

Managers and Workers in Labor and Marriage Markets*

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Abstract

Differences in productive capabilities across individuals determine outcomes in both the labor and marriage markets. This paper develops and estimates a two-factor general equilibrium model that allows us to link differences in managerial or “leadership” skill across individuals of different observable characteristics to occupational choice, outcomes in the marriage market, and to the gender wage gap. In the labor market, managerial skill allows individuals to lead teams of workers in production as in McCann et al. (2012). In the home market, information on matching and time use allows us to identify the returns to both general skills and managerial skills in home production. We find evidence that managerial skill plays a role in home production somewhat similar to the role it plays in market production but also that spouses cannot easily substitute their managerial skills in home production. When we apply our model to explaining the gender wage gap we find that, consistent with much of the recent literature on gender pay differentials, gender differences in managerial or “leadership” skill account for a large share – about 60% – of the gender wage gap, especially at the top of the wage distribution.

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1 Introduction

Most production in modern economies is carried out by teams consisting of a manager and one or more workers. Managing a team of workers requires skill, often referred to as “leadership” skill, which is theoretically distinct from cognitive or other types of general skill typically associated with education.¹ Using Canadian data, and in particular a novel measure of occupation based on subjective reporting about managerial activity, we show that about one third of workers identify as “managerial” but that the link between managerial employment and education is small, with a correlation coefficient between years of schooling and self-assessed managerial status of .12. We take this as evidence that managerial skill can be considered a distinct productive factor. Leadership skill has also recently linked the gender wage gap – particularly at the top end of the wage distribution.² In line with this literature, we find that the wage premium enjoyed by male self-assessed managers in 2010 was 60% higher than the wage premium for self-assessed female managers.

Given the important role of managerial status in the labor market, a natural question concerns its implications for sorting and returns in the marriage market. There are two reasons why leadership skill is potentially important. First, if output of couples depends on both market and home production, returns to these skills will affect spouses’ outside contributions to household welfare and outside options. Second, general and managerial skills are likely to be valuable as direct inputs into home production. Thus, an interesting question is whether or not the role of leadership skill in the home mirrors the role of leadership skill in the labor market. Are monogamous households happiest when when a

¹The large literature on non-cognitive ability, beginning with Jencks et al. (1979) and recently Weinberger (2014), has found measures of leadership, related to locus of control, to be important to occupational status and earnings later in life, conditional on education, family background, and measures of cognitive skill. In general, it is more predictive of later life outcomes than other non-cognitive measurements such as conscientiousness which affects educational attainment but not post-education outcomes conditional on schooling.

²Cattan (2013) finds that men’s advantage in “leadership” can explain much of the gender wage gap in the high end of the wage distribution, while Fortin (2008) finds evidence that the gender gap in attitudes toward work (including “the chance to be a leader”) closed between 1979 and 2000 and account for some of historical closing of the gender wage gap among younger workers over this period. Its importance rivals that of conscientiousness, though it manifests differently, mainly by allowing individuals to sort into high-status managerial occupations. The importance of leadership, and the possible gender imbalance in this attribute, is also found in the popular literature on the gendered workplace, e.g. Sandberg (2013).

“manager” spouse coordinates a “worker” spouse, suggesting negative assortative mating on leadership skill? Or do we see evidence that like matches with like both on education and leadership skill? To a first pass, our data supports the second option. We see no evidence that self-assessed managers are more likely to marry at all; however, managers who do marry are more likely to marry other managers, conditional on education, labor supply, and wages. The interpretation, however, is complicated by the fact that we are interested in returns to productive skills, but neither general skill nor managerial skill of individuals in the data is directly observable. Instead, we observe individuals’ education and occupation, their choice of partner and allocation of time across sectors. These outcomes are only noisily, and endogenously, related to the underlying productive factors.

To explore these questions further, we develop a simple structural model of the labor and marriage markets that links observable characteristics (education, occupational choice and labor supply) to unobservable productive traits in a setting that combines a Roy model (for occupational choice) to a Becker-style matching model with endogenous allocation of time across markets. Information on the joint distribution of observables across households allows us to identify the underlying distribution of underlying productive factors, and to back out their role as inputs into home production, and their final returns in the marriage market. The model also offers a rationale for imperfect assortative mating on different traits³: first, as stated above, the observables on which individuals are observed to sort serve as noisy measures of underlying, unobservable productive traits on which individuals truly sort and allocate their time. Second, even with respect to the “true” underlying traits, interactions between the traits mean that only certain combinations of traits are optimal, causing individuals to trade off one for another in seeking a marriage partner.

Our paper builds on and is complementary with several recent papers in the matching

³The economics literature has produced two broad classes of model to explain imperfect sorting in the marriage market. The first model relies on search frictions which impose delay costs to search and is a staple of the dynamic collective literature on households (Mazzocco et al. (2014)). The second type of model, first introduced in Choo and Siow (2006) and Bruze et al. (2012) in a dynamic framework, assumes sorting based on heterogeneous idiosyncratic preferences across types. Our approach is different from either of these.

literature. Hurder (2013) focuses on education decisions made in expectation of time use in later life. Since more skilled jobs require greater time commitment on average, high skilled workers prefer partners who specialize in home production. In her framework, this can lead to non-assortative or imperfectly assortative matching on education, even when education is complementary in household utility. Our paper builds on this insight but focusses more explicitly on sorting with respect to managerial ability, which is only weakly related to education. To this end, we make use of a very useful feature of the SLID in which individuals are asked directly about whether their job entailed supervisory and/or managerial components (two separate questions) which allows us to identify managers in the data without recourse to occupational data which is necessarily more coarse and may also confound success at work (related to general skill) with true managerial ability.

In another very closely related paper to ours, McCann et al. (2012) (hereafter, MSSW) develop a structural model of the labor market in which people sort into occupations (worker/manager) and firms (of different size) based on their post-education endowments of cognitive skill and managerial skill. The managerial skill acts a measure for ones ability to work with others and produce in teams by taking on coordinating roles, a structure we directly adopt for our own model of the labor market. In contrast to MSSW, however, we treat educational choices as exogenously given and instead focus on simultaneous matching in, and division of time between, the labor and the marriage markets. To estimate the returns to skill in the marriage market, we model the effects of both types of skill in the home market in a very flexible way that allows us to back out the implied degree of substitutability of spouses' managerial and general skill and the contributions of these traits to household productivity. We also back out the distribution of skills by gender in the population; the links between unobservable skills and education; and the utility that different types of agents receive in the marriage market as a function of their productivity and the scarcity of their skill set in the market.

Once we have estimated the model and explored its implications for the distribution and productivity of general and managerial skills, we use it to explore returns in the marriage market. Specifically, we are interested in whether the returns from marrying

are spread homogenously across the population or localized and type-specific. If marriage yields large welfare gains that are independent of spouses' types, for instance due to gains from home production that are largely independent of productive factors in the labor market, then declining marriage rates may be driven by changing social norms and may create private welfare losses. If, on the other hand, returns to marriage are heterogenous and match-specific, then policies to encourage marriage are likely to be ineffective or even welfare-reducing. To summarize our findings, we find evidence that marriage market rents are fairly broad-based, but sensitive to introducing search frictions. The highest skill individuals receive large returns from marriage on average, but these returns are match-specific. When we introduce even small search frictions into the model estimated to match the 2010 data, marriage rates fall, with an elasticity of about $-.2$. Much of this is driven by the fact that leadership skill is very productive in home production of marrieds but not easily substitutable between spouses.

Finally, in the last part of the paper, we use our model to shed light on the gender wage gap. In our model, a gender wage gap can arise because women have different distributions of k or n than men, or there may be a differential in the time women can devote to work due, for instance, to energy differentials (Becker (1991)) or to loss of time due to the menstrual cycle: a pure gender effect for which we find indirect evidence in time use data. We decompose the gender wage gap into these three channels by adjusting each element one by one in our estimated model so that women have the same distributions of skills and time as men. Consistent with the bent of much recent literature on non-cognitive skills and gender, we find that women's lower levels of n explain most of the gender wage gap: the gap shrinks by about 60% when we replace women's distribution of n with men's distribution of n . By contrast, women's and men's distributions of k are very similar in the 2010 data and do not contribute to the gender wage gap. Giving women the male distribution of disposable productive time decreases the gender wage gap by 6%, in line with (though smaller than) estimates of the biological gender gap from Ichino and Moretti (2009).

The rest of the paper proceeds as follows. In section 2, we examine data from the SLID

to illustrate the connections between occupation choices, time use and marriage partners. In section 3, we develop our model of labor and home production, while section 4 outlines the method of estimating and solving the general equilibrium model. We present the main results from our estimation and from our application to the gender wage gap in sections 5-7. Section 8 concludes.

2 Data: Survey of Labour and Income Dynamics

The main data source for this paper is the restricted cross sectional version of the SLID for 2010, which is available through Statistics Canada’s Data Research Center. The SLID contains household level information on labor supply, occupational status (about which more below), educational attainment, fertility, wages, earnings and incomes. The sample for analysis consists of males and females between the ages of 35 and 50, specifically non-widowed singles and heterosexual couples. We exclude families in which at least one member spent any part of the reference year as a full time student and those in which no adult member of the one-person or two-person family is a labor force participant in the reference year, which we take to mean that at least one member of the households reports being employed for at least 50 hours a year.⁴ This allows us to focus on a marriage market that should be close to equilibrium and excludes individuals who may be disabled or dependent on government benefits. These sample restrictions leave us with about 6000 females and 5800 males in 2010.⁵

For the analysis, we bundle educational attainment into four categories: (1) high school and less, (2) some postsecondary including university attendance without a degree, two-year or technical college, or other non-university certifications, (3) undergraduate degrees including bachelor degrees, and (4) anything greater than a bachelor’s degree.

⁴“Usual” hours means total hours worked excluding unplanned leaves from work.

⁵To make sure we have a balanced panel of spouses, the criterion for selection of marrieds at the individual level is that (1) at least one spouse is between 35 and 50 and (2) for couples in which one partner falls out of this range, the age difference between the spouses does not exceed five years. For singles, individuals must be non-widows between 35 and 50. The exact sample size is repressed to comply with Statistics Canada disclosure guidelines. In the rest of this section, and in the simulations, we report results based on a sample of 6000 males and 6000 females, with the statistics based on the SLID 2010 cross-sectional weights.

Table 1 shows population statistics by gender and educational attainment in 2010 for the restricted sample. Around 70% of individuals obtained a college education or less. As well, the gender ratio in educational attainment is close to 1:1, though the male education distribution is more concentrated in the lowest and highest education categories. This gender parity in educational attainment represents one point in a long trend of increasing relative female educational attainment. In corresponding data from 1995 (not shown), women aged 35-50 were still five percentage points less likely than men to have completed a bachelor degree or higher. Since we exclude households with no working members, men have nearly 100% participation rates regardless of educational attainment.

Table 1: Summary Statistics: by gender and education

	Pop share	Marriage rate	Part time rate	Full time rate	Share of managers	Log wages
Women						
HS	0.198	0.76	0.30	0.53	0.20	2.70
College	0.483	0.76	0.28	0.62	0.27	2.84
Bachelor	0.241	0.78	0.22	0.66	0.32	3.23
> Bachelor	0.079	0.77	0.24	0.69	0.35	3.31
Men						
HS	0.239	0.76	0.15	0.82	0.33	2.9
College	0.477	0.74	0.13	0.85	0.43	3.1
Bach	0.191	0.77	0.12	0.85	0.52	3.39
> Bachlor	0.094	0.79	0.10	0.88	0.59	3.56

The last two columns – showing occupational status rates and log wages – are of special interest for our analysis. Following MSSW, we classify workers into “managers” (i.e. instructors or team leaders) and “workers” (subordinates who take instruction from others). The SLID provides information on occupation at the 3-digit level using Statistics Canada’s S-NOC, very close to the SOC classification of occupations. More useful for our purposes, however, it also contains variables reporting the answer to the questions “Is your [main] job perceived as managerial?” and “Does your job entail direct supervision of other workers?”. For individuals answering yes to the second question, the SLID further records how many employees the respondent supervises. For our main analysis, we use the union of affirmative responses to the managerial and supervisory questions as our measure of being a manager. The benefit of these subjective questions over formal

occupational classification is that, even at the three-digit level, it is impossible to capture all managerial activity as understood in the MSSW framework,⁶ and indirect evidence suggests that an indicator for being in a “managerial profession” reflects career success and is a measure of competence rather than a measure of being in true leadership position in which tasks at work are concentrated on coordinating team members.

Based on our subjectively reported occupational measure, we see that, even as of 2010, men are still substantially more likely – by around 63% – than women to report being in managerial jobs at all levels of education. Overall, the share of managers in the population of employed individuals is relatively high, at 45% of employed men and 31% of employed women and increasing in education for both genders, though not overwhelmingly. Finally, the last two columns report average log wages for each of our four educational levels. The raw gender wage gap is around 24 log points, smallest for the lowest educated (high school or less, whose wages may be bounded downward by minimum wage laws) and constant over the rest of the education distribution.

