ECO210: Mathematical methods for economic theory
Fall 2020

Overview

Course website: https://mjo.osborne.economics.utoronto.ca/index.php/course/index/13/index

Instructor: Martin J. Osborne

This course covers mathematical methods commonly used in economic theory. In conjunction with MAT223 (Linear Algebra I), it is designed to be sufficient preparation for third- and fourth-year economics courses.

The topics covered are multivariate calculus (focusing on the tools used in economics), concavity and convexity, constrained optimization, and differential equations. Illustrative examples are taken from economics, but the purpose of the course is to teach mathematical methods, not economic theory.

The main aim of the course is to teach you the techniques commonly used to solve the mathematical problems that arise in economics. A secondary aim is to teach you how to make rigorous mathematical arguments. The ability to make such arguments deepens your understanding of the techniques and also allows you to modify the techniques when they do not exactly fit a problem you have to solve. With the second aim in mind, I will lead you through some proofs.

The course content is sufficient mathematical preparation for a Master's degree in Economics. If you plan to continue to a PhD, you should consider taking more rigorous courses, like MAT235 or MAT237 and higher-level math courses. (These courses do not cover all the topics in this course, but are at a higher theoretical level.) Alternatively, you could take this course and take higher-level math courses later.

Text

All the material for the course is covered in an online book designed specifically for this course. (This resource bills itself as a "tutorial", but to avoid confusion with the weekly tutorials for the course, I call it a "book".) This book is intended to be self-contained, but if you wish to refer to another book, my favorite is *Mathematics for economic analysis* by Knut Sydsæter and Peter J. Hammond (Prentice Hall, 1995). Unfortunately this book is out of print. (The authors have written another related book, *Essential mathematics for economic analysis*, which does not fit the course as well (and is expensive).) If you are relatively comfortable with the material you could look at a somewhat more advanced book, *Mathematics for economists* by Carl P. Simon and Lawrence Blume (Norton, 1994). This book is a bit more advanced than the course, but if you are comfortable with a formal approach you might like it.

Prerequisites and exclusions

The prerequisites for the course are ECO100Y1(67%)/(ECO101H1(63%), ECO102H1(63%))/ ECO105Y1(80%); MAT133Y1(63%)/(MAT135H1(60%), MAT136H1(60%))/ MAT137Y1(55%)/ MAT157Y1(55%), the corequisites are ECO200Y1/ ECO204Y1/ ECO206Y1, and the exclusions are MAT235Y1, MAT237Y1.

I expect you to be familiar with basic mathematical concepts and the following topics, which are covered in the prerequisite courses.

1. Basic logic.
2. Matrices and solutions of simultaneous linear equations (including determinants and Cramer's rule). *Note*: **If you have not studied matrices previously, you need to do so before taking this**

course. (You can either learn the material independently, or take a basic math course that covers them.)
3. One variable calculus (differentiation and integration, including exponential and logarithmic functions).
4. Basic multivariate calculus (partial differentiation).
5. Curve and set sketching.
6. Basic optimization for functions of a single variable (finding maxima and minima using calculus).

The first five topics are covered briefly in the first section of the online book; I will briefly review them in the first class. To check your knowledge, you should do all the exercises in the first section of the tutorial:

- Exercises on logic
- Exercises on matrices: determinants, inverses, and rank
- Exercises on solving systems of linear equations: Cramer's rule and matrix inversion
- Exercises on intervals and functions
- Exercises on calculus: one variable
- Exercises on calculus: many variables
- Exercises on graphical representation of functions.

You are prepared for the course if and only if you have little or no difficulty with these exercises.

I will cover the last topic with which you should be familiar (basic optimization) later in the course.

If you need to review the material, you can refer to the text used in the prerequisite course, or read the book by Sydsæter and Hammond, or consult the first section of the on-line book. The following sections of Sydsæter and Hammond are relevant.

- Material you should know, very little of which I will review:
  - Chs. 1, 2, 3, 12, 13.
- Material you should know, some of which I will review:
  - Ch. 4 (but only the idea, not the details, of limits)
  - Ch. 5 except 5.4 (covered in the course), 5.5, and 5.6.
  - Ch. 6 through 6.5 (6.1--6.3: basic ideas only)
  - Section 7.1 (7.2 is covered in the course)
  - Ch. 8 through 8.4
  - Ch. 9 through 9.4
  - Ch. 10
  - Ch. 11 through 11.2
  - Ch. 15 through 15.6

Class schedule

I will post lecture videos, slides, and tutorial and homework problems as the semester progresses. The schedule of topics for future weeks is tentative.

