

# Exporting under Quality Uncertainty

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## Abstract

It is, by now, well accepted that potential exporters face significant fixed exporting costs. Our understanding of these costs is, however, limited. In this paper, we propose one channel through which these fixed costs could arise. We develop a dynamic, two-country model where a foreign consumer learns the quality of a home product only after consuming it. As a result, home exporters need to signal their product quality in the foreign market. In equilibrium, firm-specific fixed exporting cost arises endogenously. These costs have two components: (i) the cost of signaling quality in the foreign market and (ii) the opportunity cost of exporting due to the choice of sub-optimal quality in the home market. The model generates a non-monotonic relationship between firm size and export status, and is also consistent with the presence of many small exporters and observed export dynamics.

KEYWORDS : Quality, uncertainty, asymmetric information, signaling, trade cost.

JEL CLASSIFICATION : D83, F10, F19, L15.

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# 1 Introduction

Uncertainty about product quality is an endemic problem in trade. It is often not possible to assess the true quality of a product simply by observing it. Sometimes, it may not be possible for a producer to credibly signal his product quality either. This problem of asymmetric information becomes more severe when products are traded across international boundaries.

In a highly influential paper, [Rauch \(1999\)](#) showed that proximity, common language and colonial ties are more important for differentiated products than for products traded on organized exchanges.<sup>1</sup> Under the hypothesis that differentiated products are those for which quality varies a lot, a possible interpretation of this result is that information facilitates trade. In other words, the absence of perfect information may create impediments to trade, especially in goods with varying quality, over and above the standard ones such as distance and policy barriers. How quality uncertainty affects trade is still an open question, however. In particular, how does uncertainty affect demand? What barriers does uncertainty create for exporting firms? Is the pricing decision of firms affected? How is the choice of quality affected, vis-à-vis a world with full information? And finally, does uncertainty lead to selection of some firms into exporting?

In this paper, we put uncertainty about the quality of products in the export market at the forefront of our analysis of exporting behavior.<sup>2</sup> In [Section 2](#), we develop a two-country, overlapping generations model where firms with heterogeneous capability live for two periods and produce different varieties of a quality-differentiated good. The quality of a firm's product is known in the home market, but unknown in the foreign market. Once a foreign consumer consumes the product, he learns its true quality. His knowledge is then transmitted to the rest of the population, albeit imperfectly, so that the quality of the product is known in the next period. Given this constraint, a firm chooses the quality of its product, as well as, the price in the different markets in both periods. In equilibrium, which we study in [Section 3](#), fixed exporting costs arise endogenously. These costs have two components: (i) the cost of signaling quality in the foreign market and (ii) the opportunity cost of choosing a sub-optimal quality in the

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<sup>1</sup>Other papers to provide evidence of informational asymmetry in international trade, although not necessarily about product quality, include [Gould \(1994\)](#), [Head and Ries \(1998\)](#) and [Rauch and Trindade \(2002\)](#). [Gould](#) shows that immigrant links to the home country have a strong positive effect on both exports and imports for the U.S. while [Head and Ries](#) find the same for Canada. [Rauch and Trindade](#) find that for differentiated goods, the presence of ethnic Chinese networks in both the trading partners increases trade. [Portes and Rey \(2005\)](#) run a standard gravity equation and find that informational flows, proxied by telephone call traffic and multinational bank branches, have significant explanatory power for bilateral trade flows.

<sup>2</sup>By now, it is well established that quality plays an important role in international trade. Research has shown that not only do rich countries export higher quality goods on average ([Schott, 2004](#); [Hummels and Klenow, 2005](#); [Khandelwal, 2010](#)), but even within narrowly defined sectors, firms produce and export goods of different quality ([Verhoogen, 2008](#); [Kugler and Verhoogen, 2009](#); [Hallak and Sivadasan, 2010](#)).

home market. These components, in turn, are firm-specific. Our model thus provides a micro-foundation for firm-specific fixed costs that have been used, for example, by [Das, Roberts and Tybout \(2007\)](#) to explain the exporting behavior of Colombian firms.

The model is also consistent with a number of empirical regularities that have been uncovered recently. First, there are many exporters with small sales. This is consistent with the existence of many small French exporters in the data ([Eaton, Kortum and Kramarz, 2011](#)). The reason behind the existence of small exporters is, however, very different from the one in [Arkolakis \(2008\)](#). Unlike in [Arkolakis](#), the small exporters in our model are firms at a certain stage of their life-cycle. Second, the model is consistent with the finding that in a given year, a significant fraction of Chilean exporters are new exporters who export very little compared to those who have exported for at least a year ([Blum et al., 2009](#)). As we show in the paper, even the most capable firms optimally export little in their first year as exporters. Finally, in our model, although all exporters start off small, only the most capable among them experience a growth in sales. Thus, the model is consistent with observed export dynamics of Colombian exporters ([Eaton, Eslava, Kugler and Tybout, 2007](#)).

The model also provides an explanation for the non-monotonic relationship between size of firms and their export status. [Melitz \(2003\)](#) makes a stark prediction that all exporters are larger than all non-exporters. Of course, we do not expect to see this relationship in the data. But [Hallak and Sivadasan \(2010\)](#) show that the non-monotonicity between firm size and export status is systematic - the fraction of exporters increases with firm size. In our model, uncertainty in the export market distorts the product quality of the exporting firms. This results in some of the exporters earning a lower revenue from home sales compared to some non-exporters. If the revenue from foreign sales is small enough in period one, there would be an overlap between young exporters and non-exporters, at least over some firm size interval. By imposing some restrictions on the underlying capability distribution, our model generates a relationship between firm size and export size similar to the one uncovered by [Hallak and Sivadasan](#).

The difficulty in determining the true quality of a product induces consumers to engage in costly processes to obtain information about product characteristics. One such process is “search” whereby consumers try to evaluate a product before it is purchased. But when this way of acquiring information is too expensive (for example, when a new product is introduced in the market), consumers might try to determine quality by purchasing or “experiencing” the product. The sale of such experience goods fundamentally changes the behavior of firms too.<sup>3</sup>

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<sup>3</sup>That exporters face difficulty in selling quality differentiated products is consistent with evidence presented in the Survey of Business and Innovation Statistics (2009) conducted by Statistics Canada in 2009. This survey shows that about 17 percent of Canadian firms believe that meeting the quality requirements of consumers is one

Although firms could potentially sell low quality products and make short-term profits, they are dissuaded from doing so by the dynamic nature of demand. Because consumers try to obtain information about a product through consumption, the seller of an experience good essentially bundles information about his product with the product itself. Selling a low quality product means providing adverse information about the product that reduces the future stream of profits (Shapiro, 1983b). In such a setting, a firm choosing to sell a high quality product might even have to incur a loss initially to send a credible signal.<sup>4</sup> This is the strategy that we highlight in this paper.

Our paper is related to two strands of the existing literature. In industrial organization, there is a large literature on experience goods. See, for example, Schmalensee (1978), Smallwood and Conlisk (1979), Klein and Leffler (1981), Shapiro (1983a) and Riordan (1986). Most of the papers in this literature focus on a single market. By allowing firms to sell in two different markets with different informational structures, our model can explain why some firms might choose to serve only one market, i.e., not export even without any exogenous cost of exporting. As we show in this paper, it is the need to signal quality in the presence of asymmetric information that generates a cost that acts as a ‘fixed’ exporting cost.

We also contribute to the theoretical literature on quality in international trade. Many of the papers in this literature have been concerned with explaining the pattern of trade in vertically differentiated goods (Flam and Helpman, 1987; Stokey, 1991; Murphy and Shleifer, 1997; Fajgelbaum et al., 2009), or the systematic difference in unit value of exports across firms (Baldwin and Harrigan, 2007; Kugler and Verhoogen, 2009; Hallak and Sivadasan, 2010; Johnson, 2010). Our work is closely related to those papers that have explored the role of asymmetric information about product quality in an international trade context (Grossman and Horn, 1988; Bagwell and Staiger, 1989; Raff and Kim, 1999; Chisik, 2003).

Finally, our paper contributes to the literature on uncertainty in international trade. Most of the papers in this literature focus on how countries formulate trade policy in the presence of uncertainty (See Ruffin, 1974, for example). Our paper is closer in spirit to Chisik (2012) and Handley and Limao (2012) who also study uncertainty in trade policy, but focuses on firms’

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of the main barriers to trade.

<sup>4</sup>Of course, there are alternative ways of getting around the problem. For example, firms could provide warranties; if full warranties are given, there will not be any incentive for firms to deviate and quality will be the same as in the complete information case. The problem with full warranties, however, is that it introduces a moral hazard on the consumers’ side (Allen, 1984). Since it is difficult to establish whether a product malfunctioned because it was of low quality or whether it was poorly maintained, a full warranty reduces consumers’ incentives to use the product properly. Consequently, firms will only give partial warranties and hence reputation will matter for the firm’s choice of quality. In a recent paper, Roberts (2011) shows that guaranties cannot substitute for reputation in a decentralized on-line market.

response to such uncertainty. Finally, [Dasgupta and Mondria \(2012\)](#) build on the framework in this paper to explore the role of trade intermediaries in alleviating information frictions in international trade.

The rest of the paper is organized as follows. We present a model of quality uncertainty in [Section 2](#) and go on to study the equilibrium and its properties in [Section 3](#). In this section, we also discuss how our model is consistent with some of the evidence that have been uncovered recently. [Section 4](#) concludes.

