ECO210: Mathematical methods for economic theory

Fall 2021

Overview

Course website: https://mjo.osborne.economics.utoronto.ca/index.php/course/index/14/index

Instructor: Martin J. Osborne

This course covers mathematical methods commonly used in economic theory. In conjunction with MAT223 (Linear Algebra I), it is designed to be sufficient preparation for third- and fourth-year economics courses.

The topics covered are multivariate calculus (focusing on the tools used in economics), concavity and convexity, constrained optimization, and differential equations. Illustrative examples are taken from economics, but the purpose of the course is to teach mathematical methods, not economic theory.

The main aim of the course is to teach you the techniques commonly used to solve the mathematical problems that arise in economics. A secondary aim is to teach you how to make rigorous mathematical arguments. The ability to make such arguments deepens your understanding of the techniques and also allows you to modify the techniques when they do not exactly fit a problem you have to solve. With the second aim in mind, I will lead you through some proofs.

The course content is sufficient mathematical preparation for a Master's degree in Economics. If you plan to continue to a PhD, you should consider taking more rigorous courses, like MAT235 or MAT237 and higher-level math courses. (These courses do not cover all the topics in this course, but are at a higher theoretical level.) Alternatively, you could take this course and take higher-level math courses later.

Text

All the material for the course is covered in an online book designed specifically for this course. (This resource bills itself as a "tutorial", but to avoid confusion with the weekly tutorials for the course, I call it a "book".) This book is intended to be self-contained, but if you wish to refer to another book, my favorite is *Mathematics for economic analysis* by Knut Sydsæter and Peter J. Hammond (Prentice Hall, 1995). Unfortunately this book is out of print. (The authors have written another related book, *Essential mathematics for economic analysis*, which does not fit the course as well (and is expensive).) If you are relatively comfortable with the material you could look at a somewhat more advanced book, *Mathematics for economists* by Carl P. Simon and Lawrence Blume (Norton, 1994). The level of this book is above the level of the course, but if you are like a formal approach you might find the book helpful.

Prerequisites and exclusions

The prerequisites for the course are ECO100Y1(67%)/(ECO101H1(63%), ECO102H1(63%))/ECO105Y1(80%)/ECO100Y5(67%)/(ECO101H5(63%), ECO102H5(63%))/(MGEA02H3 (67%), MGEA06H3 (67%)); MAT133Y1(63%)/(MAT135H1(60%),

MAT136H1(60%))/ MAT137Y1(55%)/MAT157Y1(55%) the recommended corequisites are ECO200Y1/ ECO204Y1/ ECO206Y1, and the exclusions are MAT235Y1, MAT237Y1.

I expect you to be familiar with basic mathematical concepts and the following topics, which are covered in the prerequisite courses.

1. Basic logic.
2. Matrices and solutions of simultaneous linear equations (including determinants and Cramer's rule). *Note*: **If you have not studied matrices previously, you need to do so before taking this course.** (You can either learn the material independently, or take a basic math course that covers them.)
3. One variable calculus (differentiation and integration, including exponential and logarithmic functions).
4. Basic multivariate calculus (partial differentiation).
5. Curve and set sketching.
6. Basic optimization for functions of a single variable (finding maxima and minima using calculus).

The first five topics are covered briefly in the first section of the online book, which is the material in the first week of the class. To check your knowledge, you should do all the exercises in the first section of the tutorial:

- Exercises on logic
- Exercises on matrices: determinants, inverses, and rank
- Exercises on solving systems of linear equations: Cramer's rule and matrix inversion
- Exercises on intervals and functions
- Exercises on calculus: one variable
- Exercises on calculus: many variables
- Exercises on graphical representation of functions.

You are prepared for the course if and only if you have little or no difficulty with these exercises.

I will cover the last topic with which you should be familiar (basic optimization) later in the course.

Class schedule

I will post lecture videos, slides, and tutorial and homework problems as the semester progresses.

Your solutions to each week's homework should be uploaded to Crowdmark. I will send you a link each week.

Week 1 (September 9 – September 15)

Topics

Logic
Matrices
Systems of linear equations
Intervals and functions
Calculus: one variable
Calculus: many variables

Monday September 13, 4pm – 5pm: Tutorial.

