ECO210: Mathematical methods for economic theory

Fall 2019

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This course covers mathematical methods commonly used in economic theory. In conjunction with MAT223 (Linear Algebra I), it is designed to be sufficient preparation for third- and fourth-year economics courses.

The topics covered are multivariate calculus (focusing on the tools used in economics), concavity and convexity, constrained optimization, and differential equations. Illustrative examples are taken from economics, but the purpose of the course is to teach mathematical methods, not economic theory.

The main aim of the course is to teach you the techniques commonly used to solve the mathematical problems that arise in economics. A secondary aim is to teach you how to make rigorous mathematical arguments. The ability to make such arguments deepens your understanding of the techniques and also allows you to modify the techniques when they do not exactly fit a problem you have to solve. With the second aim in mind, I will lead you through some proofs.

The course content is sufficient mathematical preparation for a Master's degree in Economics. If you plan to continue to a PhD, you should consider taking more rigorous courses, like MAT235 or MAT237 and higher-level math courses. (These courses do not cover all the topics in this course, but are at a higher theoretical level.) Alternatively, you could take this course and take higher-level math courses later.

Text

All the material for the course is covered in an tutorial designed specifically for this course. When I last taught the course, most students said that they did not need to consult a book, but if you wish do so, my favorite is *Mathematics for economic analysis* by Knut Sydsæter and Peter J. Hammond (Prentice Hall, 1995). Unfortunately this book is out of print. (The authors have written another related book, *Essential mathematics for economic analysis*, which does not fit the course as well (and is expensive).) If you are relatively comfortable with the material you could look at a somewhat more advanced book, *Mathematics for economists* by Carl P. Simon and Lawrence Blume (Norton, 1994). This book is a bit more advanced than the course, but if you are comfortable with a formal approach you might like it.

Prerequisites

The prerequisites for the course are ECO100Y1(67%)/(ECO101H1(63%), ECO102H1(63%))/ ECO105Y1(80%); MAT133Y1(63%)/(MAT135H1(60%), MAT136H1(60%))/ MAT137Y1(55%)/ MAT157Y1(55%) and the corequisites are ECO200Y1/ ECO204Y1/ ECO206Y1.

I expect you to be familiar with basic mathematical concepts and the following topics, which are covered in the prerequisite courses.

1. Basic logic.

2. Matrices and solutions of simultaneous linear equations (including determinants and Cramer's rule). *Note*: If you have not studied matrices previously, you need to do so before taking this course. (You can either learn the material independently, or take a basic math course that covers them.)
3. One variable calculus (differentiation and integration, including exponential and logarithmic functions).
4. Basic multivariate calculus (partial differentiation).
5. Curve and set sketching.
6. Basic optimization for functions of a single variable (finding maxima and minima using calculus).

The first five topics are covered briefly in the first section of the on-line tutorial; I will briefly review them in the first class. To check your knowledge, you should do all the exercises in the first section of the tutorial:

- Exercises on logic
- Exercises on matrices: determinants, inverses, and rank
- Exercises on solving systems of linear equations: Cramer's rule and matrix inversion
- Exercises on intervals and functions
- Exercises on calculus: one variable
- Exercises on calculus: many variables
- Exercises on graphical representation of functions.

You are prepared for the course if and only if you have little or no difficulty with these exercises.

I will cover the last topic with which you should be familiar (basic optimization) later in the course.

If you need to review the material, you can refer to the text used in the prerequisite course, or read the book by Sydsæter and Hammond, or consult the first section of the on-line tutorial. The following sections of Sydsæter and Hammond are relevant.

- Material you should know, very little of which I will review:
  - Chs. 1, 2, 3, 12, 13.
- Material you should know, some of which I will review:
  - Ch. 4 (but only the idea, not the details, of limits)
  - Ch. 5 except 5.4 (covered in the course), 5.5, and 5.6.
  - Ch. 6 through 6.5 (6.1--6.3: basic ideas only)
  - Section 7.1 (7.2 is covered in the course)
  - Ch. 8 through 8.4
  - Ch. 9 through 9.4
  - Ch. 10
  - Ch. 11 through 11.2
  - Ch. 15 through 15.6

Problem sets

The **only** way to learn the material in this course is to do lots of problems! After each section of the tutorial is completed, you should do the exercises for that section. Some of the problems on the term tests will be similar to exercises in the tutorial.

Tutorials

Every week (including the first week), the TA will conduct a tutorial. I will assign problems specifically for each tutorial. During each tutorial, you will solve these problems. The TA will give you some guidance, if necessary, but **you** will be expected to actively solve the problems during the tutorial. You will not be expected to have tried to do the problems before the tutorial; each week I will post the questions when I post the slides for that week's lecture, at latest by the morning of the day of the class.

Class schedule

Class: Class R2-4 in BL205 (Martin J. Osborne). Tutorial R4-5 in BL205.

I will post slides for each class by the morning of the day of the class. The *compact* versions are best for printing, the *complete* ones best for viewing on a screen.

Week 1 (September 5)
: Review of logic, matrices, systems of linear equations, intervals and functions, one-variable calculus, and basic multivariate calculus. The chain rule and implicit differentiation.

