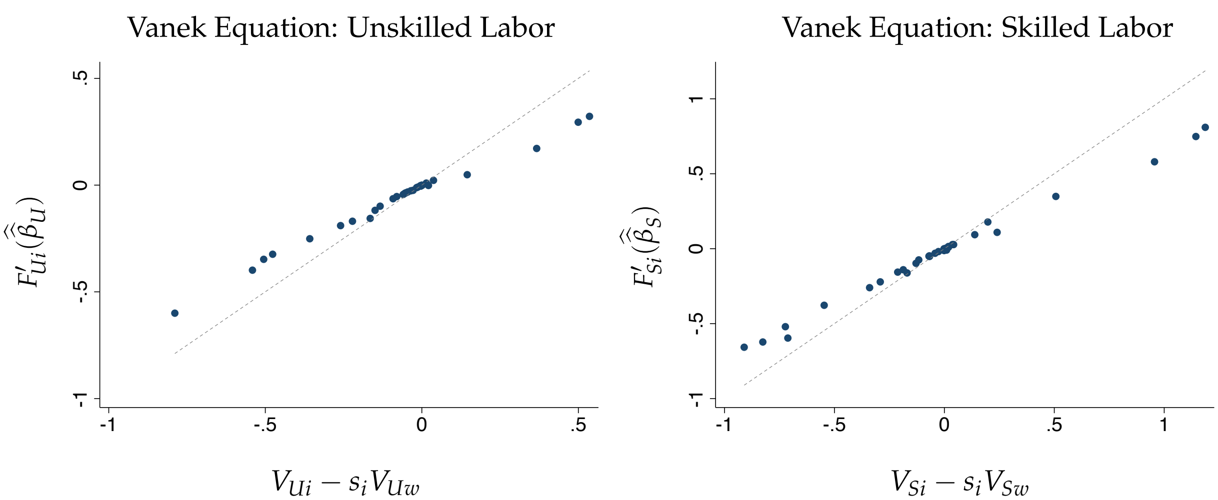
This section contains additional empirical results. The top row of figure A1 presents the same results for the Vanek equation as in figures 3 and 6 except that the observations for China and the United States are removed for visual exposition. The top row of figure A2 presents results for the Vanek equation placing all weight on the Wage equation, and then bottom row of figure A2 presents results for the Vanek equation placing all weight on the Techniques equation. Figure A3 presents results for the Wage equation, Techniques equation, Vanek equation, and disaggregated techniques equation when we use three types of labor (high skilled, medium skilled, and low skilled). Table A1 presents test statistics associated with these three types of labor analogous to Table 1. Figure A4 presents results for the Wage equation, Techniques equation, Vanek equation, and disaggregated techniques equation for capital and labor. Table A2 presents test statistics associated with this specification.

Figure A1: The Vanek Equation with Outliers (US and China) not Displayed

Panel A. Two-Equation Approach ($\widehat{\beta}_f$, Equations W and T)