Table 2 provides a closer look at the relationship between wages, education, occupation and gender. We report the coefficients from the following log-linear regression:

$$Y_i = \alpha_j + \beta_2^j x_i^2 + \beta_3^j x_i^3 + \beta_4^j x_i^4 + \kappa^j M_i + \nu^j G_i + \delta^j P_i + \eta C_i + \epsilon_i \quad (1)$$

where j indexes different populations (males, females, and a pooled sample) and i indexes individuals. Y_i is the log hourly wage in 2010, x_k^j is a dummy variable for education level k , M_i is a dummy variable for being a self-assessed manager/supervisor, G_i is a dummy variable that takes a value of one for females, P_i is a dummy variable indicating part time work (which we defined as less than 32 hours a week on average during the year, ignoring unplanned absences due to illness or other unexpected life events) and C_i is a marriage dummy. Column (1) reports results for equation (1) in which j is the entire population of

⁶For instance, even at the four-digit level, both the shift manager and the counter and kitchen staff at McDonalds during a standard shift would likely be classified as 6711, although the shift manager is in charge of managing the shift’s production of service. At the other end of the occupational distribution, a general physician who runs a medical office office and her office partner who shares or rents office space and staff from her will both have the same occupational classification – 3112 – but are likely to provide different answers to the subjective managerial/supervision questions since only the first doctor manages the office staff.

workers. Columns (2) and (3) separate the population of workers into women and men.

From column (1), we see that education affects wages in the expected manner: individuals with college earn 17% more per hours more than individuals with less than college, while individuals with undergraduate university degrees earn 30% more per hour than what college attendees earn, and post-graduates earn 27% more per hour of undergraduate degree holders. The gender wage gap, controlling for full time/part time status, education, and marital status, falls to 18%, consistent with the fact that women have lower average labor supply, which be reflected in lower accumulated firm-specific skill (Hirsch (2005)). When the estimation sample is separated by gender in columns (2) and (3), we see a fairly gender-neutral educational gradient: women receive a greater reduced-form return to undergraduate university while men receive a greater reduced-form return to college and to postgraduate work. However, men earn a much higher premium than women to being married, at least in reduced form, and – most notably for our purposes – also receive a much higher return to self-assessed managerial status. Specifically, men’s wage premium to being a self-perceived manager is 160% of women’s managerial premium conditional on education, marital status and full time status.

We now turn to assessing how education and managerial status affect outcomes in the marriage market. Table 4 investigates the relationship between managerial status and marital status. To facilitate interpretation, we report odds ratios and z -scores from logit regressions of likelihood of identifying as a manager on marital status and other observed variables by gender.

$$P(a_i|X_i, C_i) = \frac{\exp(\alpha + \beta X_i + \eta C_i + \delta P_i + \zeta \bar{W}_i)}{1 + \exp(\alpha + \beta X_i + \eta C_i + \delta P_i + \zeta \bar{W}_i)}, \quad (2)$$

where $\beta X_i = \beta_2 x_i^2 + \beta_3 x_i^3 + \beta_4 x_i^4$

In (2), a_i is a dummy variable equal to one if the individual is a manager, X_i is a dummy vector for the level of education, and C_i and P_i indicate being married and being a part time worker as before. \bar{W}_i is the average log wage of the individual over the period (up to six years) he or she is observed as a worker in the SLID, which we include in the even-

Table 2: Log hourly wage

	All (1)	Women (2)	Men (3)
Schooling			
College (β_2)	0.193*** (0.015)	0.147*** (0.024)	0.186*** (0.023)
Bach. (β_3)	0.521*** (0.019)	0.505*** (0.028)	0.423*** (0.028)
> Bach (β_4)	0.696*** (0.026)	0.545*** (0.037)	0.604*** (0.035)
Manager (κ)	0.184*** (0.013)	0.151*** (0.020)	0.258*** (0.019)
Part-time (δ)	-0.091*** (0.016)	-0.094*** (0.020)	-0.056*** (0.028)
Female (ν)	-0.178*** (0.013)		
Married(η)	0.074*** (0.015)	0.026*** (0.020)	0.147*** (0.021)
_constant (α)	2.80*** (0.018)	2.72*** (0.027)	2.73*** (0.026)
R-squared	0.156	0.115	0.130

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

numbered columns (2) for women and (4) for men, keeping in mind that it is potentially endogenous. Consistent with the evidence from table 3, the odds ratios reported in the first three rows of the table suggest that increasing education is associated with a greater likelihood of being a self-assessed manager; though the relationship is stronger for men than for women and is weakened substantially once we control for human capital in the form of \overline{W} . Men with post-graduate education are 2.8 times more likely than male high school graduates to be managers; this drops to 1.8 times once we control for \overline{W} , though it remains significant. Among women, for whom \overline{W} is less correlated with managerial status, the relationship between education and managerial status is weaker, and becomes (marginally) insignificant at conventional levels once we include \overline{W} . For both genders, part time workers are less likely to be managers than full time workers by around 60% for women and 70% for men. Finally, and notably, neither for men nor for women does being married raise the likelihood of managerial status, regardless of whether we control

for \bar{W} .

Table 3: Logit estimation for being a manager when employed

		Women		Men	
		odds ratio	odds ratio	odds ratio	odds ratio
School					
	College(β_2)	1.32** (2.14)	1.23 (1.56)	1.55*** (4.35)	1.36*** (2.90)
	Bach(β_3)	1.72*** (3.78)	1.27 (1.56)	2.18*** (5.85)	1.60*** (3.30)
	> Bach(β_4)	1.79*** (3.10)	1.27 (1.07)	2.84*** (5.96)	1.83*** (3.25)
Married(η)		1.07 (0.53)	1.06 (0.49)	1.13 (1.16)	1.04 (0.38)
Part-time(δ)		0.41*** (-7.74)	0.43*** (-7.15)	0.31*** (-7.36)	0.33*** (-6.97)
Avg log wage (ζ)			1.77*** (4.56)		1.67*** (4.67)
constant (α)		0.392*** (-6.59)	0.081*** (-7.25)	0.536*** (-5.21)	0.064*** (-8.82)
pseudo R-squared		.0334	.0477	.0429	.0706

Dependent variable: indicator for being a manager

z-scores in parentheses

*** p<0.01, ** p<0.05, * p< 0.1

Table 4 reports results from a similar set of logit regressions in which the dependent variable is an indicator not for being a manager, but for having a managerial spouse. Most of our observable variables are weak predictors of the likelihood of marrying a manager. The effects of education are fairly weak determinants of being married to a manager and disappear for both genders once we control for own human capital in the form of \bar{W} . Female spouses of managers are marginally more likely to be part-time workers than full time workers or non-participants but this effect is not robust in the sample of women for whom we can calculate and include \bar{W} . By contrast, we do find a quite strong and significant assortative matching of managers to managers, for both genders, despite the fact that, as shown in table 3, managers are not more likely to be married overall than non-managers. That this result persists (again for both genders) when we control for earning ability (\bar{W}), suggests that it is more than a spurious correlation due to matching on earnings or on career success. It also suggests that managerial ability manifests differently in the home than it does in the labor market, where managers are

Table 4: Logit Estimates for the likelihood of being married to a manager

		Female		Male	
		(1)		(2)	
		odds ratio	odds ratio	odds ratio	odds ratio
Schooling	College	1.17 (1.32)	1.05 (0.36)	1.11 (0.87)	1.09 (0.69)
	Bach	1.59*** (3.31)	1.24 (1.43)	0.94 (-0.36)	0.90 (-0.58)
	> Bach.	1.53** (2.15)	1.30 (1.31)	1.07 (0.32)	1.02 (0.11)
Manager		1.49*** (3.66)	1.46*** (3.47)	1.44*** (3.42)	1.39*** (3.06)
Full time		0.79 (-1.57)	0.87 (-0.57)	0.74 (-1.14)	0.84 (-0.47)
Part time		1.10 (0.57)	1.21 (0.77)	0.70 (-1.09)	0.78 (-0.64)
Avg log wage			1.13 (1.37)		1.11 (1.15)
Constant		0.654*** (-2.93)	0.473** (-2.28)	0.385*** (-3.47)	0.255*** (-3.11)
pseudo R-squared		.0120	.0114	.0064	.0068

Dependent variable: indicator for being married to a manager

z-scores in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

– by definition – most productively paired with subordinate workers.

To summarize, our estimates suggest that occupation type (specifically “manager” vs “worker”) is only very imperfectly correlated with education, and that both education and managerial status independently raise wages. This is consistent with a two-factor model of the labor market in which managerial or leadership skill operates differently than the general productive skills associated with education. Furthermore, evidence for the independence of the two skills carries over into the marriage market. The correlation of education within couples is well known. Using our four-category measure within our sample, it is .45. However, managers (and therefore workers) tend to marry each other even conditional on education and earning ability; the raw correlation of manager status within couples is .08. In additional regressions, omitted for space, we find the familiar results that married men tend to work more than single men while the reverse is true for women, and that, among women, hours worked increase in education. Finally, there

is a major gender difference in returns to managerial skill that is not (or much less) observable with respect to education. Although women are much less likely than men to report being managers at work, the increase in managerial status with education is much smaller for women than for men and their wage returns to being a manager, conditional on education, are 60% lower than men's. We will explore these patterns in the context of a two-factor model of the labor and marriage markets in which general skill k and leadership skill n are productive both at home and at work and are imperfectly reflected in education and occupation outcomes.

3 Model

To examine how general and managerial skills influence matching and time use, we develop a static transferable-utility general equilibrium matching model in which both types of skill are potentially productive both at home and in the labor market. Individuals sort themselves into the marriage, occupation, and firm (team) that maximize their private payoff. The job and marriage markets are both perfectly competitive so that the privately optimal decisions of individuals over sorting and time use are also socially efficient. We borrow our model of the labor market from MSSW (2014) with two important differences. First, we simplify our model relative to theirs in that we take education to be exogenously given and related to both general/worker ability and leadership/managerial ability according to a reduced-form correspondence function that we estimate in the model. Second, we extend their model by relaxing the fixed time allocation in MSSW so that people freely divide their time between the labor market and home production so as to maximize their overall payoff in the marriage market. To pin down individuals' optimal time use across different household types, and because we are agnostic on the role of general and managerial skill in the home, we introduce a more general flexible gender- and marital-status specific production function for home production which also takes as inputs each household member's time at home, general skill, and managerial skill. This production function allows us to explore how closely the productivity of inputs at home

mirrors those at work. We estimate its parameters as part of the model.

Within this setting, each person has four attributes which serve as state variables: managerial skill n , general skill k , preference for home-produced goods relative to market goods a , and schooling e . Only e is directly observable to the econometrician, but only k and n are productive. (a is introduced to provide an additional aspect of sorting in the marriage market based on preferences. We ignore it for now.) Managerial skill is bounded between \underline{n} and \bar{n} , general skill is bounded between \underline{k} and \bar{k} and (exogenous) schooling is bounded between \underline{e} and \bar{e} . In theory, k and n can be continuous on their feasible interval while e is discrete, and we will use this assumption in developing the model in this section before discussing the numerical implementation, that will require us to formally discretize all of the variables.

For each individual i of gender G , k and n are related to e as follows:

$$\begin{aligned} k_i &= \kappa_0^G e_i^{\kappa_1^G} + \epsilon_i \\ n_i &= \nu_0^G e_i^{\nu_1^G} + \eta_i \\ \begin{pmatrix} \epsilon_i \\ \eta_i \end{pmatrix} &\sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\epsilon^2 & \rho^G \\ \rho^G & \sigma_\eta^2 \end{pmatrix} \right] \end{aligned} \quad (3)$$

which implies that $\mathbb{E}(\eta_i|\epsilon_i) = \rho^G \left(\frac{\sigma_\eta}{\sigma_\epsilon} \right) \epsilon_i$ and $Var(\eta_i|\epsilon_i) = (1 - \rho^G)^2 \sigma_\eta^2$. In (3), the reduced-form “returns” to education are subject to a pair of stochastic shocks which are heteroskedastic in education and correlated by gender-specific parameter ρ . This specification allows for individual heterogeneity (e.g. innate talent or conscientiousness) that may jointly determine the adult levels of both types of adult skill. Of course, since we do not observe innate talents (or pre-education endowments of k and n), we cannot estimate the causal effects of education on adult k and n , only their correlations.

Given their k and n , adult individuals allocate their time use between home and work given market wages and the value of their skill set in home production. In the labor market, managerial skill improves one’s productivity (and wages) only when one

is in a position to supervise others. The general skill modifies the quality or quantity of production at work, regardless of one's occupation. The labor market is an unconstrained perfectly competitive matching market allowing for many-to-one matching. The wage an individual $\{k_i, n_i\}$ receives per efficiency unit of time therefore gives his marginal social product as a worker in the steady state economy. In contrast, the home market, while also perfectly competitive, allows for at most one-to-one matching (bachelorhood or monogamy). The value of an individual's time in home production, the 'household wage', is not unobservable (and therefore cannot serve as an estimation target) but can be recovered from the shadow price of each type of individual in the home market. Given the assumption of perfectly transferable utility within couples, it too reflects the individual's marginal social product in the marriage market.

Final output and welfare is determined in the home sector. As in Becker et al. (1977), labor production is only an intermediate input into the household utility function, along with the production value of time spent at home. Single individuals allocate their time to maximize utility derived from personal market income and home production given their relative tastes for each as captured by a . Couples pool their market income and allocate their time between home and work to maximize their joint output and private utility, again as valued according to their tastes a_l and a_g .

3.1 Labor Market Production

In order to incorporate managerial skills into production, the labor market has a general joint production process consisting of two tasks, a productive task P and a coordinating task C , both of which are required for production. Consider two workers, person i and person j with respective skills (k_i, n_i) and (k_j, n_j) and time committed to collaborating in team production h_i and h_j , respectively. Each person can dedicate time to working in task P and task C . When person i spends time in task P he is coordinated by individual j in task C . Reciprocally, when person i spends time in task C it is dedicated to coordinating individual j 's time in task P . Assuming that they cannot use leftover time elsewhere, the two will allocate their time across the tasks to maximize their total output.

