Statistical Tables

While the simplest way to calculate P-Values is to use the XLispStat cumulative density functions, the test-statistics for unit root and cointegration tests do not follow standard distributions. Accordingly, the next three pages contain the relevant statistical tables for Dickey-Fuller and Phillips-Perron unit root tests, for Engle-Granger cointegration tests based on unit root tests of regression residuals, and for Johansen cointegration tests.

The critical values for the unit root tests in the table that follows were calculated using Monte Carlo methods by David Dickey and Wayne A. Fuller and were obtained from their paper "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica*, Vol. 49, July 1981, pages 1062 and 1063, and from Walter Enders, *Applied Econometric Time Series*, Wiley Series in Probability and Mathematical Statistics, John Wiley & Sons, 1995, pages 223, 419 and 421, and from James D. Hamilton, *Time Series Analysis*, Princeton University Press, 1994, page 763. These critical values are based on sample sizes of 100 and remain unchanged when the Dickey-Fuller estimating equations are augmented by inclusion of lagged values of Δy_t to improve the fit as indicated by the AIC and SBC. Larger sample sizes will result in critical values that are slightly smaller in absolute value and smaller sample sizes will result in somewhat larger critical values.

STATISTICAL TABLES FOR UNIT ROOT TESTS

True Model Used to Generate the Data: $y_t = y_{t-1} + \epsilon_t$

1. Model Estimated: Dickey-Fuller $\Delta y_t = a_0 + a_1 y_{t-1} + a_2 t + \epsilon_t$
Phillips-Perron $y_t = \tilde{a}_0 + \tilde{a}_1 y_{t-1} + \tilde{a}_2 (t - n/2) + \tilde{\epsilon}_t$

Hypothesis	Test Statistic	Critical Values 10%	5%	1%
$a_1 = 0$, $\tilde{a}_1 = 1$	t-based	-3.15	-3.45	-4.04
$a_0 = 0$, $\tilde{a}_0 = 0$	t-based	2.73	3.11	3.78
$a_2 = 0$, $\tilde{a}_2 = 0$	t-based	2.38	2.79	3.53
$a_1 = a_2 = 0$, $\tilde{a}_1 = 1$ & $\tilde{a}_2 = 0$	F-based	5.47	6.49	8.73
$a_0 = a_1 = a_2 = 0$, $\tilde{a}_0 = \tilde{a}_2 = 0$ & $\tilde{a}_1 = 1$	F-based	4.16	4.88	6.50

2. Model Estimated: Dickey-Fuller $\Delta y_t = a_0 + a_1 y_{t-1} + \epsilon_t$
Phillips-Perron $y_t = \tilde{a}_0 + \tilde{a}_1 y_{t-1} + \tilde{\epsilon}_t$

Hypothesis	Test Statistic	Critical Values 10%	5%	1%
$a_1 = 0$, $\tilde{a}_1 = 1$	t-based	-2.58	-2.89	-3.51
$a_0 = 0$, $\tilde{a}_0 = 0$	t-based	2.17	2.54	3.22
$a_0 = a_1 = 0$, $\tilde{a}_0 = 0$ & $\tilde{a}_1 = 1$	F-based	3.86	4.71	6.70

3. Model Estimated: Dickey-Fuller $\Delta y_t = a_1 y_{t-1} + \epsilon_t$
Phillips-Perron $y_t = \tilde{a}_1 y_{t-1} + \tilde{\epsilon}_t$

Hypothesis	Test Statistic	Critical Values 10%	5%	1%
$a_1 = 0$, $\tilde{a}_1 = 1$	t-based	-1.61	-1.95	-2.60

CRITICAL VALUES FOR REGRESSION-RESIDUAL BASED COINTEGRATION TESTS

Estimated Cointegrating Regression Residual:

$$z_t = y_t - \beta_0 - \beta_1\, x_{1t} - \beta_2\, x_{2t} - \beta_3\, x_{3t} - \ldots\ldots - \beta_N\, x_{Nt}$$

Number of Variables $N+1$	Sample Size	Critical Values 10%	5%	1%
2	50	3.28	3.67	4.32
	100	3.03	3.37	4.07
	200	3.02	3.37	4.00
3	50	3.73	4.11	4.84
	100	3.59	3.93	4.45
	200	3.47	3.78	4.35
4	50	4.02	4.35	4.94
	100	3.89	4.22	4.75
	200	3.89	4.18	4.70
5	50	4.42	4.76	5.41
	100	4.26	4.58	5.18
	200	4.18	4.48	5.02
6	500	4.43	4.71	5.28

Notes and Sources: Standard Dickey-Fuller and Phillips Perron unit-root tests are applied to the regression residuals using the critical values above instead of those on the previous page, focussing entirely on the coefficients of the laggeed residual. Except for the case of 6 variables, these critical values were calculated using Monte Carlo methods by Robert F. Engle and Byung Sam Yoo and obtained from their paper "Forecasting and Testing in Co-Integrated Systems," *Journal of Econometrics*, Vol. 35, 1987, page 157. The critical values for the case of 6 variables using 500 observations were calculated by Peter C. B. Phillips and S. Ouliaris, "Asymptotic Properties of Residual Based Tests for Cointegration," *Econometrica*, Vol. 58, 1990, 165-93, and were obtained from James D. Hamilton, *Time Series Analysis*, Princeton University Press, 1994, page 766, Case 2. The complete set of Phillips-Ouliaris critical values distinguish between whether or not a constant and trend are included in the cointegrating regression. These values are so similar in the three cases to the ones calculated by Engle and Yoo, based on the inclusion of a constant but not trend, that the complexities of including them here are avoided.

CRITICAL VALUES FOR JOHANSEN COINTEGRATION TESTS

	Probability that Statistic Exceeds Entry					
$n-h$	0.10	0.05	0.01	0.10	0.05	0.01
	Unrestricted Estimation: Trend Drift in Data					
	L-max			Trace		
1	2.816	3.962	6.936	2.816	3.962	6.936
2	12.099	14.036	17.936	13.338	15.197	19.310
3	18.697	20.778	25.521	26.791	29.509	35.397
4	24.712	27.169	31.943	43.964	47.181	53.792
5	30.774	33.178	38.341	65.063	68.905	76.955
	Unrestricted Estimation: No Trend Drift in Data					
	L-max			Trace		
1	6.691	8.083	11.576	6.691	8.083	11.576
2	12.783	14.595	18.782	15.583	17.844	21.962
3	18.959	21.279	26.154	28.436	31.256	37.291
4	24.917	27.341	32.616	45.245	48.419	55.551
5	30.818	33.262	38.858	69.956	69.977	77.911
	Estimation and Data: No Trend Drift & Constant in Cointegrating Vector					
	L-max			Trace		
1	7.563	9.094	12.740	7.563	9.094	12.741
2	13.781	15.752	19.834	17.957	20.168	24.988
3	19.796	21.894	26.409	32.093	35.068	40.198
4	25.611	28.167	33.121	49.925	53.347	60.054
5	31.592	34.397	39.672	71.471	75.328	82.969

Notes and Sources: n is the number of variables and h is the number of cointegrating vectors under the null hypothesis. The critical values in the table are copied from Walter Enders, *Applied Economic Time Series*, Wiley Series in Probability and Statistics, 1995, page 420. The top two sections are identical to those found in James D. Hamilton, *Time Series Analysis*, Princeton University Press, 1994, pages 767 and 768, Cases 2 and 3.