Quality Uncertainty and Intermediation in International Trade *

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Abstract

Uncertainty about product quality is endemic in international trade. We develop a dynamic, two country model where home producers differ in terms of the quality of the products they sell. This quality is imperfectly observed by foreign consumers initially but known once the product is consumed. We show that this uncertainty generates an information cost of exporting, over and above the usual fixed costs used in standard heterogeneous firm models. We use the model to examine the role played by intermediaries in alleviating quality uncertainty. An intermediation technology involving higher marginal cost but lower fixed cost arises endogenously in our model. We analyze the sorting of exporters into different exporting modes. In the process, we uncover a novel externality of using intermediaries. We go on to study how the equilibrium depends on the degree of product heterogeneity, the level of information and the measure of available intermediaries.

KEYWORDS : Intermediaries, quality, uncertainty, screening, asymmetric information.

JEL Classification : D83, F10, F19, L15.

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1 Introduction

The role played by trade intermediaries - firms that facilitate the exchange of goods between producers and final consumers - cannot be emphasized enough. Spulber (1996) documents that in 1995, (i) intermediaries accounted for about a quarter of U.S. gross domestic product, and (ii) close to two million firms operated in the U.S. intermediation industry, providing various services to producers and consumers. Spulber suggests that among other things, intermediaries arise when there is uncertainty about demand and supply, or un-observability of the buyers' and sellers' characteristics. Given that such problems of asymmetric information are more acute when goods cross national borders, it is all the more surprising that intermediaries have been relegated to the background in the study of international trade.[1] But that is changing.

Recently, a number of papers have shed light on the prevalence of intermediaries in international trade (E.g. Feenstra and Hanson, 2004; Bernard, Jensen, Redding and Schott, 2010; Blum, Claro and Horstmann, 2010; Ahn, Khandelwal and Wei, 2011; Bernard, Grazzi and Tomasi, 2011; Akerman, 2012). This literature makes two important observations: First, a significant fraction of international trade is routed through intermediaries. Second, there are systematic variations in the mode of export (i.e., using intermediaries or exporting directly) not only across firms within an industry, but also across industries and destinations. Although the above mentioned papers have highlighted the importance of intermediaries in international trade, they differ in their view of the exact role being performed by these intermediary firms. The lack of a consensus is partly because intermediaries perform a wide variety of roles.

One such role that trade intermediaries perform is providing quality assurance. Uncertainty about product quality is endemic in international trade; this uncertainty creates familiar problems of adverse selection and, when firms choose the quality of their products, moral hazard (Tirole, 1988). Intermediaries alleviate this problem by screening the quality of products and then revealing it to consumers (something that would be prohibitively expensive for individual consumers to carry out). Examples abound. Li & Fung, a multinational trading and sourcing firm based in China, claims that they are committed to meeting the demands of international business through "impeccable quality; reliable, on-time delivery; and the highest standards of service".[2] Similarly, Home Depot, the world's largest home improvement retailer, has a Quality Assurance (QA) program in place which "evaluates supplier performance in the areas of fac-

[1] Evidence on informational asymmetry in international trade has been provided by Gould (1994), Head and Ries (1998), Rauch (1999), Rauch and Trindade (2002), Portes and Rey (2005), Paravisini et al. (2010), Allen (2011) among others

[2] `www.lifung.com/eng/business/efficiency.php`

tory, product, and packaging quality".[3] Feenstra and Hanson (2004) argue that one of the factors behind Hong Kong's success in intermediating trade between China and the rest of the world is information; Hong Kong's traders have an informational advantage in identifying Chinese producers who can meet foreign quality standards.[4]

In this paper, we develop a model to understand the role of intermediaries in facilitating trade in the presence of asymmetric information about product quality. We begin in Section 2 by proposing a model of quality uncertainty. Our model has two countries, home and foreign, and we focus on the home producers' decisions to export to the foreign country. Producers live for two periods and are heterogeneous in terms of the quality of the good they produce. But their qualities are imperfectly observed in the foreign market in the first period; if they sell a positive amount in the first period, their quality is fully revealed at the beginning of the second period.

Given this setting, we solve for a pooling equilibrium where every exporter charges a common price $\bar{p}$ in period one and exports the same quantity. In this equilibrium, every producer above a threshold level of quality exports in both periods while the rest never export. Due to imperfect observability of quality, producers have to forego a part of their full information profits, with this part being producer-specific. Thus, quality uncertainty generates an "information cost" of exporting, thereby lowering the profits of the exporters relative to a full information world.

In Section 3, we introduce intermediaries. The latter are profit-maximizing foreign firms that differ in terms of their ability to screen quality. After screening the quality of a home firm, an intermediary could reveal this quality to foreign consumers. We derive the condition under which truth-telling is optimal for the intermediaries. Hence, intermediaries solve the problem of quality uncertainty in the first period. Given the limited supply of intermediaries, however, producers compete for intermediaries; in equilibrium, the price paid by the intermediaries to the producers adjusts to clear the market for intermediaries.

We show that in the presence of intermediaries, there exists a semi-separating equilibrium where every exporter above a threshold level of quality exports directly because they benefit relatively less from using intermediaries; the high quality exporters signal their quality by *not using* intermediaries. These exporters continue to charge the price $\bar{p}$. But the quantity sold by

[3] `https://suppliercenter.homedepot.com/wps/portal`

[4] An alternative way to mitigate the problem of quality uncertainty is the introduction of a third party which evaluates the quality of a product. Examples include Moody's bond ratings or the *U.S. News & World Report's* ranking of college. See Dranove and Jin (2010) for a survey on the state of the theoretical and empirical literature on quality assurance.

these exporters is higher than in Section 2. This is because the entry of intermediaries creates a positive externality for the direct exporters: in the presence of intermediaries, the low quality exporters choose to export through intermediaries, thereby raising the average quality of direct exporters. Because foreign consumers have rational expectations, they correctly anticipate this increase in average quality resulting in higher demand for the goods sold by direct exporters.

In our model, intermediaries allow some exporters to signal the true quality of their goods to foreign consumers. These exporters do not have to incur the information cost of exporting. But selling through intermediaries also raises the final price of the goods, partly due to the markup charged by intermediaries, resulting in lower sales for some exporters. Intermediation has been modeled in the literature as a technology that allows producers to export with a lower fixed cost but higher variable cost (See Ahn et al., 2011; Bernard, Grazzi and Tomasi, 2011; Akerman, 2012); we provide a micro-foundation for such a technology.

We use the model to study how the equilibrium, and in particular the share of producers exporting through intermediaries, depends on the degree of product differentiation, the availability of information and the measure of intermediaries. First, we show that when goods are more horizontally differentiated, a greater share of producers export through intermediaries. A higher degree of horizontal differentiation arises when consumers care less about quality. Because producers signal higher quality by exporting directly, there is less need to incur this costly signal when quality matters less. Our prediction is consistent with the findings of Feenstra and Hanson (2004), Blum et al. (2009) and Tang and Zhang (2011).

Second, we look at how the equilibrium is affected when foreign consumers have less information about home producers. With less information, the share of producers exporting through intermediaries goes up. But more importantly, a change in the composition of exporters changes the average price charged by the exporters. We show that the average price charged by the exporters goes up when foreign consumers have less information. If distance is a measure of information cost, our results imply that intermediaries should be more important for exporting to destinations that are further away. This is consistent with the findings of Ahn et al. (2011), Tang and Zhang (2011) and Akerman (2012). At the same time, our results suggest that for the same good, average export price should increase with the distance of the destination. Evidence for this phenomenon has been provided by Hummels and Skiba (2004) and Baldwin and Harrigan (2011).

Finally, we examine the change in equilibrium when the measure of intermediary firms increases. As expected, a greater share of producers export through intermediaries when there are more of the latter. At the same time, more and more direct exporters switch into exporting through intermediaries. Because the price received by exporting through intermediaries is, on average, higher, some of the producers receive higher prices for their exports when the number of intermediaries increases. If the intermediation sector of rich countries is more developed relative to poor countries, then our result could partly explain why firms set higher prices for the same product in richer markets (Manova and Zhang, 2012).

Our paper contributes to the recent but growing literature examining the role of intermediaries in international trade.[5] Motivated by the recent backlash against intermediaries in the developing world, Antràs and Costinot (2011) present a model where intermediaries provide market access to farmers. But matching is random, with the rate at which farmers and intermediaries match being determined endogenously. Antràs and Costinot show that depending on the kind of integration being considered, intermediation could raise or lower welfare of farmers in developing countries. Search frictions also feature in the model of Blum, Claro and Horstmann (2010), who are motivated by the finding that within the universe of Chilean exporters and Colombian importers, there are no small matches - matches between small importers and small exporters do not occur in the data. In their model, firms have an exogenous cost of finding consumers, with this cost being larger for small firms. In equilibrium, small firms match with large intermediaries while the large firms match directly with consumers (who can be thought of as small importers). Facilitating international matches, however, is not the only role of intermediaries that has been studied. Ahn, Khandelwal and Wei (2011) and Akerman (2012) argue that intermediaries arise primarily to overcome trade costs. Both of these papers augment the Melitz (2003) model by adding an intermediation technology; firms have the option of direct exporting or exporting through an intermediary, with the latter entailing a lower fixed cost but a higher per unit cost. In a related paper, Chaney (2011) presents a model of network formation in international trade. Although not formally introducing trade intermediaries, Chaney argues that the mechanism presented in his paper could be a way in which intermediaries connect exporters with consumers.

To our knowledge, the only papers that theoretically study uncertainty about product quality and the role of intermediaries in alleviating this uncertainty in an international trade context are Bardhan, Mookherjee and Tsumagari (2010) and Tang and Zhang (2011). Bardhan et al. develop a model where an intermediary's (broadly defined) concern for reputation allows it to solve a quality moral hazard problem. They, however, are interested in the impact of trade liberalization on income distribution. Incomplete contracts lie at the heart of the model of trade

[5] Our paper is also related to several papers that study the role of intermediaries in a closed economy like Gehrig (1993), Spulber (1996), Rubinstein and Wolinsky (1987), Biglaiser (1993) and Shi and Siow (2011).

intermediaries developed by Tang and Zhang. In their model, exporting through intermediaries entails a lower fixed cost. But in an incomplete contracting environment, intermediaries underinvest in costly quality verification, effectively raising the unit cost of firms exporting through intermediaries. The resultant differences in cost between the two modes of export lead to a productivity sorting of firms.[6]

The rest of the paper is organized as follows. We present the model without intermediaries in Section 2. Intermediaries are introduced in Section 3 and the equilibrium is compared with the one in the previous section. We also perform comparative statics with respect to some of the parameters of our model in this section. Section 4 concludes.

2 A Model of Quality Uncertainty

In this section, we develop a framework that will allow us to study the problem of exporters in the presence of asymmetric information about product quality. This framework will also serve as a benchmark against which we can evaluate the effect intermediaries have on firm's decision to export, as well as, the effect on quantity, price and average quality.

2.1 A Benchmark Model

Preferences. Consumer preferences are captured by an additive random utility model. Suppose there are J varieties of a differentiated good available to consumers, with the quality of a variety lying between $\underline{q}$ and $\bar{q}$. Each consumer can consume a single unit of the differentiated good. Given quality q_j and price p_j of a variety j, a consumer i derives the following utility from consuming a unit:

$$u_j^i = \theta q_j - p_j + \epsilon_j^i,$$

where θ captures the preference for quality and ϵ_j^i is an idiosyncratic shock drawn from a cumulative distribution $\Phi^i(\epsilon_1^i, \epsilon_2^i, ..., \epsilon_J^i)$.[7] θ typically depends on the income of consumer i. To ease

[6] Other theoretical contributions to the literature include Rauch and Watson (2004), Felbermayr and Jung (2008) and Petropoulou (2008). Rauch and Watson (2004) study the formation of intermediation networks and how intermediaries provide access to their network of contacts to other interested parties. Matching and endogenous network building by intermediaries also feature in the work by Petropoulou (2008) while in Felbermayr and Jung (2008), trade intermediaries arise endogenously due to contractual frictions.