The effectiveness of each individual in the coordinating task depends on their level of managerial skill n . Using a generalization of MSSW, we assume that managerial skill modifies the time required to help one's teammate accomplish task P , while, conditional on the coordinator's n , productivity of the team is given by CES function of the pair's general skills. The team-based production function is:

$$Y^J(\theta_i^P, \theta_i^C; k_i, n_i, h_i, \theta_j^P, \theta_j^C; k_j, n_j, h_j) = (\alpha_1 k_i^{\alpha_2} + (1 - \alpha_1) k_j^{\alpha_2})^{\frac{1}{\alpha_2}} \min\{\theta_i^P, \alpha_0 n_j \theta_j^C\} \\ + (\alpha_1 k_j^{\alpha_2} + (1 - \alpha_1) k_i^{\alpha_2})^{\frac{1}{\alpha_2}} \min\{\theta_j^P, \alpha_0 n_i \theta_i^C\} \quad (4)$$

such that both individuals' time constraints hold: $\theta_m^P + \theta_m^C \leq h_m$ for $m = i, j$. Here α_1 represents the share of the task coordinator's cognitive skills in production and $\frac{1}{1-\alpha_2}$ is the elasticity of substitution between the coordinator and the worker's general skills. α_0 gives a measure of the productivity of managerial skill n , which will be important for capturing average team size in the economy.

If i and j were the only two individuals in the labor market, then in general they would not both specialize since specialization would leave time left over for one member of the team. However, if the member's left over time can be contributed to a different team, then specialization will, as we show below, become optimal. Suppose individual j is a very good coordinator (has very high n). Then he will coordinate all of i 's time and will have $h_j - \frac{h_i}{n_j}$ left over after coordinating all of individual i 's time to work with another individual outside the team.⁷ Firms in the model can be thought of as a matching of teams. Since in practice there is no constraint on how many workers a coordinator can match with per unit of time, optimal matching will in general be many-to-one. Since the time units we are working with are arbitrary and production per unit time depends only on a worker's endowment of skills, firms are indifferent across individuals, or more specifically across time supplied by individuals, provided that two individuals have the same effective skill set and efficiency units of time. Since we assume a perfectly elastic

⁷In theory, he could work alone coordinating himself. However, if the time spent producing and coordinating must be carried out simultaneously, this may not be possible. We assume in this exposition that one cannot be a worker and a manager simultaneously.

supply of workers, labor market equilibrium has firms making zero profits, and wages $\omega(k, n)$ adjust to satiate demand of firms for the time of all individuals of type (k, n) . In equilibrium, the wages will be determined from the solution to the competitive market problem, as the marginal social problem of each type of worker to the economy.

In this setting, under the assumption that n and k are continuous on their support, labor market equilibrium will be characterized by full specialization of individuals into tasks.

Proposition 3.1 *Within a given team, profits are maximized when all employees allocate their time exclusively to a specific task, either P or C , for the entire production process.*

The proof follows from the example of a firm trying to produce X units of output by hiring employees of type (k_i, n_i) and (k_j, n_j) . For simplicity, let $\alpha_0 = 1$, $\alpha_2 = 0$, and $X = k_j^{\alpha_1} k_i^{1-\alpha_1}$. Efficient production requires that when employee- j coordinates, employee- i is hired for 1 unit of time and employee- j is hired for $1/n_j$ units of time. Similarly, when employee- i coordinates, the firm needs $\left(\frac{k_i}{k_j}\right)^{1-2\alpha_1}$ of time from employee- j and $\left(\frac{k_i}{k_j}\right)^{1-2\alpha_1} \frac{1}{n_i}$ from employee- i . For a pairing of i and j , the firm will allocate person i to task P so long as:

$$\underbrace{\text{Cost of } i \text{ in task } P}_{\omega(k_i, n_i)} - \underbrace{\left(\frac{k_i}{k_j}\right)^{1-2\alpha_1} \omega(k_j, n_j)}_{\text{Cost of } j \text{ in task } P} \leq \left(\underbrace{\left(\frac{k_i}{k_j}\right)^{1-2\alpha_1} \frac{\omega(k_i, n_i)}{n_i}}_{\text{Cost of } i \text{ in task } C} - \underbrace{\frac{\omega(k_j, n_j)}{n_j}}_{\text{Cost of } j \text{ in task } C} \right) \quad (5)$$

which says that each individual is put to the task in which they are comparatively less costly to the firm. This result is a simple application of Ricardo efficiency.

Firms look for optimal matches between types to maximize profits taking wages as given. Individuals also take wages as given and sort into the tasks and teams that maximize their take home pay. In the remainder of the paper, individuals who sort into task P are labeled ‘workers’ – w – and those who sort into task C we call ‘managers’ – m . The resulting labor market equilibrium is characterized by the following additional rules:

Corollary 3.2 1. *Wages of workers are independent of their social skills.*

2. *For a given k , there is a cut-off $\hat{n}(k)$ below which everyone below becomes a worker and above which everyone becomes a manager.*

3. *The span of control of a manager is proportional to her managerial skill n .*

- *Managers with social skill n and h hours of time can control hn hours of workers.*

4. *Higher cognitively skilled managers match with higher cognitively skilled workers.*

Formal proofs of each point using a slightly less general functional form for production are given in MSSW. Intuitively, however, the positive assortative matching on k is due to the complementarity of the CES production function (for $\alpha_2 < 1$) of k_w and k_m . Since workers do not use their n in production, an obvious requirement of competitive equilibrium that they not experience any return to n , since the marginal social product of two workers with the same k and different n is the same. Given the Leontief nature of the production function with respect to n , it cannot be efficient for manager j to supervise less than n_j workers per unit of time. Finally, the existence of a single cut-off $\hat{n}(k)$ for each k above which all individuals are managers follows from the fact that the managerial wage is strictly increasing in n , the worker's wage is independent of n , and individuals must be indifferent between any matches that their type realizes in equilibrium, which only occurs at the level of n where the managerial wage just equals the worker wage. Whether $\hat{n}(k)$ is increasing or decreasing in k depends on the relative scarcity of n and k in the population and on the parameters of the production function, α .

With these features, the production of a manager of type (k_m, n_m) who contributes one unit of time to producing with a team of workers of type (k_w, n_w) produces output:

$$f(n_m, k_m, n_w, k_w) = \alpha_0 n_m (\alpha_1 k_m^{\alpha_2} + (1 - \alpha_1) k_w^{\alpha_2})^{\frac{1}{\alpha_2}} \quad (6)$$

3.2 Household Production

Individuals in the marriage market are distinguished on three characteristics: k and n which are potentially productive in home production as well as in market production, and a , which determines the relative preference of home produced goods. Differences in a across individuals allow us to account for differences in sorting and time use that cannot be explained by productivity differences alone. For individual i with education e , a_i is drawn from a truncated normal distribution:

$$a_i = \min(\max(\bar{a}(e) + \epsilon_a, 0), 1) \quad (7)$$

$$\epsilon_a \sim N(0, \sigma_a^2)$$

and

$$\bar{a}(G, e) = \underline{a}_0 + \underline{a}_1 e$$

which simply says that we allow the mean of a to vary by education.

Following the classic model of Becker (1985), household output is generated by a CES function of market goods and home-produced goods. Market goods are bought with income earned in the labor market with an implicit price of one, while home-produced goods are generated through effective time spent in home production. There are two types of families in the economy: singles and married couples. We begin with the more complicated problem for marrieds.

Married households. For couples consisting of male- (k_g, n_g, a_g) and female- (k_l, n_l, a_l) , household output is given by:

$$Z^M(\cdot) \equiv F(e_l, e_g) + \max_{(t_g^w, t_l^w, t_g^h, t_l^h)} \sum_{i=f,m} \left[a_i \left(H(k_g, n_g, k_l, n_l, \mathbf{t}_g^h, \mathbf{t}_l^h) \right)^x + (1 - a_i) \left(t_g^w \omega_g(k_g, n_g) + t_l^w \omega_l(k_l, n_l) \right)^x \right]^{\frac{1}{x}} \quad (8)$$

where $\frac{1}{1-x}$ is the elasticity of substitution between home production H and market goods, $\omega(k, n)$ is the competitive wage per effective hour of work in the labor market. $F(\cdot)$ is a non-pecuniary benefit to marriage that accrues to all married couples, due, for instance, to

social approbation. We assume it is an equal-shares Cobb-Douglass function of partners' education levels divided by the average level of education, which allows us to match the education level in the economy: $F = F_0 e_l^5 e_g^5$. Partners' labor market incomes are assumed to be perfect substitutes when buying market goods, and a_i is spouse i 's relative preference for home goods vs. market income. Equation (8) therefore gives the value of married output of the couple according to their specific preferences. Once produced, however, output is perfectly transferable (through the use of side payments) so that $U_g^M = \varrho Z^M$ and $U_l^M = (1 - \varrho)Z^M$ where ϱZ^M is the husband's marginal contribution to output in the household sector and is determined endogenously in the efficient sorting.

A main question we want to address using our model is whether home production is similar to market production, and specifically whether n and k have similar interpretations at home as at work. At the same time, the exact nature of home production, and its relationship to productive skills, is largely a "black box" and so, in contrast to market production, we do not want to impose too much structure on the roles of n and k at home. Consider first a version of equation (4) re-conceived for home production:

$$H(\cdot) = \left((\gamma_1^{M,g}(k_g) \min\{\tilde{\theta}_g^P, \gamma_2^{M,g}(n_g, n_l)\tilde{\theta}_l^C\})^r + (\gamma_1^{M,l}(k_l) \min\{\tilde{\theta}_l^P, \gamma_2^{M,l}(n_l, n_g)\tilde{\theta}_l^C\})^r \right)^{\frac{1}{r}} \quad (9)$$

where for spouse $i \in \{g, l\}$, $\mathbf{t}_i^h = \{\tilde{\theta}_i^P, \tilde{\theta}_i^C\}$. Unlike the market production technology, partners' specific contributions to household production H are not likely to be either homogenous or easily converted to cash equivalents in a competitive market. Therefore, we do not assume that they are perfect substitutes in the production of output ($r \leq 1$). As well, in contrast to labor market production, we assume that a spouse's individual contribution to home production depends only on her own general productive skill and not on their partner's. Otherwise, if $\gamma_2^{M,l} = n_g$ and $\gamma_2^{M,g} = n_l$ then (9) is a direct analog of (4). The interpretation in this case relies on that fact that, because partners do not have exactly the same tastes, partner i will need to direct partner $-i$ to produce goods she will enjoy. Her efficiency in doing so depends on her leadership skill n . For example if the husband does the shopping and the wife does the cooking, but the couple are inefficient

communicators, the wife may have to go to the store with the husband (or revisit the store later) and the husband may have to remain in the kitchen during meal preparation to prevent the wife from adding more salt than he likes to the food.

Of course, the roles of “manager” and “worker” are not well-defined roles within monogamous households the way they are in modern firms, and managerial skill at work may not necessarily translate at home. For that reason, γ_2^M may depend not only on the partner’s but also on one’s own n (or, potentially, on neither partner’s n). Flexible forms for the γ s that allow for all these possibilities are defined below. Here, we note that if the production function at home mirrors that at work with $\gamma_2^{M,i}(n_i, n_{-i}) = n_{-i}$ for partners i and $-i$ it will not have the same implications for assortative mating. In the labor market, worker-manager teams are strongly negatively sorted on n (conditional on k) since in general higher n individuals become managers. In the home, however, high n partners would match with other high n spouses because otherwise the time freed up by efficient supervision of the spouse would be wasted.

Because we are summing over preferences and because the spouses’ problem involves two choices per capita rather than one (after imposing time constraints), we are not able to express it as a nested CES and derive direct analytical solutions for time use. However, letting $\tilde{\theta}_i^P + \tilde{\theta}_i^C = t_i^h$ for partner i , it is easy to show that the optimal division of home production time between producing and coordinating will always satisfy:

$$\tilde{\theta}_i^P = \frac{\gamma_2^{M,i}(t_{-i}^h - \gamma_2^{M,-i}t_i^h)}{1 - \gamma_2^{M,i}\gamma_2^{M,-i}} \quad (10)$$

which is defined and unique whenever $\gamma_2^{M,i}\gamma_2^{M,-i} \neq 1$. Solutions are feasible whenever $\tilde{\theta}_i^P \in (0, t_i^h)$ for both partners $i = g, l$. Where the solution for at least one spouse is not feasible, we can discard the proposed allocation since it implies that, at the closest feasible solution, time must be wasted in home production which could be used in the labor market instead. It is therefore relatively easy to approximate the optimal choice of t_i^h and t_m^h numerically.