## 2 A Model of Quality Uncertainty

In this section, we develop a framework that will allow us to study the problem of exporters in the presence of asymmetric information about product quality. There are two symmetric countries - home and foreign. Without loss of generality, we shall carry out our analysis with respect to the home country. Each country is populated by a unit mass of individuals. Each individual owns one unit of labor that he supplies inelastically in return for a wage of  $w$ .

**Preferences:** There are  $J$  varieties of a differentiated good available to consumers, with the quality of a variety  $q \in \mathbb{R}$ . A consumer can consume only one unit of the differentiated good - he chooses the one with the highest utility. Given quality  $q_j$  and price  $p_j$  of a variety  $j$ , a consumer  $i$ 's utility from consuming a unit is given by:

$$u_j^i = \theta q_j - p_j + \epsilon_j^i,$$

where  $\theta$  captures the preference for quality and  $\epsilon_j^i$  is an idiosyncratic shock drawn from a cumulative distribution  $\Phi^i(\epsilon_1^i, \epsilon_2^i, \dots, \epsilon_J^i)$ . To ease exposition, we assume that the  $\epsilon_j^i$ s are independently and identically distributed across individuals, i.e.,  $\Phi^i = \Phi$ . Therefore, we can drop the superscript  $i$  from the preferences. The probability that a consumer chosen at random selects variety  $j$  is given by

$$f_j = \text{Prob}(u_j = \max_s u_s), \quad s = 1, 2, \dots, J.$$

By assuming that  $\Phi$  is a Type I Extreme Value Distribution with variance  $\sigma^2$  ([McFadden, 1973](#)), the choice probability becomes

$$f_j = \frac{\exp\{\frac{1}{\sigma}(\theta q_j - p_j)\}}{\sum_{s=1}^J \exp\{\frac{1}{\sigma}(\theta q_s - p_s)\}}.$$

The derivation of the choice probability from the indirect utility function is quite standard (see [Anderson et al., 1992](#)). From the Law of Large Numbers it follows that  $f_j$  is also the fraction of consumers who demand variety  $j$ . As the number of available varieties becomes arbitrarily large (the set of varieties converging to a continuum in the limit),  $f_j$  reduces to

$$f(j) = \frac{\exp\{\frac{1}{\sigma}(\theta q(j) - p(j))\}}{\int_{s \in \Omega} \exp\{\frac{1}{\sigma}(\theta q(s) - p(s))\} ds}, \quad (1)$$

where  $\Omega$  is the subset of varieties that are available to the consumers ([Anderson et al., 2001](#)). Note that conditional on price, consumers demand more of higher quality varieties. This property captures the vertical differentiation aspect of the differentiated good. Furthermore, there is positive demand for every variety as long as  $\sigma > 0$  and the price does not go to infinity. This property reflects the horizontal differentiation aspect of the differentiated good and ensures that in equilibrium, there is a positive demand for every variety.

**Production:** Every period, a measure one of firms enter the market, produce for two periods and then exit.<sup>5</sup> Therefore, there is a “young” and an “old” cohort of firms in every period. Firms are endowed with some asset that allows them to hire workers at a wage of  $w$  and carry out production - there is no free entry.<sup>6</sup> Assuming that producing a new variety is costless, every firm produces a unique variety, and accordingly, has some market power. Firms choose (i) what quality to produce and (ii) how many units to produce (or conversely, what price to charge).

Firms are endowed with capability  $\lambda$  drawn from an exogenous distribution  $G(\lambda)$  with density  $g(\lambda)$  and support  $[\lambda_L, \lambda_H]$ . The technology for quality is given by

$$q = \lambda + \log n,$$

where  $n$  is the number of workers used for producing one unit of quality  $q$ .<sup>7</sup> If a firm chooses to produce quality  $q$ , the above technology results in a per unit cost of

$$c(q; \lambda) = we^{q-\lambda}.$$

<sup>5</sup>The assumption of firms living for two periods is not essential for our results. We could easily generalize to more than two periods. Adding more periods, however, does not affect our analysis in any significant way.

<sup>6</sup>In the heterogeneous firm literature following [Melitz \(2003\)](#), the standard assumption has been that any firm can carry out production by incurring a cost upfront. As shown by [Evans and Jovanovic \(1989\)](#), however, entrepreneurship may not be an option for many individuals because of liquidity-constraints.

<sup>7</sup>The term “capability” was introduced by [Sutton \(2007\)](#); capability is similar to productivity, but in a world where goods have heterogeneous quality. Although, capability has two dimensions in [Sutton’s](#) work, we reduce it to a single dimension captured by  $\lambda$  (See [Kugler and Verhoogen, 2009](#), for a similar treatment).

Higher  $\lambda$  allows firms to produce higher quality using the same resources, or the same quality using fewer resources.<sup>8</sup> We also assume that producers choose the same quality across time and space. The need for assuming same quality in both periods will be clear once we specify the information structure. Requiring firms to choose the exact same quality in both the home and foreign markets is without loss of generality. If we allowed firms to choose different qualities in the two markets, all our results would hold as long as there is some linkage between the two qualities.<sup>9</sup>

**Information:** Home consumers observe all firm-specific variables of domestic firms. In particular, the quality of the varieties produced by home firms is observed by home consumers in both periods. But there is no international spillover of information. Home consumers do not pass on their knowledge about products to their foreign counterparts - the two markets are completely segmented.

Unlike home consumers, foreign consumers do not observe the quality of a variety produced by a “young” home exporter. Quality is revealed in the second period when the firm is “old”, provided that there has been some consumption of the variety in the previous period.<sup>10</sup> But the revelation of quality in the second period depends on the excess demand for the good in the foreign market in the first period.

Let the amount of variety  $j$  that a young exporting firm chooses to supply to the foreign market be  $x^1(j)$ . Let the demand facing the firm be  $x_d^1(j)$ . In the second period, foreign consumers receive a noisy signal about each variety: with probability  $\beta$ , they meet an individual who has consumed a variety in period one and knows the true quality of the good, where  $\beta = \beta(x^1(j)|x_d^1(j))$ . We make the following assumptions about  $\beta$ :

ASSUMPTION 1: (a)  $\beta' > 0$  and  $\lim_{x^1 \rightarrow 0} \beta' = \infty$ ; (b)  $\beta'' < 0$ ; (c)  $\beta(0|x_d^1(j)) = 0$ ; (d)  $\beta(x^1(j)|x_d^1(j)) = 1 \forall x^1(j) \geq x_d^1(j)$ .

Therefore, with probability  $\beta$ , consumers know the true quality of a variety and demand accordingly. If the firm does not satisfy its entire first period demand (i.e.,  $x^1 < x_d^1$ ) however, there

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<sup>8</sup>The functional form for  $c(q; \lambda)$  is convenient, but not necessary. What is necessary is that the marginal cost function displays the following property:  $\frac{\partial^2 c}{\partial \lambda \partial q} < 0$ .

<sup>9</sup>Firms would choose the same quality in both the periods if it is very costly to change quality midway through production. The linkage between the qualities of the varieties produced for the home and the foreign markets would arise if, for example, the production lines cannot be separated completely.

<sup>10</sup>It should now be clear why we need more than one period. Since exporters sell an experience good, with only one period, the export market would suffer from Akerlof’s lemons problem (Akerlof, 1970); only the low quality producers would export. This problem can, however, be mitigated even in a one period model if there are some informed consumers (see Cooper and Ross, 1984).

will be some unsatisfied consumers. These consumers perceive the firm as providing very poor service. Accordingly, these unsatisfied consumers spread the message that the firm is unreliable and its product should not be consumed.<sup>11</sup> The higher is the excess demand, the greater are the chances of meeting an unsatisfied customer and receiving a negative signal about the variety.<sup>12</sup>

By applying the Law of Large Numbers, we then conclude that a fraction  $\beta$  of consumers know the true quality of a good. In particular, among those consumers who *would have* consumed the good in the event that everyone knew the true quality in the second period, only a fraction  $\beta$  would actually want to consume the good. Hence, the demand faced by an old exporter in the foreign market is

$$x_d^2(j) = \beta(x^1(j)|x_d^1(j))f(j). \quad (2)$$

The only difference between the above expression and equation 1 lies in  $\beta$ . Asymmetric information scales down demand, as long as young exporters do not meet their demand in the foreign market. A natural question to ask is, why a young exporter would want to supply less than the demand it faces. This possibility would arise if the firm is making losses per unit sold. Then it would ideally want to sell as little as possible. But selling less in period one results in reduced demand and profits in period two.<sup>13</sup> Hence, a young exporter that incurs a loss in the foreign market faces a trade-off.

At this point, it is instructive to point out the difference with [Arkolakis \(2008\)](#). In his paper, exporters have to bear a positive cost of reaching even the first consumer in any market. Our model suggests that reaching a single consumer may not be optimal for an exporter. Even though the cost of selling one unit in the foreign market is negligible, the foregone profits could be large, depending on the capability of the exporter.

The timing of the model is depicted in [Figure 1](#). Given the informational structure, it should now be clear why we require firms to produce the same quality across time. If exporters could separately choose their quality in the foreign market in the two periods, they would always choose the lowest possible quality when they were old. Anticipating this, foreign consumers'

<sup>11</sup>For example, [Matsa \(2011\)](#) argues that a stockout, which can be interpreted as a measure of product quality, is costly for retailers as it creates consumer dissatisfaction, leading some consumers to switch to other retailers. He shows that faced with competition, U.S. supermarkets reduce the incidence of stockouts.

