# AIMING FOR THE GOAL:

# CONTRIBUTION DYNAMICS OF CROWDFUNDING*


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**Abstract**

We study reward-based crowdfunding, a new class of dynamic contribution games where a private good is produced only if the funding goal is reached by a deadline. Buyers face a problem of coordination rather than free-riding. A long-lived donor may alleviate this coordination risk, signaling his wealth through dynamic contributions. We characterize platform-, donor-, and buyer-optimal equilibrium outcomes, attained by Markov equilibria with simple donation strategies. We test the model's predictions using high-frequency data collected from the largest crowdfunding platform, Kickstarter. The model fits the data well, especially for predictions concerning comparative statistics, donation dynamics, and properties of successful campaigns.



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# 1 Introduction

Reward-based crowdfunding is a growing industry that allows entrepreneurs to raise funds directly from customers prior to product launch. A typical campaign specifies a product or reward, a price, a funding goal, and a deadline. Customers only receive the product and pay the price if the goal amount is raised by the deadline—often called an all-or-nothing mechanism. Crowdfunding platforms also allow supporters to donate to campaigns and receive nothing in return. In fact, we estimate that on Kickstarter, the largest crowdfunding platform, donations constitute 28% of all funds raised. Donations occur gradually over time, decline after a campaign reaches the goal, and are essential for 73% of campaigns that succeed at the deadline.

Motivated by crowdfunding, this paper introduces a new class of dynamic contribution games and examines it both theoretically and empirically. First, we allow agents to have different preferences: Buyers care about obtaining the product, and a representative donor simply cares about success of the campaign.[1] Second, unlike in classic dynamic contribution games, including Admati and Perry (1991) and Fershtman and Nitzan (1991), we do not study public goods. In reward-based crowdfunding, goods are excludable, as buyers cannot free-ride on others' contributions.[2] The dynamics of the game are driven by two sources of uncertainty: Buyers arrive randomly, and the donor wealth—which determines future donations—is unknown to buyers. Arriving buyers face a dynamic coordination problem as it is costly to commit to purchasing if the campaign is unlikely to succeed. The donor can alleviate some of this coordination risk by contributing over time. We characterize equilibria of this game, derive model predictions, and test the predictions with newly collected high-frequency data from Kickstarter.

We model a campaign as a finite-horizon, all-or-nothing contribution game. Randomly arriving buyers make a one-shot decision to either pledge to buy the product or choose an outside option. If the campaign fails, buyers receive their pledges back, but bear the opportunity cost of not exercising

[1] "Many backers are rallying around their friends' projects. Some are supporting a new effort from someone they've long admired. Some are just inspired by a new idea, while others are motivated to pledge by a project's rewards...." Source: https://help.kickstarter.com/hc/en-us/articles/115005047933-Why-do-people-back-projects-, accessed December 4, 2018.

[2] Incentives to free ride arise in classic contribution games. See Varian (1994), Marx and Matthews (2000), Campbell et al. (2014) and Cvitanić and Georgiadis (2016), for example. In Bonatti and Hörner (2011), agents may free ride on each other's effort. An excludable good eliminates these incentives for buyers. By assuming a representative donor, we also abstract away from potential free riding among donors. The donor, however, free rides on buyer contributions.

their outside option.[3] The long-lived donor, representing the network of family and friends of the entrepreneur, can choose to donate any amount, in any period.[4] If the campaign fails, donations are returned. If it succeeds, the donor receives a benefit equal to the donor's value, or wealth put aside for the campaign, less donations made. The donor wealth is unknown to buyers. As buyers make their pledging decision based on their assessment of the probability of success of the campaign, the donor can influence these expectations dynamically by signaling his wealth with gradual donations, while also avoiding unnecessary donations.

We are interested in perfect Bayesian equilibria (PBE) of this game that are ex-ante optimal for each of the three involved parties: the platform, the donor, and the buyers. Equilibrium analysis is challenging because of the rich state space and multiplicity of equilibria. We show that all optimal equilibrium outcomes can be attained by PBE with a common simple structure: Markov equilibria in which the donor plays a "pooling-threshold strategy." In these equilibria, a donor with wealth above the equilibrium-specified wealth threshold donates just enough to incentivize the next buyer to buy given the state of the campaign. Donor types with sufficient wealth play the same strategy, while donor types who cannot meet the threshold stop donating. Once the donor runs out of funds, buyers are no longer willing to pledge and the campaign "dies." The common structure of these equilibria allows us to derive testable predictions that are valid for the continuous-time limit of platform-, donor-, and buyer-optimal equilibria. Thus, we can remain agnostic about the specific equilibrium being played when testing the model with data.

We construct these equilibria using an innovative induction on the number of additional buyers necessary for the campaign to succeed, given current donations. Within each induction step, we embed an induction in time. To characterize optimal equilibrium outcomes for the platform and the donor, we recast their optimization problem, and we maximize over buyer beliefs instead of over all possible PBEs. The platform wants buyer beliefs to be "maximally optimistic" to ensure as many purchases as possible, regardless of the donation amount. Therefore, we consider reduced histories that ignore donation amounts and only keep track of whether a donation incentivizes the next potential buyer to purchase. In the relaxed problem, the platform chooses probabilities of reaching each of these reduced histories induced by buyer beliefs. We solve it recursively and verify a platform-

[3] The buyer may incur a cost, e.g., a transaction cost of typing in the credit card information or an opportunity cost of foregoing a currently available purchasing opportunity.

[4] Most crowdfunding platforms do not allow the entrepreneur to donate directly. This motivates our assumption on the payoffs of the donor. Equilibrium outcomes remain unchanged if the entrepreneur was the donor. See Section 7.3.

optimal outcome is attained by a Markov equilibrium with pooling-threshold donor strategies with the lowest wealth threshold possible. We also show that the outcome is unique in the continuous time limit.

Donor and platform incentives would be perfectly aligned if the donor were refunded excess contributions at the deadline. This may suggest that the donor “over-donates" in the platform-optimal equilibrium and can do better by trading off lower donations with a lower probability of success. We establish that the donor cannot benefit in this way, and the unique continuous-time limits of platform- and donor-optimal equlibria coincide. Beliefs in the buyer-optimal equilibrium are “maximally pessimistic” with the highest possible wealth threshold. In this equilibrium, buyers act as if no additional donations will occur, in contrast to the platform- and donor-optimal equilibrium where buyers must form more sophisticated expectations about donor wealth.