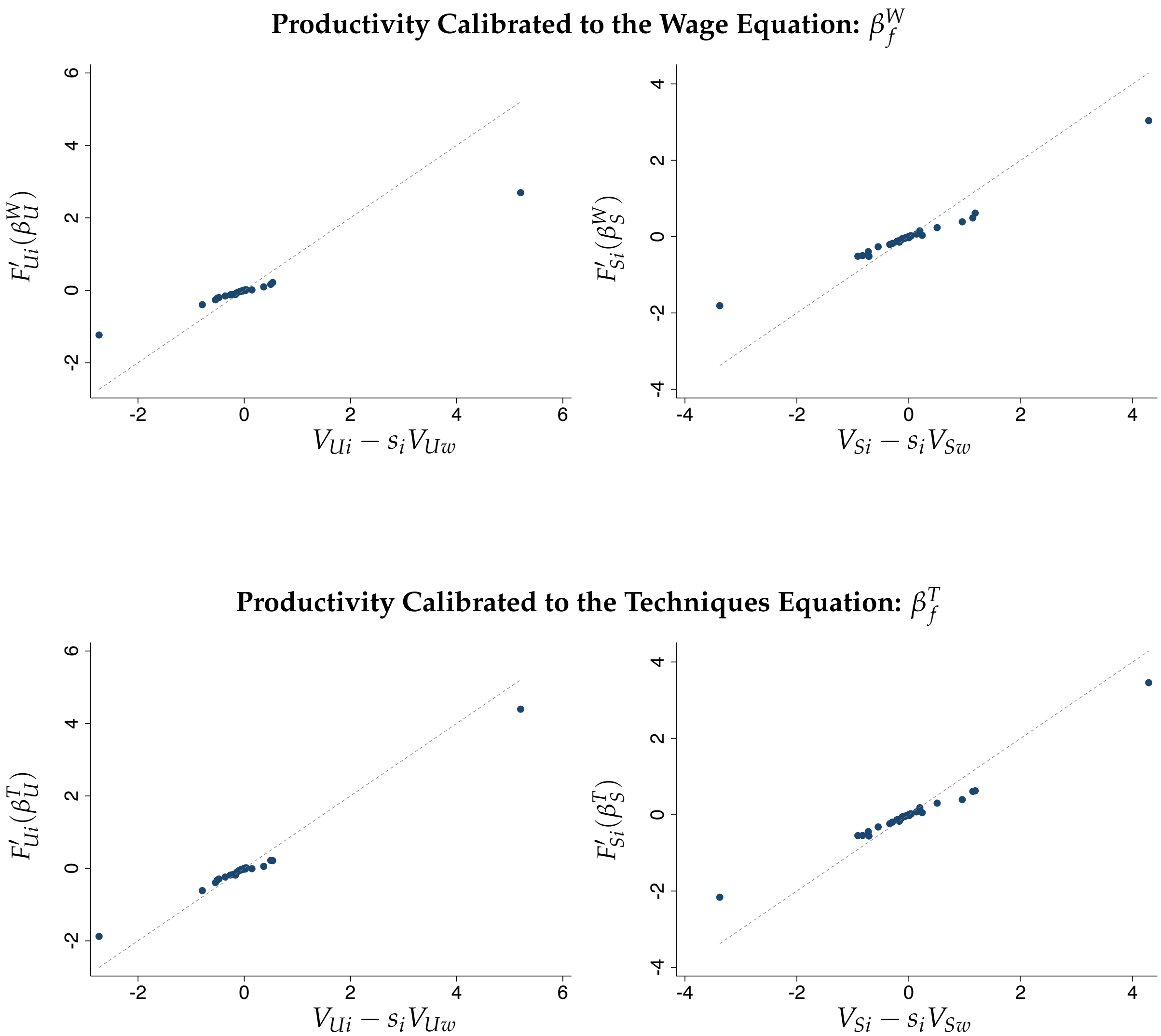


Panel B. Three-Equation Approach ($\widehat{\widehat{\beta}}_f$, Equations W, T and V)