<sup>12</sup>The world wide web is a platform through which agents meet each other and exchange information about goods. After consuming a good, both satisfied and unsatisfied consumers post their experience on websites and on-line forums. Higher is the proportion of negative reports about a good, greater is the probability that a potential consumer will believe that the good is of low quality. [Chevalier and Mayzlin \(2006\)](#) discusses the relevance of word-of-mouth communication among consumers and how it affects the sell of books online.

<sup>13</sup>If an exporter makes losses in both periods, it should simply exit the export market.





**Figure 1:** *Timing of the model*

demand in period two would correspond to the demand for a variety with the lowest quality.

### 3 Equilibrium

We begin by studying the equilibrium in the closed economy. We then go on to study the equilibrium in the open economy featuring asymmetric information.

#### 3.1 Closed Economy

Every firm produces a unique variety and solves the following problem:

$$\max_{p,q} 2(p - c(q; \lambda))x(q),$$

where  $x(q)$ , the demand for a variety with quality  $q$ , is given by equation 1. We multiply the expression by 2 because of the two periods. Profit-maximization yields the following expression for quality and price:

$$q_D(\lambda) = \lambda + \log \frac{\theta}{w}, \quad (3)$$

$$p_D(\lambda) = \sigma + we^{q_D(\lambda) - \lambda} = \sigma + \theta, \quad (4)$$

where the subscript  $D$  stands for serving only the domestic market. Although more capable firms produce higher quality varieties, every firm that serves only the home market charges the same price. The price charged is a constant markup  $\sigma$  over marginal cost  $we^{q_D(\lambda) - \lambda}$ , where  $we^{q_D(\lambda) - \lambda}$  happens to be equal to a constant  $\theta$ . Essentially, there are two forces that determine how marginal cost changes with capability. More capable firms face a lower cost of producing a *given* quality of a variety. But more capable firms also choose higher quality, which raises

their marginal cost. Given the functional forms used in this paper, these two effects cancel each other out exactly, whereby marginal cost is independent of  $\lambda$ . In general, whether marginal cost rises or falls with capability will depend on which of these two effects dominate.

The price charged also depends on  $\sigma$ , the dispersion in the taste of the consumers. Intuitively, a high  $\sigma$  increases the mass on the tails of the distribution of  $\epsilon$ . Because the demand for a particular quality comes from individuals who value it highly (i.e., those with realized values for  $\epsilon_j$  lying in the upper tail of the marginal distribution of  $\epsilon_j$ ), fattening of the upper tail raises demand, while fattening of the lower tail has no effect on demand. The price is also higher if consumers attach higher value to quality (higher  $\theta$ ). Replacing the value of  $q_D(\lambda)$  and  $p_D(\lambda)$  in the expression for profit, we get

$$\pi_D(\lambda) = \frac{2\sigma}{C} \exp\left\{\frac{1}{\sigma}(\theta\lambda + \theta \log \frac{\theta}{w} - \sigma - \theta)\right\}, \quad (5)$$

where  $C = \int_{\Omega} \exp\left\{\frac{1}{\sigma}(\theta q(j) - p(j))\right\} dj$  is a measure of aggregate demand. We set  $C$  as the numeraire. The above equation suggests that  $\pi_D(\lambda)$  is equal to  $\tilde{C} \exp\left\{\frac{\theta}{\sigma}\lambda\right\}$ , where  $\tilde{C}$  is a constant from the firm's perspective. In equilibrium, more capable firms sell higher quality at the same price and face higher demand, resulting in higher profits. Although firms charge the same price, high  $\lambda$  firms have a lower *quality adjusted* price. Observe that if a firm chooses to serve only the home market, both quality and price charged by the firm will remain unchanged even in the open economy.

### 3.2 Open Economy

Before formally defining the equilibrium in the open economy, let us point out the difference between the problem faced by foreign consumers and young home firms in our model and the standard "lemons" problem. In the standard model, the principal values an asset that the agent provides. The value of the asset, however, varies across agents, which the principal does not observe. Hence, the latter offers an average price for the asset, where the price depends on the belief of the principal regarding the distribution of assets. The strategy for the agents is to accept or reject the price. In our model, on the other hand, firms are not price-takers. They choose their price in the foreign market in both periods. In particular, young firms can use their price to signal their quality in the foreign market. Upon observing the price charged by a young firm, foreign consumers update their belief about that firm's quality,  $\mu(q|p)$ . Based on the price and the updated belief about quality, demand for the variety  $x_d^1$  is generated.

Given the structure of the problem, we use the concept of a perfect Bayesian equilibrium

(PBE). PBE requires a strategy profile for the agents and posterior beliefs about the type of the agents. In this model, the strategy for a consumer (both home and foreign) is to demand a variety, while the strategy for a firm is to choose a quality and prices for each market. The posterior belief,  $\mu(q|p)$ , is about the quality of the variety sold by a firm. Formally,

**Definition 1.** *A PBE of the model consists of strategies for the consumers and firms, and posterior beliefs such that:*

(a) *Consumers maximize utility,*

(b) *Firms maximize profits,*

(c)  $\mu(q|p)$  *is formed from the prior distribution using Bayes' rule whenever possible.*

We shall first consider the problem of a potential exporter. It should be clear that in equilibrium, if a firm chooses to export in the first period, it will do so in the second period too. In the second period, the quality of an exporter is revealed, whereby consumers solve a full information problem. The full information profit is always positive. Hence, irrespective of the quality of goods sold, if a firm has exported in the first period, it will always want to export in the second period. It is not possible for a firm to export in the second period but not the first. If a firm does not export in period one, it faces the problem of a young exporter in period two. Hence, if exporting was not profitable in period one, it cannot be profitable in period two either.

The problem of an exporter then reduces to choosing a quality for both markets, price in each market and a quantity for the foreign market in period one. The price charged by exporters in period one foreign market, however, depends on the posterior belief held by the foreign consumers about its quality. In this paper, we focus on pooling equilibria, where every young exporter charges the same price,  $\bar{p}$ , in the foreign market. We also assume that consumers have rational expectations. At the beginning of period one, foreign consumers correctly anticipate the expected quality of the young exporting firms,  $\bar{q}$ , although they cannot observe the quality of individual firms. For  $\bar{p}$  to be a pooling equilibrium price, foreign consumers must have the following beliefs:  $\mu(q = \bar{q}|p = \bar{p}) = 1$ . What happens if consumers observe a price not equal to  $\bar{p}$ ? PBE allows consumers to assign any posterior belief whenever  $p \neq \bar{p}$ . This leeway in specifying off-the-equilibrium-path beliefs generates multiple equilibria (Fudenberg and Tirole, 1991). We assume that consumers hold the following beliefs:  $\mu(q = \bar{q}|p < \bar{p}) = 1$  and  $\mu(q = -\infty|p > \bar{p}) = 1$ . When an exporter charges a price above  $\bar{p}$ , consumer demand for his product drops to zero. Given these beliefs, it is easy to see that a home exporter has no incentive to charge a price other than  $\bar{p}$ . Furthermore, if  $\bar{p}$  is below the marginal cost of production, the exporter would incur a loss in the first period. This can be interpreted as a costly signal. For the remainder of the paper, we assume that  $\bar{p}$  is such that every exporter incurs a per unit loss in

the export market in the first period. A low price by a young exporter transmits the following message to the foreign consumers: “I am incurring a loss now, but because my quality is good, I will recoup my losses in the future from repeat purchases” (Kirmani and Rao, 2000).

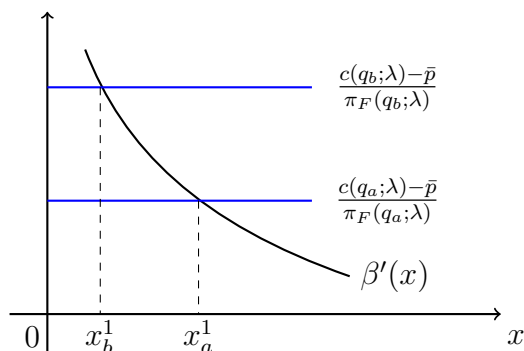
We solve the exporter’s problem in two steps. First, for a given quality  $q$  and price  $p$ , home firms choose  $x^1$ . Second, home firms choose the optimal  $q$  and  $p$ . In the first step, for a given  $q$ , the firm’s choice of first period exports  $x^1$  maximizes total export profits which has no impact on domestic profit. Specifically, conditional on quality, the problem that the firm faces in the foreign market is

$$\max_{x^1} (\bar{p} - c(q; \lambda))x^1 + \beta(x^1 | x(\bar{q}))\tilde{\pi}(q; \lambda),$$

where  $x(\bar{q}) = \exp\{\frac{1}{\sigma}(\theta\bar{q} - \bar{p})\}$  denotes the expected demand for a home firm’s variety in the foreign market,  $p_X$  is the optimal price in period two and  $\tilde{\pi}(q; \lambda) = (p_X - c(q; \lambda))\exp\{\frac{1}{\sigma}(\theta q - p_X)\}$  is the full information profit for some quality  $q$  in the second period export market. Observe that  $p_X$  and  $\tilde{\pi}(q; \lambda)$  are also the optimal price and profit in the home market. The first-order condition for profit-maximization yields

$$\beta'(x^1) = \frac{c(q; \lambda) - \bar{p}}{\tilde{\pi}(q; \lambda)}, \quad (6)$$

where  $c(q; \lambda) - \bar{p} > 0$  by assumption. Figure 2 below shows how  $x^1$  is determined in equilibrium. The downward-sloping line represents  $\beta'(x)$ , which is the same for all producers. It can be interpreted as the increase in the second period export profit due to a marginal increase in the first period quantity. The solid horizontal line represents  $\frac{c(q_b; \lambda) - \bar{p}}{\tilde{\pi}(q_b; \lambda)}$ , which measures the loss from selling an additional unit in period one, normalized by  $\tilde{\pi}(q; \lambda)$ . The value of  $x^1$  that maximizes profit occurs at the intersection of these two lines.