We test several model predictions using novel data collected from Kickstarter. Using a web scraper, we collect all available information on every campaign, every twelve hours, for over a year. The granularity of the data allows us to measure donations separately from purchases—we recover donations by tracking total revenue changes and comparing them with the total amount from buyers of each product. The difference yields donations, after adjusting for unobserved shipping costs.[5] Our samples contains more than than 3.9 million observations.

We show that donations constitute a sizable portion of revenue across campaign categories. More importantly, we show that two patterns noted in prior research—significant bunching of funds raised at exactly the goal amount, and the U-shaped pattern of contributions over time—are driven by a changing composition of contributors over time. Donations are essential for the success of campaigns that finish close to the deadline, whereas campaigns that finish early are driven by buyers.

The model predictions match the data well, both for within-campaign dynamics and across-campaign statistics. We find particularly strong support for predictions concerning comparative statistics, donor dynamics, and properties of unsuccessful campaigns. Support holds for both the average campaign and in the distribution of campaigns. For example, campaigns that succeed prior to the deadline are driven by buyers, whereas most late-finishing campaigns would have failed absent donations. We link the probability of success to goal amount and campaign length, and show

[5] Kickstarter counts shipping charges towards the goal, but we do not directly observe the amounts as we do not have individual level data. We impute shipping costs for every purchase by scraping shipping charges for every reward to every country.

that early donations vary with goal amount and campaign length, as predicted by the model. We also use an empirical model of campaign dynamics to explore properties for unsuccessful campaigns. We find that many campaigns are likely to never succeed both near the start and near the deadline, matching the bi-modal prediction of the theory.

We consider several extensions. We develop a stylized model of social learning, in which buyers receive private signals about the quality of a campaign and can learn from contributions of other buyers. This introduces a new trade-off: Although donations keep buyers optimistic about donor wealth, they also make success a weaker signal of quality because fewer buyers are needed for success. The structure of equilibria remains similar, however, some comparative statics change. For example, having a longer time horizon or a smaller goal is not necessarily better in terms of campaign outcomes.

We also explore the role of some features of the platform design. We show that a mechanism that allows donations only at the start or only at the deadline is strictly worse than the Kickstarter mechanism for the platform, the donor, and buyers. Further, not disclosing the campaign progress to buyers—effectively making the game static—can increase or decrease the probability of success.

Our dynamic framework is relevant for studying dynamic contribution games outside of reward-based crowdfunding, where there exist uncertainty in aggregate contributions and agents with heterogeneous preferences. This includes fundraising for projects such as conservation, museums, and higher education, where there are benevolent wealthy supporters or corporate donors, as well as self-serving small players— individuals who receive private benefits that relate to the cause or simply enjoy name recognition. Our framework can also be applied to situations studied by dynamic coordination games of regime change, in which small players play a coordination game in the presence of aggregate uncertainty, as in Angeletos et al. (2007). As in our game, a single informed benevolent player can dynamically help small players to coordinate more effectively.

We begin with institutional details on reward-based crowdfunding platforms and introduce our data set (Section 2). Our summary analysis motivates the model (Section 3) and equilibrium analysis that follows (Section 4). We test the model in Section 5; we discuss model extensions in Section 6; finally, we discuss features of platform design in Section 7.

## 1.1 Literature on Crowdfunding

Our paper is related to a new but growing literature on crowdfunding. Strausz (2017) solves a mechanism design problem in which there is uncertainty about the number of interested buyers, but not their purchasing decision, and the entrepreneur suffers from a moral hazard problem. We take the mechanism as given and study the contribution dynamics absent moral hazard concerns.[6] Ellman and Hurkens (2019b) also consider a mechanism design approach, absent moral hazard, with a focus on the incentives to discriminate between buyers with idiosyncratic valuations.[7] Bagnoli and Lipman (1989) show in a classic static public good environment with complete information that an all-or-nothing mechanism attains the first best.[8] All the above papers abstract from within-campaign dynamics, which is the focus of our work.

We share some basic features with the model of Alaei et al. (2016), who consider contribution dynamics but do not allow for agents with different incentives. Our work also relates to Sahm (2020); Liu (2018); Chakraborty and Swinney (2019); Chemla and Tinn (2020); Chakraborty and Swinney (2020). However, we add donations and examine the full dynamics of contributions.

We also contribute to the empirical literature on crowdfunding (Mollick, 2014; Sorenson et al., 2016; Kim et al., 2017; Babich et al., 2020), that has largely ignored the role of donations. Kuppuswamy and Bayus (2018) identify “family" contributions using historical data and show that they tend to occur in the first or last few days.[9] The role of family and friends has also been noted by Agrawal et al. (2015) in equity crowdfunding, where contributors obtain shares of a business. They argue that friends and family have valuable private information about the campaign.[10] Van de Rijt et al. (2014) use an experiment to show that donations affect the probability of receiving subsequent contributions on Kickstarter. Dai and Zhang (2019) argue that contributions prior to success are driven by pro-social incentives. To the best of our knowledge, we are the first to empirically examine the dynamic interactions of buyers and donors in crowdfunding.

[6]Ellman and Hurkens (2019a) add nuance to the model by relaxing the ex-post individual rationality constraint. Belavina et al. (2020) further distinguish between funds misappropriation and performance opacity.

[7]Chang (2020) analyzes the entrepreneur’s choice of a funding mechanism in a similar setup.

[8]Agrawal et al. (2014) call this a “provision point mechanisms” in their survey paper.

[9]Kickstarter used to show names of contributors before 2012. Kuppuswamy and Bayus (2018) mark a contributor as "family" if the last name matches the creator.

[10]Belleflamme et al. (2014) compare the effectiveness of reward-based and equity-based crowdfunding theoretically. Lee and Persson (2016) also highlight the importance of informal financing and donations in a classic investment setting. Other recent work in this area include Grüner and Siemroth (2017), Abrams (2017), and Li (2017).