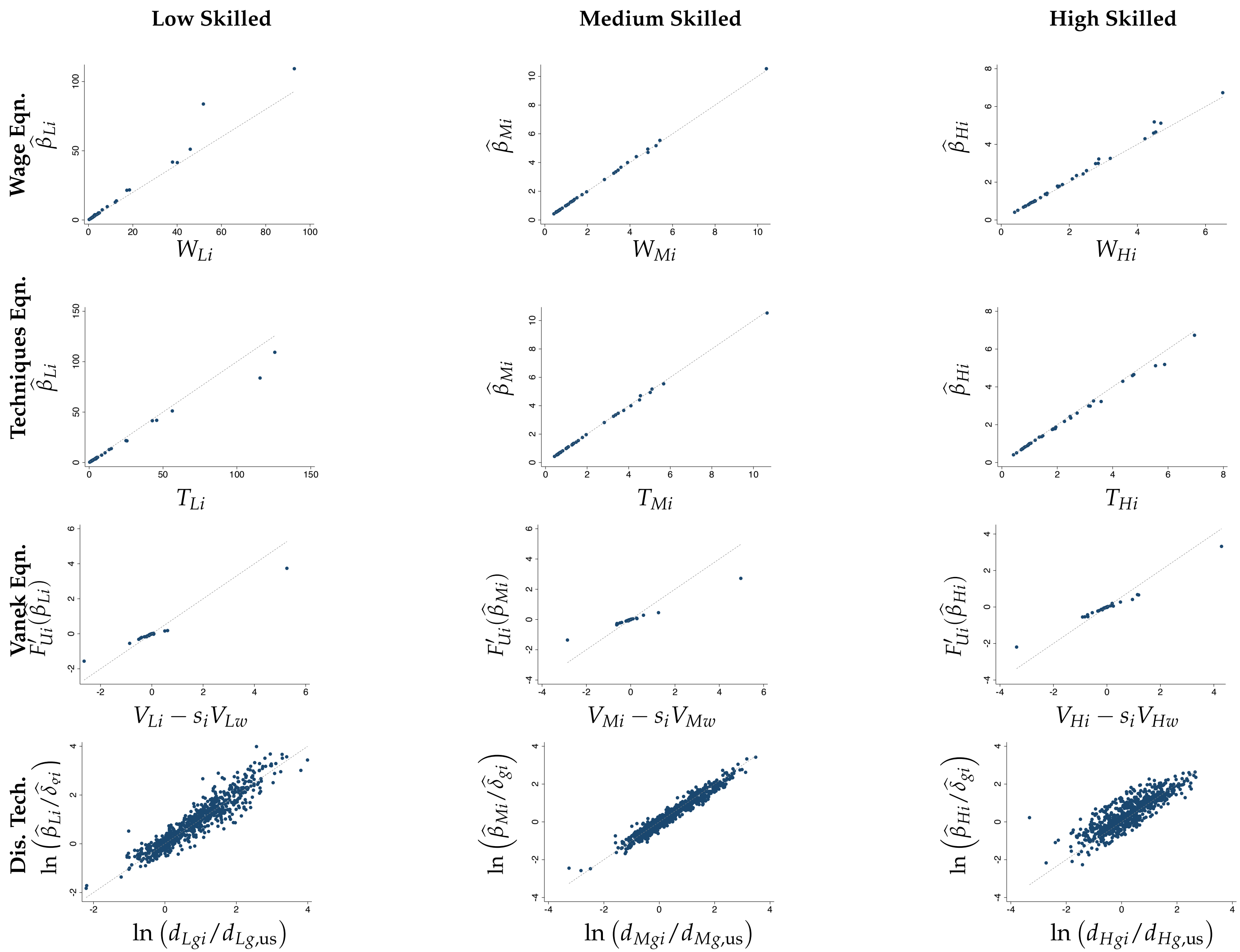
Notes: These plots are the same as those appearing in the main text except that the two outliers (China and United States) are not displayed so as to 'unpack' the remaining observations. Panel A corresponds to the bottom row of figure 3. It plots $V_{fi} - s_i V_{fw}$ against the Government Services adjusted factor content of trade (evaluated at the two-equation estimate of β_f, $\widehat{\beta}_f$). Panel B corresponds to the unskilled and skilled figures in the bottom row of figure 6. It plots $V_{fi} - s_i V_{fw}$ against the Government Services adjusted factor content of trade (evaluated at the three-equation estimate of β_f, $\widehat{\widehat{\beta}}_f$). The left panels are for unskilled labor and the right panels are for skilled labor. All lines are 45° lines.