Wednesday September 15, 5pm: Homework 1 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 2 (September 16 – September 22)

Topics

Chain rule
Implicit differentiation
Differentials and comparative statics
Homogeneous functions

Monday September 20, 4pm – 5pm: Tutorial.

Wednesday September 22, 5pm: Homework 2 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 3 (September 23 – September 29)

Topics

Concave and convex functions of a single variable

Monday September 27, 4pm – 5pm: Tutorial.

Wednesday September 29, 2:10pm–2:55pm: Term Test 1. *Time is tentative. It may have to be changed, depending on room availability.*

Wednesday September 29, 5pm: Homework 3 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 4 (September 30 – October 6)

Topics

Quadratic forms: definitions
Quadratic forms: conditions for definiteness
Quadratic forms: conditions for semidefiniteness

Monday October 4, 4pm–5pm: Tutorial.

Wednesday October 6, 5pm: Homework 4 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 5 (October 7 – October 13)

Topics

Concave and convex functions of many variables
Quasiconcavity and quasiconvexity

Wednesday October 13, 5pm: Homework 5 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 6 (October 14 – October 20)

Topics

Optimization: introduction

Optimization: definitions
Existence of an optimum
Necessary conditions for an interior optimum

Monday October 18, 4pm–6pm: Tutorial. [Note: 2 hours.]

Wednesday October 20, 2:10pm–2:55pm: Term Test 2. *Time is tentative. It may have to be changed, depending on room availability.*

Wednesday October 20, 5pm: Homework 6 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 7 (October 21 – October 27)

Topics

Local optima
Conditions under which a stationary point is a global optimum

Monday October 25, 4pm–5pm: Tutorial.

Wednesday October 27, 5pm: Homework 7 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 8 (October 28 – November 3)

Topics

Optimization with an equality constraint: necessary conditions for an optimum for a function of two variables
Optimization with an equality constraint: interpretation of Lagrange multipliers
Optimization with an equality constraint: sufficient conditions for a local optimum for a function of two variables
Optimization with an equality constraint: conditions under which a stationary point is a global optimum

Monday November 1, 4pm–5pm: Tutorial

Wednesday November 3, 5pm: Homework 8 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 9 (November 4 – November 6 and November 14 – November 17)

Topics

Optimization with equality constraints: n variables, m constraints
The envelope theorem

Monday November 15, 4pm–5pm: Tutorial.

Wednesday November 17, 2:10pm–2:55pm: Term Test 3. *Time is tentative. It may have to be changed, depending on room availability.*

Wednesday November 17, 5pm: Homework 9 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 10 (November 18 – November 24)

Topics

Optimization with inequality constraints: the Kuhn-Tucker conditions
Optimization with inequality constraints: the necessity of the Kuhn-Tucker conditions
Optimization with inequality constraints: the sufficiency of the Kuhn-Tucker conditions
Optimization with inequality constraints: nonnegativity conditions
Optimization: summary of conditions under which first-order conditions are necessary and sufficient

Monday November 22, 4pm–5pm: Tutorial.

Wednesday November 24, 5pm: Homework 10 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 11 (November 25 – December 1)

Topics

Differential equations: introduction
First-order differential equations: existence of a solution
Separable first-order differential equations
Linear first-order differential equations
Differential equations: phase diagrams for autonomous equations

Monday November 29, 4pm–5pm: Tutorial.

Wednesday December 1, 2:10pm–2:55pm: Term Test 4. *Time is tentative. It may have to be changed, depending on room availability.*

Wednesday December 1, 5pm: Homework 11 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Week 12 (December 2 – December 8)

Topics

Second-order differential equations
Systems of first-order linear differential equations

Monday December 6, 4pm–5pm: Tutorial.

Wednesday December 8, 5pm: Homework 12 due. (To be submitted via Crowdmark. You will receive an email notification, with a URL.)

Finals (December 10 – December 21)

Final exam/assessment to be arranged by university

Components

Each week, you are expected to learn the material scheduled for the week. (The semester starts on a Thursday, so that is the day on which each week starts.) In the middle of each week, on Monday, the TA will hold a tutorial, and at the end of each week, on Wednesday at 5pm, the homework for the week is due (see the EVALUATION PAGE).