[7] The indirect utility function can be obtained from a problem where a consumer allocates his income between a homogeneous numeraire good and one unit of a differentiated good of quality q_j and price p_j. Consider the following representation of the consumer's utility function: $U_j^i = v(Y^i - p_j) + q_j + e_j^i$, where $Y^i > p_j$ is the consumer's income. Taking a first-order Taylor series expansion, we have $U_j^i \approx v(Y^i) - v'(Y^i)p_j + q_j + e_j^i$. Defining $\theta = 1/v'(Y^i)$ and normalizing the other variables gives us $u_j^i \approx \theta q_j - p_j + \epsilon_j^i$ (See Tirole, 1988).

exposition, we assume that the ϵ_j^is are independently and identically distributed across individuals, i.e., $\Phi^i = \Phi$. Hence, we can drop the superscript i from the preferences. The probability that a consumer chosen at random selects variety j is given by

$$f_j = Prob(u_j = \max_s u_s), \quad s = 1, 2, ...J.$$

Following McFadden (1973), we assume that Φ is a Type I Extreme Value Distribution with variance σ^2. The choice probability then becomes

$$f_j = \frac{exp\{\frac{1}{\sigma}(\theta q_j - p_j)\}}{\sum_{s=1}^{J} exp\{\frac{1}{\sigma}(\theta q_s - p_s)\}}.$$

The derivation of the choice probability from the indirect utility function is quite standard (See Anderson et al., 1992). From the Law of Large Numbers it follows that f_j is also the fraction of consumers who demand variety j. The total demand for variety j is then given by $f_j L$, where L is the size of the population .[8] Now, as the number of available varieties becomes arbitrarily large (the set of varieties converging to a continuum in the limit), f_j reduces to

$$f(j) = \frac{exp\{\frac{1}{\sigma}(\theta q(j) - p(j))\}}{\int_{s\in\Omega} exp\{\frac{1}{\sigma}(\theta q(s) - p(s))\}ds}, \tag{1}$$

where Ω is the subset of varieties that are available to the consumers (Anderson et al., 2001). $f(j)$ is now the choice density. Two features of the above demand function are noteworthy and play an important role in our analysis later on. First, conditional on price, consumers demand more of higher quality varieties. This property captures the vertical differentiation aspect of the differentiated good. Second, as long as $\sigma > 0$, there is positive demand for every variety, provided that its price does not go to infinity. This property reflects the horizontal differentiation aspect of the differentiated good and ensures that an equilibrium exists with producers selling varieties of different quality and different consumers having strict preference for different varieties.

Production. Every period, a measure one of producers enter the market, produce for two periods and then exit. Therefore every period, there is a "young" and an "old" cohort of producers.

[8]Notice that f_j is the familiar multinomial logit demand. This demand structure is quite common in quality based trade models (See, for example Verhoogen, 2008; Khandelwal, 2010; Fajgelbaum et al., 2011).

After entering, each producer draws a quality q, where q is distributed according to $G(q)$. Assuming that producing a new variety is costless, every producer produces a unique variety, and accordingly, has some market power. The cost of producing each unit of a variety is $\bar{c}$, which is the same across firms.[9] Producers choose what price to charge, which in turn determines the demand they face. For the remainder of the paper, we shall refer to a producer with quality q simply as producer q.

Exporting. Exporting is costly. Following Melitz (2003), we assume that producers have to pay a one time fixed cost F in order to access the foreign market. This cost could reflect administrative costs like filling up forms and obtaining export licenses, among other things. Importantly for our purpose, an exporter has to bear this cost *irrespective* of the mode of export.

Quality Uncertainty. Under asymmetric information, the quality of a product is not perfectly observed by foreign consumers in the first period. Once a product is consumed, its quality is known. Accordingly, we are back to the full information world in period two. In period one, foreign consumers observe the true quality with some probability. The likelihood of producer q's true quality being observed is captured by $1 - \alpha(q)$, where

$$\alpha'(q) < 0; \lim_{q \to \underline{q}} \alpha(q) = 1; \lim_{q \to \bar{q}} \alpha(q) = 0.$$

We term $\alpha(q)$ the "visibility" of producer q. We offer multiple interpretations of $\alpha(q)$ later on in the paper.

2.2 Equilibrium

To prepare for the analysis of equilibrium with asymmetric information, we first solve for the full information equilibrium. We consider trade between two countries - home and foreign, where the former is a small country. Throughout, our focus is on the problem faced by a home producer in the foreign market.

Full Information. Every producer produces a unique variety and maximizes period profits.

[9] A constant per unit cost can be derived from a model where the marginal cost of production is increasing in quality and firms choose quality optimally. To see this, suppose firms draw capability ϕ and let the technology for producing one unit of quality q be given by $q = \phi + log(n - \bar{n})$, where $\bar{n}$ is the minimum units of labor required to produce one unit of *any* quality while n is the actual units of labor used. If w is the cost of labor, then the marginal cost of each unit of quality q is $c(q) = w(\bar{n} + e^{q-\phi})$. Profit-maximization then gives the optimal choice of quality as $q = \phi + \log \frac{\theta}{w}$. Plugging this into the marginal cost yields $c(q) = \theta + w\bar{n}$, which is a constant.

The full information profit, $\pi_F(q)$, is given by the following:

$$\pi_F(q) = \max_p (p - \bar{c}) exp\{\frac{1}{\sigma}(\theta q - p)\} / \int_{\Omega^*} exp\{\frac{1}{\sigma}(\theta s - p(s))\} ds,$$

where Ω^* is the set of varieties available in the foreign country and the foreign population has been set equal to one. Profit-maximization yields the following:

$$p_F(q) = \sigma + \bar{c},$$

where $p_F(q)$ is the full information price. Although every producer sells a unique variety, the price charged is a constant markup σ over marginal cost $\bar{c}$, and hence is a constant itself. Note, however, that quality adjusted price is decreasing in q.

The price charged depends on σ, the dispersion in the taste of the consumers. Intuitively, an increase in σ increases the mass on the tails of the distribution of ϵ_j. Since the demand for a particular variety comes from individuals who value it highly (i.e., those who draw ϵ_j from the upper tail of the distribution), fattening of the upper tail raises demand, while fattening of the lower tail has no effect on demand. Replacing the value of p_F in the expression for profit, we get

$$\pi_F(q) = \frac{\sigma}{C} exp\{\frac{1}{\sigma}(\theta q - \sigma - \bar{c})\}.$$

where $C = \int_{\Omega^*} exp\{\frac{1}{\sigma}(\theta s - p(s))\} ds$ is a measure of competition. Because home is a small open economy, home producers take C as given. We set C as the numeraire. In equilibrium, producers exporting higher quality varieties charge the same price but face higher demand, resulting in higher profits. Total (or lifetime) profits of a producer in the export market is then simply $2\pi_F(q)$. For future reference, let us denote the full information quantity by $x_F(q)$.

A producer exports if $2\pi_F(q) \geq F$. If F is neither too large nor too small,[10] there exists a unique, interior quality level q_F^* such that producers with $q \geq q_F^*$ export. q_F^* solves

$$2\pi_F(q_F^*) = F. \tag{2}$$

In the presence of asymmetric information about product quality, the set of exporters will typically change, as we explore next.

Asymmetric Information. The timing of the game is as follows: (i) Producers post their prices, (ii) Foreign consumers update their beliefs about producers' qualities and place their

[10] A sorting equilibrium exists as long as $2\pi(\bar{q}) > F > 2\pi_F(\underline{q})$.

demand, (iii) Varieties are exported and consumed, (iv) Qualities are revealed, (v) Second period price and quantity are determined. Asymmetric information and the sequential nature of moves means that this is a dynamic game of incomplete information. Accordingly, the solution concept we shall use is Perfect Bayesian Equilibrium (PBE).

Within the class of PBE, we focus on a pooling PBE. Let q_U^* be the quality of the marginal producer who exports. We propose an equilibrium where every exporter whose quality has not been revealed charges a price $\bar{p} \in [\bar{c}, \sigma + \bar{c}]$ and foreign consumers, who have rational expectations, believe that the exporters have quality $\bar{q}(q_U^*)$, where

$$\bar{q}(q_U^*) = E[q|q \geq q_U^* \text{ and } q \text{ not revealed}].$$

Henceforth, we drop the argument from $\bar{q}(q_U^*)$ and simply write it as $\bar{q}$. As is well known, a PBE allows the foreign consumers to assign any posterior belief whenever $p \neq \bar{p}$. This leeway in specifying off-the-equilibrium-path beliefs generates multiple equilibria (Fudenberg and Tirole, 1991). We assume that foreign consumers have the following off-equilibrium beliefs: $\mu(q = q_L|p > \bar{p}) = 1$ and $\mu(q = \bar{q}|p < \bar{p}) = 1$, where $q_L < \bar{q}$. In words, if a foreign consumer observes any price higher than $\bar{p}$, he thinks that the producer is selling a variety that has low quality. The underlying reason behind such beliefs is the expectation that high quality producers should offer low introductory prices to signal their quality.[11] Let the first period profit of an exporter selling at a price $\bar{p}$ be denoted by $\pi_{PE}(q_U^*)$, where PE stands for pooled equilibrium. Define the corresponding quantity demanded $exp\{\frac{1}{\sigma}(\theta\bar{q} - \bar{p})\}$ as $x_{PE}(q_U^*)$. $\pi_{PE}(q_U^*)$ is then given by

$$\pi_{PE}(q_U^*) = (\bar{p} - \bar{c})exp\{\frac{1}{\sigma}(\theta\bar{q} - \bar{p})\}. \tag{3}$$

To ensure the existence of a sorting equilibrium, we assume that $\pi_{PE}(\underline{q}) < \pi_F(\underline{q})$. For an exporter (whose quality has not been revealed) to charge $\bar{p}$ in equilibrium, he must not be able to earn more than $\pi_{PE}(q_U^*)$ by deviating. As we formally show in the Appendix, for any q_U^*, there always exists q_L low enough such that an exporter would never want to deviate from $\bar{p}$. In other words, charging $\bar{p}$ is an incentive compatible strategy for all exporters. We conclude that under quality uncertainty, the expected first period profit of an exporter is given by

$$\pi_D(q) = \alpha(q)\pi_{PE}(q_U^*) + (1 - \alpha(q))\pi_F(q). \tag{4}$$

where the D subscript denotes Direct exporters. Although direct exporting is the only mode of

[11] Consumers could, instead, expect the producers to charge higher prices to signal higher quality. See the discussion at the end of Section 2.

exporting in this section, this will change once we introduce intermediaries in the next section. The behavior of $\pi_D(q)$ depends on $\pi_{PE}(q_U^*)$ and $\pi_F(q)$, as well as, $\alpha(q)$. Given that $\alpha(q)$ is decreasing in q, $\pi_D(q)$ is close to $\pi_{PE}(q_U^*)$ for low quality producers and close to $\pi_F(q)$ for high quality producers. Figure 1 shows $\pi_D(q)$ under two different situations.