Figure A2: Performance of the Vanek Equation Using β_{fi}^W and β_{fi}^T



Notes: Each panel plots $V_{fi} - s_i V_{fw}$ against the factor content of trade $F'_{fi}(\beta_f)$ from equation (21) i.e., adjusted for nontradable Government Services. In the top row, the β_f that makes the Wage equation fit perfectly (β_f^W) is plugged into the Vanek equation. This yields a very good fit of the Vanek equation for both unskilled labor (left panel) and skilled labor (right panel). In the bottom row, the β_f that makes the Techniques equation fit perfectly (β_f^T) is plugged into the Vanek equation. This again yields a very good fit of the Vanek equation. Each point is a country and all lines are 45° lines.

Figure A3: Three Types of Labor



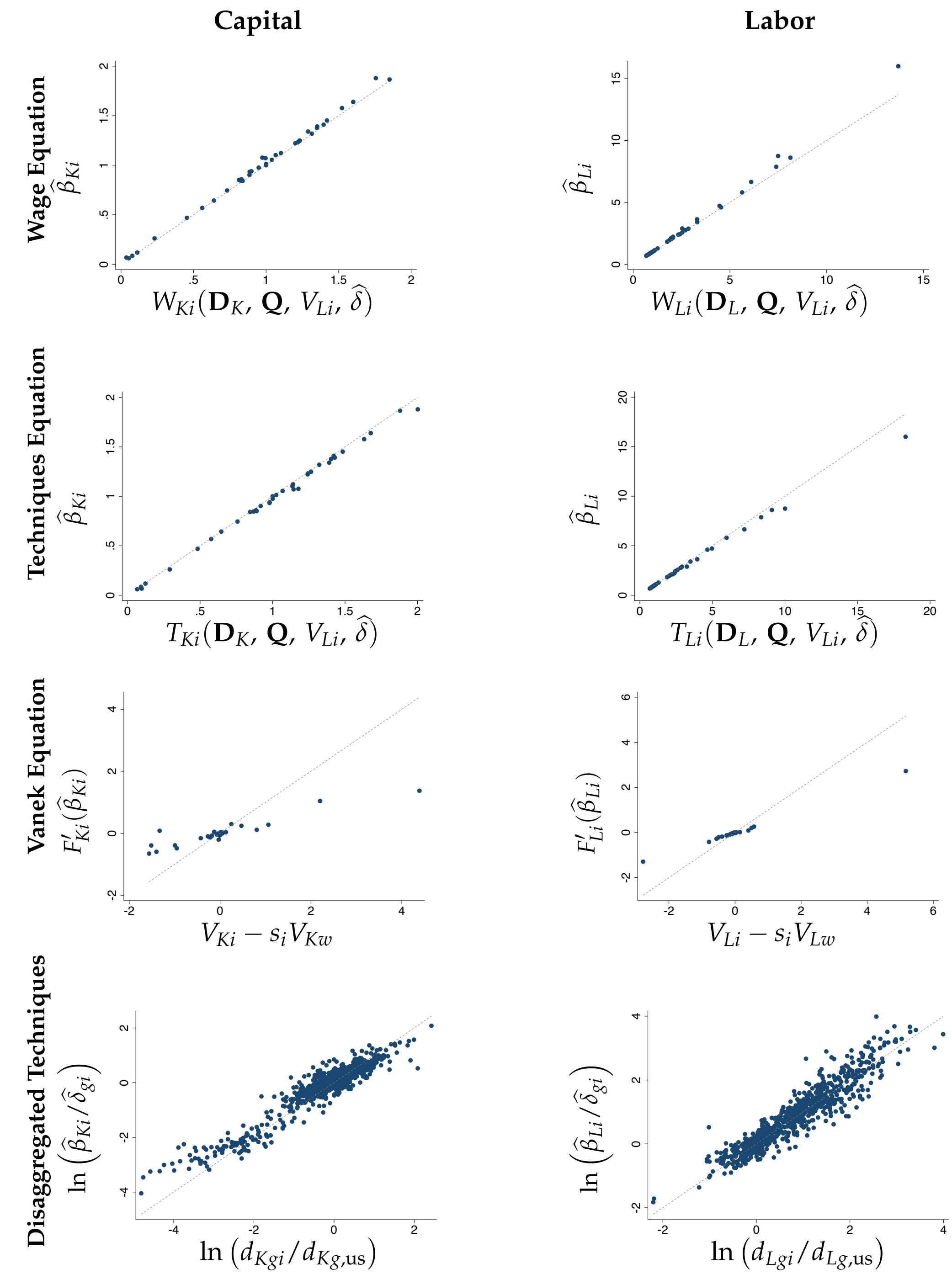
Notes: The left-hand column plots are for least skilled labor, the middle column plot is for medium skilled labor, and the right column plots are for high skilled labor. The top row is the Wage equation (W), the second row is the Techniques equation (T), and the third row is the Vanek equation (V), and the fourth row are relative factor demands from equation (13), namely, $\ln(d_{fgi}/d_{fg,\text{us}}) = \ln(\beta_{fi}/\delta_{gi})$. All equations are evaluated at the two-equation estimates of the β_{fi} and calibrated values of the δ_{gi}. Each observation is a factor and country except for the bottom row which is factor-country-industry specific. The 45° line is displayed in each panel.