The course offers the material in the following formats.

Online book

The book ("tutorial") is available (for free) here.

Lectures

Pre-recorded lectures will be linked from the SCHEDULE PAGE.

Slides

The slides I use in the lectures will be linked from the SCHEDULE PAGE.

To learn the material, use whatever combination of these formats works for you.

The **only** way to master the material in this course, like other technical material, is to do lots of exercises. Just as you cannot read a manual about a programming language and then expect to be able to write a perfect program without any practice, you cannot expect to master the material in this course without putting it to work by doing exercises. So after you go through the relevant sections of the book, the slides, and/or the lectures for the week, you should put the material to work by engaging in the following activities.

Problems in the book

Each section in the book has a page of exercises. Each week, you should work through the exercises for the pages covered that week.

Tutorial

Every week (including the first week of class) the TA will hold a tutorial. I will assign problems specifically for each tutorial. (There will be a link from the SCHEDULE PAGE.) During each tutorial, *you* will solve these problems. The TA will give you some guidance, if necessary, but *you* will be expected to actively solve the problems during the tutorial. (You will not be expected to have tried to do the problems before the tutorial.)

Homework

Every week, I will assign homework problems, which will be due on Wednesday at 5pm. You will write your solutions on paper, scan them, then upload the scan to Crowdmark. (I will send you a link each week.) I will post answers to each homework soon after the deadline for its submission. The homeworks will be linked from the SCHEDULE PAGE, which will also have information about how to submit them. (See the EVALUATION PAGE for information about how your mark in the course will be determined.)

Term tests

There will be four 45-minute term tests. For details, see the SCHEDULE PAGE and the EVALUATION PAGE. Some of the problems on the tests and final assessment will be similar to exercises in the book, to tutorial exercises, and to homework exercises.

Final assessment/exam

See the EVALUATION PAGE.

See the DELIVERY PAGE for more information about the delivery methods for the components and the technology requirements for the course.

Delivery

Lectures

The lectures will be pre-recorded; you can watch them at any time. Links are posted on the SCHEDULE PAGE.

Tutorials

The tutorials may be delivered in person or online, depending on the university's rules and the TA's preferences. When the delivery format has been determined, the details will be posted on the SCHEDULE PAGE.

Office hours

Office hours will be held online. For details, see the HELP PAGE.

Tests and final exam

See the EVALUATION PAGE.

Technology requirements

You must have access to a **laptop or desktop computer** with a **strong, stable internet connection**. A phone is not an acceptable substitute — some required components may not be accessible on smartphones. A **camera and microphone** will be required for online office hours and, if the university does not allow in-person testing, for tests and exams. *High speed broadband access is highly recommended.*

Sometimes, of course, computers malfunction. You are responsible for ensuring that you maintain regular backup copies of your files, use antivirus software (if using your own computer), and schedule enough time when completing an assessment to allow for delays due to technical difficulties. Computer viruses, crashed hard drives, broken printers, lost or corrupted files, incompatible file formats, faulty internet, and similar mishaps are common issues when using technology; they are not acceptable grounds for deadline extensions.

Time zone

All times posted are in local Toronto time. If you are in a different time zone, please make sure you are aware of Toronto time in relation to your timezone. Errors in calculations are not an acceptable reason to miss tests or deadlines.

Evaluation

Academic integrity

Academic integrity is essential to the pursuit of learning and scholarship in a university, and to ensuring that a degree from the University of Toronto is a strong signal of each student's individual academic achievement. As a result, the University treats cases of cheating and plagiarism very seriously. The University of Toronto's Code of Behaviour on Academic Matters) outlines the behaviours that constitute academic dishonesty and the processes for addressing academic offences. All suspected cases of academic dishonesty will be investigated following procedures outlined in the Code of Behaviour on Academic Matters. If you have questions or concerns about what constitutes appropriate academic behaviour or appropriate research and citation methods, you are expected to seek out additional information on academic integrity from your instructor or from other institutional resources.

Your mark in the course

Your mark in the course will be based on your marks in the weekly homeworks (1% per complete homework submitted on time), four term tests (the best three of which count 18% each), and a

final assessment (34%).