Single households. The problem for singles is analogous but simpler. A single

individual with gender G has the following household production function:

$$Z_G^S(k, n, a) \equiv \max_{\mathbf{t}^h, \mathbf{t}^w} \left(a(H(k, n, \mathbf{t}^h))^x + (1-a)(\omega(k, n)t^w)^x \right)^{(1/x)} \quad (11)$$

where t^j is the total disposable productive time allocated to activity j and the other variables are the same as for marrieds. For singles, H is produced analogously to marrieds as

$$H(\cdot) = \gamma_1^{S,G}(k) \min\{\tilde{\theta}^P, \gamma_2^S(n)\tilde{\theta}^C\}$$

and again $\mathbf{t}^h = \{\tilde{\theta}^P, \tilde{\theta}^C\}$ and $\tilde{\theta}^P + \tilde{\theta}^C = t^h$. Since singles produce alone, it is reasonable to expect that $\frac{\partial \gamma_2^S}{\partial n} \approx 0$, something we can use the model to test.⁸ For singles it is easy to show that optimal allocation of time at home to productive activity P satisfies

$$\frac{\tilde{\theta}^P}{t^h} = \frac{\gamma_2^S}{1 + \gamma_2^S}$$

This formulation allows us to rewrite home production as $H(\cdot) = \gamma^S(k, n)t^h$ where $\gamma^S(\cdot) = \frac{\gamma_1^S \gamma_2^S}{1 + \gamma_2^S}$. This in turn allows us to solve explicitly for the optimal ratio of time spent in home and market production:

$$\frac{t^h}{t^w} = \left(\frac{a}{1-a} \right)^{\frac{1}{1-x}} \left(\frac{\gamma^S}{\omega} \right)^{\frac{x}{1-x}} \equiv \Gamma^S \quad (12)$$

which holds as an interior solution whenever $x < 1$. Letting T_G be the total amount of disposable time available to a gender- G individual (see section 3.3 below) we can further solve analytically for t^h (or t^w):

$$t^h = T_G \frac{\Gamma^S}{1 + \Gamma^S} \quad (13)$$

Finally, since singles simply consume their output, $U^S(k, n) = Z^S(k, n)$.

Function forms for γ . In the above analysis, much depends on the shape of $\gamma_1^{ms,G}$

⁸See the corresponding production function for workers who produce alone in MSSW equation (x). We rule out working alone (self employed non-managers) in our paper.

for marital status ms and gender G , γ_2^S for singles and $\gamma_2^{M,i}$ for married spouse i , with partner $-i$. We define them as follows:

$$\begin{aligned}\gamma_1^{ms,G}(k) &= \pi_0 k^{\pi_1}; \\ \text{where } \pi_j &= \pi_{j0} + \pi_{j1}\mathbb{I}_{G=l} + \pi_{j2}\mathbb{I}_{ms=M} + \pi_{j3}\mathbb{I}_{G=l}\mathbb{I}_{ms=M}, \quad j = \{0, 1\} \quad (14) \\ \gamma_2^S(n) &= \tau_{1,S} + n^{\mu_S} \\ \gamma_2^{M,i}(n_i, n_{-i}) &= \tau_{1,M} + (\tau_2 n_i^{\tau_3} + (1 - \tau_2) n_{-i}^{\tau_3})^{\frac{\mu_M}{\tau_3}}\end{aligned}$$

In (14), we allow π , which governs the productivity of k in home production, to differ by gender and marital status. This choice reflects the fact that men and women may in general devote themselves to different bundles of home production tasks, especially since the model abstracts from fertility. If managerial skills are productive at home as well as at work, then we expect μ_M (and possibly also μ_S) > 0 . However, if production at home resembles production at work, then managerial skills should have relatively small effect on the home production of singles who produce alone (and know their own tastes), implying that $\mu_S < \mu_M$, and also that own managerial skills are less important than the coordinating partner's managerial skills in one's personal contribution to home production: $\tau_2 < .5$. If managerial skills are relatively substitutable by "team members" within the household (unlike in a market production team), then $\tau_3 \rightarrow 1$ and $\tau_2 \rightarrow .5$. Overall, we believe this relatively flexible specification of home production across types of household allows us to gain insight into how our two factors interact in generating unobservable utility, where the empirical evidence on matching and time use serves as identification.

3.3 Effective time and the gender wage gap

Time use data from the 2012 General Social Survey on time use suggest that Canadian women between 35 and 50 have on average 30 minutes a day less disposable productive time than men, after sleep, passive resting leisure, and personal care after subtracted

(results available upon request). As well, Ichino and Moretti (2009) suggest that women supply less effective time at work (at the monthly or more aggregated level) than men due to the menstrual cycle, and that this difference can explain at least 14% of the 20% gender wage gap between white collar workers at a large Italian firm. In order to study this aspect of the gender wage gap, we therefore allow for women's effective time at work t^w to differ from men's by a factor ι . That is, if women choose clock hours of work \hat{t}^w , their actual effective hours of work are given by t^w , and the time constraint gives $\hat{t}^w + t^h = 1$, where we normalize each agent to have one unit of clock time to dispose of. Due to CRTS in labor market production, this implies that a woman with skill set $\{k_l, n_l\}$ will earn $\iota\omega(k_l, n_l)$ of her male counterpart's wage, allowing for a pure gender difference in wages that cannot be explained by differences in endowments of k and n across genders.

4 Linear Programming

Because there are no frictions and no cross-market externalities in the two-sector economy developed in section 3, we solve the utilitarian social planner problems associated to each market, which is equivalent to the decentralized equilibrium in each market. Each market yields a matrix matching problem that can be solved through linear programming. For the marriage market, this is simply the standard marriage matching model as explicated in Becker (1991) where individuals (of each gender) are distinguished along three traits: k , n , and taste home-produced goods a . For the labor market, the problem is slightly more complicated: first because individuals must first choose whether to be a worker or a manager and then sort into their optimal match with individuals from the other side of the market⁹; and second because worker-manager matches can be many-to-one. This latter complication is much simplified by employing the Leontief production function from MSSW, which completely pins down the number of workers assigned to each manager with given n , and so the team sizes in production. Once we have solved the labor and marriage market problems, we iterate between the two markets until time use decisions at

⁹In the marriage market, this would be like choosing first whether to be a husband or a wife and then choosing the optimal spouse of the opposite gender.

home generate the correct labor supply in the labor market and incomes received in the labor market generate the same optimal division of time in the home market. Because the separate markets could in principle be projected into a single high-dimension marriage market in which spouses directly sort on occupational status and firm type as well as characteristics, we know there exists a unique solution to the problem for any set of parameters. Finally, at the efficient solution, the Lagrangian multipliers attached to the social planner's constraints in the two markets give the shadow prices that emerge in the corresponding competitive equilibrium for each person of type (k, n, a) within each market. In the labor market, this shadow price is the wage per unit of clock time that the individual earns in his best-fit job (the job in which he has the highest social marginal product), and in the marriage market it is the income or utility he receives in his marriage, or his individual output if he is single.

4.1 Competitive Labor Market

In the competitive labor market firms take wages as given and maximize profits.

The labor market is only concerned with the distribution of effective labor market time supplied by types (k, n) , so let $\Omega(k, n)$ be the number of hours that all the individuals of type (k, n) supply. The values of $\Omega(k, n)$ are determined simultaneously with time use decisions in the household sector. For the allocation of workers and managers, let $\epsilon(k_m, n_m, k_w, n_w)$ be the number of hours a manager of type (n_m, k_m) devotes to supervising workers with (k_w, n_w) as attributes. Efficiency requires that the workers in the match devote $n_m \epsilon(k_m, n_m, k_w, n_w)$ units of effective time to the match. This gives the labor supply constraint for the time of type (k, n) as:

$$\int_A \epsilon(k, n, dk_w, dn_w) + \int_A n_m \epsilon(dk_m, dn_m, k, n) \leq \Omega(k, n) \quad (15)$$

Then using the production function (6), the social planner's constrained problem

simply becomes:

$$\max_{\lambda} \int_{A \times A} f(k_m, n_m, k_w, n_w) \lambda(dk_m, dn_m, dk_w, dn_w) \quad (16)$$

where $\lambda(k_m, n_m, k_w, n_w)$ is the total measure of matches between worker $(k_w, n_w) \in A$ and manager $(k_m, n_m) \in A$. The shadow value of the effective time (supply) constraint for each type of individual in A is the wage that would emerge in the competitive market that serves as the decentralized dual of the planner's problem.¹⁰ By extracting this shadow value from the solution to the linear programming problem, we have the wages per unit of effective time, $\omega(k, n)$, that then enter the household matching and time use problem.

4.2 Home Market

The home market is characterized one-to-one and one-to-zero matching, i.e. monogamy and singlehood. Unlike in the labor market, however, the one-to-one matching constraint is defined with respect to the number of people in a given match rather than to the effective time they offer in the household sector. The marriage market brings together a supply of males of type (k, n, a) of $\Gamma(k, n, a)$ and a supply of females of type (k, n, a) of $\Psi(k, n, a)$. For each possible marriage, there are optimal levels of time spent in home production t_g^* and t_l^* (which we solve for numerically in a preliminary step) and effective time in the labor market $1 - t_g^*$ and $\iota(1 - t_l^*)$. The total share of each type of marriage is given by $\varphi(k_g, n_g, a_g, k_l, n_l, a_l)$. Finally, there is a subset of the population that chooses to remain single and spend t_s^{**} producing at home alone according to equation (12) where $t^w = 1 - t_s^{**}$ for men and $t^w = \iota(1 - t_s^{**})$ for women. The number of singles of type (k_s, n_s, a_s) is captured by $\varsigma_s(k_s, n_s, a_s)$ where $s = g, l$ for males and females respectively. Taken together, these equations generate the population constraint for males

¹⁰Recall that the wage per unit of effective time is not the same as the wage per unit of total physical time; if women have lower effective time at work for given clock time, they will be observed with lower wages per unit clock time.

in the marriage market:

$$\int_A \int_{[\underline{a}, \bar{a}]} \varphi(k, n, a, da_l, dk_l, dn_l) + \varsigma_g(k, n, a) \leq \Gamma(k, n, a) \quad (17)$$

and similarly for females in the marriage market:

$$\int_A \int_{[\underline{a}, \bar{a}]} \varphi(dk_g, dn_g, da_g, k, n, a) + \varsigma_l(k, n, a) \leq \Psi(k, n, a) \quad (18)$$

where $[\underline{a}, \bar{a}]$ is the support of taste for home production a .

The social planner maximizes the sum of utilities in the home market given the population of males (17) and females (18). That is, the planner allocates individuals across marriages (defined by the types and time-allocations of the partners) to find:

$$\begin{aligned} & \sup_{\varphi, \varsigma_g, \varsigma_l} \int_A \times \int_A \int_{[\underline{a}, \bar{a}] \times [\underline{a}, \bar{a}]} Z^M(k_g, n_g, a_g, k_l, n_l, a_l) \varphi(dn_g, dk_g, da_g, dn_l, dk_l, da_l) \quad (19) \\ & + \int_A \int_{[\underline{a}, \bar{a}]} Z^{S_g}(k_g, n_g, a_g) \varsigma_g(dn_g, dk_g, da_g) + \int_A \int_{[\underline{a}, \bar{a}]} Z^{S_l}(k_l, n_l, a_l) \varsigma_l(dn_l, dk_l, da_l) \end{aligned}$$

where Z^M is defined by (8) and Z^S is defined by (11). The dual of the planner's problem – finding the set of prices that generate a stable competitive equilibrium – represents the decentralized competitive marriage market. Accordingly, again, the type-specific shadow prices of (17) and (18) that emerge from the planner's problem represent respectively the marginal contribution to social output of a male or female of given type. Within a marriage, these shadow prices govern the distribution of marital surplus.

The solution to the planner's problem gives not only a distribution of income, time use, share of marital surplus and partner traits, but also the supply of labor by (n, k) given market wages. Equilibrium in the whole economy requires that the demand for the services of each type of participant (worker and manager of type (k, n)) in the labor market just equals the optimal time supplied to the labor market by each type given their optimal partner and opportunities for production at home. That is:

$$\begin{aligned}\Omega(k, n) \equiv & \int_{[\underline{a}, \bar{a}]} \left(\int_A \int_{[\underline{a}, \bar{a}]} (1 - t_g^*) \varphi(k, n, a, dk_l, dn_l, da_l) + (1 - t_g^{**}) \varsigma_g(k, n, a) \right) da \\ & + \int_{[\underline{a}, \bar{a}]} \left(\int_A \int_{[\underline{a}, \bar{a}]} \iota(1 - t^*) \varphi(dk_g, dn_g, da_g, k, n, a) + \iota(1 - t_l^{**}) \varsigma_l(k, n, a) \right) da\end{aligned}$$

4.3 Estimation

We solve the model using the following iterative process, which searches for a fixed point between the home and market sectors of the economy at which decisions in the home sector produce the labor supply that generates the distribution of wages in the labor market that in turn reproduces the same time-use decisions in the home sector.

1. Solve the planner's labor market problem with a arbitrary baseline labor supply Δ .
2. Use these wages to solve the planner's household market problem to get matches across type and the time use of the population.
3. Use the time use from the solution to the household market given the current wage distribution to generate a new labor supply.
4. Solve the planner's labor market problem with the new supply of time for a new set of wages.
5. Repeat from (2) until wages and time use converge for each (k, n, a) type individually by gender.

The problem is much simplified by choice of technology (6), which precisely pins down the match between managers and workers within a team and thus eliminates match-specific spillover effects (although the model generalizes without much difficulty other than increased computational burden to the more general case). Nevertheless, we cannot rule out the possibility of multiple equilibria in our simulated economy arising from different starting distributions of hours worked (step 1 above). The reason is that the matching market theorem that assures that the only stable equilibrium is the planner's

solution (Becker (1991) Chapter 4) relies on the fact that, at this equilibrium, no coalition can form who would choose to deviate from the efficient solution, by leaving their current firm or spouse to form a new firm or marriage. However, by iterating between the labor and marriage markets rather than solving them simultaneously, we are explicitly ignoring the possibility of *cross-market* coalitions. To circumvent this problem while preserving computational feasibility, we add an additional loop to the program, choosing a finite number of Δ s as starting points, iterating to convergence, and choosing the fixed point that generates the highest return. In practice, the economy nearly always converges to the planner’s solution immediately from any initial choice of Δ , implying that the functional form of the model we have chosen generates a fairly well-behaved problem.