Table A1: Test Statistics for the Fit of the Vanek Equation

	Least Skilled Labor			
Factor Content of Trade	Rank Corr.	Variance Ratio	Sign Test	Slope Test
	(1)	(2)	(3)	(4)
1. F_{fi}	0.980	0.040	0.974	0.199
2. $F_{fi}(\widehat{\beta}_f)$	0.983	0.120	0.974	0.344
3. $F'_{fi}(\widehat{\beta}_f)$	0.962	0.463	0.921	0.677
4. $F_{fi}(\widehat{\beta}_f)$	0.982	0.162	0.974	0.400
5. $F'_{fi}[\iota]$	0.493	0.000	0.002	0.005
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.998	0.618	0.974	0.784
	Medium Skilled Labor			
Factor Content of Trade	Rank Corr.	Variance Ratio	Sign Test	Slope Test
	(1)	(2)	(3)	(4)
1. F_{fi}	0.972	0.033	0.895	0.181
2. $F_{fi}(\widehat{\beta}_f)$	0.968	0.035	0.895	0.185
3. $F'_{fi}(\widehat{\beta}_f)$	0.995	0.271	1.000	0.518
4. $F_{fi}(\widehat{\beta}_f)$	0.964	0.072	0.921	0.264
5. $F'_{fi}[\iota]$	0.405	0.005	0.553	0.064
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.998	0.462	1.000	0.679
	High Skilled Labor			
Factor Content of Trade	Rank Corr.	Variance Ratio	Sign Test	Slope Test
	(1)	(2)	(3)	(4)
1. F_{fi}	0.974	0.016	0.947	0.127
2. $F_{fi}(\widehat{\beta}_f)$	0.941	0.044	0.842	0.202
3. $F'_{fi}(\widehat{\beta}_f)$	0.992	0.496	0.947	0.699
4. $F_{fi}(\widehat{\beta}_f)$	0.902	0.206	0.842	0.421
5. $F'_{fi}[\iota]$	0.179	0.036	0.500	0.154
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.995	0.642	0.974	0.799

Notes: These tables presents test statistics for the fit of the Vanek equation (V) for different specifications of the factor content of trade. The top six rows are for least skilled labor. In row 1, the actual factor content of trade is used. In row 2, the factor content of trade is calculated using $\widehat{\beta}_f$ (the two-equation estimate of β_f) and equation (16). In row 3, the factor content of trade is adjusted for nontraded Government Services using equation (21). In row 4, the nontraded Government Services adjustment is put on the right-hand side of the Vanek equation as in part 1 of lemma 2 and as in Davis and Weinstein (2001). In row 5, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but all elements of the vector $\widehat{\beta}_f$ are set to 1. In row 6, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but the estimate of β_f is from the three-equation approach. The middle six rows repeat this exercise for medium skilled labor. The bottom six rows repeat it for high skilled labor. 'Rank Corr.' is the rank or Spearman correlation between the factor content of trade and $V_{fi} - s_i V_{fw}$. 'Variance Ratio' is the variance of the factor content of trade divided by the variance of $V_{fi} - s_i V_{fw}$. 'Sign Test' is the proportion of observations for which the factor content of trade and $V_{fi} - s_i V_{fw}$ have the same sign. 'Slope Test' is the OLS slope estimate from a regression of the factor content of trade on $V_{fi} - s_i V_{fw}$.