**Figure 2:** Determination of  $x^1$

From (6), the solution for  $x^1$  is given by

$$x^1 = \beta^{t-1} \left( \frac{c(q; \lambda) - \bar{p}}{\tilde{\pi}(q; \lambda)} \right) = \phi(q). \quad (7)$$

The properties of the function  $\phi(q)$  depend on the properties of  $\beta$  and how the ratio  $(c(q; \lambda) - \bar{p})/\tilde{\pi}(q; \lambda)$  changes with  $q$ . As  $q$  increases, both  $c(q; \lambda)$  and  $\tilde{\pi}(q; \lambda)$  rise. Intuitively, as  $q$  rises, there are two forces that act on  $x^1$ . On the one hand, a higher  $c(q; \lambda)$  implies that *per unit* losses in period one are higher. The firm tends to lower these losses by reducing  $x^1$ . On the other hand, a higher  $\tilde{\pi}(q; \lambda)$  implies that the marginal benefit of increasing  $x^1$  is now higher. An increase in  $x^1$  raises demand faced by the firm in period two, which is more valuable for high quality firms. This tends to raise  $x^1$ . Given the functional forms we have assumed, the former effect dominates. Accordingly,  $x^1$  is decreasing in  $q$ . This is shown in Figure 2 as a decline from  $x_a^1$  to  $x_b^2$  as quality increases from  $q_a$  to  $q_b$ .

After incorporating the equilibrium value of  $x_1$  in exporter  $\lambda$ 's total profit, the second step of the exporter's problem is given by

$$\pi_X(\lambda) = \max_{p,q} \left[ [2 + \beta(\phi(q))] (p - c(q; \lambda)) \exp\left\{ \frac{1}{\sigma}(\theta q - p) \right\} + (\bar{p} - c(q))\phi(q) \right].$$

Note that total profit consists of two parts - the profits in the three full-information markets and the profit in the first period export market. The first-order condition with respect to price yields:

$$p_X(\lambda) = \sigma + we^{q-\lambda}.$$

As in the closed economy, the optimal price is a constant mark-up over marginal cost and is the same in all the full information markets. The first order condition with respect to quality yields

$$[2 + \beta(\phi(q))] \exp\left\{ \frac{1}{\sigma}(\theta q - p_X(\lambda)) \right\} (\theta - we^{q-\lambda}) = we^{q-\lambda} \phi(q), \quad (8)$$

where we have used the Envelope Theorem to replace  $\beta'(\phi(q))$  using (6). The left-hand side of (8) is the sum of marginal profits in the full information home and foreign markets. Therefore, the left-hand side represents the marginal benefit of increasing quality. The right-hand side is the corresponding marginal cost - it is the increase in the first period loss in the foreign market due to higher marginal cost of production. Let us denote the optimal value of  $q$  by  $q_X(\lambda)$ .

Replacing the equilibrium values of  $p$  and  $q$  in the exporter's profit yields

$$\pi_X(\lambda) = [2 + \beta(\phi(q_X(\lambda)))] \bar{\pi}(q_X(\lambda); \lambda) + (\bar{p} - c(q_X(\lambda); \lambda)) \phi(q_X(\lambda)). \quad (9)$$

The second term on the right-hand side of the above equation is a loss that a firm has to incur if it decides to export. In other words, it is a costly signal of a firm's quality. This, however, is not the only cost that exporters have to bear. The assumption that firms produce goods of the same quality for all the markets implies that exporting affects the choice of quality in the home market. Producing  $q_D(\lambda)$  maximizes profits if the firm were only serving the home market. In an open economy, however,  $q_D(\lambda)$  is no longer jointly optimal for both markets. In equilibrium, exporters end up choosing  $q_X(\lambda) < q_D(\lambda)$ . There is an opportunity cost of exporting. The following proposition characterizes  $q_X(\lambda)$ .

**Proposition 1.** *If  $\sigma > \theta$ ,  $q_X(\lambda)$  is increasing in  $\lambda$  and  $q_X(\lambda) < q_D(\lambda)$ . Moreover, if  $q_X(\lambda)$  does not increase too fast with  $\lambda$ ,  $\phi(q_X(\lambda))$  is increasing in  $\lambda$ .*

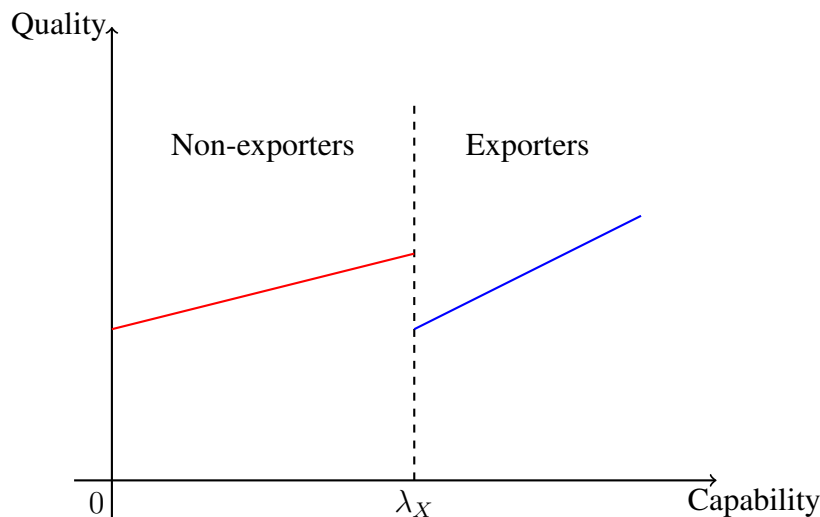
Proposition 1 also suggests that if  $q_X(\lambda) - \lambda$  does not increase too fast,<sup>14</sup> then more capable exporters also export more in period one. The intuition for this result can be obtained by looking at (6). Conditional on quality, higher  $\lambda$  firms have a lower per unit loss (normalized by full information profits). This tends to raise  $\phi(q)$ . At the same time, high  $\lambda$  firms end up choosing higher quality,  $q_X$ , which tends to raise these normalized losses, thereby reducing  $\phi(q)$ . As long as  $q_X$  does not increase too fast with  $\lambda$ , the right-hand side of (6) decreases with  $\lambda$ , resulting in a positive relation between  $\phi(q)$  and  $\lambda$ .<sup>15</sup>

The quality distortion due to asymmetric information creates an interesting outcome - an overlap in quality between non-exporters and exporters. This is shown in Figure 3. The non-monotonicity in quality contrasts with the monotone relation between capability and quality in Baldwin and Harrigan (2007); Verhoogen (2008); Kugler and Verhoogen (2009); Johnson (2010). Our analysis indicates that there are some firms producing high quality products that do not export. At the same time, there are some exporters who sell low quality products.

Potential exporters compare their profits from exporting  $\pi_X(\lambda)$  with those from not exporting  $\pi_D(\lambda)$ . For the less capable firms, the additional profit from exporting in period two is not sufficient to cover the loss from (i) signaling costs and (ii) sub-optimal quality at home. Hence, they choose to stay domestic and produce the full-information quality. For a range of parameter values, there exists a capability  $\lambda_X$  in equilibrium such that all firms with capability more than

<sup>14</sup>It can be shown that  $q_X(\lambda) - \lambda$  is increasing in  $\lambda$ .

<sup>15</sup>There is an implicit assumption here that foreign consumers do not observe the supply of home producers.



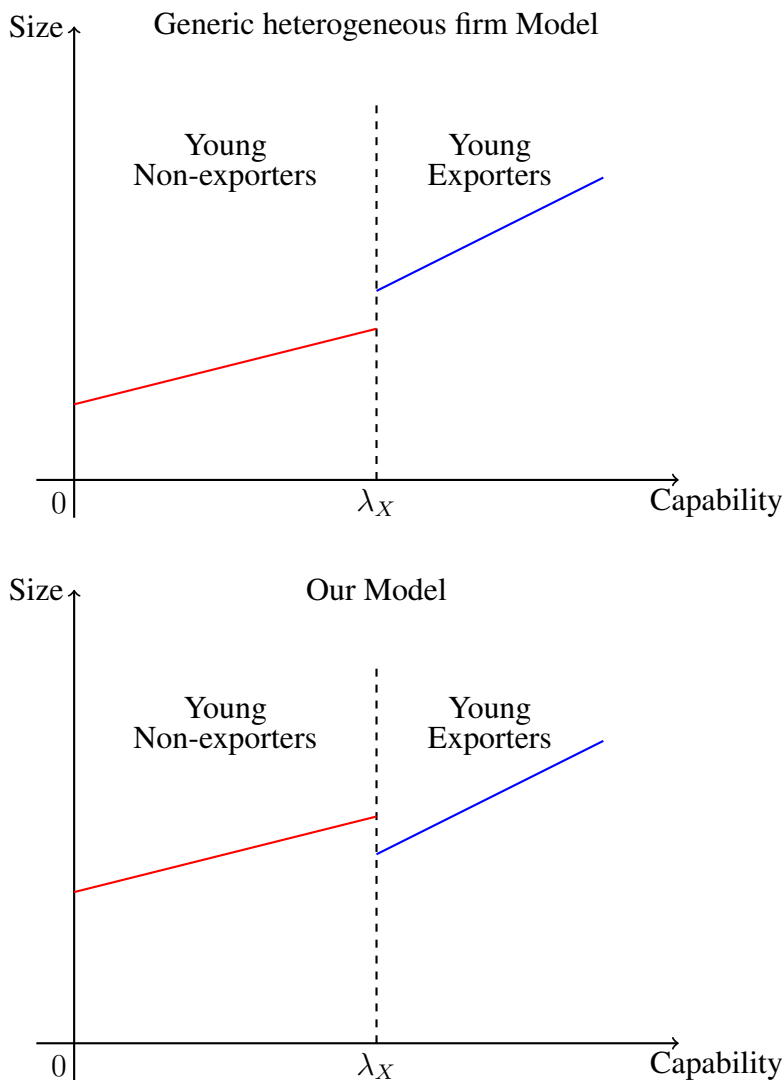
**Figure 3:** *Choice of quality by firms*

$\lambda_X$  export, while the rest only sell domestically. We state this result formally in the proposition below.