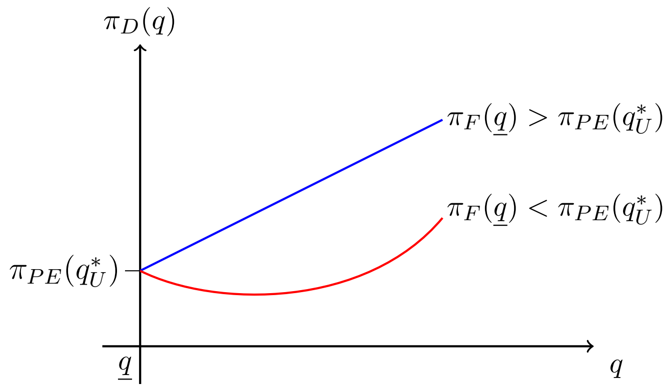


Figure 1: *First period profit as a function of q*

Notice that a producer who is deciding whether to export or not, compares only the total profit from exporting, $\pi_D(q) + \pi_F(q)$, with F. Because $\pi_D(q)$ is increasing in q for q large enough (see Figure 1), the total profits are also increasing for q large enough. We seek a cut-off q_U^* such that every producer with quality above q_U^* chooses to export. q_U^* solves

$$\alpha(q_U^*)\pi_{PE}(q_U^*) + (2 - \alpha(q_U^*))\pi_F(q_U^*) = F \tag{5}$$

where $\pi_{PE}(q_U^*)$ is increasing in q_U^*. How the left-hand side of the above equation changes with q_U^* depends on $\alpha(q)$. We make the following assumption:

ASSUMPTION 1: $|\alpha'(q)|$ is small for all q.

Under Assumption 1, the left-hand side of Equation 5 is increasing in q_U^*.[12] Furthermore, under our assumptions on F and $\pi_{PE}(\underline{q})$, the left-hand side is less than F for small q_U^* and exceeds F for large q_U^*. Accordingly, there exists a unique q_U^* for which the equation is satisfied.

How does q_U^* compare with the export cut-off in the full information world, q_F^*? In Equation 3, when $\bar{p}$ is close enough to $\bar{c}$, $\pi_{PE}(q_U^*)$ is close to zero. Then, for $q_U^* = q_F^*$, total profits under quality uncertainty is close to $(2 - \alpha(q_F^*))\pi_F(q_F^*)$. As long as $\alpha(q_F^*) > 0$, these profits are less than $2\pi_F(q_F^*)$, which, by Equation 2, equals F. Hence, the the exporter with quality q_F^*

[12]Denoting $\Omega = \alpha(q_U^*)\pi_{PE}(q_U^*) + (2-\alpha(q_U^*))\pi_F(q_U^*)$, we have $\frac{\partial\Omega}{\partial q_U^*} = \alpha(q_U^*)\frac{\partial\pi_{PE}(q_U^*)}{\partial q_U^*} + (2-\alpha(q_U^*))\frac{\partial\pi_F(q_U^*)}{\partial q_U^*} - \alpha'[\pi_F(q_U^*) - \pi_{PE}(q_U^*)]$. Now, irrespective of the sign of $\pi_F(q_U^*) - \pi_{PE}(q_U^*)$, $\frac{\partial\Omega}{\partial q_U^*} > 0$ if $|\alpha'|$ is small.

would be incurring a loss under quality uncertainty, implying that the marginal exporter in this scenario must have higher quality, i.e., $q_F^* < q_U^*$ as shown in Figure 2.

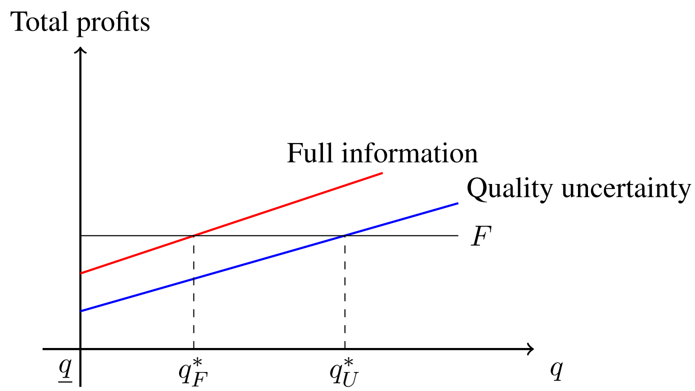


Figure 2: *Export cutoffs under different informational structure*

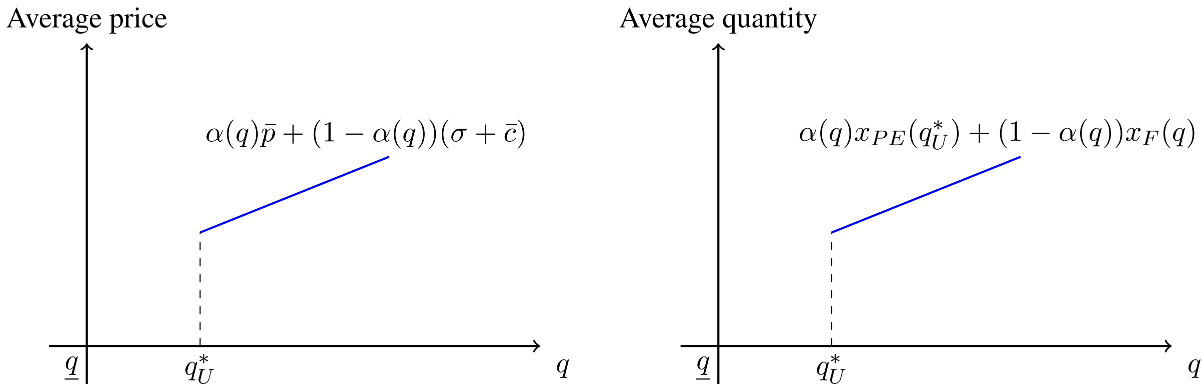
Quality uncertainty has asymmetric effects on the exporting profits of producers. On the one hand, when quality is not revealed, high quality producers lose out because foreign consumers believe that they are of average quality. On the other hand, the likelihood of a firm's quality being revealed itself depends on own quality. To understand the role played by quality uncertainty, let us re-write the firm's net profit from exporting under uncertainty as

$$\begin{aligned} \textit{Net profit} &= \alpha(q)\pi_{PE}(q_U^*) + (2-\alpha(q))\pi_F(q) - F. \\ &= 2\pi_F(q) - [F + \underbrace{\alpha(q)(\pi_F(q) - \pi_{PE}(q_U^*))}_{\text{Information cost}}]. \end{aligned} \tag{6}$$

As the above equation shows, relative to the full information world, quality uncertainty creates an additional cost of exporting - the "information cost" - which is captured by $\alpha(q)(\pi_F(q) - \pi_{PE}(q_U^*))$. Due to imperfect observability of quality in period one, producers have to incur a cost that takes the form of foregone profits. The information cost enters the producer's net profit expression in the same way as the exogenous fixed cost F. At the same time, it is specific to a producer through its dependence on $\alpha(q)$ and $\pi_F(q)$. Note that the information cost may not be monotone in quality. Higher quality producers have larger foregone profits. But the probability that they have to *actually forego* any profit is relatively smaller; for producers with very high quality, the information cost is negligible because their qualities are almost always revealed.

Although exporters charge the price $\sigma + \bar{c}$ if their quality is revealed, and $\bar{p}$ if not, the average price, $\alpha(q)\bar{p} + (1-\alpha(q))(\sigma+\bar{c})$, is increasing in quality as shown in Figure 3 (because $\bar{p} < \sigma + \bar{c}$). Similarly, the quantity sold by the exporter whose quality has been revealed,

$x_F(q)$, is increasing in quality, while it is a constant $x_{PE}(q_U^*)$ for the others. In this case, the average quantity sold by exporters could be non-monotonic in quality depending on how $\alpha(q)$ changes with quality. It is easy to show that if Assumption 1 is satisfied, the average quantity is increasing in quality.



(a) *Average price* **(b)** *Average quantity exported*

Figure 3: *Average price and quantity of exporters*

To summarize, in the presence of quality uncertainty, there exists a quality level q_U^* such that producers with quality above q_U^* export. Observe that in equilibrium, if a producer chooses to export in the first period, he will do so in the second period too. On the other hand, if a producer does not export in the first period, he does not export in the second period either. By not exporting in the first period, a producer faces the problem of a young exporter in period two; if exporting was not profitable in the first period, it cannot be profitable in the second period either.

2.3 Discussion

The analysis presented above does not critically depend on our choice of a pooling equilibrium. We could have, instead, solved for a partial pooling (or semi-separating) equilibrium characterized by multiple quality intervals for exporters. In such an equilibrium, exporters in the same interval would charge the same price (and sell the same quantity). The key insight derived above, that the presence of asymmetric information creates an information cost of exporting, would not have changed however. In such an equilibrium, the information cost faced by individual producers would be smaller. Although the high quality producers would still be treated as average quality, this average quality itself would differ across intervals; the average quality of an interval with many high quality producers would be higher than one with many low quality producers. Nevertheless, the information cost would not disappear. The behavior of average price and average quantity would be qualitatively similar too. Because a partial pooling equilibrium would complicate the analysis without much gain in insight, we chose to focus on a pooling equilibrium.

As already discussed, the off-equilibrium belief held by foreign consumers, $\mu(q = q_L | p > \bar{p}) = 1$, is motivated by the expectation that a producer who introduces a new *experience* good should charge a low price to signal high quality. The underlying reasoning is that a low price today implies lower profits (or even losses) today and only a high quality producer, who can earn large profits in the future when his quality is revealed, can benefit from such a strategy. Thus, loosely speaking, charging a low price is an incentive compatible strategy for a high quality producer. In the real world, however, it is quite often observed that producers signal high quality by charging a high price. Why do we not pick such a strategy for the producers and the corresponding beliefs?

Observe that a strategy where only high quality producers charge high price can never be incentive compatible in our model - a low quality producer will always have an incentive to mimic the high quality producer. For such a strategy to be incentive compatible, one would require the presence of heterogeneous income groups, something that is absent in our model (recall that income enters our model through the preference parameter θ). When rich consumers have a greater willingness to pay for high quality and poor consumers have a greater willingness to pay for low quality, producers face a trade-off. A low quality producer who mimics a high quality producer gains the market share of rich consumers but loses the market share of poor consumers.[13] Although we could easily introduce heterogeneity among consumers in terms of their income, we chose not to as it would distract us from analyzing the role of intermediaries, the main objective of this paper.

The assumption that $\alpha(q)$ is decreasing in quality is clearly a simplification, but it can be theoretically justified on several grounds. First, it can be justified by assuming that producers are multi-product firms with higher quality producers selling more products in any market.[14] Under this assumption, if a high quality producer that sells a number of products in the foreign market introduces a new product, then consumers are more likely to find out its quality

[13] See Kirmani and Rao (2000) for a discussion of the conditions under which one can get different pricing strategies of firms selling experience goods.