Figure A4: Capital and Labor



Notes: The left-hand side plots are for capital, the right-hand side plots are for labor. The top panels are the Wage equation (W). The upper middle panels are the Techniques equations (T). The lower middle panels are the Vanek equations (V). The bottom panels are relative factor demands from equation (13), namely, $\ln(d_{fgi}/d_{fg,\text{us}}) = \ln(\beta_{fi}/\delta_{gi})$. All equations are evaluated at the estimated values $\widehat{\beta}_{fi}$ and calibrated values $\widehat{\delta}_{gi}$. In the top and middle panels, each observation is a factor and country (f,i) while in the bottom panels each observation is a factor, industry and country (f,g,i). The 45° line is displayed in each panel.

Table A2: Test Statistics for the Fit of the Vanek Equation: Capital and Labor

Factor Content of Trade	Capital Rank Corr. (1)	Capital Variance Ratio (2)	Capital Sign Test (3)	Capital Slope Test (4)	Labor Rank Corr. (5)	Labor Variance Ratio (6)	Labor Sign Test (7)	Labor Slope Test (8)
1. F_{fi}	0.845	0.059	0.789	0.228	0.948	0.036	0.816	0.189
2. $F_{fi}(\widehat{\beta}_f)$	0.741	0.093	0.763	0.187	0.933	0.065	0.842	0.253
3. $F'_{fi}(\widehat{\beta}_f)$	0.816	0.126	0.868	0.332	0.957	0.261	0.868	0.509
4. $F_{fi}(\widehat{\beta}_f)$	0.685	0.124	0.711	0.246	0.927	0.107	0.868	0.324
5. $F'_{fi}[\iota]$	-0.027	0.020	0.553	-0.052	0.074	0.000	0.447	0.006
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.973	0.315	0.947	0.554	0.980	0.453	0.947	0.673

Notes: This table presents test statistics for the fit of the Vanek equation (V) for different specifications of the factor content of trade. In row 1, the actual factor content of trade is used. In row 2, the factor content of trade is calculated using $\widehat{\beta}_f$ (the two-equation estimate of β_f) and equation (16). In row 3, the factor content of trade is adjusted for nontraded Government Services using equation (21). In row 4, the nontraded Government Services adjustment is put on the right-hand side of the Vanek equation as in part 1 of lemma 2 and as in Davis and Weinstein (2001). In row 5, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but all elements of the vector $\widehat{\beta}_f$ are set to 1. In row 6, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but the estimate of β_f is from the three-equation approach. 'Rank Corr.' is the rank or Spearman correlation between the factor content of trade and $V_{fi} - s_i V_{fw}$. 'Variance Ratio' is the variance of the factor content of trade divided by the variance of $V_{fi} - s_i V_{fw}$. 'Sign Test' is the proportion of observations for which the factor content of trade and $V_{fi} - s_i V_{fw}$ have the same sign. 'Slope Test' is the OLS slope estimate from a regression of the factor content of trade on $V_{fi} - s_i V_{fw}$.