The *compact* versions of the slides are best for printing, the *complete* ones best for viewing on a screen.

Week 1 (September 10 – September 16)

Topics

Logic
Matrices
Systems of linear equations
Intervals and functions
Calculus: one variable
Calculus: many variables

Thursday September 10, 4pm – 5pm: Tutorial. In this tutorial, in addition to guiding you to solve some problems, the TA will also show you how to combine photos of several pages into a single file, so that you know how to submit your homework.

Tuesday September 15, 5pm: Homework 1 due.

Week 2 (September 17 – September 23)

Topics

Chain rule
Implicit differentiation
Differentials and comparative statics
Homogeneous functions

Thursday September 17, 4pm – 5pm: Tutorial.

Tuesday September 22, 5pm: Homework 2 due.

Week 3 (September 24 – September 30)

Topics

Concave and convex functions of a single variable: definitions
Concave and convex functions of a single variable: differentiable functions
Concave and convex functions of a single variable: strict concavity and convexity

Thursday September 24, 3pm – 4pm: Practice session for test (attendance mandatory).

Thursday September 24, 4pm – 5pm: Tutorial.

Monday September 28, 4:10pm – 4:50pm: Test 1 (covers weeks 1 and 2).

Tuesday September 29, 5pm: Homework 3 due.

Week 4 (October 1 – October 7)

Topics

Quadratic forms: definitions
Quadratic forms: conditions for definiteness
Quadratic forms: conditions for semidefiniteness

Thursday October 1, 4pm – 5pm: Tutorial

Tuesday October 6, 5pm: Homework 4 due.

Week 5 (October 8 – October 14)

Topics

Concave and convex functions of many variables
Quasiconcavity and quasiconvexity

Thursday October 8, 4pm – 5pm: Tutorial

Tuesday October 13, 5pm: Homework 5 due.

Week 6 (October 15 – October 21)

Topics

Optimization: introduction
Optimization: definitions
Existence of an optimum

Thursday October 15, 2:10pm – 2:50pm: Test 2 (covers weeks 3 and 4).

Thursday October 15, 4pm – 5pm: Tutorial

Tuesday October 20, 5pm: Homework 6 due.

Week 7 (October 22 – October 28)

Topics

Necessary conditions for an interior optimum
Local optima
Conditions under which a stationary point is a global optimum

Thursday October 22, 4pm – 5pm: Tutorial.

Monday October 26, 4:10pm – 4:50pm: Test 3 (covers weeks 5 and 6).

Tuesday October 27, 5pm: Homework 7 due.

Week 8 (October 29 – November 4)

Topics

Optimization with an equality constraint: necessary conditions for an optimum for a function of two variables
Optimization with an equality constraint: interpretation of Lagrange multipliers
Optimization with an equality constraint: sufficient conditions for a local optimum for a function of two variables
Optimization with an equality constraint: conditions under which a stationary point is a global optimum

Thursday October 29, 4pm – 5pm: Tutorial.

Tuesday November 3, 5pm: Homework 8 due.

Week 9 (November 5 – November 8 and November 16 – November 18)

Topics

Optimization with equality constraints: n variables, m constraints
The envelope theorem

Thursday November 5, 4pm – 5pm: Tutorial.

Monday November 16, 4:10pm – 4:50pm: Test 4 (covers weeks 7 and 8).

Tuesday November 17, 5pm: Homework 9 due.

Week 10 (November 19 – November 25)

Topics

Optimization with inequality constraints: the Kuhn-Tucker conditions
Optimization with inequality constraints: the necessity of the Kuhn-Tucker conditions
Optimization with inequality constraints: the sufficiency of the Kuhn-Tucker conditions
Optimization with inequality constraints: nonnegativity conditions
Optimization: summary of conditions under which first-order conditions are necessary and sufficient

Thursday November 19, 4pm – 5pm: Tutorial.

Tuesday November 24, 5pm: Homework 10 due.

Week 11 (November 26 – December 2)

Topics

Differential equations: introduction
First-order differential equations: existence of a solution
Separable first-order differential equations
Linear first-order differential equations
Differential equations: phase diagrams for autonomous equations

Thursday November 26, 4pm – 5pm: Tutorial.

Monday November 30, 4:10pm – 4:50pm: Test 5 (covers weeks 9 and 10).

Tuesday December 1, 5pm: Homework 11 due.

Week 12 (December 3 – December 10)

Topics

Second-order differential equations
Systems of first-order linear differential equations

Thursday December 3, 4pm – 5pm: Tutorial.