**Proposition 2.** *If  $3(\frac{3}{2}e^{\frac{2}{3}} - 1)w > \sigma > \theta$ , then there exists  $\lambda_X$  such that firms with  $\lambda > \lambda_X$  export while the rest only sell domestically.*

Under what conditions do we have a sorting equilibrium? As Proposition 2 states, a perfect sorting equilibrium is more likely if  $\sigma$ , which captures the dispersion in tastes of consumers, lies in a bounded interval. Intuitively, the difference between  $\pi_D(\lambda)$  and  $\pi_X(\lambda)$  depends on the extent of the distortion in quality, i.e., by how much  $q_X$  falls short of  $q_D$ . This distortion, in turn, depends on the value of  $\sigma$ . On the one hand, firms would like  $q_X$  to be as close to  $q_D$  as possible. On the other hand, higher  $q_X$  implies higher marginal cost, which increases the losses (or reduces profits) in period one foreign market. The extent to which firms can recover these losses from the full information markets depends on how high a price they can charge in these markets. The higher is  $\sigma$ , the greater is the heterogeneity in tastes meaning that firms can raise their price without affecting their demand too much. In other words, if  $\sigma$  is too small, the losses from exporting would be high for every firm, with the result that all firms might end up not exporting. This creates a lower bound for  $\sigma$ . An analogous argument shows that if  $\sigma$  is too large, exporting is attractive for every firm, with the result that all firms end up exporting. This, in turn, creates an upper bound for  $\sigma$ . The likelihood of a perfect sorting equilibrium also depends on  $w$  lying in a bounded interval. The wage  $w$  determines the marginal cost of production. A higher  $w$  increases the first period losses in the foreign market, thereby reducing

the attractiveness of exports. Like  $\sigma$ , a low enough  $w$  would make all firms export while a high enough  $w$  would prevent any firm from exporting.



**Figure 4:** Firm size of the “young” cohort

**Relation between size and export status:** The uncertainty in quality breaks a standard feature of the generic heterogeneous firm model - the monotonic relation between size and capability, where size is measured in terms of sales. In the standard model, there is a discontinuity in size at the threshold  $\lambda_X$ ; the size of the marginal exporter is higher than the marginal non-exporter because exporting entails a fixed cost. This is shown in the top panel of Figure 4. For the young cohort of firms in our model, there is a similar discontinuity at  $\lambda_X$ , as shown in the bottom panel of Figure 4. Unlike the standard model, under certain conditions, the size of the marginal



exporter is *lower*.

This opposite outcome arises again due to a sub-optimal choice of quality by the marginal exporter in the home market. To see why this is the case, observe that sales in the home market depend on two components: the price,  $p_X(\lambda)$ , and the quantity,  $\exp\{\frac{1}{\sigma}(\theta q - p_X(\lambda))\}$ . As the marginal exporter's quality is below  $q_D(\lambda_X)$ , two effects come into play. On the one hand, conditional on price, home consumers demand less of his product. On the other hand, his price is low because of lower marginal cost. This has both a direct negative effect on sales, as well as, an indirect positive effect (by raising demand). One can show that when  $q_D(\lambda) - q_X(\lambda)$  is large enough, the marginal exporter's sales in the home market are lower than the marginal non-exporter. If the sales from the foreign market are small enough (which would be the case if  $\bar{p}$  is small),<sup>16</sup> the total sales of the marginal exporter, and by continuity some of the smaller exporters, is less than the marginal non-exporter. For some capability distributions, the lower sales of the young exporters generates a well-behaved relation between percentage of exporters and size quantiles of home firms. In the following proposition, we show that if the distribution for capability is a truncated Pareto, the fraction of exporters in each size quantile increases as we move to higher quantiles.<sup>17</sup>

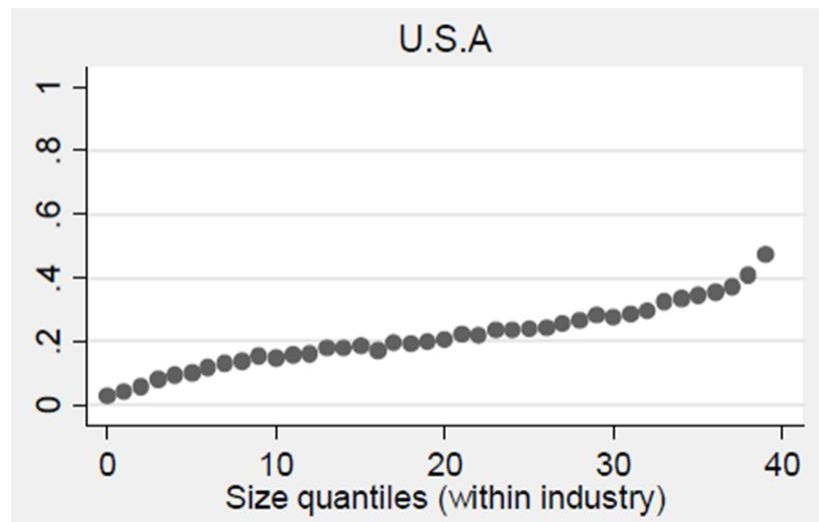
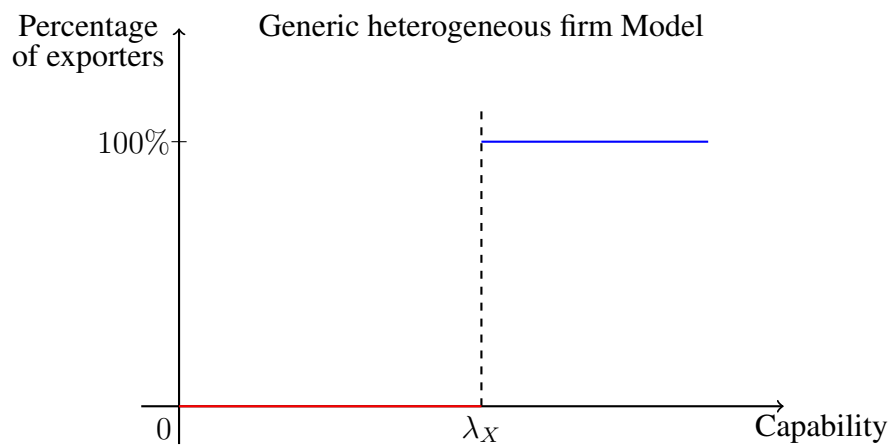
**Proposition 3.** *If the capability distribution  $G(\lambda)$  is a truncated Pareto, then the percentage of young exporters is an increasing function of the firm size quantiles.*

The increasing relation between the fraction of exporters and size quantiles contrasts sharply with the perfect sorting of export status by size in the standard model, shown in the top panel of Figure 5. In contrast, the bottom panel, reproduced from Hallak and Sivadasan (2010), which presents evidence on the relationship between percentage of exporters and size quantiles from the U.S. Hallak and Sivadasan uncover a similar pattern in the data for Chile, India and Colombia. Two key features of their model are that (i) firms are required to meet minimum quality standards in order to export, and (ii) there is a second degree of heterogeneity (on top of the heterogeneity in capability) in firms' ability to produce quality. The quality constraint effectively "distorts" a firm's choice; combined with two dimensional heterogeneity, this generates a non-monotonic relationship between size (determined by capability) and export status (determined by quality). Although firms in our model are differentiated with respect to a single attribute, capability, the cause for non-monotonicity is a distortion caused by the requirement to signal quality (which is costly). Hence, although the two models are quite different, the cause

<sup>16</sup>First period sales of the marginal exporter in the foreign market is  $\bar{p}\beta(\phi)$  where  $\phi$  is a function of  $\lambda_X$ .

<sup>17</sup>A truncated Pareto distribution can be justified on the grounds that the largest firms in an industry already have a good reputation in the export market and hence, the problem of uncertain quality does not apply to them.

for non-monotonicity is, at a deeper level, quite similar.