[14] A number of recent papers have documented the dominance of multi-product firms in international trade (See, for example, Baldwin and Gu, 2009; Arkolakis and Muendler, 2010; Mayer et al., 2010; Bernard, Redding and Schott, 2011).

relative to the producer who has not sold any other product in that market before. $\alpha(q)$ then is related to the reputation of the producer. [15] Second, it can be justified in a model with marketing/advertisement costs. In such a model, producers use advertisement expenses to signal their quality. Higher the quality of a producer, higher is the expenditure on advertisement, making it more likely that his quality is revealed.[16]

3 The Model with Intermediaries

In this section, we introduce trade intermediaries in the model of quality uncertainty developed above. Intermediaries are profit-maximizing firms located in the foreign country, that buy goods from the home producers and sell them to foreign consumers.[17] Intermediaries vary in terms of their ability to screen the quality of products. We denote this ability by λ. Each intermediary is potentially infinitely lived but faces an exogenous probability of exit in each period equal to $1-\delta$. As intermediaries exit, they are replaced by new intermediaries. Without modeling the underlying entry and exit process, we assume that in the steady state, λ is distributed according to $H(\lambda)$, $\lambda \in [\underline{\lambda}, \bar{\lambda}]$. Furthermore, there is a measure $M < 1$ of intermediaries in the steady-state. We take M as given, and later on perform comparative statics with respect to M. Henceforth, we refer to an intermediary with ability λ simply as a λ intermediary.

3.1 Intermediation Technology

Each intermediary can screen the quality of goods from multiple producers. The number of producers whose quality an intermediary can screen, the capacity of an intermediary, depends on its own ability, as well as, the qualities of the producers. We assume that an intermediary can match with only one type of producer. Let this capacity be denoted by $n(q, \lambda)$ with the following properties:

[15] In a recent paper, Bernard, Blanchard, Van Beveren and Vandenbussche (2012) document that a large number of Belgian manufacturing firms export products that they do not produce. One possible explanation is that it is easier for a firm to export a new product when it has already established a reputation selling other products in a market.

[16] Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) among others theoretically predict a positive correlation between unobserved product quality and advertising expenses. Based on the insights from these models, Thomas et al. (1998) find a positive relationship between advertisement expenditures and product quality using data from the U.S. automobile industry. Arkolakis (2010) develops a model where more productive (higher quality) producers spend more on advertisement, although his model has search goods and not experience goods.

[17] Blum et al. (2009) provide evidence that while import intermediaries are very important, export intermediaries carry very little of Chile's exports, except in agricultural and food products.

$$n_\lambda > 0; n_q < 0; n_{\lambda q} > 0.$$

$n(q, \lambda)$ captures in a reduced-form way the dependence of the intermediary's capacity on the characteristics of the intermediary and producers. Conditional on quality, this capacity is increasing in the intermediary's ability. One can think of higher ability intermediaries having more resources (for example, extensive networks of agents) that allow them to match with more producers of a given quality.[18] The capacity is also decreasing in the quality of each producer. A natural interpretation of this assumption is that higher quality goods have more attributes or features and an intermediary has to spend more resources in order to test each one of those features. Hence, for a given endowment of resources, screening higher quality implies screening fewer producers.[19] The third property is a complementarity condition which states that the negative effect of quality on capacity is decreasing in the intermediary's ability. This could be because intermediaries with higher ability have relatively more expertise in screening higher quality.[20] As we shall see later on, this complementarity between quality and ability determines the equilibrium matching between producers and intermediaries.

3.2 Intermediary's problem

Foreign consumers cannot observe the quality of home goods sold by the intermediaries in any given period. Consequently, the intermediary could lie about its quality. Given this set up, the intermediary makes two independent choices: what quality to announce to final consumers, and what quality to actually sell.

To discourage intermediaries from lying, consumers can punish an intermediary who lies about the quality of the goods it is selling. Notice the key difference between a producer and an intermediary: even if a producer lies about its quality, its demand never drops to zero. Once the true quality is revealed to the consumers, given the preferences, there will always be some demand for the good in the second period. The problem faced by the intermediary is different, however, because an intermediary sells different goods in two consecutive periods. This follows

[18]Heterogeneity among intermediaries also features in the model of Rauch and Watson (2004). In their model, intermediaries randomly draw the size of their networks.

[19]The assumption that an intermediary can match with only one type of producers can be justified by assuming some sort of complementarity between the qualities of the different producers who are being screened by the same intermediary. Such complementarity could arise due to learning: when an intermediary screens a product with $q = q'$, it becomes more efficient in screening other products with $q = q'$ but not $q \neq q'$.

[20]Another way of saying this is that high ability intermediaries have an absolute advantage in screening all qualities but a comparative advantage in screening high qualities.

from the fact that a producer uses an intermediary for only one period (a producer does not need an intermediary once its quality is known to foreign consumers).[21] Consequently, an intermediary has to rely on its reputation to sell goods.

The market for intermediaries is perfectly competitive. There is an equilibrium producer price schedule, $p_s(q)$, which gives the price an intermediary has to pay to purchase goods of any quality. An intermediary takes $p_s(q)$ as given and purchases the optimal quality. Let us first consider the case where an intermediary tells the truth. Denoting the intermediary's profit by $r(\lambda)$, a λ intermediary's problem is given by

$$r(\lambda) = \max_{\tilde{p}_I(q,\lambda),q} (\tilde{p}_I(q,\lambda) - p_s(q)) exp\{\frac{1}{\sigma}(\theta q - \tilde{p}_I(q,\lambda))\} n(q,\lambda).$$

where $\tilde{p}_I(q,\lambda)$ is the final price charged by a λ intermediary selling products of quality q to the foreign consumers. Maximizing $r(\lambda)$ with respect to $\tilde{p}_I(q,\lambda)$ yields

$$\tilde{p}_I(q,\lambda) = \sigma + p_s(q). \tag{7}$$

Like the producer under full information, the intermediary charges a price that is a constant mark-up over its marginal cost, which is simply the price it pays to the producer. Replacing the value of $\tilde{p}_I(q,\lambda)$ in $r(\lambda)$ and differentiating with respect to q, we get the first-order condition for optimal quality:

$$\frac{\partial p_s(q)}{\partial q} = \theta + \sigma \frac{\epsilon_q}{q}, \tag{8}$$

where ϵ_q is the elasticity of $n(q,\lambda)$ with respect to q. The above equation can be interpreted as follows: the left-hand side denotes the cost incurred by an intermediary if he tries to purchase from a producer with a marginally higher quality. The right-hand side denotes the corresponding *net* marginal benefit. A higher quality is more valuable, the higher is the willingness to pay on the part of the consumers (high θ). The marginal benefit, however, falls with quality as it reduces the capacity of an intermediary, and accordingly, his total sales (ϵ_q is negative). We assume that $n(q,\lambda)$ is such that $|\frac{\epsilon_q}{q}| < \frac{\theta}{\sigma}$ for all q. Therefore, $\frac{\partial p_s(q)}{\partial q} > 0$. Let us denote the optimal quality under truth-telling by $q(\lambda)$. Because matches are one-to-one (as we prove later), we can invert the optimal quality function as $\lambda = \lambda(q)$ Replacing λ in Equation 7, we

[21] Ahn et al. (2011) provide evidence that in Ghana, many firms start exporting through intermediaries but later transition into direct exporting. In a related paper, Macchiavello (2010) shows that when they start exporting to UK, Chilean wineries initially sell through distributors who have an advantage in discovering new wineries. Once they have established a reputation, these wineries move on to other distributors.

have

$$p_I(q) = \sigma + p_s(q). \tag{7a}$$

If the intermediary chooses to lie, it can sell a quality different from what it announces. What quality it actually sells in equilibrium will of course depend on consumers' beliefs and the punishment from lying. We assume that if an intermediary lies, consumers stop purchasing from him for T periods.[22] Furthermore, every consumer observes the full history of all intermediaries. Given our assumption that foreign consumers observe the ability of the intermediaries, the consumers know what the optimal quality of each intermediary, $q(\lambda)$, is. We propose an equilibrium where a λ intermediary announces and sells $q(\lambda)$. Under what condition would this be an equilibrium? As the following lemma shows, for large T, the intermediary always tells the truth.

Lemma 1. *For T large enough, there exists a truth-telling equilibrium where a λ intermediary screens and sells goods of quality $q(\lambda)$.*

For the remainder of the paper, we assume that T is such that the λ intermediary always sells $q(\lambda)$. Finally, we characterize the matching problem. As the next lemma shows, higher ability intermediaries match with higher quality producers, i.e., there is positive assortative matching (PAM) in equilibrium. This is a consequence of the complementarity between intermediary's ability and product quality in determining capacity.

Lemma 2. *There is one-to-one positive assortative matching between intermediaries and producers.*

Lemma 2 allows us to define a matching function $m(q) : q \to \lambda$ with $m'(q) > 0$. A corollary is that $m(q)$ is invertible. The exact relation between q and λ will depend on the distributions G and H, as well as, equilibrium prices, which we explore next.

3.3 Equilibrium

In this section, we characterize the full equilibrium of the model. In the first period, home producers have a choice of exporting directly or exporting through intermediaries, where the profit from the latter is given by

$$\pi_I(q) = (p_s(q) - \bar{c})exp\{\frac{1}{\sigma}(\theta q - \sigma - p_s(q))\}. \tag{9}$$

[22] Note that this is a form of *trigger* strategy that agents typically use to sustain co-operation in repeated games (Gibbons, 1992).

Notice that although a producer receives $p_s(q)$ per unit that he sells, the quantity sold depends on $p_I(q)$, the price charged by the intermediaries to the final consumers. An exporter compares $\pi_I(q)$ with $\pi_D(q)$, the first period profit from direct exporting, and chooses the maximum; the second period profits are the same irrespective of the mode of exporting in period one. Let us denote producers who use intermediaries as I exporters and those exporting directly as D exporters. An equilibrium allocation is defined as:

Definition 1. *An equilibrium of this model consists of*
(i) Sorting of producers into non-exporters, I exporters and D exporters;
(ii) An assignment of I exporters to intermediaries;
(iii) A producer price schedule $p_s(q)$; such that
(a) utilities of consumers and profits of intermediaries and producers are maximized, (b) consumer's beliefs about quality are consistent with the sorting of producers, and (c) the market for intermediaries clears.

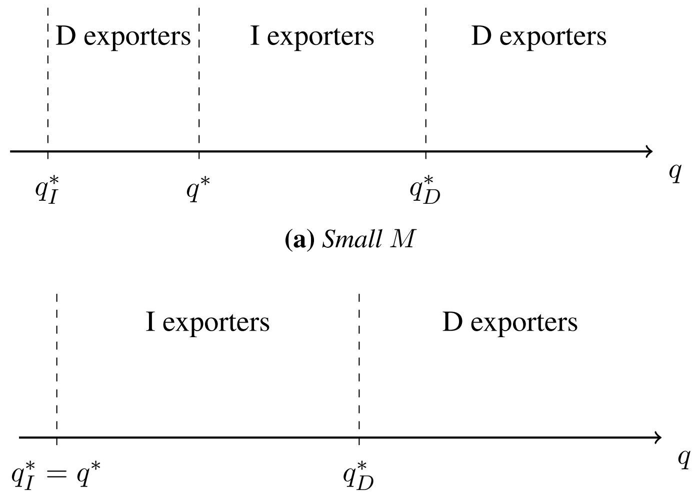
The following lemma provides some characterization of the equilibrium allocation.

Lemma 3. *In equilibrium, the set of producers using intermediaries is connected. Furthermore, the high quality producers sell directly.*

Lemma 3 simplifies the analysis of equilibrium. According to this lemma, if producers with qualities q_1 and q_2 export through intermediaries, then all producers with qualities between q_1 and q_2 will also use intermediaries. Furthermore, the highest quality producers sell directly because they have the highest visibility - it is very likely that their quality will be revealed. Depending on the parameters of the model, there are two possible equilibrium configurations as shown in Figure 4. q_I^* denotes the export cutoff under intermediation; $[q^*, q_D^*]$ in Figure 4a and $[q_I^*, q_D^*]$ in Figure 4b denote the set of I exporters. Depending on the measure of intermediaries, we could either have a situation where exporters with intermediate levels of quality (Figure 4a) or the exporters with the lowest quality (Figure 4b) export through intermediaries.[23] How does q_I^* compare with q_U^*, the export cutoff without intermediation?