# 2 Institutional Details, Data, and Summary Analysis

## 2.1 Crowdfunding Background

In reward-based crowdfunding, backers support the creation of a new product. The largest platforms are Indiegogo and Kickstarter. There are also many local platforms, such as Startnext for German-speaking countries, and Wishberry in India. Kickstarter has helped fund over 194,000 campaigns, raising over 5.5 billion dollars from 19 million people.[11]

In a reward-based crowdfunding campaign, an entrepreneur posts a campaign that specifies a funding goal ($G$), a funding deadline ($T$), prices for rewards ($p$) and if desired, capacity limits for each reward. Individuals can pledge to buy at a particular reward level, or pledge to simply donate any amount and receive no reward in return. Platforms also allow individuals to purchase the reward and contribute in excess of the reward amount.[12] Entrepreneurs may offer different versions of the product (versioning) or may offer different quantities of the product (quantity discounting). Reward-based crowdfunding uses an all-or-nothing model, which means that transactions are realized if and only if the funding goal is reached by the deadline.[13]

Both Indiegogo and Kickstarter limit the length of a campaign to at most two months, but most campaigns last thirty days. Once a campaign goes live, the core features cannot be changed: For example, $G, T$ and $p$ are fixed. The entrepreneur can take the campaign to a draft mode in order to edit the text, but this does not pause the stopwatch to the deadline. She can also provide updates to existing and potential backers, and backers can post comments during and after the campaign.

Campaigns are diverse, with products ranging from documentary films to high-tech products. Individuals and startups are common creators of campaigns. Perhaps the most well-known product that originated on Kickstarter is the Peloton Bike. Peleton posted their campaign in 2013; the campaign saw 297 backers pledge $307,332 on a $250,000 goal.[14] Five years later, the company

[11] Source: https://www.kickstarter.com/help/stats. Accessed on December 27, 2020.

[12] Some entrepreneurs request that buyers pledge an amount greater than the price if they are interested in obtaining additional product features. These are called “add-ons" or "optional buys." Some campaigns have stretch goals, which means that the entrepreneur informally adjusts the goal, and if met, adjusts the final product. Since these instances affect our interpretation of a donation, we remove any campaign in our sample that includes words related to add-ons, optional buys, and stretch goals. However, all our results are robust to including these campaigns.

[13] In practice, pledging may result in credit card authorization holds that become a charge if the goal is met, or drop off in case of failure.

[14] https://www.kickstarter.com/projects/568069889/the-peloton-bike-bring-home-the-studio-cycling-exp/description

had a valuation of $4 billion.[15]

## 2.2 Data Sample

We create a new data set on all campaigns posted on Kickstarter from March 2017 to September 2018. Using a web scraper, we track the progress of each campaign, every twelve hours, from its inception to its ending time. The panel structure of the data allows for studying campaign dynamics. In total, the data sample contains roughly 3.9 million observations.

Whereas prior research on Kickstarter uses data reporting an aggregate measure of contributions, our data collection scripts capture some information on buyers and donors separately.[16] By processing the source code of the campaign web pages, we recover the count of backers for each reward as well as the count of donors who did not purchase a reward.[17] We also capture total revenues and revenues coming from the purchase of rewards alone. The difference comprises donor contributions—coming from either the donate option, or from buyers paying more than the posted price—plus any incremental shipping revenue from purchases.[18] This is because shipping costs are included in the progress towards the goal but are not included in the base prices listed on the platform. That is, we see both left-hand-side variables in the equation below, but only the sum of the right-hand-side variables.

$$\text{Total Revenue}_t - \text{Buyer Revenue}_t = \text{Donor Revenue}_t + \text{Shipping Costs}_t.$$

Shipping costs are not known exactly as we do not have individual purchase data. To address this issue, we collect shipping costs for every campaign reward-country combination. We then assign a shipping cost to each observed purchase. The most frequently seen shipping options are free shipping, single-rate shipping, or worldwide shipping with region-specific or country-specific

[15] https://www.nytimes.com/2018/08/03/technology/pelotons-new-infusion-made-it-a-4-billion-company-in-6-years.html

[16] Kuppuswamy and Bayus (2018) identify “family" contributions using last name matching as contributor names were visible before 2012. Other work has mostly utilized data from Kicktraq, Kaggle, or constructed data with similar features, that do not allow for the dynamic investigation of buyers and donors separately. Instead, revenues are recorded as the sum of revenues coming from both buyers and donors, divided by the total number of buyers and donors in each period.

[17] This implies that we observe a lower bound on the count of donors because if a buyer also donates, we observe this as a single contributor. The bound is informative since over a third of donations occur in periods in which there are no purchases. It is also true that the change in the number of total backers (purchases and donations) exceeds the number of reward purchases, so it is not the case that most donors are also purchasers.

[18] Our data do not allow us to separate pure donations from donations occurring from contributions from buyers above the posted price.

prices. We complete our analyses under three assignments: (i) least-expensive shipping, (ii) all buyers are located in the United States, and (iii) most-expensive shipping.[19] Specifications (i) and (iii) are important because they provide lower and upper bounds of donations. In our baseline results, we use (ii); the other specifications appear in Appendix C. In our calculations, we also incorporate the bound

$$\text{Shipping Costs}_t \leq \text{Total Revenue}_t - \text{Buyer Revenue}_t,$$

since donations are positive contributions to campaigns. After subtracting shipping charges, we recover the flow of donations to every campaign, every twelve hours.[20]

We do not observe individual purchases or individual-specific donations. However, since the median number of donors per period is one, we can usually capture the actual donation amount after subtracting shipping cost estimates. We define a buyer to be an individual who pledges for any reward; however, some rewards may better be classified as a donation. An example would be if the lowest reward is a thank-you card. Another example for a disguised donation is an expensive reward that allows the buyer to meet with the entrepreneur. We repeat all our analyses treating the most-expensive and least-expensive rewards as donations (see Online Appendix C).

Table 1 shows summary statistics for the 30,897 campaigns included in the sample.[21] Only 51.6% of campaigns are successful.[22] The table shows overall sample means and means for unsuccessful (Uns.) and successful (Suc.) campaigns, respectively. There is a positive correlation between campaign length and goal ($\rho = .16$). The average goal amount is \$15,346 , while the median goal amount is \$5,800. Unsuccessful campaigns tend to have higher goals and more revenues coming from donations than successful campaigns.