Hallak & Sivadasan (2009)

**Figure 5:** *Percentage of exporters in each size quantile (data)*

**Quality upgrading:** Our analysis of the equilibrium quality as a function of capability indicates that in the cross-section, we should observe an overlap in quality between non-exporters and exporters. Instead, one could interpret this result as a statement about the behavior of quality across time periods. Our model would then suggest that high capability firms experience a quality *downgrading* when they start to export. Evidence from Mexico provided by [Iacovone and Javorcik \(2010\)](#) shows that firms that enter the export market experience an increase in the unit value of their products in anticipation of exporting. Similarly, [Verhoogen \(2008\)](#) finds that

firms that expand their exports following the peso devaluation crisis see an increase in ISO 9000 certification. Both pieces of evidence are suggestive of some sort of quality upgrading due to better exporting opportunities.

We have two arguments to defend what seems like a counter-factual prediction of our model. First, there are no *switchers* in our model - none of the firms switch from being a non-exporter to an exporter or vice versa. Switching implies that a firm is a non-exporter when young and an exporter when old. As already argued, if exporting is not optimal in period one, then it is not optimal in period two either. Thus, firms have the same export status and the same quality throughout their lives - there is no scope for quality upgrading or downgrading within the strict confines of our model.

Second, there are two main theories behind quality upgrading - (a) the hypothesis of [Alchian and Allen \(1964\)](#) that rests on per unit transportation costs and (b) a higher willingness to pay for quality by foreign consumers, possibly due to lower marginal utility of income, as espoused by [Linder \(1961\)](#). Our model features zero transportation costs and symmetric countries. Once we drop either one of these two assumptions, our model would also generate quality upgrading by some, if not all, exporters. To see this, suppose the foreign consumers have a higher marginal utility to pay for quality.<sup>18</sup> When the difference between marginal willingness ( $\theta$ ) in the two countries is large enough, the quality under exporting would be higher than that under not exporting. This is stated below formally.

**Proposition 4.** *If the marginal willingness to pay for quality is significantly higher for foreign consumers relative to home consumers,  $q_X(\lambda) > q_D(\lambda)$ .*

Note, however, that even if some firms engaged in quality upgrading, they would continue to incur an opportunity cost of exporting. It is the assumption that firms are constrained to choose the same quality in all markets, and not quality downgrading *per se*, that makes exporting costly.

**Endogenous fixed and sunk costs of exporting:** The model generates endogenous firm-specific fixed exporting costs ([Das et al., 2007](#); [Eaton et al., 2011](#)). One part of these costs take the form of foregone revenues in the home market. Note that this cost depends on quality and accordingly, affects the firm's choice of quality. But the cost is "fixed" in the sense that conditional on quality, an additional unit produced by a firm is not affected by this cost. Furthermore, the cost of signaling acts as an additional part of the fixed cost, and like the opportunity cost, depends on the quality chosen by the firm.

<sup>18</sup>This assumption is realistic when considering exports from developing countries like China to developed countries like the U.S.

In this model, firms do not face any uncertainty. Even though foreign consumers do not observe the quality of varieties sold by home exporters in the first period, the quality is known exactly in period two. Consequently, firms know the demand for their product in period two foreign market for every quality. The model could generate sunk costs of exporting if we consider the following modification: suppose foreign consumers receive a public signal that a variety has very low quality with some positive probability at the beginning of period two. Firms then face the uncertainty that some of them will have very low export demand in period two.<sup>19</sup> In this situation, both the opportunity cost and the signaling cost act like sunk costs. A firm has to incur these costs if it wants to export, but these are sunk once it has entered the foreign market. The signaling cost has to be incurred in period one and cannot be recovered if the firm decides not to export in period two. As for the opportunity cost, since the firm cannot change quality in period two, the revenue in period two home market continues to be lower (relative to the full information case), even if the firm chooses not to export.

**Abundance of small exporters:** In the Melitz model, there is a fixed cost of exporting that is the same across firms. In order to export, firms should be able to generate large enough sales in the export market. This implies that there is a discrete jump in size at  $\lambda_X$  as shown in the top half of Figure 4, and that only the most capable firms in an industry export. [Eaton et al. \(2011\)](#) find the existence of many small French exporters in the data, which is not consistent with the Melitz model. One possible explanation has been provided by [Arkolakis \(2008\)](#). He argues that firms do not face a uniform fixed cost, but a fixed cost that depends on the number of consumers reached by the firm. The small exporters observed in the data are firms that are capable enough to reach the first foreign consumer, but not capable enough to reach too many of them. Consequently, their sales are low.

Our model provides an alternative explanation for the abundance of small exporters in the data. Young exporters do not need to have high sales in the foreign market to cover some fixed exporting cost. As discussed earlier, the fixed exporting costs are partly foregone sales at home. Rather, these exporters have low enough sales in period one in order to signal the quality of their product in the foreign market. This mechanism is partly responsible for the discrete drop in size at  $\lambda_X$  in the bottom half of Figure 4. Therefore, when looking at a cross-section of exporters, one would always observe many small firms. These are simply the young firms that are trying to signal their quality.

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<sup>19</sup>In this model, because  $\beta$  captures the probability of private signals, the demand facing a firm is deterministic by the Law of Large Numbers. Although some foreign consumers might get a negative signal about that the quality of a variety, others will know the true quality. With public signals, however, every consumer has the same belief. Of course, we are assuming that consumers ignore their private signals, if there are any.

**Exporter dynamics:** Our model has implications for how the volume of exports changes over time. In particular, our model predicts that the more capable firms will increase their exports between periods one and two. Since the quality of exporters is unknown in the first period, the initial demand facing the more capable exporters is low. But once their true quality is revealed, the demand for their product rises. Work by [Eaton et al. \(2007\)](#) on Colombian firms seems to suggest that there are indeed firms that experience growth in the export markets over time.<sup>20</sup> The same study provides evidence that it is much more likely for a firm to export in a particular period if it has exported in the previous period.<sup>21</sup> Our model provides a rationale for this finding. The problem faced by a firm that has already exported is quite different from the others. The former has already revealed its quality in the foreign market and is more likely to export in the future. The dynamic behavior of firms in our model is also consistent with the finding in [Blum et al. \(2009\)](#) that in a given year, a significant fraction of Chilean exporters are new exporters who export very little compared to those who have exported for at least a year.

## 4 Conclusion

In this paper, we have presented a framework to study uncertainty about product quality in international trade. The main assumption is that home consumers observe the quality of home goods perfectly, but foreign consumers do not. As a result, home firms have to signal the quality of their products in the foreign market. Fixed costs of exporting arise endogenously in our model due to signaling and the choice of sub-optimal quality in the home market. These distortions cause an overlap in quality between non-exporters and exporters. A key insight of the paper is that it is costly to reach the first consumer in a market because a firm optimally would not sell to a single consumer. Despite being quite stylized, our model is consistent with a number of features in the data.

A key parameter of our model is  $\bar{p}$ , the pooling equilibrium price faced by home exporters in the first period foreign market. Throughout our analysis, we have remained agnostic about how  $\bar{p}$  is determined. If we believe that  $\bar{p}$  depends on the foreign consumers' perception of quality of home firms, then a decrease in  $\bar{p}$  can lead to a negative perception and act as a significant barrier to entry in the export market.<sup>22</sup> In fact, if the average capability of home firms is not

<sup>20</sup>[Rauch and Watson \(2004\)](#) cite evidence indicating that developed country buyers start small when purchasing from developing country suppliers and then increase order size as the suppliers' ability to meet quality is revealed.

<sup>21</sup>Similar findings are reported in [Roberts and Tybout \(1997\)](#) and [Bernard and Jensen \(2004\)](#).

<sup>22</sup>For example, [Morawetz \(1981\)](#) argues that the decline of Colombian export of clothing in the mid-70s was partly due to the gradual realization by U.S. buyers that Colombian suppliers are not reliable due to their inability

too high, a low enough  $\bar{p}$  could result in zero exports. In this case, even the most capable home firms would rather sell only at home than incur losses in trying to establish reputation in the foreign market. Thus, a negative prior belief about the quality of exports could be sustained, suggesting path dependence in the exporting behavior of firms. This is an important theme that we hope to examine further in our future work.

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to deliver orders on time.

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## Appendix

*Proof of Proposition 1:* For simplicity, we write  $q(\lambda)$  simply as  $q$ . Differentiating the profit function under exporting with respect to  $q$  and evaluating the derivative at  $q = q_D$ ,

$$\begin{aligned} \frac{\partial \pi_X(\lambda)}{\partial q} \Big|_{q=q_D} &= -w.e^{q_D-\lambda} \phi(q_D) \\ &< 0 \end{aligned}$$

Therefore, the profit-maximizing quality under exporting,  $q_X$ , must be less than  $q_D$ . To see that  $q_X$  is increasing in  $\lambda$ , differentiate equation 8 with respect to  $\lambda$ . Collecting terms, we have

$$\left[ \left(1 - \frac{\theta}{\sigma}\right) + we^{q_X-\lambda} \left(\frac{1}{\sigma} + \frac{1}{\theta - we^{q_X-\lambda}}\right) \right] \frac{dq_X}{d\lambda} = 1 + we^{q_X-\lambda} \left(\frac{1}{\sigma} + \frac{1}{\theta - we^{q_X-\lambda}}\right)$$

Now,  $q_D > q_X$  implies that  $\theta = we^{q_D-\lambda} > we^{q_X-\lambda}$ . So, if  $\theta < \sigma$ ,  $\frac{dq_X}{d\lambda} > 0$ .  $\square$

*Proof of Proposition 2:* We shall use the intermediate value theorem to prove this proposition. We shall also prove the result for the following functional form for  $\beta$ :  $\beta = 1$  if  $x(q) = x(\bar{q})$  and zero otherwise. Then, by continuity, the result will go through for  $\beta' > 0$ . We proceed in steps. First, we find the conditions under which  $\frac{\partial \pi_X(\lambda)}{\partial \lambda} \geq \frac{\partial \pi_D(\lambda)}{\partial \lambda} \forall \lambda$ . Second, we find the conditions under which  $\pi_X(\lambda) < \pi_D(\lambda)$  for  $\lambda = 0$ . And finally, we find the conditions under which  $\pi_X(\lambda) > \pi_D(\lambda)$  for  $\lambda \rightarrow \bar{\lambda}$ .