With a method to solve the planner’s problem for any set of parameters, we estimate the model’s parameters to match the SLID data on the distributions of education, occupational status, and labor supply (full-time work, part-time work and non-participation) within and across households, and the conditional wage distributions by gender, education, managerial status, and education, taken from section 2. From these moments we are able to back out two important sets of unobservables: the correlations of education with adult n and k , and the contributions of n and k to output in the home sector using the flexible parameterizations introduced in section 3. We can then use our results to analyze how couples sort on unobservables, how n and k are rewarded in the marriage markets in 1995 and 2010, and how the post-education distributions of n and k , along with gender differences in disposable time, contribute to the gender wage gap.

4.4 General Method of Moments

We use a generalized method of moments estimator to compute the model, searching over the parameter space using a simulated annealing variation on the Nelder-Meade method to minimize the loss function generated between 146 moments of the model, described below, and their empirical counterparts. For each draw of the parameter set, we solve the model using the iterative process described in section 4.3 using the CMPL linear programming package to solve the labor and marriage market matching problems,

Fortran to generate the market inputs and perform aggregation, and Stata to calculate the population statistics in our simulated economy. Worker and managerial ability k and n , and preferences a , are unobserved in the data, but we use educational attainment, occupational status (worker vs. manager), and labor for each individual in our SLID sample, as well as information on the disposable time of individuals by education, marital status, and managerial status, to identify the parameters.

We use five sets of moments to estimate the model. First, we take as moments the gender-specific log wage regressions from Table 2 columns (2) and (3), which give us the reduced form earnings returns to education, managerial status, marital status, and part-time status for men and women, a total of 16 targets (we include the R^2 s to help pin down σ_{me}^w). These targets allow us to estimate the parameters of the production function in the labor market and provide identifying information on the relationship between productive factors n and k and education across gender. Second, assuming a sample of 6000 men and 6000 women and using the SLID weights to create the cells, we calculate the number of marriages by education attainment of the partners (16 cells), occupation of the partners (8 cells, where the categories are non-worker, worker, and manager, and the cell with both partners non-workers is excluded) and labor force attachment of the partners (8 cells, where the categories are non-worker, part-time, and full-time, and the cell with both partners non-workers excluded), for a total of 32 moments. These moments provide important information on returns to productive skills that are correlated with education and occupation in the home sector. Third, we use a set of moments capturing the correlations between education, occupation and labor force attachment for the single and married, male and female, populations, which provides further information on the relationships between skills and education, and the productive potential of skills at home, and allows us to separately estimate the home production returns of singles and marries, the π s, τ s and μ s. Specifically, we construct tables where each cell gives the count, in our population of 6000 women and 6000 men distributed according to the SLID weights, of education by labor force attachment, education by occupation, and labor force attachment by occupation for each gender/marital status combination. Excluding combinations that

are redundant (i.e. no occupation and part time *lfs*) and recalling that singles can never work zero hours, this gives 98 additional moments.

We minimize a loss function computed by a weighted sum of the squared differences between our 146 moments calculated from the SLID and the same 146 moments generated in the model using parameter set Θ :

$$\min_{\Theta} \sum_i w_i (x_i - e_i(\Theta))^2 \quad (20)$$

where the weight w_i associated to moment i is chosen to make the magnitudes of all the moments similar.¹¹ The parameter set Θ estimated in the model corresponds to those introduced in section 3. The 37 elements of Θ are summarized in table 5.

Finally, since the model is a matching model solved by linear programming methods, in practice all the variables, not only education and occupational status, must be discretized. To that end, we allow k to take seven and six n values: $k \in \{3, 6, 9, 12, 15, 18, 21\}$ and $n \in \{1, 2, 3, 4, 5, 6\}$. The α_0 will then adjust to capture the average team size in the economy.¹² We discretize the realizations of ϵ_a to take two values, which are spaced σ_a above and below zero. Because a is not separately identified from the γ s for either men or women, we also normalize \underline{a}_0 so that $\bar{a}(g, e = 2.4) = .5$: that is, the average value of a , which is also the conditional expectation of a at the mean level of education in the population is .5.

¹¹We multiply the coefficients of the wage regressions and time use regressions by 1000. For cell-based moment, we choose a weight of 1.

¹²The median team size based on the “number of people supervised” question in the SLID is 2.5 in 2010, which we take as an upper bound on team size since individuals are unlikely to undercount the number of people supervised but may not supervise them all at once. The ratio of “managers” to “workers” in the 2010 SLID is about 1.9. We take this to be a lower bound since participants are asked only if their main job has managerial aspects, not whether it is the sole or major component of the job. In multi-level firms, for instance, employees may devote time to coordinating and coordinated tasks in the context implied by our model, or may work secondary jobs that are not managerial. Indeed, recent research on the role of managers in firms suggests a higher span of control than implied by the raw SLID numbers: Hoffman and Tadelis (2016) report an average span of control per manager of about 6 at the monthly level in a large US service firm while Fredriksen et al. (2016) report an average span of control of about 8 per manager at the annual level in a Scandinavian financial sector firm. Neither of these measures are directly comparable to ours since they do not measure the average span of control per unit of time worked. Nevertheless, in keeping with the suggestive evidence that our implied span of control is too low, we assume in the model that any individual whose characteristics make them indifferent between managing and working in labor market equilibrium spends time doing both and is therefore reported as being “manager” in the data.

Parameter	Description
$\mathbb{E}(k G = l) = \kappa_0^l e^{\kappa_1^l}$	Mean growth of k with education: women
$\mathbb{E}(n G = l) = \nu_0^l e^{\nu_2^l}$	Mean growth of n with education: women
$\mathbb{E}(k G = g) = \kappa_0^g e^{\kappa_1^g}$	Mean growth of k with education for men
$\mathbb{E}(n G = g) = \nu_0^g e^{\nu_2^g}$	Mean growth of n with education: men
σ_ϵ^2	Variance of returns to education in terms of k
σ_η^2	Variance of returns to education in terms of n
ρ^l	Correlation of ϵ and η in (3) for women
ρ^g	Correlation of ϵ and η in (3) for men
α_0	TFP of managerial skill in production
α_1	Share of manager's general skills in production
$\frac{1}{1-\alpha_2}$	Elasticity of substitution between manager's and worker's k
$\frac{1}{1-x}$	Elasticity of substitution between home and market goods
$\frac{1}{1-r}$	Elasticity of substitution between husband and wife's home production
$\gamma_1^{ms,G}(k) = \pi_0^{ms,G} k^{\pi_1^{ms,G}}$	Productivity of k in home production for singles and couples
$\gamma_2^S(n) = \tau_{1,S} + n^{\mu_S}$	Return to n in home production: singles
$\gamma_2^{M,i}(n_i, n_{-i}) = \tau_{1,M} + (\tau_2 n_i^{\tau_3} + (1-\tau_2) n_{-i}^{\tau_3})^{\frac{\mu_M}{\tau_3}}$	Effect of spouse i and $-i$'s n on spouse i 's home production: couples
$a = \underline{a}_0 + \underline{a}_1 e \pm \sigma_a$	Distribution of individual-level preferences for market vs home- produced goods
F_0	Non-pecuniary benefit of marriage
σ_w^{me}	Measurement error in log wages
\mathcal{T}_0	Gender difference in disposable time

Table 5: Parameters for estimation

5 Results

In this section we discuss the estimates of the model and briefly explore the implications for returns to education, for the role of general and managerial skill in home production, and the differences in market and home technologies. In section 6 we examine in more detail returns in the marriage market, and whether returns to marriage are broad-based or match specific. Finally, in section 7 we turn to a study of the gender wage gap. Specifically we examine how much the observed gap can be attributed to differences in managerial skills vs. general skills and pure gender factors (in our context, differences in disposable productive time between genders). Then, we re-estimate the model to match the comparable SLID population in 1995 and explore how the components of the gender wage gap have changed over time, using the growth decomposition proposed by Kassenboehmer and Sinning (2014), building on Firpo et al. (2007) and Firpo et al. (2009).

5.1 Estimation

Table 6 reports the parameter values from the model fit to the 2010 SLID. Table 7 reports some reduced-form indications of the model's goodness of fit. Tables 12-14 in appendix B show the complete set of household cells by education, labor supply, and occupation targeted by the model. Given that the model is highly over-identified, it fits the data fairly well. The top panel of table 7 shows that the model replicates the reduced-form wage equations from section 2. Specifically: returns to education are increasing in schooling attainment, with the biggest marginal return accruing to the completion of an undergraduate degree, while wage premiums from marriage and from managerial status are larger for men than for women. The bottom panel summarizes the model's ability to capture sorting and time use in the marriage market: the model captures the strong but imperfect assortative matching between spousal educations and log wages (adjusted for measurement error) and the weaker but positive assortative mating on managerial status among working married couples. Even as late as 2010, some intra-

Table 6: Estimated parameters

$\mathbb{E}(k G = l)$	$9.62e^{0.294}$	$\mathbb{E}(k G = g)$	$10.16e^{0.169}$
$\mathbb{E}(n G = l)$	$2.67e^{0.312}$	$\mathbb{E}(n G = g)$	$3.12e^{0.304}$
σ_ϵ^2	1.738	σ_η^2	1.063
ρ_l	0.055	ρ_g	0.094
α_0	0.569		
α_1	0.229		
α_2	-1.326		
x	0.183		
r	0.677		
a	$0.289 + 0.088e \pm 0.113$		
$\gamma_{1,l}^S(k)$	$2.90k^{0.923}$		
$\gamma_{1,g}^S(k)$	$3.38k^{0.848}$		
$\gamma_{1,l}^M(k)$	$2.71k^{0.908}$		
$\gamma_{1,g}^M(k)$	$2.15k^{0.949}$		
$\gamma_2^S(n)$	$3.037 + n^{0.510}$		
$\gamma_{2,i}^M(n_i, n_j)$	$2.29 + (0.37n_i^{-0.12} + 0.63n_j^{-0.12})^{-6.22}, i = \{g, l\}, j = \{l, g\}$		
F_0	1.821		
σ_w^{me}	0.144		
\mathcal{T}_0	1.002		

household specialization occurs, as evidenced by the weak negative correlation of spouse labor supplies.¹³

5.2 Wages and education

Figure 1 shows the distributions of hourly wages generated by the model, which matches the hump-shaped wage distribution with a long right tail that characterizes the empirical wage distribution. Indeed, the estimated variance and skewness of log wages in 2010 (both untargeted) are .354 and 1.58, both fairly close to their empirical counterparts of .275 and 2.08. The distribution of wages for managers is shifted to the right relative to the corresponding distribution for workers, reflecting the manager wage premium,

¹³Here we compare average weekly hours worked in the SLID to time at work in the model where a labor supply of 40 hours a week corresponds to 50% of disposable productive time spent at work in the model. Note, however, that this correlation is sensitive to sample selection. It would increase to about .10 if we included married households in our age range in which neither spouse works. As discussed in section 2, we omit these households to avoid the confounding effects of disability and/or reliance on government benefits.

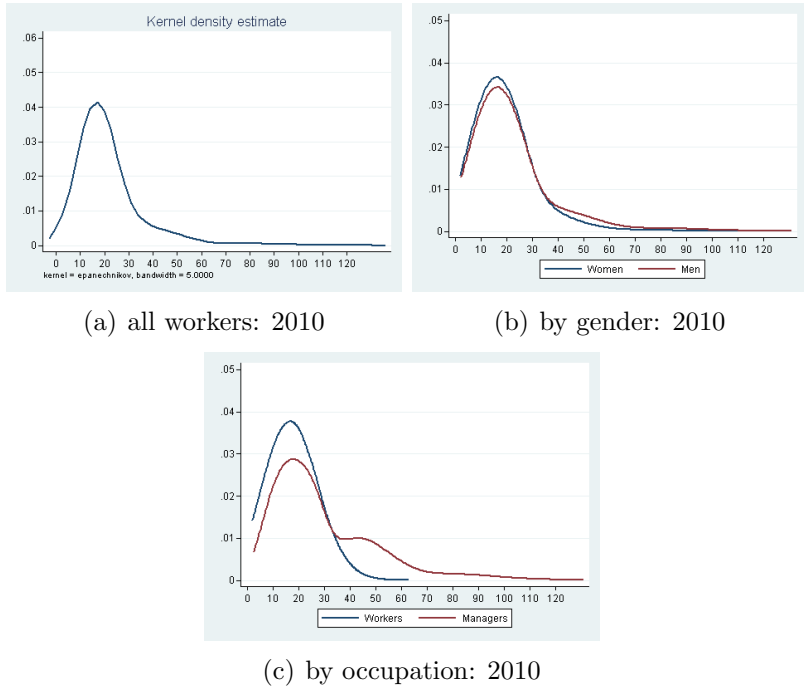
Table 7: Data fit: summary

	Target	Estimate	Target	Estimate
	Female log wages		Male log wages	
Constant	2.720 (0.031)	2.687 (0.020)	2.730 (0.026)	2.735 (0.020)
College	0.147 (0.024)	0.219 (0.018)	0.186 (0.023)	0.182 (0.017)
Bachelor	0.505 (0.034)	0.535 (0.020)	0.422 (0.028)	0.381 (0.022)
Post-grad	0.545 (0.043)	0.664 (0.028)	0.604 (0.035)	0.529 (0.027)
Married	0.026 (0.029)	-0.037 (0.016)	0.147 (0.021)	0.054 (0.017)
Manager	0.151 (0.019)	0.186 (0.015)	0.258 (0.018)	0.309 (0.015)
Part time	-0.094 (0.035)	-0.164 (0.016)	-0.056 (0.028)	-0.089 (0.020)
R-sq	0.112	0.311	0.132	0.291
Spousal correlations			Target	Estimate
Husband and wife's education			0.450	0.516
Husband and wife's wages			0.380	0.346
Husband and wife's managerial status			0.086	0.066
Husband and wife's labor supply			-0.032	-0.080

and the distribution for male wages lies to the right of the distribution of female wages reflecting the gender wage gap. The gender wage gap in the model, as in the data, is more pronounced at the high end of the wage distribution.