All campaigns have a donation option, and most campaigns offer several different reward levels

[19] With USA shipping, we assume all products are delivered to the United States. This is motivated by the fact that most campaigns originate from the United States.

[20] If we were able to track campaigns continuously, we would observe every single purchase and/or donation, but we would still need to address the shipping-cost issue. We chose twelve-hour increments to keep data collection manageable.

[21] We winsorize the sample by dropping the bottom 0.5% and the top 0.5% of campaigns in terms of the goal amount. This removes campaigns with low \$1 goals and campaigns with several million dollar goals (one in the billions). These extreme values do impact some means, such as average goal, but medians are unchanged. In addition, we drop campaigns that were removed by the creator and campaigns under copyright dispute.

[22] This number is higher than the statistic reported by Kickstarter (38%) for all campaigns since the platform's inception. One reason is that we exclude campaigns with optional buys and add-ons. These campaigns have a lower probability of success. After including these campaigns, the probability of success is 41%, suggesting the probability of success has increased since Kickstarter's inception. Source: https://www.kickstarter.com/help/stats.

Table 1: Summary Statistics for the Data Sample

| Variable | Mean (All) | Mean (Uns.) | Mean (Suc.) | Median | 5th % | 95th % |
|---|---|---|---|---|---|---|
| Project Length | 32.3 | 34.1 | 30.6 | 30.0 | 15.0 | 60.0 |
| Goal ($) | 15,346.2 | 21,696.7 | 9,393.6 | 5,800.0 | 350.0 | 58,093.6 |
| Number of Rewards | 7.7 | 7.1 | 9.6 | 7.0 | 1.0 | 17.0 |
| Donor Revenue (per period) | 25.9 | 4.8 | 48.1 | 0.0 | 0.0 | 65.0 |
| Buyer Revenue (per period) | 171.2 | 19.5 | 329.7 | 0.0 | 0.0 | 577.3 |
| Percent Donations at Deadline | 28.0 | 32.8 | 23.4 | 16.0 | 0.0 | 100.0 |
| Number of Projects | 30,897 | 14,949 | 15,948 | – | – | – |

Note: Statistics are calculated for the 30,897 campaigns included in the sample after data cleaning. A period is twelve hours. Means are computed for all campaigns (All) - unsuccessful campaigns (Uns.) and successful campaigns (Suc.).

(or buckets). The average number of buckets in any campaign is 7.7. Successful campaigns have more reward levels. We find that 65.4% of buckets do not have a capacity limit and when capacity limits are used by entrepreneurs, only 17.3% of buckets ever reach their limit. In the model, we abstract from multiple reward levels and implications of versioning or price discrimination.

Table 1 also shows that donors are important: Donations constitute 28.0% of all total contributions made on Kickstarter, with the median campaign receiving 16% of revenues from donations. We calculate average revenues of $171.2 and $25.9 per twelve hours for buyers and donors, respectively. Both distributions are heavily skewed toward smaller amounts. The median donation is $33.3. Donations also occur infrequently. For successful campaigns, donations are zero in 92.9% of observations. Unsuccessful campaigns receive lower revenues from both sources. Finally, we observe that roughly one-third of donations occur in periods in which there are no buyers.

Table 2 presents the same summary statistics for the top four categories, as measured by the number of campaigns: design, film and video, music, and technology. The table shows there is rich heterogeneity in the types of campaigns and that donations are important across diverse categories. For example, music campaigns are twice as likely to succeed as technology campaigns. Music campaigns have one-fourth the average goal amount of technology campaigns. The table also shows that donations constitute at least 16.6% of total revenue for all four categories. Design has the lowest percentage of revenue from donations across all categories in the data. The categories with the

Table 2: Top Category Summary Statistics

| | Design | Film & Video | Music | Technology |
|---|---|---|---|---|
| Project Length | 33.7 | 31.7 | 32.3 | 35.3 |
| | (10.6) | (11.5) | (11.4) | (11.2) |
| Goal ($) | 19,078.2 | 16,562.7 | 8,445.8 | 34,602.8 |
| | (29628.7) | (32,235.9) | (17,806.4) | (52,263.2) |
| Number of Rewards | 8.4 | 8.0 | 7.9 | 6.7 |
| | (5.1) | (5.8) | (6.0) | (4.8) |
| Donor Revenue | 35.3 | 36.7 | 21.8 | 27.7 |
| (per period) | (378.2) | (341.6) | (181.4) | (363.8) |
| Buyer Revenue | 409.5 | 76.1 | 60.1 | 321.7 |
| (per period) | (2,767.9) | (475.7) | (308.2) | (2,123.6) |
| Percent Donations | 16.6 | 41.9 | 36.8 | 27.5 |
| at Deadline | (23.4) | (29.7) | (29.4) | (33.6) |
| Percent Donations | 21.8 | 28.3 | 30.3 | 11.6 |
| of Goal | (68.7) | (59.4) | (121.7) | (39.6) |
| Percent Successful | 52.5 | 55.5 | 66.3 | 34.0 |
| Number of Projects | 4,343 | 3,329 | 3,033 | 3,235 |

Note: Summary statistics for the top four Kickstarter categories, based on the number of campaigns within a category. Standard deviation reported in parentheses.

highest fraction of donations are dance and journalism—with donations between 42-51% of total revenue.