First, consider the case where there are few intermediaries (small M) so that $q_I^* < q^*$. In this case, the expected quality of D exporters would typically be different relative to the case without intermediation. As long as this expected quality is higher in the presence of intermediaries, the information cost of entering the export market falls for the D exporters (See Equation 6). This would cause some of the non-exporters close to q_U^* to switch into exporting; in equilibrium, $q_I^* < q_U^*$.

[23]The equilibrium allocation also depends on the abilities of intermediaries.



(a) *Small M*

(b) *Large M*

Figure 4: *Equilibrium allocation of exporters*

Second, consider the case where there is a large number of intermediaries (large M) so that $q_I^* = q^*$. In this case, the expected quality of D producers is necessarily higher in the presence of intermediaries. This is because, the distribution of D exporters is a truncation of the distribution of exporters in the absence of intermediaries, with the truncated distribution having a higher lower bound ($q_D^* > q_U^*$).[24] Hence, the information cost of some of the non-exporters close to q_U^* falls. Of course, in equilibrium, these producers find it optimal to export through intermediaries. For the remainder of the paper, we shall focus on the case where M is large.

Proposition 1. *As long as there are a large number of intermediaries, the entry of intermediaries causes some non-exporting producers to switch into exporting.*

When there are many intermediaries, the introduction of intermediaries has two effects: (a) it creates a *positive externality*, whereby the information cost facing the D exporters are reduced, and (b) it leads to a separating equilibrium in which the high quality producers signal their quality by *not using* intermediaries.[25] The low quality firms, on the other hand, find it optimal to export through intermediaries - exporting directly is too costly.

The existence of a separating equilibrium, where high quality producers export directly while the low quality ones use intermediaries, requires the incentive compatibility constraint to be satisfied for both low and high quality producers. When would such an allocation be an

[24]If, on the other hand, $q_D^* < q_U^*$, then obviously there must be some non-exporters who switch into exporting in the presence of intermediaries.

[25]To be technically correct, it is a semi-separating equilibrium.

equilibrium? When a producer exports directly, his quality is revealed with some probability, with this probability increasing in the producer's quality. Accordingly, a low quality producer choosing to export directly is more likely to receive $\pi_{PE}(q_D^*)$, which would typically be less than what it can earn by using intermediaries, $\pi_I(q)$. This leads the low quality firms to use intermediaries. For the high quality firms on the other hand, using intermediaries is relatively more costly. The quality of these firms is revealed with a high probability if they export directly. If they use intermediaries, this probability increases by a small amount; the benefit of using intermediaries is small. At the same time, because the supply of intermediaries is fixed, producers have to compete with each other for intermediaries. At the margin, high quality producers are willing to pay less to the intermediaries (demand a higher $p_s(q)$) compared to the low quality producers. This causes the high quality firms to export directly.

Recall that the supply of producers and intermediaries is given by the distributions $G(q)$ and $H(\lambda)$ respectively. Since the market for intermediaries must clear, we have

$$\int_{q_I^*}^{q} dG(s) = \int_{m(q_I^*)}^{m(q)} n(m^{-1}(s), s) M dH(s) \qquad \forall q < q_D^*. \tag{10}$$

The left-hand side of the above expression captures the demand for intermediaries coming from producers with qualities between q_I^* and some q. Because each producer needs one intermediary, this is the same as the measure of producers in that quality interval. The right-hand side captures the supply of intermediaries. A s intermediary has a capacity of $n(m^{-1}(s), s)$ producers - hence, the supply of intermediaries is the measure of intermediaries augmented by their capacity.

Differentiating Equation 10 with respect to q and re-arranging we have,

$$m'(q) = \frac{g(q)}{n(q, m(q))h(m(q))M}, \tag{11}$$

where $g(.)$ and $h(.)$ are the density functions of $G(.)$ and $H(.)$ respectively. Equation 11 represents a differential equation which, along with the two boundary conditions $m(q_I^*) = \underline{\lambda}$ and $m(q_D^*) = \bar{\lambda}$ allows us to solve for the equilibrium matching function. The thresholds q_I^* and q_D^*, in turn, are the solutions to $\pi_I(q_I^*) + \pi_F(q_I^*) = F$ and $\pi_I(q_D^*) = \pi_D(q_D^*)$ respectively. Figure 5 shows the equilibrium allocation of producers along with their profits.

Period one quantity. The q_D^* producer is indifferent between using intermediaries and selling directly. Let the quantity sold when using intermediaries, $exp\{\frac{1}{\sigma}[\theta q - \sigma - p_s(q)]\}$, be denoted

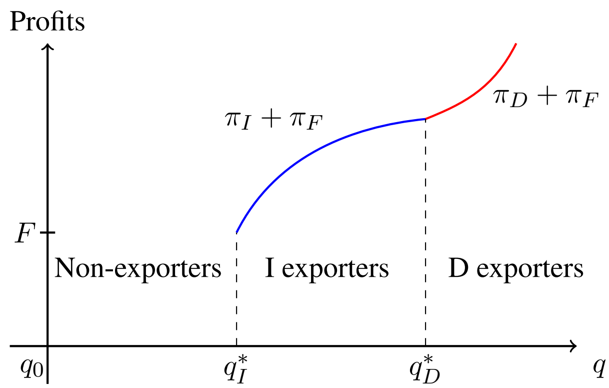


Figure 5: *Equilibrium profits*

by $x_I(q)$. Note that $x_I(q_D^*)$ is less than both $x_{PE}(q_D^*)$ (the pooled equilibrium quantity) and $x_F(q_D^*)$ (the full information quantity). Accordingly, $x_I(q_D^*)$ is less than $\alpha(q_D^*)x_{PE}(q_D^*) + (1 - \alpha(q_D^*))x_F(q_D^*)$, where the latter is the expected quantity sold by the producer if he had exported directly. Thus, there is a discontinuity in quantity at q_D^*. Furthermore, differentiating $x_I(q)$ with respect to q, we have

$$\begin{aligned}\frac{\partial x_I(q)}{\partial q} &= \frac{1}{\sigma}exp\{\frac{1}{\sigma}[\theta q - \sigma - p_s(q)]\}(\theta - \frac{\partial p_s(q)}{\partial q}) \\ &> 0\end{aligned}$$

where the inequality follows from Equation 8. The quantity sold by I producers is increasing in quality. But in spite of the intermediaries revealing the true quality of the producers, the double marginalization has a negative effect on demand (relative to direct exporting), with this effect becoming stronger as quality increases. For producers with quality less than q_D^*, the expected quantity sold when exporting directly is still relatively low; accordingly, these producers find it optimal to use intermediaries. But for high levels of quality, selling less through intermediaries is too costly. The equilibrium quantity as a function of quality is shown in Figure 6. The discontinuity in quantity exported at q_D^* suggests that a positive measure of producers using intermediaries sell less relative to if they had exported directly. This is consistent with the predictions of a model where producers using intermediaries have a higher marginal cost. We shall come back to this issue later. We summarize the findings in the following proposition:

Proposition 2. *The expected quantities sold by both I and D exporters are increasing in quality. There is a discontinuity in quantity at the quality level at which the exporter is indifferent between using intermediaries or not.*

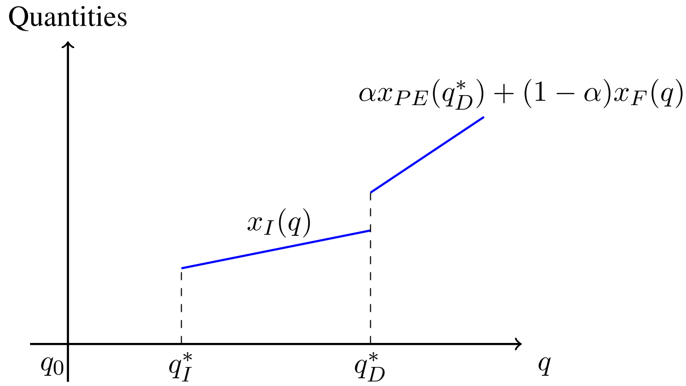


Figure 6: *Equilibrium quantities*

Period one price. Observe that in this model, there are two price schedules of interest: the price received by the producers and the price paid by the final consumers. We have already established that both the price received by the I exporters, $p_s(q)$, and the average price received by the D exporters, $\alpha(q)\bar{p} + (1 - \alpha(q))(\sigma + \bar{c})$, are increasing in q. In the following lemma, we derive an additional property of $p_s(q)$.

Lemma 4. *The price received by the I exporters is greater than $\bar{p}$ for all q.*

By using an intermediary, a producer ensures that he does not have to export at a price that is too low in the event his quality is not revealed. Therefore, intermediaries provide some sort of insurance to the exporters. Combining Lemma 4 with Equation 7a, we have

$$\begin{aligned} p_I(q) &= \sigma + p_s(q) \\ &> \sigma + \bar{c} \end{aligned}$$

where the second line follows from the fact that $\bar{p} > \bar{c}$. When sold through an intermediary, a home product sells for a higher price in period one relative to period two, when its quality is revealed. The higher price paid by final consumers for a product sold through an intermediary also arises when there is double marginalization.[26] Of course, whether the average price received by an I exporter, if he chooses to export directly, is lower or higher than $p_s(q)$ will depend on $\alpha(q)$. Notice that at q_D^*, it must be true that $\pi_I(q_D^*) = \pi_D(q_D^*)$. Expanding the terms,

[26] Note, however, that double marginalization occurs when the mark-up charged by the intermediary adds to the mark-up charged by the producer. The price charged by the I exporters in our model is not simply a markup over marginal cost, but is endogenously determined in the market for intermediaries.

we have

$$[p_s(q_D^*) - \bar{c}]x_I(q_D^*) = \alpha(q_D^*)[\bar{p} - \bar{c}]x_{PE}(q_D^*) + (1 - \alpha(q_D^*))\sigma x_F(q_D^*).$$

Because $x_I(q_D^*) < x_{PE}(q_D^*)$ and $x_I(q_D^*) < x_F(q_D^*)$, we can write

$$p_s(q_D^*) - \bar{c} > \alpha(q_D^*)\bar{p} + (1 - \alpha(q_D^*))(\sigma + \bar{c}) - \bar{c}.$$

We conclude that $p_s(q_D^*) > \alpha(q_D^*)\bar{p} + (1 - \alpha(q_D^*))p_F(q_D^*)$; the price received by the marginal direct exporter q_D^* and, by continuity, some of the I exporters close to q_D^*, if they export through an intermediary, is greater than the price if they export directly.

By exporting through intermediaries, an exporter solves the asymmetric information problem. Because his quality is revealed, he does not have to incur the information cost. At the same time, the price paid by the final consumers for any variety sold through intermediaries is higher relative to the full information price. It is *as if* the per unit costs of these goods have gone up. It has been assumed in the literature that exporting through intermediaries involves a lower fixed cost but a higher marginal cost (Ahn et al., 2011; Bernard, Grazzi and Tomasi, 2011; Akerman, 2012) . Our model provides an explanation of why that might be the case. We summarize the above results in the following proposition.