Homeworks

Each week, homework will be due on Wednesday at 5pm. You will get 1 point for each completed homework you submit by this deadline. (There will be no exceptions.) Homework is "completed" if you provide a serious attempt to answer every question, showing how you obtained your answers. Simply stating an answer does not count as a "serious attempt".

Term tests

There will be four 45-minute term tests.

The emphasis in each test will be the material listed below. However, much of the later material depends on the earlier material, so the material for the tests is effectively cumulative.

Test 1
: Wednesday September 29, 2:10pm-2:55pm. Coverage: material in weeks 1 and 2.

Test 2
: Wednesday October 20, 2:10pm-2:55pm. Coverage: material in weeks 3, 4, and 5.

Test 3
: Wednesday November 17, 2:10pm-2:55pm. Coverage: material in weeks 6, 7, and 8.

Test 4
: Wednesday December 1, 2:10pm-2:55pm. Coverage: material in weeks 9 and 10.

No aids, including calculators, will be permitted in any term test or in the final assessment.

Your best three marks in these four tests will each receive a weight of 18%.

Final exam

Coverage: whole course, with more weight on meterial in weeks 11 and 12. Date and time to be arranged by the university. This exam will receive a weight of 34%. **If it is not in person, it may contain a significant oral component.**

Principles used in marking tests and exams

- You must give reasoning to get credit for an answer. If you give the right answer without any explanation you will get 0. For a problem whose solution requires a mathematical argument, an "explanation" must contain words that indicate how your mathematical arguments are linked, and how they answer the question.

  If, for example, a question asks you to find the minimizer of the function x^2, it is not sufficient to write "$2x = 0$, $x = 0$" or something like that. Instead, you need to incorporate your calculations into regular English sentences. You could write something like

  "The function is convex (because ...), so its minimizers are the values of x for which the derivative is zero. Differentiating with respect to x we obtain $2x$, so the derivative is zero if

and only if $x = 0$. Thus the minimizer of the function is $x = 0$."

Or you could use more mathematical notation and write something like

"Define the function f by $f(x) = x^2$. Then f is convex (because ...), so its minimizers are the values of x for which $f'(x) = 0$. We have $f'(x) = 2x$, so $f'(x) = 0$ if and only if $x = 0$. Thus the minimizer of the function is $x = 0$."

- You get 0 if you give two answers to a problem, one right and one wrong.
- If you give the right answer and the right reasoning, but in addition add some incorrect reasoning, you will get less than full credit.
- I do not take off points for poor English per se, **but** if the meaning of what you write is not clear you will lose points.
- I do not penalize small errors in algebra **unless** they lead to arguments that are simpler than the ones that arise in their absence.
- If you formulate a problem incorrectly, but use the correct methods correctly, you will not be penalized heavily **unless** your formulation leads to an analysis that is simpler than or very different from the one for the correct formulation.

Missed tests

If you miss a term test because of illness, you have the option to write a makeup test. There will be **one** makeup test, on **Wednesday December 8 from 2:10pm to 2:55pm**. This test will cover **all the material for the course** (regardless of which test you missed). If you do not write the makeup test, you will receive a mark of 0 for the missed test; there will be no makeup for the makeup.

Help

If you have a question about the material in the course, I strongly encourage you to post it on the FORUM. (Note that you can, if you wish, do so anonymously.) Doing so has several advantages.

- In formulating the question precisely, you may well find that you can answer it yourself.
- You'll probably get a pretty quick response.
- Everyone will see the question and responses.
- You'll get a certified written answer.
- There will be a nice written record of all the questions, which will help me improve the course.

Note that you will receive credit for particularly helpful responses to class members' questions on the FORUM. To receive such credit, your post must show your name — it cannot be anonymous.

For questions that are not suitable for the forum, we hold office hours.

TA office hours

The TA, Alexandre Lehoux, will hold office hours on the day before each term test and final exam. Details to be announced.

Instructor office hours

Martin Osborne holds office hours every Tuesday from 3pm to 4pm during the semester. Log in to the course website to make an appointment.

Economics Study Center

The Economics Study Center offers help for this course (and others). Here is their Canvas page: https://q.utoronto.ca/courses/188930.