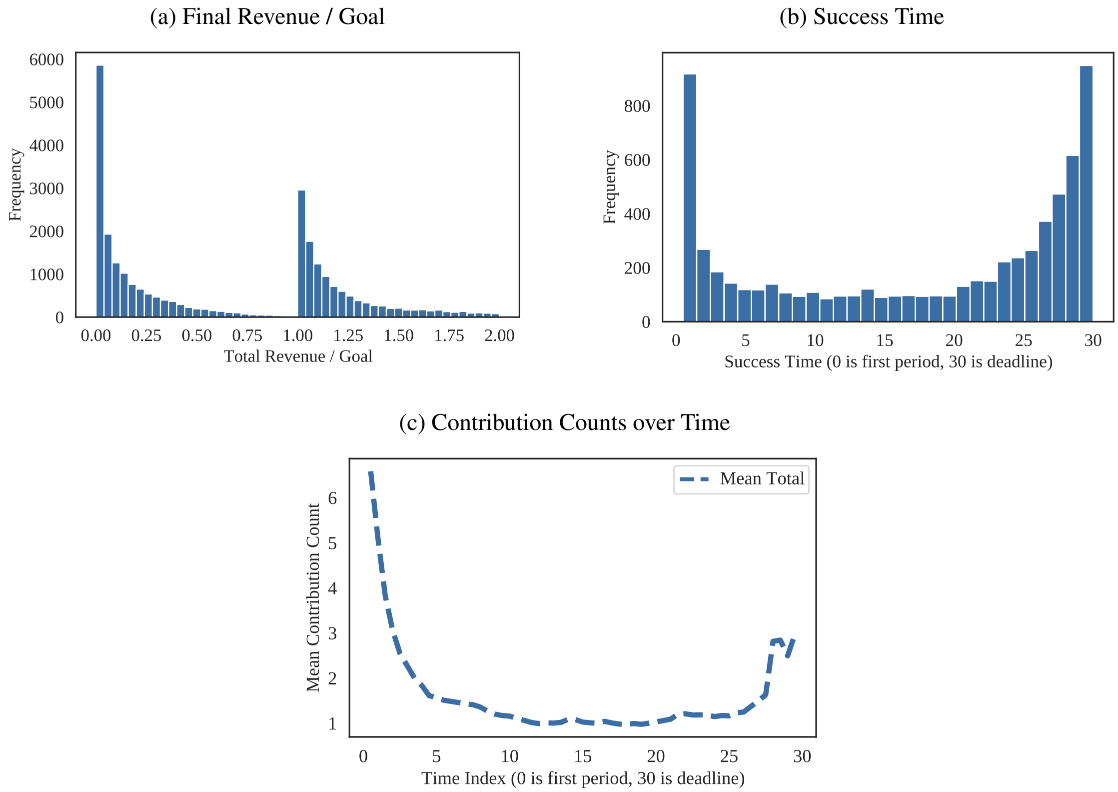
### 2.3 Descriptive Analysis

We start with some stylized facts about buyer and donor contribution dynamics to motivate our model. After presenting the model and conducting equilibrium analysis, we revisit the data and test several model predictions.

**Campaign Revenues, U-Shaped Contributions, and Success Times**

Figure 1 presents summary data on campaign outcomes. The top row of the figure contains histograms of two metrics: (a) revenue relative to goal ($R/G$) at the end of the campaign and (b) the period in which successful campaigns reach their goal. Henceforth, we refer to the period in which a

Figure 1: Frequency Histograms of Final Revenue and Success Time




Note: (a) Total campaign revenue is the sum of donations and purchases. The fraction is defined as total revenue at the deadline divided by the campaign goal. (b) For 30-day campaigns only. (c) The mean count of contributions (donations and purchases) for 30-day campaigns. $t = 0$ corresponds to the first day of the campaign. $t = 30$ corresponds to the time at which the campaign ends.

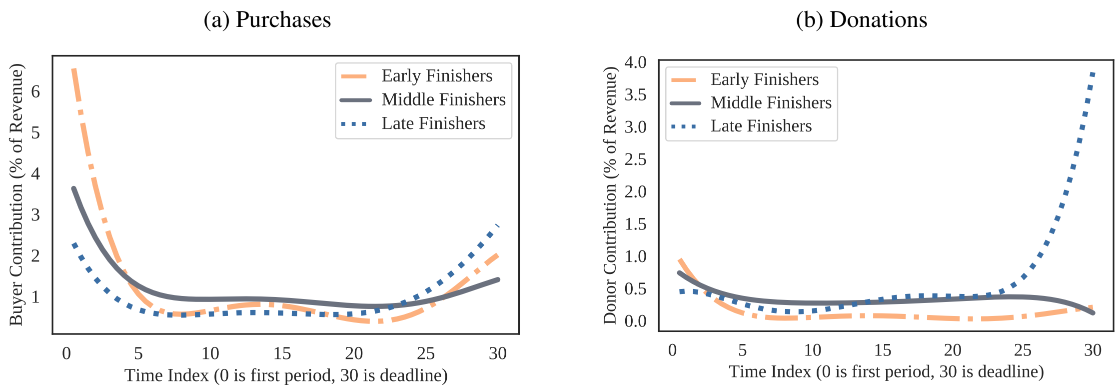
campaign reaches its goal as the *success time*. The top-left panel (a) shows considerable bunching at zero and one. In other words, most campaigns are either unsuccessful with very little money raised or they raise exactly their goal. There is a thin and long tail beyond $R/G = 2$ that is not shown. The top-right panel (b) shows that success times are bimodal, with many campaigns succeeding both close to the start and close to the deadline. In the figure, we focus on 30-day campaigns in order to keep the time index constant.[23] The time index $t = 0$ corresponds to the first day the campaign is offered and $t = 30$ corresponds to the end time. This horizon will also be used with all subsequent time graphs unless specifically noted. Finally, panel (c) depicts another U-shaped pattern noted in Kuppuswamy and Bayus (2018), that the average number of contributors over time spike at the beginning and end of campaigns.

[23] Recall that more than half of campaigns utilize this time horizon.

### Different roles of buyers and donors

Our novel data allow us to explain an important driver of the patterns documented above: The proportion of buyer and donor contributions varies by success time and time remaining until the deadline. That is, investigating average contributions masks an important compositional change in contributions over time.

Figure 2: Average Contributions of Buyers and Donors over Time for Successful Campaigns




Note: Percentage of Revenue is defined as the amount of purchases (donations) in a period, divided by the total amount of revenue (donations plus purchases) at the deadline. There are three lines, corresponding to: 1) campaigns that are successful within the first three days of posting, 2) campaigns that are successful in the last three days before the ending time, and 3) campaigns that are successful in days 3-27. Only 30-day campaigns are graphed. $t = 0$ corresponds to the first day of the campaign; $t = 30$ corresponds to the ending time.