Step 1: From (5), the slope of  $\pi_D$  can be calculated as

$$\frac{\partial \pi_D(\lambda)}{\partial \lambda} = 2\theta.exp\left\{\frac{1}{\sigma}(\theta\lambda + \theta \log \frac{\theta}{w} - \sigma - \theta)\right\}.$$

The slope of  $\pi_X$  can be calculated from (9) as

$$\frac{\partial \pi_X(\lambda)}{\partial \lambda} = we^{q_X-\lambda} exp\left\{\frac{\theta\bar{q}}{\sigma}\right\} + 2\sigma.exp\left\{\frac{1}{\sigma}(\theta q - \sigma - we^{q_X-\lambda})\right\} \left(\frac{1}{\sigma} we^{q_X-\lambda}\right).$$

Substituting from the first-order condition and a bit of algebra yields

$$\frac{\partial \pi_X(\lambda)}{\partial \lambda} = exp\left\{\frac{\theta\bar{q}}{\sigma}\right\} \frac{\theta}{\frac{\theta}{we^{q_X-\lambda}} - 1}.$$

Therefore, for  $\pi^X$  to be steeper than  $\pi^D$  for all values of  $\lambda$ , we need the following :

$$\frac{2}{C.exp\left\{\frac{\theta\bar{q}}{\sigma}\right\}} \left(\frac{\theta}{we^{q_X-\lambda}} - 1\right) exp\left\{\frac{1}{\sigma}(\theta\lambda + \theta \log \frac{\theta}{w} - \sigma - \theta)\right\} \leq 1$$

A sufficient condition for the above inequality to be satisfied is that

$$2\left(\frac{\theta}{we^{q_X-\lambda}} - 1\right) \leq 1 \tag{A1}$$

and

$$\frac{1}{C \cdot \exp\{\frac{\theta \bar{q}}{\sigma}\}} \exp\{\frac{1}{\sigma}(\theta \lambda + \theta \log \frac{\theta}{w} - \sigma - \theta)\} \leq 1 \quad (\text{A2})$$

Inequality (A1) can be re-written as

$$q^X - \lambda \geq \log\left(\frac{2\theta}{3w}\right)$$

Now, the LHS of the above inequality is increasing in  $\lambda$  (to be shown). Hence, if we can show that this inequality holds for  $q^X(0)$ , we are done. For  $q^X(0)$  to be greater than  $\log(\frac{2\theta}{3w})$ , we must have  $\frac{\partial \pi^X(0)}{\partial \lambda} > 0$  when  $q = \log(\frac{2\theta}{3w})$ . Now,

$$\frac{\partial \pi^X(0)}{\partial \lambda} \Big|_{q=\log(\frac{2\theta}{3w})} = \frac{\theta}{3} \left[ \frac{3}{C} \exp\left\{\frac{1}{\sigma}\left(\theta \log\left(\frac{2\theta}{3w}\right) - \frac{2\theta}{3}\right) - 1\right\} - 2 \exp\left\{\frac{\theta \bar{q}}{\sigma}\right\} \right]$$

The above expression can be made positive by choosing  $\sigma$  small enough, as long as  $\theta \log(\frac{2\theta}{3w}) - \frac{2\theta}{3} > 0$ . The required condition is

$$\theta > \frac{3}{2} w e^{\frac{2}{3}} \quad (\text{A3})$$

Inequality (A2) can be re-written as

$$\exp\left\{\frac{1}{\sigma}(\theta \lambda + \theta \log \frac{\theta}{w} - \sigma - \theta)\right\} \leq C \cdot \exp\left\{\frac{\theta \bar{q}}{\sigma}\right\} \quad (\text{A4})$$

The above inequality essentially puts an upper bound on  $\bar{\lambda}$ . Therefore, as long as (A3) is satisfied and  $\bar{\lambda}$  is such that (A4) is satisfied,  $\frac{\partial \pi^X(\lambda)}{\partial \lambda} \geq \frac{\partial \pi^D(\lambda)}{\partial \lambda} \forall \lambda$ .

Step 2: We shall proceed by deriving the condition under which  $\pi^X(0, q^X(0)) < \pi^D(0, q^X(0))$ . Then the above inequality follows. We first show that if  $\theta < \frac{3}{2}$ ,  $q^X - \lambda$  is increasing in  $\lambda$ . Defining  $q^X - \lambda$  as  $\Phi$  and replacing in (8), we have

$$w e^\Phi \exp\left\{\frac{\theta \bar{q}}{\sigma}\right\} = \frac{3\sigma}{C} \exp\left\{\frac{1}{\sigma}(\theta \Phi + \theta \lambda - \sigma - w e^\Phi)\right\} \left[\frac{1}{\sigma}(\theta - w e^\Phi)\right].$$

Differentiating w.r.t.  $\lambda$  and collecting terms,

$$\left[ w e^\Phi \exp\left\{\frac{\theta \bar{q}}{\sigma}\right\} + \frac{3}{C} R (w e^\Phi - \frac{1}{\sigma}(\theta - w e^\Phi)^2) \right] \frac{d\Phi}{d\lambda} = \frac{3}{\sigma C} R \theta (\theta - w e^\Phi)$$

where  $R = \exp\left\{\frac{1}{\sigma}(\theta \Phi + \theta \lambda - \sigma - w e^\Phi)\right\}$ . For  $\frac{d\Phi}{d\lambda}$  to be positive, we need

$$\theta^2 < [(2\theta + \mu) - w e^\Phi] w e^\Phi$$

for all  $\lambda$ . Now,  $(2\theta + \sigma) - w e^\Phi > 0$  because  $w e^\Phi < w e^{q^D - \lambda} < \theta$ . Hence, if  $\Phi$  is increasing in  $\lambda$  and the above expression holds for  $\lambda = 0$ , then we are done. Because  $\sigma > \theta$ , we know that,

$$[(2\theta + \sigma) - w e^\Phi] w e^\Phi > \theta^2 w e^\Phi$$

Hence, if  $w e^\Phi > 1$  for  $\lambda = 0$ , then we are done. In order for this to be true, we need  $q^X(0) > \log(\frac{1}{w})$ . We have already derived the condition under which  $q^X(0) > \log(\frac{2\theta}{3w})$ . Hence, if  $\theta < \frac{3}{2}$ , our claim that

$q^X - \lambda$  is increasing in  $\lambda$  is true. Now, from (5) and (9), we get the following:

$$\pi^D(0, q^X(0)) = \frac{2\sigma}{3} \exp\left\{\frac{\theta\bar{q}}{\sigma}\right\} \frac{we^{q^X(0)}}{\theta - we^{q^X(0)}}.$$

and

$$\pi^X(0, q^X(0)) = we^{q^X(0)} \exp\left\{\frac{\theta\bar{q}}{\sigma}\right\} \left(\frac{\sigma}{\theta - we^{q^X(0)}} - 1\right).$$

For  $\pi^D(0, q^X(0)) > \pi^X(0, q^X(0))$ , we need the following:

$$3(\theta - we^{q^X(0)}) > \sigma$$

If we can find an upper bound for  $q^X(0)$ , then we have a condition. Let this upper bound be 0. For this to be true,  $\frac{\partial \pi^X(0, q^X)}{\partial q^X}$  must be negative at  $q^X = 0$ . The corresponding condition is

$$\frac{1}{\exp\left\{1 + \frac{w}{\sigma}\right\}} \left(\frac{\theta}{w} - 1\right) < \frac{C}{3} \exp\left\{\frac{\theta\bar{q}}{\sigma}\right\}$$

The above condition holds for  $\sigma$  small enough. Hence, for  $\sigma$  small,  $e^{q^X(0)} < 1$ . Then, for the boundary condition to be satisfied, we must have

$$3(\theta - w) > \sigma \tag{A5}$$

Step 3: Conditions under which  $\pi^X(\lambda) > \pi^D(\lambda)$  for  $\lambda$  large enough. We shall show this by deriving the condition under which there exists a  $\lambda$  such that  $\pi^X(\lambda, q^D(\lambda)) > \pi^D(\lambda, q^X(\lambda))$ . Then the above inequality follows. From (5) and (9), we get the following:

$$\begin{aligned} \pi^X(\lambda, q^D(\lambda)) &= -w \frac{\theta}{w} \exp\left\{\frac{\theta\bar{q}}{\sigma}\right\} + \frac{3\sigma}{C} \exp\left\{\frac{1}{\sigma}(\theta\lambda - \theta \log \frac{\theta}{w} - \sigma - \theta)\right\}. \\ \pi^D(\lambda, q^D(\lambda)) &= \frac{2\sigma}{C} \exp\left\{\frac{1}{\sigma}(\theta\lambda + \theta \log \frac{\theta}{w} - \sigma - \theta)\right\}. \end{aligned}$$

Hence, the required condition is

$$\exp\left\{\frac{1}{\sigma}(\theta\lambda + \theta \log \frac{\theta}{w} - \sigma - \theta)\right\} > \frac{\theta C}{\sigma} \exp\left\{\frac{\theta\bar{q}}{\sigma}\right\}. \tag{A6}$$

Since the LHS of the above expression is increasing in  $\lambda$ , there exists a  $\lambda$  for which the above inequality holds. Suppose  $\bar{\lambda}$  is such that (A6) holds. But we know that  $\bar{\lambda}$  must also satisfy (A5). Such a  $\bar{\lambda}$  would exist if  $1 > \frac{\theta}{\sigma}$ , which we assume is true. We can summarize conditions (A3) and (A4) as

$$3\left(1 - \frac{2}{3}e^{-\frac{2}{3}}\right)\theta > 3\left(\frac{3}{2}e^{\frac{2}{3}} - 1\right)w > \sigma > \theta.$$

□

**Proof of Proposition 3:** We have shown that there is a one-to-one relation between  $\lambda$  and sales,  $r(\lambda)$ . Hence we shall prove the result for capability quantiles in stead of size quantiles. We proceed in steps.