Week 2 (September 12)
: Differentials and comparative statics, homogeneous functions

Week 3 (September 19)
: Concavity and convexity

Week 4 (September 26)
: Concavity and convexity

Week 5 (October 3)
: 2:10p-3:40p: Term test 1. 4p-5p: Concavity and convexity

Week 6 (October 10)
: Optimization, Interior optima

Week 7 (October 17)
: Optimization with equality constraints

Week 8 (October 24)
: Optimization with equality constraints

Week 9 (October 31)
: Optimization with inequality constraints

Week 10 (November 14)
: 2:10p-3:40p: Term test 2. 4p-5p: Optimization with inequality constraints

Week 11 (November 21)
: Differential equations

Week 12 (November 28)
: Differential equations

Evaluation

Your grade in the course will be based on your marks in two term tests and a three-hour final exam. The material covered by all tests will be *cumulative*. **Note that the tests will not be held in the regular classroom.**

Test 1
: Week 5: Thursday 3 October, 2:10-3:40. Location: EX 310 (Exam Center).

Test 2

Week 10: Thursday 14 November, 2:10-3:40. Location: EX 310 (Exam Center).

After each test there will be a class from 4:00 to 5:00.

No calculators will be permitted in either term test or in the final exam.

Each term test will receive a weight of 25% in the final mark, and the final exam will receive a weight of 50%.

Redemption

Low but passing marks in the term tests will be redeemable by a high mark on the final exam, and marks between 40% and 50% on the term tests will be *partly* redeemable. Specifically, for any term test on which your mark is at least 50% and is lower than your mark on the final exam, the 25% weight for the test will be transferred to the final exam. For a term test on which your mark is between 40% and 50% and is lower than your mark on the final exam, your term test mark, say x, will receive a weight of $2.5(50 - x)$% and the weight $25 - 2.5(50 - x)$% will be transferred to the final exam. (For example, if you score 45% on a test, then that test will receive a weight of 12.5% and the remaining 12.5% of the weight for that test will be transferred to the final exam.) **If you do not take a test or if you receive a mark of 40% or less, no weight for that test will be transferred to the final exam.** For example, if your mark on each term test is 60 and your mark on the final exam is 80, then your final mark will be 80, but if your mark on each term test is 40 and your mark on the final exam is 80, then your final mark will be 60.

Principles used in marking tests and exams

- You must give reasoning to get credit for an answer. If you give the right answer without any explanation you will get 0. For a problem whose solution requires a mathematical argument, an "explanation" must contain words that indicate how your mathematical arguments are linked, and how they answer the question.

  If, for example, a question asks you to find the minimizer of the function x^2, it is not sufficient to write "$2x = 0$, $x = 0$" or something like that. Instead, you need to incorporate your calculations into regular English sentences. You could write something like

  "The function is convex (because ...), so its minimizers are the values of x for which the derivative is zero. Differentiating with respect to x we obtain $2x$, so the derivative is zero if and only if $x = 0$. Thus the minimizer of the function is $x = 0$."

  Or you could use more mathematical notation and write something like

  "Define the function f by $f(x) = x^2$. Then f is convex (because ...), so its minimizers are the values of x for which $f'(x) = 0$. We have $f'(x) = 2x$, so $f'(x) = 0$ if and only if $x = 0$. Thus the minimizer of the function is $x = 0$."

- You get 0 if you give two answers to a problem, one right and one wrong.
- If you give the right answer and the right reasoning, but in addition add some incorrect reasoning, you will get less than full credit.

- I do not take off points for poor English per se, **but** if the meaning of what you write is not clear you will lose points.
- I do not penalize small errors in algebra **unless** they lead to arguments that are simpler than the ones that arise in their absence.
- If you formulate a problem incorrectly, but use the correct methods correctly, you will not be penalized heavily **unless** your formulation leads to an analysis that is simpler than or very different from the one for the correct formulation.

Sample midterm tests

Sample test 1 (2008) • Solutions
Sample test 2 (2008) • Solutions
Sample test 1 (2006) • Solutions
Sample test 2 (2006) • Solutions
Note that the material covered by these tests may differ somewhat from the material covered by the tests in the current class.

Sample final exam

Sample final exam • Sample final exam solutions
Note that the material covered by this exam may differ somewhat from the material covered by the exam in the current class.

Missed tests

If you miss the term test because of a serious illness, you must provide me with either a fully completed University of Toronto "Verification of Student Illness or Injury" Certificate, available at http://www.illnessverification.utoronto.ca/, a University of Toronto "Student health or disability related certificate", a letter from your college Registrar, or a letter from Accessibility Services. **I will not accept any other form.** Here are the rules.

1. You need to see the doctor **before** the test. I will not accept certificates that relate to visits after the test.
2. The illness must be serious enough that you are **unable** to write the exam. It is not enough that, given the illness, you would not do as well as you otherwise might. I will not accept certificates concerning minor ailments.
3. The certificate must be completed by a qualified medical doctor (not, for example, a chiropractor, acupuncturist, or other medical professional).
4. The certificate **must** include the doctor's OHIP registration number.
5. The certificate must be completed in full and must be completely legible. In particular, I will not accept a certificate unless the doctor's name, licensing body, and registration number are legible.
6. You **must** email me the day of the test, explaining the circumstances.
7. You **must** bring me the **original** certificate (not a scan or copy) in person as soon as you are better. I will not accept emailed certificates.
8. You must write a makeup test. The makeup test for the first term test will be held on **Friday October 11, 10:10am–12:00pm** and the makeup test for the second term test will be held

on **Friday November 22, 10:10am–12:00pm**. If you do not write the makeup exam for the test you missed, you will receive 0 for that test: there will be no makeup for the makeup.