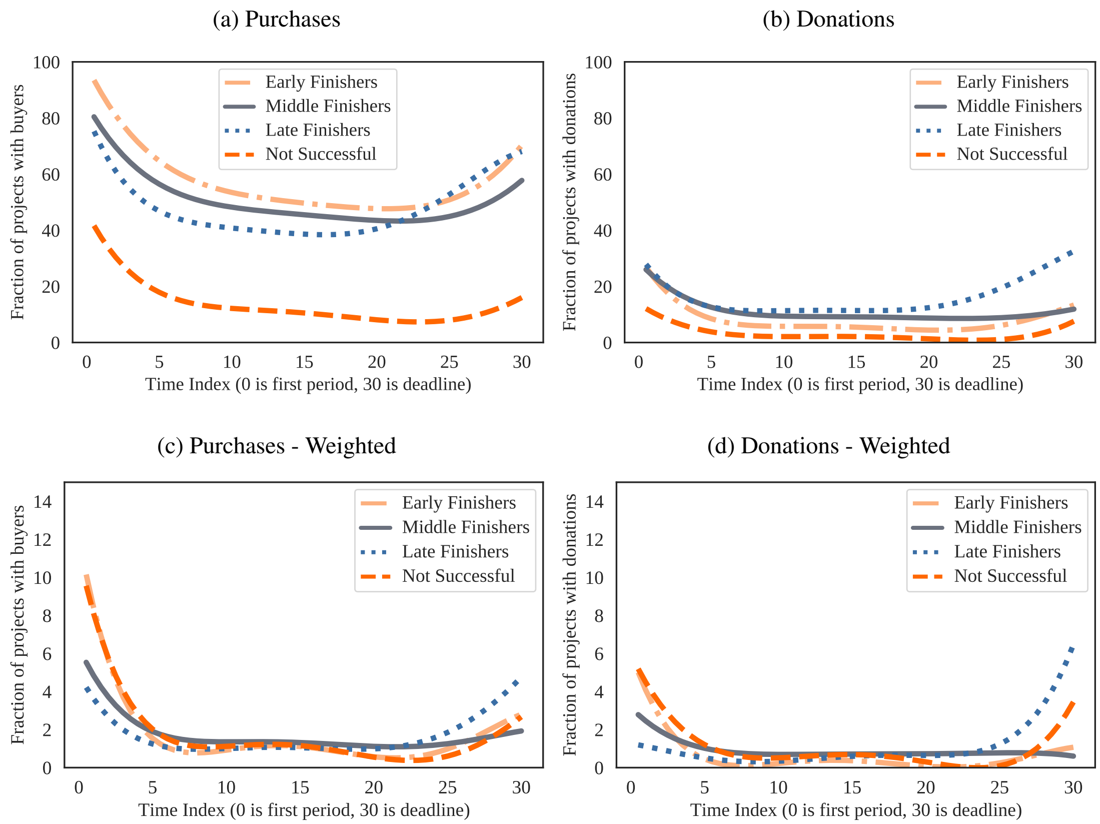
To demonstrate this, we partition campaigns into three groups: campaigns that succeed in the first three days (*early finishers*), those that succeed in days 3-27 (*middle finishers*), and those that succeed in the last three days before the deadline (*late finishers*). There are 1,371, 3,123, and 2,236 campaigns in each group, respectively. Figure 2 plots contributions (as a percentage of total revenue at the deadline) from buyers (panel a) and donors (panel b) for these three groups separately.[24] The figure shows that purchases exceed donations by a three-to-one margin for early-finishing campaigns, but donations exceed purchases for late-finishing campaigns. That is, the deadline effect noted in prior research is mostly (more than half) driven by donations. We estimate that nearly 18.3% (median, 16.0%) of total cumulative revenue is gathered within the first day for early-finishing campaigns.[25] The large mass of middle finishers see lower donations and lower

[24]Dividing by total revenue allows us to controls for the fact that some campaigns raise much more than the goal while others just reach the goal.

[25]A spike in purchases at the beginning is likely driven by the platform advertising new campaigns. The fact that a

purchases in both early and later periods.

Figure 3: Percentage of Campaigns that Receive Purchases/Donations over Time




These figures show the percentage of campaigns that have donations or purchases over time, for 30 day campaigns. 30 denotes the campaign deadline. Four lines are shown: early finishers, middle finishers, late finishers and unsuccessful campaigns. The bottom panels are weighted by within-campaign revenue. For example, if a campaign receives donations every period, the top donation graph for this single campaign would be a horizontal line at 100. However, if 90% of donations in terms of dollars occur in in the first period, the weighted graph would place 90% of the campaign weight at the first period with the remaining 10% allocated to the donations for the remaining periods.

To show that the changing composition of contributors is omnipresent, we investigate the percentage of campaigns that receive purchases and donations over time. Figure 3 plots the percentage of campaigns that see purchases or donations over time, for early-, middle-, and late finishers, and unsuccessful campaigns. In the bottom panels, we plot the percentages weighted by within campaign buyer/donor revenues. For example, if the *early finishers* contained a single campaign that experienced donations every period, the line would be horizontal at 100%. However, if total dona-

mass of campaigns finish at the beginning is not inconsistent with our model. Some campaigns likely have a very high arrival rate, and after success, the arrival rate changes. This could either be due to less promotional activity or higher prices.

tions equaled \$100 and \$99 of those donations occurred in the last period, the line would be close to zero except for the last period, where it would be close to 100%. Collectively, the figure shows that most campaigns receive purchases every period, but weighted by revenues, unsuccessful campaigns receive purchases mostly at the beginning of the campaign. The same pattern is true for donations, except that donations are less frequent than purchases. The bottom right graph shows the significance of donations for both late-finishing and unsuccessful campaigns.