Proposition 3. *The final price of the products sold by the I exporters is higher than the full information price. The price received by some of the exporters in period one is higher if they use intermediaries relative to exporting directly.*

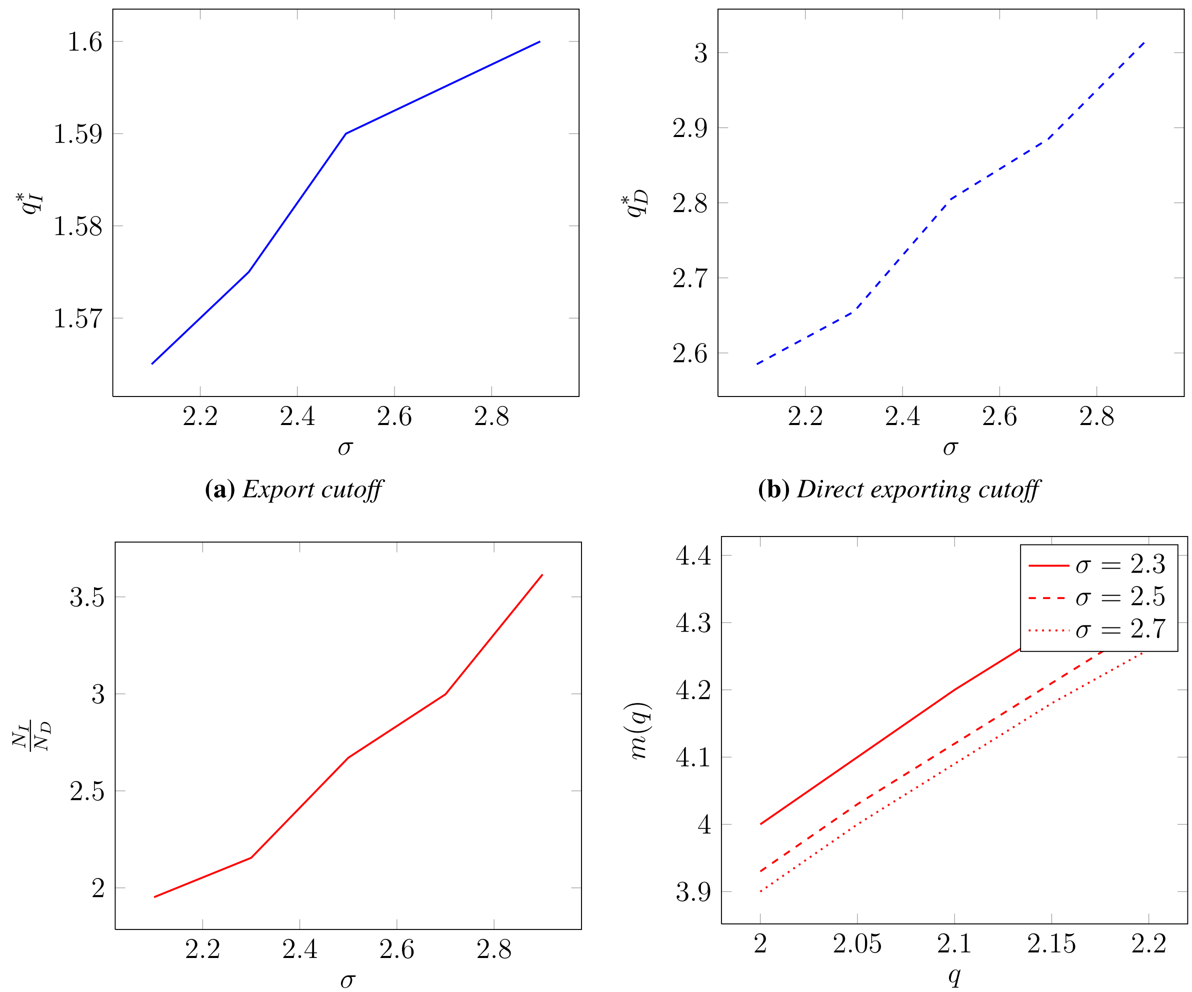
3.4 Comparative statics

Having characterized the equilibrium, we proceed to perform comparative statics exercises with respect to the following variables: σ, α and M. The results in this section are derived numerically.[27]

[27]The parameter values used for this exercise are $\theta = 1, \sigma = 2.5, \bar{c} = 1, \bar{p} = 1.05, F = 1.5, M = 0.1$. The distribution for q is a truncated exponential with $\underline{q} = 1, \bar{q} = 5$ and shape parameter equal to 1. Similarly, the distribution for λ is a truncated exponential with $\underline{\lambda} = 3, \bar{\lambda} = 5$ and shape parameter equal to 1. Finally, $\alpha(q) = 0.5 + 0.5 \cdot \frac{1}{q}$.

3.4.1 A change in σ

Parameter σ captures the heterogeneity in the consumers' tastes; a higher value of σ corresponds to more heterogeneity. We consider the effect of a change in σ on the two thresholds, q_I^* and q_D^*, and the relative importance of intermediaries in exporting. The latter is measured by the ratio of N_I, the measure of I exporters, to N_D, the measure of D exporters.



(a) *Export cutoff* **(b)** *Direct exporting cutoff*

(c) *Relative importance of intermediaries* **(d)** *Matching*

Figure 7: *Comparative statics with respect to σ*

An increase in σ causes q_D^* to rise, as shown in Figure 7b. A higher value of σ corresponds to a lower elasticity of substitution for the varieties (Anderson et al., 1992) - goods with higher σ are more differentiated. As discussed earlier, the preferences that give rise to the multinomial logit formulation of demand in Equation 1 have two dimensions: a vertical dimension captured by quality and a horizontal dimension captured by variety. When σ is small, agents' preferences are closely aligned. As a result, variety matters less and small differences in quality can lead to large differences in demand. Things are different when σ is large. Because of consumers' love for variety, a firm selling a high quality variety may not face too high a demand. Accordingly, exporters have lower incentive to signal their quality. Because high quality producers signal their quality by exporting directly, a high σ lowers their incentive to signal, causing some of them to switch to using intermediaries.

Our model's explanation of the relationship displayed in Figure 7b is different from the one that is usually offered in the context of models of quality uncertainty (Ahn et al., 2011). The latter explanation goes something like this: differentiated goods are those for which quality matters more. Because producers can reveal their quality by exporting through intermediaries, producers in more differentiated goods sectors are more likely to use intermediaries. As a result, the marginal direct exporter has higher quality in sectors with higher σ.

The switching of D exporters into I exporters has an interesting general equilibrium effect. As more exporters try to use intermediaries, the demand for intermediation services rises. With a fixed supply of intermediaries, this lowers the p_s schedule - the intermediaries pay less to the producers. This, in turn, reduces the profits of the marginal exporter (q_I^*), causing him to exit from the export market. Accordingly, we get an upward-sloping relationship between σ and q_I^*, as displayed in Figure 7a.

As σ increases, the measure of D exporters, N_D, falls. The effect on N_I is less obvious. On the one hand, some D exporters become I exporters; on the other hand, some of the I exporters become non-exporters. Nevertheless, the measure of I exporters goes down. At the heart of this result lies the matching technology between intermediaries and producers. An increase in σ improves the matches of intermediaries - each of them is matched with higher quality producers. On the flip side, each producer is now matched with a lower ability intermediary as shown in Figure 7d. Recall that the capacity of an intermediary depends negatively on the quality of the producers he serves; an increase in σ then implies that the same measure of intermediaries serves a smaller measure of producers - N_I falls. What happens then to the ratio $\frac{N_I}{N_D}$? In this example, $\frac{N_I}{N_D}$, which captures the importance of intermediaries in facilitating exports, goes up, as shown in Figure 7c. The prediction that the share of producers exporting through intermediaries is higher for more differentiated sectors is consistent with the findings of Feenstra and Hanson (2004), Blum et al. (2009) and Tang and Zhang (2011). In Blum et al. and Tang and Zhang, however, a higher degree of product differentiation raises the profits of the producers with lower quality. This result follows from these producers being the ones who tend to export through intermediaries.

3.4.2 A change in α

Next we examine the effect of a change in $\alpha(q)$ on producers' decision to export and the relative importance of intermediaries. Recall that $\alpha(q)$ measures the visibility, or the probability that the quality of a q producer is not revealed if he exports directly.

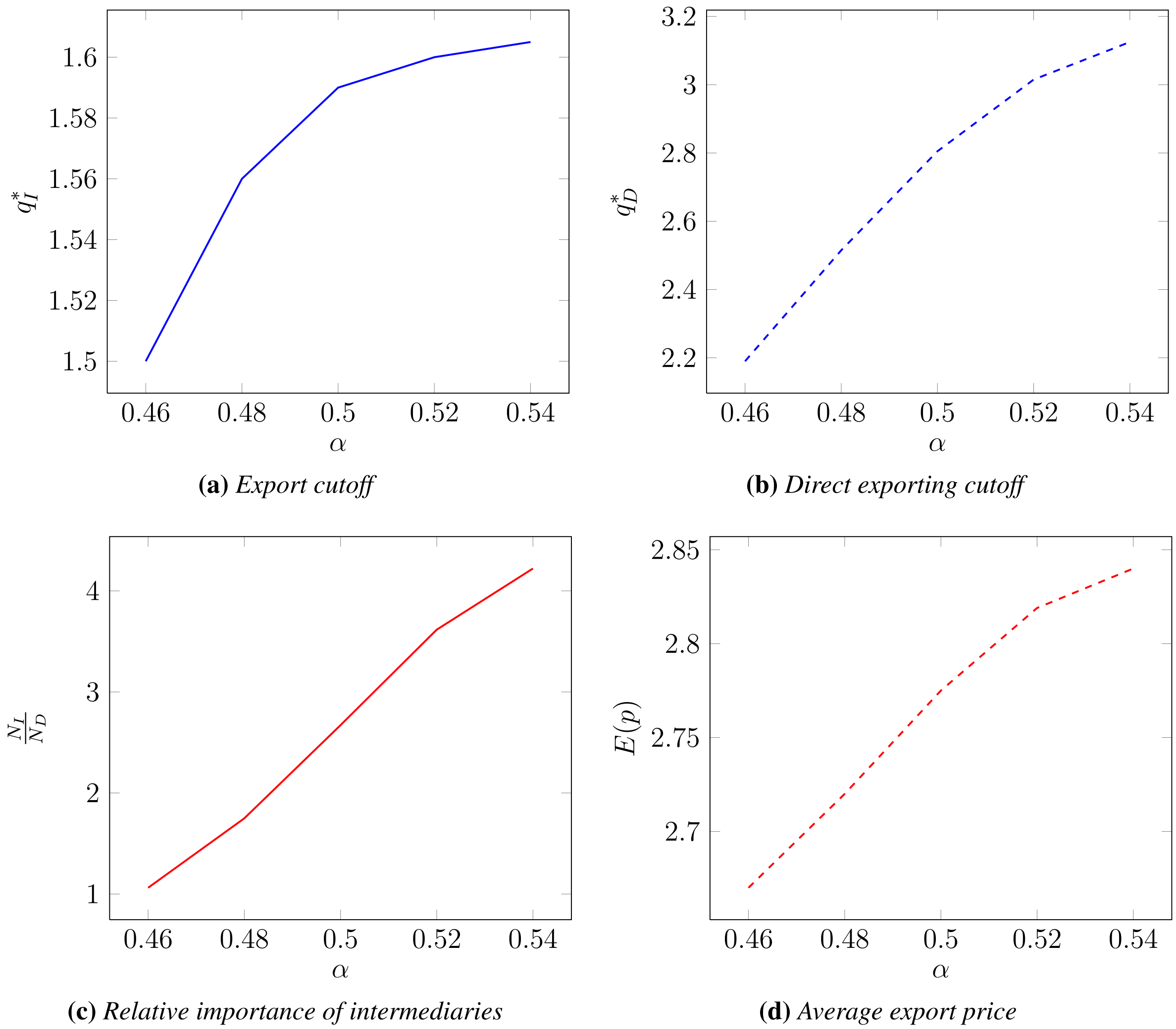


Figure 8: *Comparative statics with respect to* α

We are interested in the effects of a uniform increase in $\alpha(q)$ across producers; this can be interpreted as lower levels of information about the home products in the foreign market. With less information, direct exporting becomes a less attractive alternative. This causes the marginal D exporter to switch into exporting using intermediaries as displayed in Figure 8b. As the demand for intermediaries rises, the price schedule p_s is pushed down, causing the lowest quality exporters to exit. This is shown in Figure 8a. For the reasons mentioned above, a higher α also reduces the measure of I exporters but increases $\frac{N_I}{N_D}$, as shown in Figure 8c.