Next, Figure 2 shows box plots of how adult k and n vary with education for men and for women. While both factors increase with observed education, for women the relationship between n and education (simple correlation coefficient of .333) is weaker than the relationship between k and education (simple correlation coefficient of .404). This finding is consistent with the empirical results from section 2 which showed that female self-described managers come from all parts of the education distribution and that the conditional wage returns to university education are greater than the conditional wage returns to self-assessed managerial status. In terms of identification of the model, this is very important because it allows us to distinguish leadership skill from general or cognitive skill more commonly associated with educational attainment. For men, n is higher than for women at every level of education, and the simple correlation of n

Figure 1: Distribution of wages



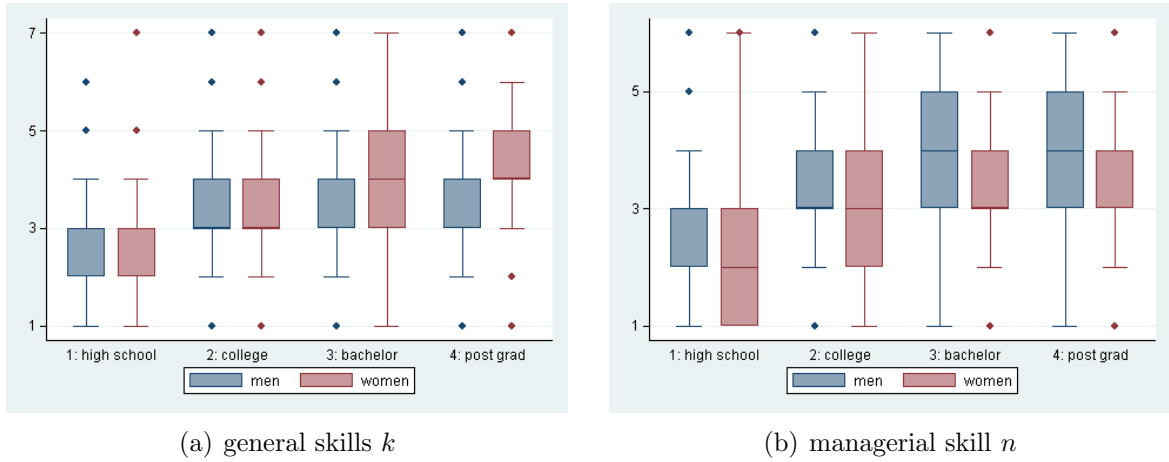
and education is larger than the correlation of k and education (.385 vs .271). This is again consistent with the facts that men are much more likely to be managers and that education is a somewhat better predictor of managerial status for men than for women (see table 1).

Given our estimate of $\alpha_0 = .57$, figure 2 (and figure 5 in section 6) also shows that the average level of n in the economy is quite high, with the average potential span of control in the labor market equal to $3.11\alpha_0 = 1.77$ per unit of time, which is only a little smaller than the span of control suggested by the manager to worker ratio in the economy. Managerial talent is quite abundant, which accounts for the relatively small wage returns to being a manager replicated in the simulation from the data given the extremely productive role it plays in the generalized MSSW production function.

5.3 Home production

Figure 3 plots the effective returns to home production estimated in the model. The upper panel plots the shares of time spent in “productive” home production for wives’ and husbands (that is $\frac{\tilde{\theta}^P(n_i, n_g, t_i^h, t_g^h)}{t_i^h} \leq 1$ for spouse i at optimal values of t_i^h and t_g^h) for

Figure 2: Returns to education by gender



every combination of $\{n_i, n_g\}$, where the average is taken over the possible values of spousal k and own and spouse a . The lower panel plots, for women on the left and men on the right, the total $\gamma = \gamma_1^{ms} \tilde{\theta}^P$ s for marrieds and singles against average wages by k , where again married γ is taken at the optimum and then averaged over k_G for gender G .

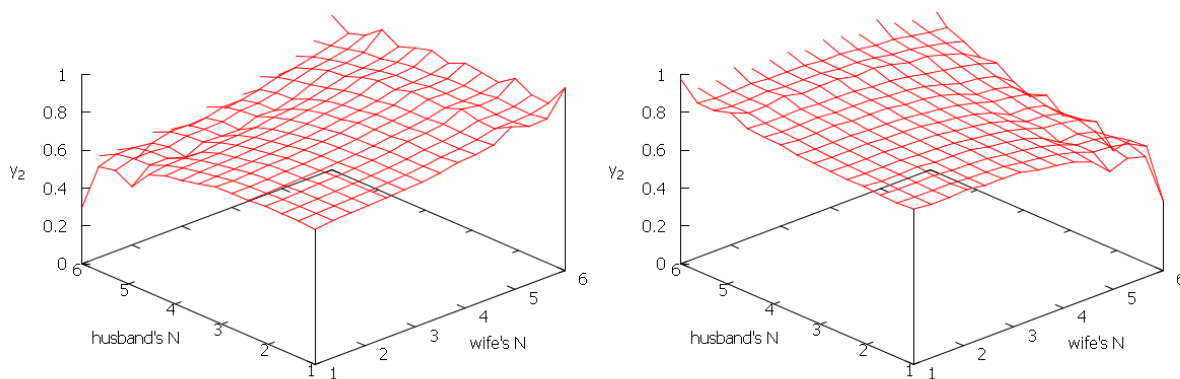
There are three main takeaways from estimates from table 6 and figure 3. First, as predicted by our conception of n as determining an individual's ability to lead or manage others, n increases the home productivity of singles substantially less than it increases the home productivity of marrieds: $\mu_S = .510$ and $\mu_M = .766$ so that home γ_2^M is increasing in n faster than γ_2^S is; since γ_2 determines how much time devoted to home production is actually productive, to a first approximation this should have the effect of making marriage relatively attractive for high- n individuals. It is important to note, however, that the attractiveness of marriage depends on the opportunity cost of managing one's spouse relative to engaging in spending that time at work.

Second, also consistent with the idea of managerial ability as inherently other-directed, the role spousal n in $\gamma_2^M(\cdot)$ is larger than the role of own n : own n has a production share of $\mu_1 = .37$. It is also clear from subfigures 3 and 3: effective home production time $\tilde{\theta}^P/t^h$ increases for both husbands and wives from about .7 when partner's n is low to about .9 when partner's n is high. Therefore, we find some evidence that n plays a somewhat similar role in the marriage market as it does in the labor market. One difference,

however, is that labor market production generally requires negative assortative mating on n within teams since high n individuals become managers to take advantage of their comparative advantage and match with (relatively) low- n workers whose productivity depends only on k . This negative assortative mating is made possible by many-to-one matching and homogenous production goods. In the marriage market, by contrast, our estimates suggest at least some degree of positive assortative mating on n will be optimal, for two reasons. First, even within own home productivity, own and spousal n are complements: the estimated elasticity of substitution between own and spousal n of $\frac{1}{1+.123} \approx .92$. Second, wife's and husband's home production are not homogenous, and both inputs are needed to produce utility. We estimate $r = .67$, implying an elasticity of substitution between *effective spouse inputs*, or spouse tasks, of about 3. That is, even though one's productivity at home depends mainly on the spouse's n in line with labor market productivity, efficient production in home tasks by *both* spouses requires that they be reasonably matched on n .

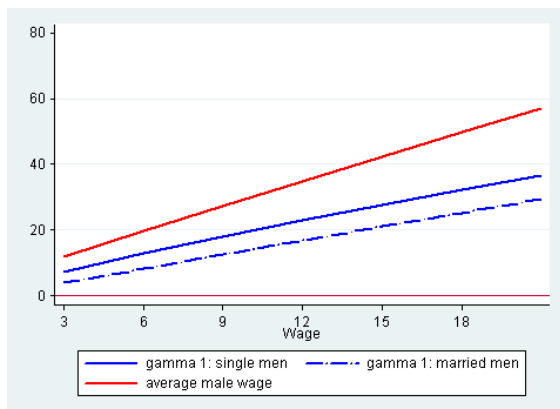
Third and finally, productivity at home γ increases in general skills k for both singles and marrieds of both genders. This is true structurally (as seen by the estimates of π determining γ_1) and conditional on the optimal choices of t^h for each type of household, which affect the "home wage" γ as shown in figures 3 and 3. Overall γ is very similar for marrieds and singles and for men and women, and that k is less productive at home than in the market over the whole distribution of k . Single men have higher returns to general skills at home than married men, but for women there is very little difference in the average "home wage" across marital status, and the small married-single gap that exists decreases in k . Although not accounted for in the model, one reason why general skills may be more productive at home for marrieds, especially for women, is that being married raises the likelihood of having children: in the 2010 SLID, about 30% (15%) of single women (men) 35-50 live with children compared to 60% of married individuals. Guryan et al. (2008) find that highly productive (by market measures) individuals substitute away from non-child based home production but not away from child-based home production, which is consistent with what we see in the estimated model.

Figure 3: Productivity at home by k and n

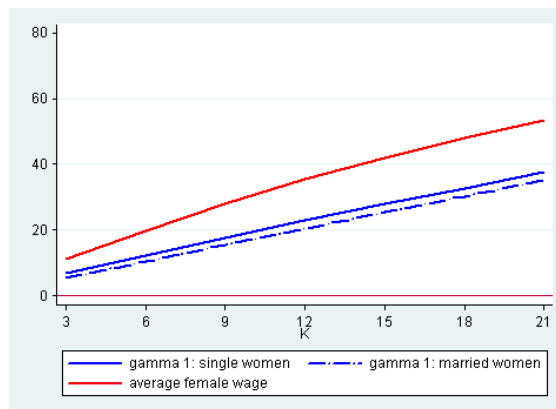


(a) husbands: $\tilde{\theta}/t^h$

(b) wives: $\tilde{\theta}/t^h$



(c) men: γ



(d) women: γ

5.4 Market production

Finally, our model generates estimates of the overall share of managerial human capital in market production as well as of the complementarity of manager and worker capital. The general skills of managers and workers within a given team are strongly complementary (more complementary than in the Cobb-Douglas production function assumed in MSSW), with an elasticity of substitution between manager's and worker's k of .43. The share of the manager's capital in production is relatively small at .23, which means that on average relatively low k as well as high n individuals will sort into managerial professions. Given the large average span of control among potential managers in the economy, this (conditional) negative occupational sorting on k and the relative abundance of n keep education-conditional returns to managerial status relatively modest, around 20% as estimated in the SLID.

6 Sorting and returns in the marriage market

Having discussed our basic model results, we now turn to exploring the implications of the model for optimal sorting in the marriage market. It is well established that, in modern economies, marriage is associated with higher welfare and better long-run outcomes. Married agents live longer (Kaplan and Kronick (2006)), accumulate greater wealth (Vespa and Painter II (2011)), have higher long-run wages (Ginther and Zavodny (2001)), and benefit from intrahousehold insurance provided by their spouse (Ortigueira and Siassi (2013)) which can even substitute for poor intertemporal choice-making due, for instance, to impatience. The question of whether the benefits of marriage are over- or understated in reduced-form analyses, however, is an open question.¹⁴ The observed returns to marriage could be driven mainly by selection of the most productive agents

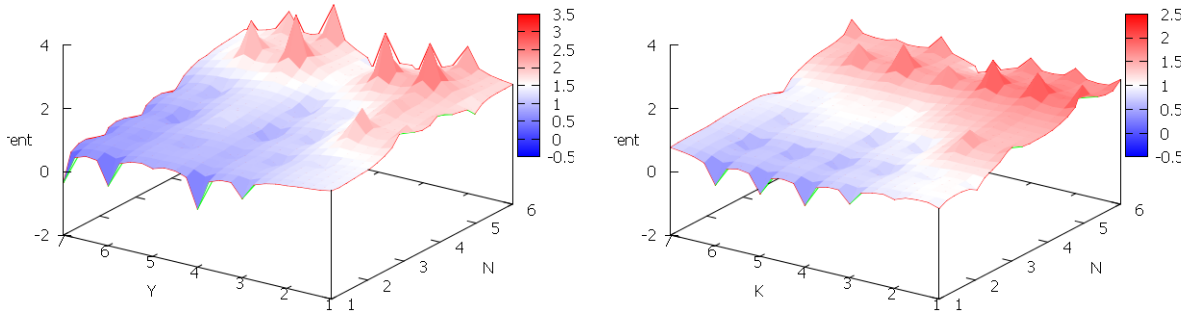
¹⁴See, for instance ?. In the context of our model, it is necessary to distinguish between the marriage wage premium and broader returns to marriage. Dougherty (2006) argues that the marriage wage premium can be considered a distributed fixed effect that is not caused by specialization within the household but instead by men (and to a lesser extent women) with greater long-run wage trajectories selecting into marriage. This is consistent with our model, which examines households who are typically at or near the top of their life cycle wage trajectories. In our model, the reduced-form relationship between wages and marital status is necessarily caused by selection, though overall benefits of marriage can be due both to selection and to higher production functions within marriage.

into marriage, even though being married raises the output and welfare of these agents only marginally above what they would get when single. On the other hand, if individuals choose to get married only when they are especially well suited to marriage, then population estimates of returns to marriage (in different dimensions) will understate the returns to being married among the married population. The latter explanation, which is in the spirit of Willis and Rosen (1979) for wage returns to education, makes most sense in a multi-factor model in which the factors have different productiveness and returns in the marriage market. Sorting into marriage is also influenced by the social valuation of marriage, captured by F_0 : if individuals are predisposed to marriage based on social approbation, then even individuals who are able to produce more private utility from single life may choose to marry due to non-pecuniary payoffs.