Table 3: Descriptive Statistics for Early, Middle and Late Finishing Campaigns

| Variable | Early | Middle | Late |
|---|---|---|---|
| Goal (\$) | 8,148.3<br>(16,306.4) | 8,382.5<br>(15,322.2) | 11,119.4<br>(16,853.8) |
| Number of Rewards | 9.4<br>(6.9) | 9.3<br>(5.8) | 10.1<br>(6.6) |
| Average Price | 260.7<br>(516.3) | 265.5<br>(414.9) | 345.8<br>(435.9) |
| $R/G$ | 7.4<br>(37.4) | 1.5<br>(1.0) | 1.1<br>(0.1) |
| $D/R(\%)$ | 10.0<br>(14.8) | 21.6<br>(21.7) | 36.8<br>(23.8) |
| $D/G(\%)$ | 52.5<br>(193.2) | 27.9<br>(27.6) | 38.9<br>(25.2) |
| Number of Projects | 1,371 | 3,123 | 2,236 |
| Top Categories | Theater<br>Design<br>Games | Theater<br>Music<br>Design | Theater<br>Film & Video<br>Music |

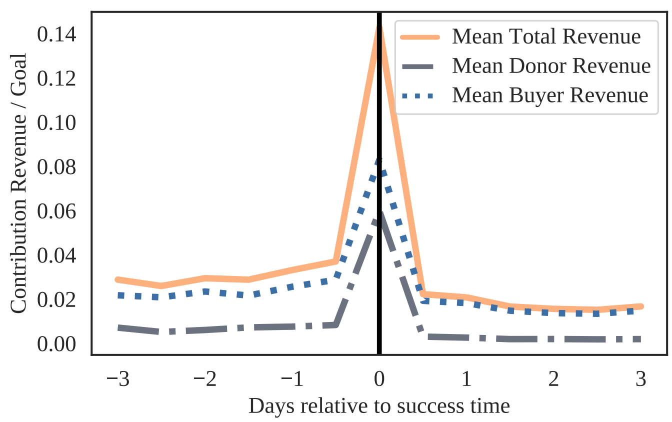
Note: Summary statistics for successful campaigns partitioned by success time. Only 30-day campaigns are included. Early finishers complete within three days. Late finishers complete in the last three days. All other campaigns are included in the middle category. Standard deviation reported in parentheses.

In Table 3, we present descriptive statistics for the three groupings of campaigns and show that early-finishing campaigns are quite different from late-finishing ones. Early finishers tend to raise much more money relative to the goal (with mean $R/G = 7.4$ for early finishers versus $R/G = 1.1$ for late finishers, and medians of 3.3 and 1.0, respectively). Early- and late-finishing campaigns tend to fall into different categories. Theater, games, and design are the top three categories for early-finishers; film and video, music, and theater are the top three categories for late-finishers.

**Contributions at and after the time of success**

We also note a change in contribution dynamics after a campaign meets its goal. In Figure 4, we plot average buyer and donor revenue flows (divided by goal) for 30-day middle-finisher campaigns, three days before and after they succeed. The plot establishes an important fact: Once a campaign reaches its goal, donations drop to a significantly lower level while buyers remain active. In fact, 77.1% of campaigns see lower donations, or donations stay at zero, after success.[26] This result can be interpreted as donors caring mainly about the campaign reaching its goal. Since donors do not receive any rewards, donations should taper off after success.

Figure 4: Donations and Purchases Relative to Success Time




Note: The success time is the closest time after the campaign reaches its funding goal for 30 day campaigns. +3 means 3 days after the goal is reached; −3 means 3 days prior to the goal being reached. This plot includes the subset of campaigns in which the success time is greater than three and less than 27 (N=3,403).

Figure 4 also shows that buyer contributions also drop after success. If buyers' decisions depended on the probability of success, then we would expect purchases to not decline after success. The drop in purchases may be explained by a drop in advertising activity, both on and off the platform, after success. The decline in purchases may also be caused by increased prices after campaigns succeed.[27]

[26] One anomalous finding is that for campaigns that finish before the deadline, there is an increase in both donations and purchases in the period of success. This is perhaps due to the platform advertising when a campaign is near success.

[27] While some entrepreneurs close the highest price buckets, they also decrease the capacity of lower buckets. We find that the percentage of filled buckets increases substantially from 1.5% to 9.8% immediately after success. As a result, conditional on looking at buckets with capacity, the mean bucket price increases by $0.8. For buckets with positive sales, the mean price increase is substantially higher at $62.0 (median, $11.7). This suggests that entrepreneurs offer a substantial discount prior to the success of the campaign. In general, a drop in purchases is plausible if success and arrivals before success are correlated events. We discuss time-, arrival- and success-dependent arrival rates in Section 6.2.

## 3 A Model of Reward-Based Contribution Games

**Crowdfunding campaign.** An entrepreneur launches a campaign to raise a goal amount $G > 0$ in time $T > 0$. She offers a product at a price $p > 0$ and also accepts donations. The entrepreneur can only receive the funds if the goal amount $G$ is raised by the deadline $T$. For each campaign, define $M_0 := \lceil \frac{G}{p} \rceil$ to be the number of buyers needed to raise $G$. Time is divided into periods of length $\Delta$. Let $\mathbb{T}^\Delta := \{\Delta, 2\Delta, \ldots, T\}$ denote the set of periods. For any period $t \in \mathbb{T}^\Delta$, it is convenient to think of the time remaining $u := T - t \in \mathbb{U}^\Delta \equiv \{T - \Delta, \ldots, \Delta, 0\}$.

**Players and payoffs.** There are two types of contributors, short-lived buyers (she) and a long-lived donor (he).[28,29] In every period $t$, a buyer arrives with probability $\Delta\lambda$. Upon arrival, she can either pledge to pay $p$ to buy the product, or choose an outside option of value $v_0 > 0$. All buyers have the same valuation $v > 0$ for the product. Accordingly, if a buyer pledges and the campaign is successful, she receives utility $v - p > 0$. If the campaign is unsuccessful, she pays nothing and receives utility 0. The outside option $v_0$ can be interpreted as the value of a short-lived purchasing opportunity, an inspection cost, a transaction cost of pledging to buy, or a disappointment cost of not receiving a product if buyers are loss-averse.[30] We assume that $v - p > v_0$.

The long-lived donor values a successful campaign at $w \geq 0$ and has put aside that amount of his wealth for potential contribution. We refer to $w$ as the donor wealth. If the campaign is successful, the donor's payoff is $w - D_T$, where $D_T$ is the total donation contributed to the campaign. If it is not successful, he receives all donations back and receives a utility of 0. Buyers do not observe $w$ and only know that it is drawn from a distribution on $[0, \overline{W}]$, with a continuously differentiable and strictly increasing cdf $F_0$, $f_0 := F_0'$, $\overline{W} \in [0, \infty]$.[31] We refer to $\mathbf{y} = (G, T, p, \lambda, v, v_0, \Delta)$ as the parameters of the game.