Appendix C. Small Changes in β_f^{VT}

We establish here that when one places the unknown productivity parameters on the 'right-hand side' as in equation (VT), the performance of the Vanek equation is extremely sensitive to small changes in the vector β_f.[50] Start by defining the *predicted* factor content of trade using this approach as $F^*_{fi}(\beta_f) \equiv \beta_{fi}^{-1}V_{fi} - s_i \sum_j \beta_{fj}^{-1}V_{fj}$. Trefler (1993a) derives a model in which F^{us}_{fi} is the measured factor content of trade with no Ricardian productivity differences and all techniques set equal to their US values, and $F^*_{fi}(\beta_f)$ is its predicted value (the 'predicted factor content'). See footnote 32. We now show that the relationship between F^{us}_{fi} and $F^*_{fi}(\beta_f)$ is extremely sensitive to differences in β_f even if those differences are very small.

Start by defining two productivity vectors for unskilled labor: β_U^W and β_U^{VT}–the latter of which makes the Vanek equation fit perfectly with productivity terms on the right-hand side such that $F^{us}_{Ui} = F^*_{Ui}(\beta_U^{VT})$–and define the (small) difference between the two as $\varepsilon_U = \left(\beta_U^W\right)^{-1} - \left(\beta_U^{VT}\right)^{-1}$. Because $F^*_{Ui}(\beta_U)$ is linear in its arguments, $F^*_{Ui}(\beta_U^W) - F^*_{Ui}(\beta_U^{VT}) = F^*_{Ui}(\varepsilon_U)$ or

$$F^*_{Ui}(\beta_U^W) - F^*_{Ui}(\beta_U^{VT}) = \varepsilon_{Ui}V_{Ui} - s_i \sum_{j=1}^{N} \varepsilon_{Uj}V_{Uj}$$

where ε_{Ui} is the ith element of ε_U. Now consider the variance of the right-hand side. Suppose that the ε_{Ui} are purely random variables with mean 0 and small variance $\sigma^2_{\varepsilon U} \approx 0.02$.[51] Then the right-hand side is 0 on average. Its variance is $\sigma^2_{\varepsilon U}\mathrm{var}[V_{Ui} - s_i \sum_j V_{Uj}]$. Let σ^2_{FU} and σ^2_{VU} be the variances of F^{us}_{fi} and $V_{Ui} - s_i \sum_j V_{Uj}$, respectively, where the variation is across observations i. Because missing trade is so severe, the variance ratio is $\sigma^2_{FU}/\sigma^2_{VU} = 0.0001$. Hence the variance of the right-hand side is $\sigma^2_{\varepsilon U}\sigma^2_{VU} = \sigma^2_{\varepsilon U}\sigma^2_{FU}/0.0001 = 200\sigma^2_{FU}$! Thus, even though $\sigma^2_{\varepsilon U}$ is small, the right-hand side has a large variance relative to the variance of what is to be explained (σ^2_{FU}). Restated, $F^*_{Ui}(\beta_U^W)$ and $F^*_{Ui}(\beta_U^{VT})$ may be equal on average, but because of missing trade, there is a large variance between the two. Right-hand side approaches are like drunk dart players: Every dart completely misses the dartboard, but if you average them you get a bull's-eye.

This establishes that the function $F^*_{fi}(\beta_f)$ is very sensitive to the choice of β_f. It thus explains the discrepancy in results between the approaches of Trefler (1993a) and Gabaix (1997b). Even though they generate similar values of the β_f, they generate very different predictions for the Vanek equation. Similarly, very large differences in the measured factor content of trade can result in very similar differences in β_f. This helps to explain the famous result of Gabaix (1997b) in which he shows that setting the measured factor content of trade to zero or setting it equal to its additive inverse affects the resulting values of β_{fi} very little. This reason for the substantial disagreements regarding the performance of RHS approaches (Trefler, 1993a, Gabaix, 1997b) is new to the literature and serves as a caveat for interpreting results.

[50] This is an additional reason to place the β_f terms on the 'left-hand side' as we do in our main analysis. Note that in figure A2, the performance of the Vanek equation is *not* sensitive to the small differences between β_f^W and β_f^T.

[51] This is the variance of the deviations between $(\beta_{Ui}^W)^{-1}$ and $(\beta_{Ui}^{VT})^{-1}$