To understand the distribution of gains to marriage in our simulated economy, we first plot these gains to marriage as a function of n and k , separately for men and women, averaged across leisure preferences a . The top panel of Figure 4 plots the estimated marriage rents – the differences in utility between marrying and remaining single – for different combinations of n and k . The counterfactual payoffs are those that would be received by the marginal agent who entered the model economy and was constrained to either remain single and produce alone (if he marries in the benchmark equilibrium) or to marries the best available partner (if he is single in the benchmark marriage market equilibrium) while everybody else’s choices remain unaffected. Some agent types may be either married or single in the benchmark equilibrium, in which case their rent from marriage is exactly zero.

Figure 4 shows that the true gains to marriage in the simulated economy are increasing in n but constant or mildly decreasing in k , consistent with the parameter estimates of the home production function. For women, the return to marrying is decreasing in k for low- n individuals and constant in k for high- n individuals. Overall, though, the returns to marriage appear to be relatively broad-based: there are no types who receive negative returns from marrying in the baseline economy, meaning that at least some individuals from nearly all types can find a spouse with whom marriage raises total utility. To

Figure 4: Marriage gains for men and women by k and n



(a) Gain to marriage: women

(b) Gain to marriage: men

Table 8: Cross sectional and structural returns to marriage

	Cross-sectional gain to marriage	Gains of marriage to the married	Cross-sectional gain to marriage	Gains of marriage to the married
	Men		Women	
Education	1.117 (0.365)	-0.008 (0.082)	1.522 (0.348)	-0.103 (0.066)
Education squared	-0.030 (0.012)	0.001 (0.003)	-0.040 (0.011)	0.004 (0.000)
Manager	1.605 (0.130)	1.262 (0.029)	1.552 (0.125)	1.384 (0.024)
Married	0.164 (0.138)	0.631 (0.031)	0.271 (0.122)	0.559 (0.023)
Constant	4.492 (2.767)	-0.304 (0.623)	0.666 (2.628)	0.518 (0.495)

explore the issue further, table 8 report results for men from regressing U on an indicator for marriage, occupational status and education dummies. The first column gives the population effect of marriage on U conditional on the other observables for men. The second column reports the actual gains from marriage for those men who choose to marry. Columns 3 and 4 repeat the exercise for women.

The results from the table show that the estimated cross-sectional return to marriage is smaller than the actual gain to marriage among the married, which is consistent with negative (on k) sorting into marriage in the population. (Note that the model overpredicts the likelihood that a manager is married in the reduced form, but by only a small amount since not all high n individuals become managers due to the production function.) The average “true” return to marriage in the population is close to .6 (the average utility in

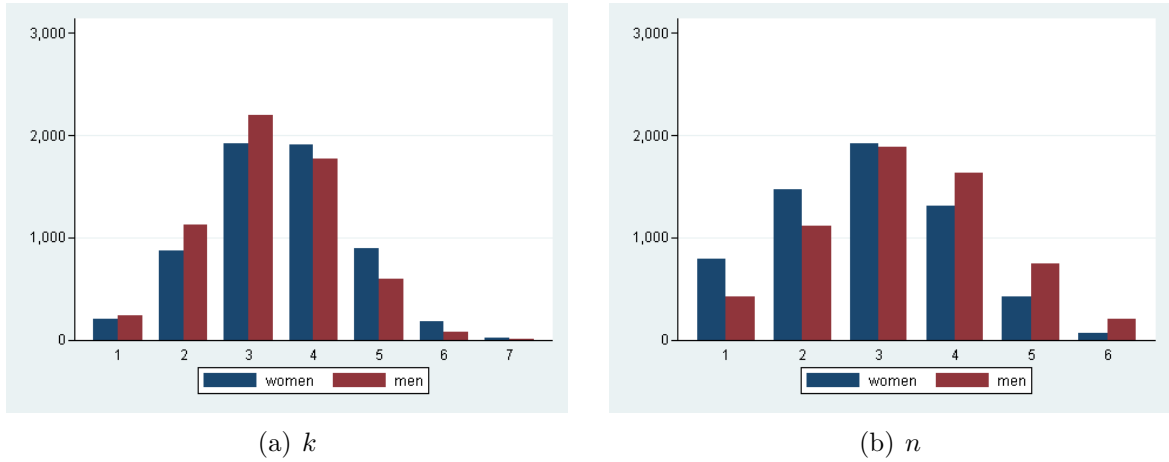
Table 9: Cross sectional and structural returns to marriage

	No frictions	With frictions	No frictions	With frictions
	Men		Women	
Share Married	0.725	0.706	0.725	0.709
Cross-sectional return to marriage	0.164 (0.138)	0.201 (0.051)	0.271 (0.122)	0.289 (0.079)
Gain from marriage to the married	0.631 (0.031)	0.626 (0.147)	0.559 (0.023)	0.511 (0.155)

the economy is approximately 20), about twice the reduced-form value.

These gains from marriage, however, depend on the ability to find an ideal match. In the model, this is simply a matter of supply and demand since the marriage and labor markets clear instantaneously. In reality, of course, marriage markets are imperfect both because searching is costly and because information about productive traits may not be observable by potential spouses as well as by the econometrician. How sensitive are marital gains to market frictions? We explore the issue by introducing a small amount of search friction into our model economy and then re-examining the marriage rates and the cross-sectional and structural returns to marriage reported in ???. Specifically, we assume that a random 10% of individuals (of each gender, separately) are unable to find their equilibrium match and must instead either accept the best match which is not an equilibrium match for their type or remain single. Table ??? reports the results, which are averaged over ten simulations, each drawing a different random 10% of the population (of the gender in the column head) who are prevented from marrying their equilibrium match. In practice, this is equivalent to capping the number of equilibrium-type marriages that are permitted in the economy, where the caps differ across the random draws. It is clear from the table that the introduction of search frictions has a major effect. Marriage rates fall about 2 percentage points in the presence of these frictions and marriage rents also fall, though estimated cross-sectional returns rise since on average the lowest k individuals opt out of marriage when they cannot marry their first choice.

Figure 5: Distributions of k and n by gender



7 Leadership skill and the gender wage gap

In this final section of the paper we explore the implications of the model for the gender wage gap and how it has changed over time.

7.1 Decomposing the 2010 gender wage gap

Figure 5 shows the estimated distributions of k and n for 2010 by gender. Consistent with much of the previous literature (assuming that k is mainly capturing cognitive skills), the gender-specific distributions of k are quite similar, in fact slightly higher for women. By contrast, women's n is lower on average than men's, with the difference in mass situated at the high end of the wage distribution.

In the model there are three ways in which women can differ from men: in k , in n , and in disposable productive time. We can therefore conduct counterfactual simulations to decompose the gender wage gap into these three components by (1) replacing women's distribution of k with men's; (2) replacing women's distribution of n with men's; and (3) replacing women's distribution of disposable productive time with men's. The odd numbered columns of table 10 report the gender wage gap measured as $\frac{\bar{\omega}_l - \bar{\omega}_g}{\bar{\omega}_g}$ that results when each of these three components is shut down, holding all other parameters constant. Even when looking at the entire population and ignoring the role of endogenous labor supply, the contributions of the three components will not generally add to one due to

Table 10: Decomposing the gender wage gap

	All individuals		All workers		Benchmark workers	
	(1)	(2)	(3)	(4)	(5)	(6)
	Wage gap	Avg	Wage gap	Avg	Wage gap	Avg
	Wage gap	Avg	Wage gap	Avg	Wage gap	Avg
benchmark	-0.147		-0.125		-0.125	
changing k	-0.146	0.004	-0.120	0.012	-0.133	-0.054
changing n	-0.068	0.559	-0.066	0.560	-0.721	0.665
changing \mathcal{T}	-0.134	0.046	-0.112	0.051	-0.193	0.064

interactive effects among the components and between them and other parameters of the model. The even numbered columns of table 10 therefore report average contributions to the “raw” gender wage gap when the components are shut down one by one. There are six paths by which the components can be down sequentially to achieve a gender wage gap of zero, entailing four different marginal contributions of component i as a share of the gender wage gap. The arithmetic average over these four marginal proportional contributions is reported in the even numbered columns.¹⁵ Columns (1) and (2) show the results using all women and men in the simulated economy. Columns (3) and (4) show results for workers in each economy, for whom the wages would be observed. Columns (5) and (6) show results for the sample of workers in the benchmark economy (based on their own and their spouse’s productive characteristics), regardless of whether they work or not in the counterfactual economy.

From table 10, it is clear that the largest contributor to the gender wage gap is the gender difference in managerial skill n , which accounts for 53% of the gender wage gap among the entire sample population between 56 and 66 of the gender wage gap between the benchmark and counterfactual simulations. By contrast, women have a similar distribution of k to men, so replacing women’s with men’s k distribution reduces the gender wage gap by very little. We estimate women’s disposable productive time to be .997 as great as men’s for each clock hour which accounts for a small amount of the

¹⁵For instance, women’s k can be replaced with men’s k in the benchmark economy (row one of the odd numbered columns), in the counterfactual economy where women have men’s n , in the counterfactual economy where women have men’s \mathcal{T} , and in the counterfactual economy where women have men’s n and \mathcal{T} . Each replacement will yield a different marginal contribution (not necessarily negative) to reducing the gender wage gap from .14 to 0.

gap, around 5%. Given the unavoidable imprecision in this type of exercise, our estimates are therefore quite in line with Ichino and Moretti (2009). Some of the gender wage gap remains due to residual factors in the model, such as different correlations of n and k conditional on education for men and women, and non-linearities between the factors.

7.2 Changes in the gender wage gap 1995-2010

[incomplete; forthcoming]

Finally, we are interested in whether and how the gender wage gap has changed over time. Table 11 shows reduced the form wage equation estimates from 2 for 2010 against the same estimates from 1995, the third year of the SLID. Over this fifteen year interval, the gender wage gap, conditional on labor force attachment and education, has shrunk only marginally from .185 to .178, which is consistent with the main findings in the economics literature on gender wage gaps for the US and Canada. Additionally, returns to managerial status have risen over the period for both men and women, but much more for men, such that the gender wage gap in returns to managerial status has emerged over the past fifteen years. Indeed, the difference across genders between wage returns to self-assessed managerial status are significant at the 1% level in 2010 but insignificant at the 10% level in 1995. By contrast, wage returns to marriage and to education below the post-graduate level demonstrate no changes over the period. Among post-graduates, the return in terms of wages has risen for men and fallen for women, which may be due to the relative increase in female post-graduate degree holders. Part time wage penalties have also fallen for both genders, but especially for men.

Table ?? in the appendix reports corresponding numbers from table ?? for 1995. A systematic exploration of changes over time in marriage and labor markets in the context of the model is left to future work. Here, we want to focus only on the growth of the gender wage gap over the fifteen year interval at different points in the distribution of wages. To do so, we partially re-estimate the model to fit the estimates from the 1995 SLID, fixing the parameters governing household output at their 2010 levels but allowing both population supplies of the two factors (which incorporates their relationship

Table 11: Log hourly wage: 2010 and 1995

	All		Women		Men	
	2010 (1)	1995 (2)	2010 (3)	1995 (4)	2010 (5)	1995 (6)
Schooling						
College	.193*** (.015)	.192*** (.016)	.147*** (.024)	.149*** (.023)	.186*** (.023)	.176*** (.025)
Bach.	.521*** (.019)	.504*** (.024)	.505*** (.028)	.489*** (.032)	.423*** (.028)	.432*** (.034)
> Bach	.696*** (.026)	.607*** (.032)	.545*** (.037)	.676*** (.044)	.604*** (.035)	.542*** (.043)
Manager	.184*** (.013)	.091*** (.016)	.151*** (.020)	.096*** (.023)	.258*** (.019)	.123*** (.023)
Part-time	-.091*** (.016)	-.153*** (.018)	-.094*** (.020)	-.158*** (.021)	-.056*** (.028)	-.139*** (.035)
Female	-.178*** (.013)	-.185*** (.015)				
Married	.074*** (.015)	.081*** (.019)	.026 (.020)	0.002 (.025)	.147*** (.021)	.129*** (.029)
_constant	2.80*** (.018)	2.731*** (.022)	2.72*** (.027)	2.68*** (.028)	2.73*** (.026)	2.72*** (.031)
R-squared	.156	.1418	.115	.1388	.130	.0958

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

to education), and the parameters governing production technology (hence wages) to vary. The assumption that home production technology and returns to marriage were stable between 1995 and 2010 is obviously questionable, but changes in home production technology over the fifteen year period will have only second order effects (through labor supply and general equilibrium effects) on the estimated gender wage gap; also, regressions run on the 1995 sample suggest little change in the degree of matching on education or managerial status over the period.

Table ?? reports the results from 10 applied to our 1995 estimated economy.