Tuesday December 8, 5pm: Homework 12 due.

Thursday December 10, 2:10pm – 2:50pm: Test 6 (covers weeks 11 and 12). **NOTE NONSTANDARD DAY AND TIME!**

Finals (December 11 – December 22)

Date to be arranged: Final assessment.

Components

Each week, you are expected to learn the material scheduled for the week. The lectures will be pre-recorded, but the nominal class time is Thursday 2-4, and Thursday 4pm each week should be your target for working through the material. The material is offered in the following formats.

Online book
The book ("tutorial") is available (for free) here.

Slides
Slides will be linked from the SCHEDULE PAGE.

Lectures
Pre-recorded lectures, which will be linked from the SCHEDULE PAGE.

To learn the material, use whatever combination of these formats works for you.

The **only** way to master the material in this course, like other technical material, is to do lots of exercises. Just as you cannot read a manual about a programming language and then expect to be able to write a perfect program without any practice, you cannot expect to master the material in this course without putting it to work by doing exercises. So after you go through the relevant sections of the book, the slides, and/or the lectures for the week, you should put the material to work by engaging in the following activities.

Problems in the book
Each section in the book has a page of exercises. Each week, you should work through the exercises for the pages covered that week.

Tutorial
Every week (including the first week of class) the TA will hold an online tutorial Thursday 4pm-5pm. I will assign problems specifically for each tutorial. (There will be a link from the SCHEDULE PAGE.) During

each tutorial, *you* will solve these problems. The TA will give you some guidance, if necessary, but *you* will be expected to actively solve the problems during the tutorial. (You will not be expected to have tried to do the problems before the tutorial.)

Homework

Every week, I will assign homework problems, which will be due on Tuesday at 5pm. You will get 2 points for each completed homework you submit by this deadline; if you submit a completed homework after the deadline but before Wednesday at 5pm, you will get 1 point. (There will be no exceptions to these deadlines, for any reason.) Homework is "completed" if you answer every question and show how you obtained your answers; simply stating an answer is not sufficient. I will post answers to the homework each Thursday. The homeworks will be linked from the SCHEDULE PAGE, which will also have information about how to submit them. (See the EVALUATION PAGE for information about how your mark in the course will be determined.)

Term tests

There will be a 40-minute term test every second week. For details, see the SCHEDULE PAGE and the EVALUATION PAGE. Some of the problems on the tests and final assessment will be similar to exercises in the book, to tutorial exercises, and to homework exercises.

Final assessment

See the EVALUATION PAGE.

See the DELIVERY PAGE for more information about the delivery methods for the components and the technology requirements for the course.

Delivery

The course will take place completely online.

Lectures

The lectures will be pre-recorded; you can watch them at any time. Links are posted on the SCHEDULE PAGE.

Tutorials

The tutorials will be delivered online in the R4-5 timeslot. For details, see the SCHEDULE PAGE.

Office hours

Office hours will be held online. For details, see the HELP PAGE.

Tests and final exam

See the EVALUATION PAGE.

Technology requirements

You must have access to a **laptop or desktop computer** with a **strong, stable internet connection**. A phone is not an acceptable substitute — some required components may not be accessible on smartphones. A **camera and microphone** are required for tests and exams, and for online contact activities such as tutorials and office hours. *High speed broadband access is highly recommended.*

Sometimes, of course, computers malfunction. You are responsible for ensuring that you maintain regular backup copies of your files, use antivirus software (if using your own computer), and schedule enough time when completing an assessment to allow for delays due to technical difficulties. Computer viruses, crashed hard drives, broken printers, lost or corrupted files, incompatible file formats, faulty internet, and similar mishaps are common issues when using technology; they are not acceptable grounds for deadline extensions.

Time zone

All times posted are in local Toronto time. If you are in a different time zone, please make sure you are aware of Toronto time in relation to your timezone. Errors in calculations are not an acceptable reason to miss tests or deadlines.

Evaluation

Academic integrity

Academic integrity is essential to the pursuit of learning and scholarship in a university, and to ensuring that a degree from the University of Toronto is a strong signal of each student's individual academic achievement. As a result, the University treats cases of cheating and plagiarism very seriously. The University of Toronto's Code of Behaviour on Academic Matters) outlines the behaviours that constitute academic dishonesty and the processes for addressing academic offences. All suspected cases of academic dishonesty will be investigated following procedures outlined in the Code of Behaviour on Academic Matters. If you have questions or concerns about what constitutes appropriate academic behaviour or appropriate research and citation methods, you are expected to seek out additional information on academic integrity from your instructor or from other institutional resources.