One could think of distance as a proxy for $\alpha(q)$. If the foreign country is farther away, the foreign consumers might have less information about home products. In such a case, when a home producer starts to export, the probability of its quality being revealed is lower. Going by this interpretation, our model suggests a positive relation between distance and the relative importance of intermediaries. This is consistent with the findings of Ahn et al. (2011), Tang and Zhang (2011) and Akerman (2012).

Another interesting effect of a change in $\alpha(q)$ concerns the average price received by exporters. The latter is computed as

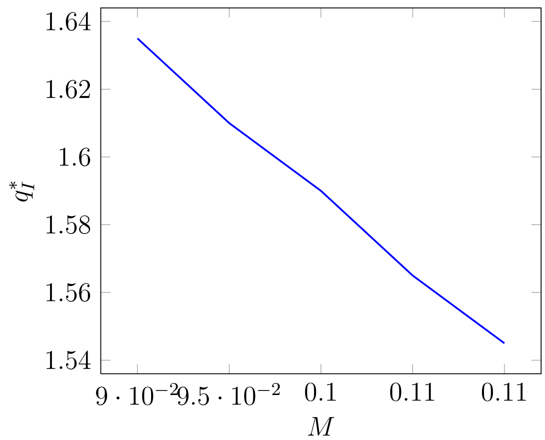
$$E(p) = \frac{1}{2}(\sigma + \bar{c}) + \frac{1}{2}(\omega \bar{p}_s + (1 - \omega)\bar{p}_D),$$

where ω is the share of I exporters in the total number of young exporters, i.e., $\omega = \frac{N_I}{N_I + N_D}$. Half of the exporters are old and hence charge the full information price $\sigma + \bar{c}$. Within the other half, a fraction ω have an average price of $\bar{p}_s$ (the average of p_s) while the remaining $(1 - \omega)$ have an average price of $\bar{p}_D$ (the average price charged by D exporters). An increase in $\alpha(q)$ affects $E(p)$ through three channels: (a) an increase in $\bar{p}_D$, because the price charged by D exporters is increasing in quality and the low quality D exporters drop out; (b) an increase in ω, as relatively more exporters start using intermediaries to export; (c) a decrease or increase in $\bar{p}_s$, because although the p_s of incumbent I exporters falls, some high quality producers switch from D to I exporters. In this example, these different forces act in a way to increase $E(p)$, as shown in Figure 8d. Our model thus provides a rationale for the finding that average export price increases with the distance of the destination, as reported in Hummels and Skiba (2004) and Baldwin and Harrigan (2011).

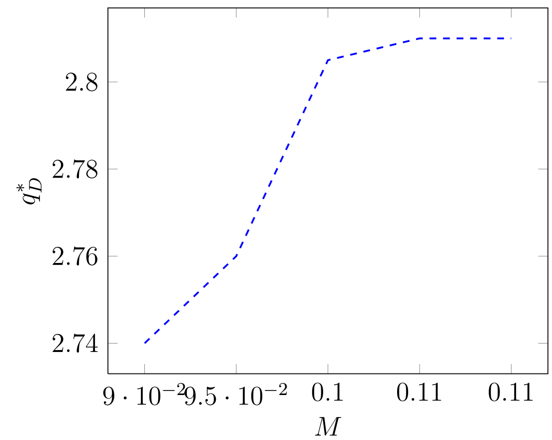
3.4.3 A change in M

Our model also allows us to examine the effects of changes in the intermediation sector on the equilibrium. We consider a change in the measure of intermediaries, M.

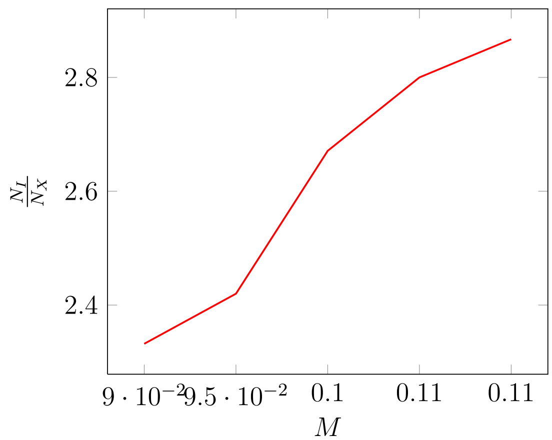
An increase in M, other things remaining the same, results in the p_s schedule shifting up as intermediaries try to compete with each other to serve home firms. Higher p_s changes the extensive margin of I exporters. This comes about due to some non-exporters, as well as, some



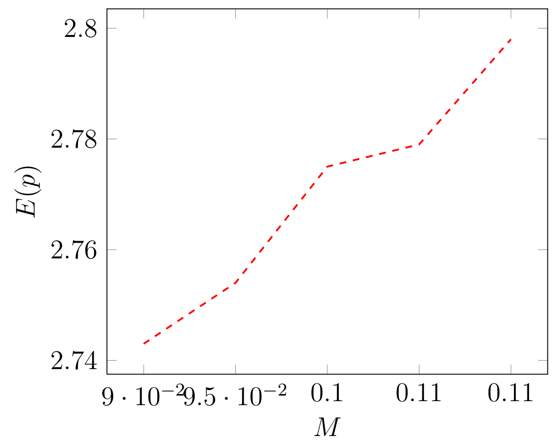
(a) *Export cutoff*



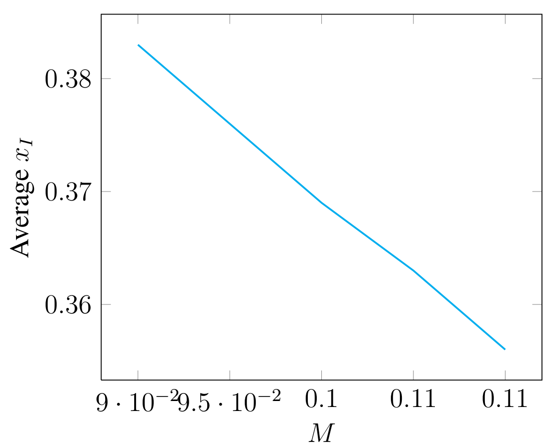
(b) *Direct exporting cutoff*



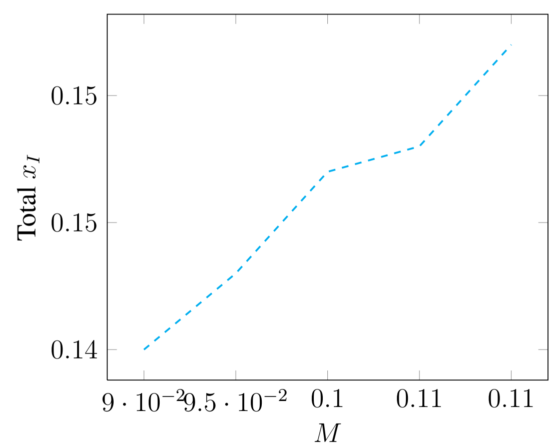
(c) *Relative importance of intermediaries*



(d) *Average export price*



(e) *Average sales of I exporters*



(f) *Total sales of I exporters*

Figure 9: *Comparative statics with respect to M*

D exporters switching into exporting through intermediaries as shown in Figures 9a and 9b. It naturally follows that N_I rises while N_D falls, raising the relative importance of intermediaries as shown in Figure 9c.

For the reasons mentioned above, an increase in M also causes the average export price, $E(p)$, to rise. This is shown in Figure 9d. In a recent paper, Manova and Zhang (2012) shows that Chinese firms price their goods differently across markets, with the prices being higher in richer destinations. Manova and Zhang argue that this is evidence that Chinese firms sell higher quality goods to richer markets. A possible explanation for this phenomenon is that rich country consumers have a higher willingness to pay for quality (high θ in our model) because of non-homothetic preferences (Flam and Helpman, 1987; Fajgelbaum et al., 2011). Our model offers an interesting alternative explanation: Suppose there are two countries - a rich country and a poor country - which import the same products from a third country. If the rich country has a more developed intermediation sector as captured by more intermediaries, then our model suggests that they should be paying higher prices, on average, for the *exact same* goods compared to the poor country. In other words, a part of the higher prices could be explained by a more developed intermediation sector in rich countries that is better able to solve the asymmetric information problem arising from uncertain quality.

Finally, we examine the effect of a change in M on the total sales of I exporters. A change in M affects total sales through both an intensive margin (change in the average sales of incumbent I exporters) and an extensive margin (change in the measure of I exporters). The average sales of the incumbent I exporter actually goes down with an increase in M (Figure 9e). This occurs because of the increase in the final price due to an increase in $p_s(q)$ (re-call that the final consumer price is a constant mark-up over $p_s(q)$). But at the same time, the number of I exporters rises sharply, resulting in an increase in total sales as displayed in Figure 9f. Therefore, the extensive margin dominates the intensive margin following an increase in the measure of intermediaries.

4 Conclusion

In this paper, we have presented a framework to examine the role of intermediaries in international trade. Home producers are heterogeneous in terms of the quality of the product they sell. The basic assumption is that foreign consumers imperfectly observe the quality of home products. This asymmetric information about product quality generates an information cost of exporting that producers have to bear. We go on to examine the role of trade intermediaries in such a setting and their effect on the price, quantity and exporting decision of producers. Trade intermediaries have a screening technology whereby they can reveal the true quality of a product. We establish the existence of an equilibrium where producers with high qualities export directly, while the low quality producers export through intermediaries. In such an equilibrium, producers exporting through intermediaries do not need to incur the information cost but effectively end up paying a higher per unit cost. We perform a number of comparative exercises; our results are consistent with evidence.

Given that a large fraction of world trade is being carried out through intermediaries, it is important that we develop a good understanding of the intermediation sector. This will not only help us better understand the choices individual exporting firms make but also the pattern of trade at a more aggregate level. By affecting prices and quantity, intermediaries could also have a large impact on welfare. Clearly, more work needs to be done.