Lastly, we are also interested in changes in the return to factors across genders at different points in the wage distribution. To explore this, we conduct a Blinder-Oaxaca style decomposition of the growth in each factor at the 25th, 50th and 75th percentiles following the method proposed in Kassenboehmer and Sinning (2014) (equation 5) applied to simulated data and the unobservable productivity factors. Specifically, let $d_{i1} = 1$ for

(simulated) observation i if he is from the 2010 simulation and zero otherwise, and let $d_{i2} = 1$ if observation i is male. Then we run the following pooled regression:

$$y_i^* = b_0 + b_1 d_{i1} + b_2 d_{i2} + b_3 d_{i1} d_{i2} + \beta^P X_i$$

where $X_i = \{k_i, n_i\}$. The parameter vector β^P is the reference parameter vector. The Oaxaca-Blinder decomposition of the estimated population gender wage gap $\mathbb{E}(y_m^*) - \mathbb{E}(y_f^*)$ is then

$$\begin{aligned} \mathbb{E}(y_m^*) - \mathbb{E}(y_f^*) &= \mathbb{E}(X_m)\beta^m - \mathbb{E}(X_f)\beta^f \\ &= (\mathbb{E}(X_m) - \mathbb{E}(X_f))\beta^P + \mathbb{E}(X_m)(\beta^m - \beta^P) - \mathbb{E}(X_f)(\beta^P - \beta^f) \\ &= \Delta_g^F + \Delta_g^R \end{aligned} \tag{21}$$

where Δ_g^F is the part of the gender wage gap attributable to changes in the factor supplies across gender and Δ_g^R is the part attributable to the gender-specific difference in the returns to the factors by gender. Note that if we were using only a single period, (21) would reduce to

$$\mathbb{E}(y_m^*) - \mathbb{E}(y_f^*) = (\mathbb{E}(X_m) - \mathbb{E}(X_f))\beta^P + \beta_0^m - \beta_0^f$$

since there is no discrimination in the model, so that factor returns are identical across genders and the \mathcal{T}_0 would simply be absorbed into the constant terms (β_0) in a log wage decomposition. In the wage growth decomposition, Δ_g^R will therefore capture changes in comparative advantage induced by overall changes in the returns to factors that benefit the gender with the more generous endowment of that factor. Taking the difference in (21) between 2010 and 1995, we get

$$\begin{aligned}
\Delta_t(\mathbb{E}(y_m^*) - \mathbb{E}(y_f^*)) &= \Delta_t(\mathbb{E}(X_m)\beta^m - \mathbb{E}(X_f)\beta^f) \\
&= \Delta_t((\mathbb{E}(X_m) - \mathbb{E}(X_f))\beta^P) + \Delta_t(\mathbb{E}(X_m)(\beta^m - \beta^P) - \mathbb{E}(X_f)(\beta^P - \beta^f)) \\
&= \Delta_t(\Delta_g^F + \Delta_g^R) \tag{22}
\end{aligned}$$

where $\Delta_t(X) = X_{1995} - X_{1990}$. (22) gives a decomposition of the difference in the gender wage gap over time. As Kassenboehmer and Sinning (2014) point out, it is in fact identical to the difference across genders in wage growth rates $\Delta_g(\Delta_t^F + \Delta_t^R)$, where $\Delta_g(X) = X_m - X_f$, which is calculated by starting with an Oaxaca decomposition of the difference in wages between the two periods for gender g and then taking the difference in these decomposed growth rates across gender.

For our mean regressions, y_i^* is simply the log wage earned by simulated individual i . For our quantile regressions, however, y_i^* the recentered influence function (RIF_{jt}) of the wage for quantile j of the wage distribution in period t (Firpo et al. (2007), Firpo et al. (2009)), which is given by

$$RIF_{jt} = q_{jt} + \frac{j - \mathbb{I}\{y \leq q_j\}}{f_Y(q_j)}$$

where q_{jt} is the value of y at the j 'th quantile of the distribution.

8 Conclusion

[tba]

References

- Becker, G. (1985). Human capital, effort, and the sexual division of labor. *Journal of labor economics* 3(1), 33–58.
- Becker, G. (1991). *A Treatise on the Family*. Harvard Univ Pr.
- Becker, G. S., E. M. Landes, and R. T. Michael (1977). An economic analysis of marital instability. *The Journal of Political Economy*, 1141–1187.
- Bruze, G., M. Svarer, and Y. Weiss (2012). The dynamics of marriage and divorce. *Journal of Labor Economics* 33(1).
- Cattan, S. (2013). Psychological traits and the gender wage gap. *The Institute for Fiscal Studies*.
- Choo, E. and A. Siow (2006). Who marries whom and why. *Journal of political Economy* 114(1), 175–201.
- Dougherty, C. (2006). The marriage earnings premium as a distributed fixed effect. *Journal of Human Resources* 41(2), 433–443.
- Firpo, S., N. Fortin, and T. Lemieux (2007). Decomposing wage distributions using recentered influence function regressions. *University of British Columbia (June)*.
- Firpo, S., N. M. Fortin, and T. Lemieux (2009). Unconditional quantile regressions. *Econometrica* 77(3), 953–973.
- Fortin, N. M. (2008). The gender wage gap among young adults in the united states the importance of money versus people. *Journal of Human Resources* 43(4), 884–918.
- Fredriksen, A., L. Kahn, and F. Lange (2016). Supervisors and performance management systems.
- Ginther, D. K. and M. Zavodny (2001). Is the male marriage premium due to selection? the effect of shotgun weddings on the return to marriage. *Journal of Population Economics* 14(2), 313–328.

- Guryan, J., E. Hurst, and M. Kearney (2008). Parental education and parental time with children. *Journal of Economic Perspectives* 22(3), 23–46.
- Hirsch, B. T. (2005). Why do part-time workers earn less? the role of worker and job skills. *Industrial & Labor Relations Review* 58(4), 525–551.
- Hoffman, M. and S. Tadelis (2016). How do managers matter? evidence from performance metrics and employee surveys in a firm.
- Hurder, S. (2013). An integrated model of occupation choice, spouse choice, and family labor supply. *Harvard University, Department of Economics*.
- Ichino, A. and E. Moretti (2009). Biological gender differences, absenteeism, and the earnings gap. *American Economic Journal: Applied Economics*, 183–218.
- Jencks, C., S. Bartlett, M. Corcoran, J. Crouse, D. Eaglesfield, G. Jackson, K. McClelland, P. Mueser, M. Olneck, J. Schwartz, S. Ward, and J. Williams (1979). *Who Gets Ahead? The Determinants of Economic Success in America*. Basic Books, New York.
- Kaplan, R. M. and R. G. Kronick (2006). Marital status and longevity in the united states population. *Journal of Epidemiology and Community Health* 60(9), 760–765.
- Kassenboehmer, S. C. and M. G. Sinning (2014). Distributional changes in the gender wage gap. *Industrial & Labor Relations Review* 67(2), 335–361.
- Mazzocco, M., C. Ruiz, and S. Yamaguchi (2014). Labor supply, wealth dynamics, and marriage decisions. *American Economic Review: Papers and Proceedings*.
- McCann, R. J., X. Shi, A. Siow, and R. P. Wolthoff (2012). Becker meets Ricardo: Multisector matching with social and cognitive skills. *Available at SSRN 2048554*.
- Ortigueira, S. and N. Siassi (2013). How important is intra-household risk sharing for savings and labor supply? *Journal of Monetary Economics* 60(6), 650–666.
- Sandberg, S. (2013). *Lean in: Women, work, and the will to lead*. Random House.

- Vespa, J. and M. A. Painter II (2011). Cohabitation history, marriage, and wealth accumulation. *Demography* 48(3), 983–1004.
- Weinberger, C. (2014). The increasing complementarity between cognitive and social skills. *Review of Economics and Statistics* 14(5).
- Willis, R. J. and S. Rosen (1979). Education and self-selection. *Journal of Political Economy* 87(5 pt 2).

A Additional equations

Recall that the FOC for the ratio of married time between husband g and wife l is:

$$\frac{t_l}{t_g} = \left(\frac{\gamma_g^M}{\gamma_l^M} \right)^{\frac{r}{r-1}} \left(\frac{\omega_l}{\omega_g} \right)^{\frac{1}{r-1}}$$

Rearranging and suppressing the M superscript gives time use by the husband in terms of the parameters:

$$t_g = \frac{\omega_g + \omega_l}{\left(\frac{\gamma_g^r}{\omega_g} \right)^{\frac{1}{r-1}} \Gamma^{\frac{x-r}{r(x-1)}} \left(\Gamma^{\frac{x(r-1)}{r(x-1)}} + 2^{\frac{x(r-1)}{r(x-1)}} \right)} \quad (23)$$

and time use of the wife

$$t_l = \frac{\omega_g + \omega_l}{\left(\frac{\gamma_l^r}{\omega_l} \right)^{\frac{1}{r-1}} \Gamma^{\frac{x-r}{r(x-1)}} \left(\Gamma^{\frac{x(r-1)}{r(x-1)}} + 2^{\frac{x(r-1)}{r(x-1)}} \right)} \quad (24)$$

where

$$\Gamma = \left(\frac{\omega_g}{\gamma_g} \right)^{\frac{r}{r-1}} + \left(\frac{\omega_l}{\gamma_l} \right)^{\frac{r}{r-1}}$$

B Model Fit

B.1 2010 SLID

Table 12: Model fit: number of matched pairs by education/labor force status/occupation

Man's <i>lfs</i>	Woman's <i>lfs</i>	Target number	Estimated number
no work	part time	92	0
no work	full time	624	608
part time	no work	50	0
part time	part time	103	0
part time	full time	1072	1215
full time	no work	99	117
full time	part time	292	428
full time	full time	2228	2193
Man's occ	Woman's occ	Target number	Estimated number
no occ	worker	399	0
no occ	manager	317	608
worker	no occ	106	0
worker	worker	1430	1497
worker	manager	1112	1147
manager	no occ	42	117
manager	worker	541	714
manager	manager	609	478
Man's educ	Woman's educ	Target number	Estimated number
HS	HS	428	385
HS	Col	389	629
HS	Uni	71	35
HS	PGr	13	18
Col	HS	522	641
Col	Col	1264	1447
Col	Uni	293	84
Col	PGr	99	48
Uni	HS	114	55
Uni	Col	358	98
Uni	Uni	441	566
Uni	PGr	202	272
PGr	HS	26	17
PGr	Col	124	28
PGr	Uni	80	148

PGr	PGr	132	90
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Table 13: Model fit: individuals by education / labor force status cell

Single women			
Education	Labor force status	Target number	Estimated number
HS	part time	107	36
HS	full time	181	94
Col	part time	164	76
Col	full time	554	593
Uni	part time	59	14
Uni	full time	265	450
PGr	part time	31	0
PGr	full time	79	176
Single men			
Education	Labor force status	Target number	Estimated number
HS	part time	75	103
HS	full time	269	199
Col	part time	163	154
Col	full time	570	539
Uni	part time	35	68
Uni	full time	224	257
PGr	part time	0	10
PGr	full time	111	109
Married women			
Education	Labor force status	Target number	Estimated number
HS	no work	208	147
HS	part time	252	329
HS	full time	443	591
Col	no work	304	287
Col	part time	636	585
Col	full time	1237	1348
Uni	no work	176	129
Uni	part time	255	262
Uni	full time	685	600
PGr	no work	29	45
PGr	part time	84	39
PGr	full time	248	199
Married men			

Education	Labor force status	Target number	Estimated number
HS	no work	51	34
HS	part time	142	111
HS	full time	902	953
Col	no work	66	47
Col	part time	197	174
Col	full time	1876	1981
Uni	no work	26	32
Uni	part time	105	109
Uni	full time	756	692
PGr	no work	0	4
PGr	part time	42	34
PGr	full time	395	390

Table 14: Model fit: individuals by education-occupation cell

Single women			
Education	Occupation	Target number	Estimated number
HS	worker	232	120
HS	manager	56	10
Col	worker	486	574
Col	manager	233	95
Uni	worker	216	360
Uni	manager	108	104
PGr	worker	65	139
PGr	manager	45	37
Single men			
Education	Occupation	Target number	Estimated number
HS	worker	232	284
HS	manager	109	18
Col	worker	426	551
Col	manager	299	142
Uni	worker	144	234
Uni	manager	112	91
PGr	worker	49	86
PGr	manager	70	33
Married women			
Education	Occupation	Target number	Estimated number
HS	no work	208	147

HS	worker	518	755
HS	manager	176	165
Col	no work	303	287
Col	worker	1331	1341
Col	manager	542	592
Uni	no work	355	129
Uni	worker	586	453
Uni	manager	355	409
PGr	no work	29	45
PGr	worker	212	95
PGr	manager	119	143

Married men			
Education	Occupation	Target number	Estimated number
HS	no work	5	34
HS	worker	686	773
HS	manager	358	291
Col	no work	66	47
Col	worker	1126	1101
Col	manager	947	1054
Uni	no work	26	32
Uni	worker	383	250
Uni	manager	478	551
PGr	no work	0	4
PGr	worker	176	87
PGr	manager	263	337

Table 15: Model fit: individuals by labor force status-
occupation cell

Single women			
Labor force status	Occupation	Target number	Estimated number
	no work	0	0
part time	worker	290	126
part time	manager	71	0
full time	worker	707	1067
full time	manager	372	246

Single men			
Labor force status	Occupation	Target number	Estimated number
	no work	0	0
part time	worker	217	335
part time	manager	69	0

full time	worker	634	820
full time	manager	520	284
Married women			
Labor force status	Occupation	Target number	Estimated number
	no work	717	608
part time	worker	1005	989
part time	manager	223	226
full time	worker	1643	1655
full time	manager	972	1083
Married men			
Labor force status	Occupation	Target number	Estimated number
	no work	150	117
part time	worker	384	307
part time	manager	101	121
full time	worker	1982	1904
full time	manager	1941	2112