Your mark in the course

Your mark in the course will be based on your marks in the weekly homeworks (2% per homework submitted on time, 1% if up to 24 hours late), six term tests (each of 5 best count 11%), and a 1.5 hour final assessment (21%). In addition, there will be a bonus of up to 2% for particularly helpful responses to other students' questions on the ONLINE FORUM.

Homeworks

You will receive 2 points for each of the 12 weekly homeworks that you submit by the deadline. The deadline for submitting each homework is Tuesday 5pm. If you submit a homework late, but before Wednesday 5pm, you will get 1 point. Homeworks submitted after Wednesday 5pm will receive no credit. (There will be no exceptions to these deadlines.) Homework is "completed" if you answer every question and show how you obtained your answers; simply stating an answer is not sufficient.

Term tests

There will be six 40-minute term tests. Each test will be invigilated. You will need a camera (your phone will suffice). The emphasis in each test will be the material for the previous two weeks. However, much of the later material depends on the earlier material, so the material for the tests is effectively cumulative.

Test 1
: Monday September 28, 4:10pm-4:50pm.

Test 2
: **Thurday** October 15, **2:10pm-2:50pm**. (Note day and time.)

Test 3
: Monday October 26, 4:10pm-4:50pm.

Test 4
: Monday November 16, 4:10pm-4:50pm.

Test 5
: Monday November 30, 4:10pm-4:50pm.

Test 6
: **Thursday** December 10, **2:10pm-2:50pm**. (Note day and time.)

No aids, including calculators, will be permitted in any term test or in the final assessment.

Each of your best five marks in the six term tests will receive a weight of 11%. (Your lowest mark in the term tests will be dropped.)

There will be no makeups for missed tests for any reason.

Final assessment

There will be a 1.5 hour invigilated final assessment, held in the exam period (December 11-22) at a time to be determined. This assessment may contain a significant oral component. It will receive a weight of 21%.

Bonus

You will receive a bonus of up to 2% for particularly helpful responses to other students' questions on the ONLINE FORUM. To receive this credit, you must post your responses under your name (not anonymously).

Principles used in marking tests and exams

- You must give reasoning to get credit for an answer. If you give the right answer without any explanation you will get 0. For a problem whose solution requires a mathematical argument, an "explanation" must contain words that indicate how your mathematical arguments are linked, and how they answer the question.

  If, for example, a question asks you to find the minimizer of the function x^2, it is not sufficient to write "$2x = 0$, $x = 0$" or something like that. Instead, you need to incorporate your calculations into regular English sentences. You could write something like

  "The function is convex (because ...), so its minimizers are the values of x for which the derivative is zero. Differentiating with respect to x we obtain $2x$, so the derivative is zero if and only if $x = 0$. Thus the minimizer of the function is $x = 0$."

  Or you could use more mathematical notation and write something like

  "Define the function f by $f(x) = x^2$. Then f is convex (because ...), so its minimizers are the values of x for which $f'(x) = 0$. We have $f'(x) = 2x$, so $f'(x) = 0$ if and only if $x = 0$. Thus the minimizer of the function is $x = 0$."
- You get 0 if you give two answers to a problem, one right and one wrong.
- If you give the right answer and the right reasoning, but in addition add some incorrect reasoning, you will get less than full credit.
- I do not take off points for poor English per se, **but** if the meaning of what you write is not clear you will lose points.
- I do not penalize small errors in algebra **unless** they lead to arguments that are simpler than the ones that arise in their absence.
- If you formulate a problem incorrectly, but use the correct methods correctly, you will not be penalized heavily **unless** your formulation leads to an analysis that is simpler than or very different from the one for the correct formulation.

Help

If you have a question about the material in the course, I strongly encourage you to post it on the FORUM. (Note that you can, if you wish, do so anonymously.) Doing so has several advantages.

- In formulating the question precisely, you may well find that you can answer it yourself.
- You'll probably get a pretty quick response.
- Everyone will see the question and responses.

- You'll get a certified written answer.
- There will be a nice written record of all the questions, which will help me improve the course.

Note that you will receive credit for particularly helpful responses to class members' questions on the FORUM. To receive such credit, your post must show your name — it cannot be anonymous.

For questions that are not suitable for the forum, we hold office hours.

TA office hours

The TA, Alexandre Lehoux, holds office hours every Thursday from 5pm to 6pm during the semester (except Reading Week). Log in to the course website for details.

Instructor office hours

Martin Osborne holds office hours every Friday from 3pm to 4pm during the semester (except Reading Week). Log in to the course website to make an appointment.