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Appendix

Proof that charging $\bar{p}$ is incentive compatible for young exporters in the absence of intermediaries: If a firm chooses $p > \bar{p}$, it will choose $p = \sigma + \bar{c}$ irrespective of the quality that consumers believe it has. The profit from this deviation is therefore,

$$\pi'(q) = \sigma exp\{\frac{1}{\sigma}(\theta q_L - \sigma - \bar{c})\}.$$

Deviating to a $p > \bar{p}$ is not profitable when $\pi'(q) < \pi(q)_{PE}$. This give us

$$exp\{\frac{\theta}{\sigma}(q_L - \bar{q})\}exp\{\frac{1}{\sigma}(\bar{p} - \sigma - \bar{c})\} < \frac{\bar{p} - \bar{c}}{\sigma}.$$

Since $\bar{p} - \sigma - \bar{c} < 0$, the required condition is $exp\{\frac{\theta}{\sigma}(q_L - \bar{q})\} < \frac{\bar{p}-\bar{c}}{\sigma}$. By choosing q_L small enough, this condition will always be satisfied. On the other hand, under the belief $\mu(q = \bar{q}|p < \bar{p}) = 1$, no firm would want to charge $p < \bar{p}$. Conditional on quality, profits of a firm are maximized when they charge $p_F(q) = \sigma + \bar{c}$. If they cannot charge this price, they will try to charge a price that is closest to $p_F(q)$, which is $\bar{p}$.

Proof of Lemma 1: Notice that given that a λ intermediary sells $q(\lambda)$, consumers' best response is to demand $x_F(q(\lambda))$. If the intermediary lies about his quality, not purchasing from him for T periods is also a best response for the consumers. This is because, one variety has measure zero; removing one variety from the consumption set has no effect on consumer utility.

Consider the intermediary. If the intermediary deviates to some quality $\hat{q}$, then given the consumers' belief, the profit of the intermediary is

$$\hat{r}(\lambda) = [p_I(q(\lambda)) - p_s(\hat{q})]exp\{\frac{1}{\sigma}(\theta q - p_I(q(\lambda)))\}n(\hat{q}, \lambda).$$

From the above equation, it is clear that $\hat{r}(\lambda)$ is decreasing in $\hat{q}$; a higher quality both raises the cost through a higher producer price and reduces the capacity, without any positive effect on demand. Accordingly, the maximum profit that an intermediary can earn is by selling the lowest possible quality $\underline{q}$.

Suppose the intermediary sells $q(\lambda)$ today. Then the present discounted value of its profit stream will be given by $r(\lambda) + \delta V$, where V is the continuation value. If selling $q(\lambda)$ is optimal, then $V = r(\lambda) + \delta V$, i.e., $V = \frac{r(\lambda)}{1-\delta}$. If the intermediary deviates, his present discounted value would be $\hat{r}(\lambda) + \delta^{T+1}V$. For the intermediary to sell $q(\lambda)$, we must have

$$r(\lambda) + \delta V > \hat{r}(\lambda) + \delta^{T+1}V.$$

Re-arranging, we have

$$\delta V(1 - \delta^T) > \hat{r}(\lambda) - r(\lambda).$$

Note that the right hand side of the above inequality is bounded above because $\hat{r}(\lambda)$ is bounded above $\forall\lambda$. Because V is increasing in δ, and δ^T can be made as small as possible by choosing a large enough T, the left-hand can be made as large as possible.

Selling $q(\lambda)$ every period is also SPNE strategy for the intermediary. To see this, observe that there are two possible histories of the game: (i) When the intermediary sold $q(\lambda)$ in the previous period, and (ii) When the intermediary deviated in the previous period. (i) is just the original game, and we have already proved that selling $q(\lambda)$ is a NE of this game. In case of (ii), the intermediary cannot sell and hence, selling $q(\lambda)$ is trivially a NE strategy.

Proof of Lemma 2: The first-order condition for the optimal choice of quality by an intermediary is $\frac{\partial r}{\partial q} = 0$. Totally differentiating the first-order condition yields,

$$\frac{\partial^2 r}{\partial^2 q}dq + \frac{\partial^2 r}{\partial q \partial \lambda}d\lambda = 0.$$

Re-arranging, we have

$$\frac{\partial^2 r}{\partial^2 q} = -(\frac{\partial^2 r}{\partial q \partial \lambda})\frac{d\lambda}{dq}.$$

Now,

$$\frac{\partial^2 r}{\partial q \partial \lambda} = \frac{\partial}{\partial q}(\frac{\partial r}{\partial \lambda}) = \frac{1}{\lambda}r\frac{\partial \epsilon_\lambda}{\partial q},$$

where ϵ_λ is the elasticity of $n(q, \lambda)$ with respect to λ and the second equality follows from the fact that $\frac{\partial r}{\partial q} = 0$. Given the assumptions on $n(q, \lambda)$, it is easy to check that $\frac{\partial \epsilon_\lambda}{\partial q} > 0$, and therefore, $\frac{\partial^2 r}{\partial q \partial \lambda} > 0$. Hence, for the second-order condition for profit-maximization to be satisfied, we must have $\frac{d\lambda}{dq} > 0$.

Positive assortative matching implies that if $q_1 \neq q_2$, then $m(q_1) \neq m(q_2)$. To prove that $m(q)$ is a function and not a correspondence, suppose that there exists intermediaries λ_1 and λ_2 ($\lambda_1 < \lambda_2$) such that producer q' matches with both intermediaries. The corresponding profits of the intermediaries are given by

$$r(\lambda_i) = \sigma exp\{\frac{1}{\sigma}(\theta q' - \sigma - p_s(q'))\}n(q', \lambda_i), i = 1, 2.$$

Without loss of generality, let us assume that q' is the optimal quality for intermediary λ_1. Then q' must satisfy,

$$\frac{\partial p_s(q')}{\partial q'} - \theta = \sigma \frac{\partial n(q', \lambda_1)}{\partial q} / n(q', \lambda_1).$$

Differentiating $r(\lambda_2)$ with respect to q yields

$$\frac{\partial r(\lambda_2)}{\partial q} = exp\{\frac{1}{\sigma}(\theta q - \sigma - p_s(q))\}[\frac{n(q, \lambda_2)}{\sigma}(\frac{\partial p_s(q)}{\partial q} - \theta) + \frac{\partial n(q, \lambda_2)}{\partial q}].$$

Evaluating the above expression at $q = q'$, we have

$$\frac{\partial r(\lambda_2)}{\partial q}|_{q=q'} = A[\frac{\partial n(q, \lambda_2)}{\partial q} / n(q, \lambda_2) - \frac{\partial n(q, \lambda_1)}{\partial q} / n(q, \lambda_1)].$$

where $A = exp\{\frac{1}{\sigma}(\theta q' - \sigma - p_s(q'))\}n(q, \lambda_2)$. Given our assumptions on $n(q, \lambda)$, the right-hand side of the above equation is positive, implying that $r(\lambda_2)$ is not maximized at $q = q'$. Hence, q' is not optimal for intermediary λ_2, yielding a contradiction.

Proof of Lemma 3: To prove the first part, suppose that, on the contrary, there are two disconnected sets of producers using intermediaries - $[q_1, q_2]$ and $[q_3, q_4]$, with $q_1 < q_2 < q_3 < q_4$. Positive assortative matching implies that $m(q_2) < m(q_3)$. But then, because the abilities of intermediaries is a continuous variable, we can always find λ' such that $m(q_2) < \lambda' < m(q_3)$ and no producer matches with intermediary λ'. Now, the profit of the intermediary $m(q_2)$ is

$$r(m(q_2)) = \sigma exp\{\frac{1}{\sigma}(\theta q_2 - \sigma - p_s(q_2))\}n(q_2, m(q_2))$$

Suppose intermediary λ' buys from producer q_2 by paying $p_s(q_2)$ and charges a final price of $\sigma + p_s(q_2)$. Its profit is then strictly higher than $m(q_2)$ because $n(q_2, \lambda') > n(q_2, m(q_2))$. In fact, intermediary λ' can pay $p_s(q_2) + \epsilon$ (ϵ small) to producer q_2 and continue to earn more than $m(q_2)$. But $r(m(q_2)) > 0$ meaning that the profits of λ' are strictly positive too. But then it cannot be that no producer matches with λ'.

To prove the second part, suppose a producer q is indifferent between using an intermediary and exporting directly. Then we must have $\alpha(q)\pi_{PE} + (1 - \alpha(q))\pi_F(q) = \pi_I(q)$. For producers with quality close to $\bar{q}$, $\alpha(q) \to 0$. Accordingly, we must have

$$\sigma exp\{\frac{1}{\sigma}(\theta q - \sigma - \bar{c})\} = [p_s(q) - \bar{c}]exp\{\frac{1}{\sigma}(\theta q - \sigma - p_s(q))\},$$

where the left-hand side denotes $\pi_F(q)$. Now, the problem $\max_p [p - \bar{c}]exp\{\frac{1}{\sigma}(\theta q - p)\}$ has the solution $p = \sigma + \bar{c}$. Therefore,

$$\begin{aligned}\sigma exp\{\frac{1}{\sigma}(\theta q - \sigma - \bar{c})\} &\geq [p_s(q) - \bar{c}]exp\{\frac{1}{\sigma}(\theta q - p_s(q))\}, \forall p_s(q) \\ &> [p_s(q) - \bar{c}]exp\{\frac{1}{\sigma}(\theta q - \sigma - p_s(q))\}.\end{aligned}$$

Therefore, for quality close to $\bar{q}$, exporting directly yields strictly higher profits.

Proof of Lemma 4: First we prove that $p_s(q_I^*) > \bar{p}$. Suppose not, i.e., suppose $p_s(q_I^*) = \bar{p}$. Then,

$$\begin{aligned}\frac{\pi_I(q_I^*)}{\pi_D(q_I^*)} &= \frac{[\bar{p} - \bar{c}]exp\{\frac{1}{\sigma}(\theta q_I^* - \sigma - \bar{p})\}}{\alpha(q_I^*)[\bar{p} - \bar{c}]exp\{\frac{1}{\sigma}(\theta \bar{q} - \bar{p})\} + (1 - \alpha(q_I^*))\sigma exp\{\frac{1}{\sigma}(\theta q_I^* - \sigma - \bar{c})\}} \\ &= \frac{[\bar{p} - \bar{c}]X}{\alpha(q_I^*) + (1 - \alpha(q_I^*))Y}.\end{aligned}$$

where $X = \frac{[\bar{p}-\bar{c}]exp\{\frac{1}{\sigma}(\theta q_I^* - \sigma - \bar{p})\}}{[\bar{p}-\bar{c}]exp\{\frac{1}{\sigma}(\theta \bar{q} - \bar{p})\}}$ and $Y = \frac{\sigma exp\{\frac{1}{\sigma}(\theta q_I^* - \sigma - \bar{c})\}}{[\bar{p}-\bar{c}]exp\{\frac{1}{\sigma}(\theta \bar{q} - \bar{p})\}}$. Note that $X < 1$, because $q_I^* > \bar{q}$ while $Y > 1$ by assumption. Accordingly, the above ratio is less than one. But this cannot be an equilibrium because we must have $\pi_I(q_I^*) \geq \pi_D(q_I^*)$. Therefore, we get a contradiction $\Longrightarrow$ We must have $p_s(q_I^*) \neq \bar{p}$. Furthermore, $p_s(q_I^*)$ must change in a way such that $\pi_I(q_I^*)$ rises. Now,

$$\begin{aligned}\frac{\partial \pi_I(q_I^*)}{\partial p_s(q_I^*)}|_{p_s(q_I^*)=\bar{p}} &= [1 - \frac{\bar{p} - \bar{c}}{\sigma}]exp\{\frac{1}{\sigma}(\theta q_I^* - \sigma - \bar{p})\} \\ &> 0.\end{aligned}$$

where the last line follows from $\bar{p} < \sigma + \bar{c}$. It follows that $p_s(q_I^*) > \bar{p}$. But then $p_s(q) > \bar{p}, \forall q$.