Step 1: The  $j$ -th p-quantile is defined as  $\int_{\lambda_j}^{\lambda_{j+1}} dG(\lambda) = p$ . If  $G(\lambda)$  is truncated Pareto with shape parameter  $\kappa$ , then

$$\int_{\lambda_j}^{\lambda_{j+1}} dG(\lambda) = \Gamma \left[ \left( \frac{1}{\lambda_j} \right)^\kappa - \left( \frac{1}{\lambda_{j+1}} \right)^\kappa \right],$$

where  $\Gamma$  is some constant. Suppose  $\lambda_{j+1} = \lambda_j + h$ , where  $h$  is a constant. Then  $\frac{d}{d\lambda_j} \Gamma \left[ \left( \frac{1}{\lambda_j} \right)^\kappa - \left( \frac{1}{\lambda_{j+1}} \right)^\kappa \right] < 0$ . Hence,  $h$  must rise as  $\lambda_j$  increases. Accordingly, let us define  $\lambda_{j+1} - \lambda_j = h(\lambda_j)$ ,  $h' > 0$ .

Step 2: In equilibrium, it must be the case that for  $\lambda \rightarrow_+ \lambda^X$ ,  $r(q^X; \lambda) < r(q^D; \lambda)$ . We shall prove this by contradiction. Suppose not. We know that for  $\lambda$  close to but greater than  $\lambda^x$ ,  $\pi(q^X; \lambda) < \pi(q^D; \lambda)$  (since  $q^X$  is the profit-maximizing quality). Therefore, if  $r(q^X; \lambda) > r(q^D; \lambda)$ , it must be the case that  $c(q^X; \lambda) > c(q^D; \lambda)$  so that  $r(q^X; \lambda) - c(q^X; \lambda) < r(q^D; \lambda) - c(q^D; \lambda)$ , i.e., we must have  $d\left(\frac{r(q; \lambda)}{c(q; \lambda)}\right)/dq > 0$ . Now,

$$\begin{aligned} \frac{r(q; \lambda)}{c(q; \lambda)} &= \frac{\sigma + we^{q^D - \lambda}}{we^{q^D - \lambda}} \\ &= 1 + \frac{\sigma}{we^{q^D - \lambda}} \end{aligned}$$

It is clear that as  $q$  rises,  $\frac{r(q; \lambda)}{c(q; \lambda)}$  falls; we have a contradiction.

Step 3: From Step 2 it is clear that there is an overlap in the Home sales between young exporters and non-exporters. But the exporters also earn revenues from the Foreign market. These revenues are directly proportional to  $\bar{p}$  and can be made as small as possible. As a result, we can get an overlap in total sales between young exporters and non-exporters. Now, for the full information case, we have  $r^D(\lambda) = K \exp\left\{\frac{\theta}{\sigma} \lambda\right\}$ , where  $K$  is some constant and  $r^D$  is the sales of a firm that sells only in the Home market. We hypothesize that the Home sales of an exporter,  $r^X$ , can be similarly written as  $r^X(\lambda) = K' \exp\left\{\frac{\theta}{\sigma} \lambda\right\}$ . Pick the capability of a non-exporter. Let us call this  $\lambda^D$ . Pick the corresponding capability level for the exporter,  $\lambda^X$ , such that  $r^D(\lambda^D) = r^X(\lambda^X)$ . Then it is easy to show that  $\lambda^X = \lambda^D + \phi$ . From Step 1, we know that  $\lambda_{j+1} = \lambda_j + h(\lambda_j)$ . Let us define,

$$\lambda_{j+1}^i = \lambda_j^i + h^i(\lambda_j^i), i = D, X$$

where the subscript  $i$  on the function  $h(\cdot)$  allows for the function to be different for exporters and non-exporters. Combining this with our previous observation, we have

$$\begin{aligned} \lambda_{j+1}^D &= \lambda_j^D + h^D(\lambda_j^D) \\ \lambda_j^X &= \lambda_j^D + \phi \\ \lambda_{j+1}^X &= \lambda_j^X + h^X(\lambda_j^X) \\ &= \lambda_j^D + \phi + h^X(\lambda_j^D + \phi) \end{aligned}$$

For any size quantile, the fraction of exporters to non-exporters is then given by

$$\xi_j = \frac{\left(\frac{1}{\lambda_j^D + \phi}\right)^\kappa - \left(\frac{1}{\lambda_j^D + \phi + h^X(\lambda_j^D + \phi)}\right)^\kappa}{\left(\frac{1}{\lambda_j^D}\right)^\kappa - \left(\frac{1}{\lambda_j^D + h^D(\lambda_j^D)}\right)^\kappa}$$

We are interested in how  $\xi_j$  changes as  $\lambda_j^D$  increases. Differentiating  $\xi_j$  with respect to  $\lambda_j^D$ , and ignoring a positive constant, we have

$$\begin{aligned} \frac{d\xi_j}{d\lambda_j^D} = & \left[ -\left(\frac{1}{\lambda_j^D + \phi}\right)^{\kappa+1} + \left(\frac{1}{\lambda_j^D + \phi + h^X(\lambda_j^D + \phi)}\right)^{\kappa+1} \left(1 + \frac{\partial h^X}{\partial \lambda}\right) \right] \left[ \left(\frac{1}{\lambda_j^D}\right)^\kappa - \left(\frac{1}{\lambda_j^D + h^D(\lambda_j^D)}\right)^\kappa \right] \\ & - \left[ -\left(\frac{1}{\lambda_j^D}\right)^{\kappa+1} + \left(\frac{1}{\lambda_j^D + h^D(\lambda_j^D)}\right)^{\kappa+1} \left(1 + \frac{\partial h^D}{\partial \lambda}\right) \right] \left[ \left(\frac{1}{\lambda_j^D + \phi}\right)^\kappa - \left(\frac{1}{\lambda_j^D + \phi + h^X(\lambda_j^D + \phi)}\right)^\kappa \right] \end{aligned}$$

Suppose  $\frac{\partial h^D}{\partial \lambda}$  and  $\frac{\partial h^X}{\partial \lambda}$  are small. Then we can ignore these values. Taking Taylor series expansion around  $h^D = 0$ ,

$$\left(\frac{1}{\lambda_j^D + h^D}\right)^\kappa = \left(\frac{1}{\lambda_j^D}\right)^\kappa - \kappa \left(\frac{1}{\lambda_j^D}\right)^{\kappa+1} h^D$$

Taking Taylor series expansion around  $h^X = 0$ , we have

$$\left(\frac{1}{\lambda_j^D + \phi + h^X}\right)^\kappa = \left(\frac{1}{\lambda_j^D + \phi}\right)^\kappa - \kappa \left(\frac{1}{\lambda_j^D + \phi}\right)^{\kappa+1} h^X$$

Similarly, we have

$$\left(\frac{1}{\lambda_j^D + h^D}\right)^{\kappa+1} = \left(\frac{1}{\lambda_j^D}\right)^{\kappa+1} - (\kappa + 1) \left(\frac{1}{\lambda_j^D}\right)^{\kappa+2} h^D$$

and

$$\left(\frac{1}{\lambda_j^D + \phi + h^X}\right)^{\kappa+1} = \left(\frac{1}{\lambda_j^D + \phi}\right)^{\kappa+1} - (\kappa + 1) \left(\frac{1}{\lambda_j^D + \phi}\right)^{\kappa+2} h^X$$

Replacing the above values in the expression for  $\frac{d\xi_j}{d\lambda_j^D}$  and again ignoring a positive constant, we have

$$\begin{aligned} \frac{d\xi_j}{d\lambda_j^D} = & \kappa^2 (\kappa + 1) h^D h^X \left(\frac{1}{\lambda_j^D + \phi}\right)^{\kappa+1} \left(\frac{1}{\lambda_j^D}\right)^{\kappa+1} \left(-\frac{1}{\lambda_j^D + \phi} + \frac{1}{\lambda_j^D}\right) \\ & > 0 \end{aligned}$$

where the second line follows from the fact that  $\frac{1}{\lambda_j^D + \phi} < \frac{1}{\lambda_j^D}$ . Therefore, the share of exporters to non-exporters is increasing in capability and hence, size quantiles.