[28] Figure 3 shows that buyer contributions occur throughout the campaign. In almost every 12-hour period, over 50% of eventually successful campaigns receive a contribution. This indicates that strategic waiting is not a first-order concern.

[29] We model the donor as a long-lived player because in the data it appears that donors use their donations at the appropriate time to encourage purchases and enable success (see Section 5.1). Our study focuses on this strategic behavior of the donor. Considering a single donor has the technical advantage that we do not have to account for free-riding between donors. Free-riding is theoretically well-understood in contribution games.

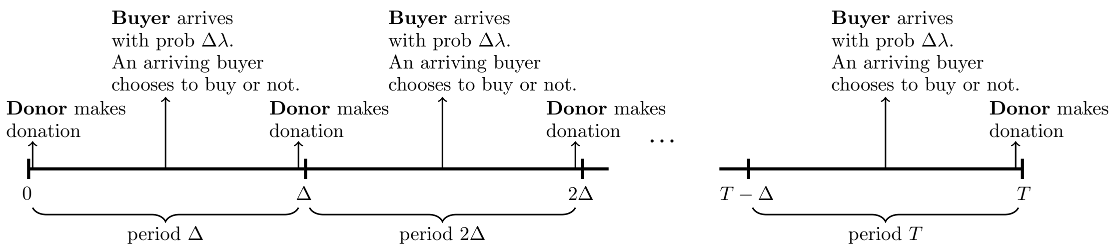
[30] Buyers may also face an opportunity cost of a credit card hold for the duration of the campaign. We consider such outside options that vary over time in Section 6.1.

[31] For simplicity, buyers cannot donate in our setting, as our focus is on the interaction between donors and buyers.

**Timing, histories, and strategies.** In every period $t$, if a buyer arrives, she first decides whether to pledge $p$ or not. Then, the donor decides whether and how much to donate. The donor also gets a chance to donate at the start of the game at time $t = 0$.

Figure 5 illustrates the timing of the game.

Figure 5: Timing within a period




We denote aggregate purchases and aggregate donations up to and including period $t$ by $N_t$ and $D_t$, respectively. The history of a buyer who arrives in period $t$ can be described by

$$\mathbf{h}_t^{B,\Delta} = \prod_{s \in \mathbb{T}^\Delta, s \leq t} (N_{s-\Delta}, D_{s-\Delta}) \in \mathscr{H}_t^{B,\Delta},$$

where $N_0 = 0$ and $\mathscr{H}_t^{B,\Delta} := \left\{ \mathbf{h}_t^{B,\Delta} \in \left(\mathbb{N} \times [0, \overline{W}]\right)^{\frac{t}{\Delta}} \;\middle|\; N_s \in \{N_{s-\Delta}, N_{s-\Delta}+1\},\ D_s \geq D_{s-\Delta} \text{ for all } s \in \mathbb{T}^\Delta, s \leq t-\Delta \right\}$. A donor history in period $t$ includes also the purchase decision in period $t$. That is,

$$\mathbf{h}_t^{D,\Delta} = \left( \prod_{s \in \mathbb{T}^\Delta, s \leq t} (N_{s-\Delta}, D_{s-\Delta}), N_t \right) \in \mathscr{H}_t^{D,\Delta},$$

where $\mathscr{H}_t^{D,\Delta} := \left\{ \mathbf{h}_t^{D,\Delta} \in \mathscr{H}_{t-\Delta}^{B,\Delta} \times \mathbb{N} \;\middle|\; N_t \in \{N_{t-\Delta}, N_{t-\Delta}+1\} \right\}$. The strategy of a buyer is a mapping $\tilde{b}^\Delta : \mathscr{H}_t^{B,\Delta} \to [0,1]$ where $\tilde{b}^\Delta(\mathbf{h}_t^{B,\Delta})$ is the probability with which a buyer buys at history $\mathbf{h}_t^{B,\Delta}$. The strategy of the donor is a mapping $\tilde{D}_+^\Delta : \mathscr{H}_t^{D,\Delta} \times [0, \overline{W}] \to \mathbb{R}$ where $\tilde{D}_+^\Delta(\mathbf{h}_t^{D,\Delta}; w) = D_t \geq D_{t-\Delta}$.[32]

**Solution concept.** A perfect Bayesian equilibrium (PBE) is given by an assessment $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, a tuple of strategies and beliefs, such that,

i) at any donor history $\mathbf{h}_t^{D,\Delta}$, $\tilde{D}_+^\Delta$ maximizes the donor's expected payoff given buyers' strategies

[32] Formally, we allow for mixed strategies. In that case, we denote by $\tilde{D}_+^\Delta(\mathbf{h}_t^{D,\Delta}; w)$ to be the random variable that describes the mixed strategy at the corresponding histories.

and beliefs;

ii) at any buyer history $\mathbf{h}_t^{B,\Delta}$, $\tilde{b}^\Delta$ maximizes the expected payoff of the buyer given her beliefs and the donor's strategy;

iii) for any buyer history $\mathbf{h}_t^{B,\Delta}$, $\tilde{F}^\Delta(\cdot;\mathbf{h}_t^{B,\Delta})$ is the distribution of the public belief of the buyer about $W$, derived from the strategies according to Bayes' Rule whenever possible.

Note that donor rationality in any PBE implies that a donor with wealth $w$ never donates more than $w$. We often refer to the sum of aggregate purchases and donations $R_t := D_t + N_t p$ up to the end of period $t$ as *period-$t$ revenue* and call $R_T = D_T + N_T p$ simply *revenue*. Further, we call the tuple $(N_t, D_t)_{t\in\mathbb{T}^\Delta}$ the *campaign outcome*. Every assessment $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, given the distribution of buyer arrivals and donor wealth distribution, induces a probability measure $\mathbb{P}$ governing the outcomes of the game. We call a campaign outcome *successful* if the goal is reached by the deadline, i.e., if $R_T \geq G$. Then, within the class of PBEs, we define notions of optimality for each of the three stakeholders in any campaign: the platform, the donor, and the buyers.

**Definition 1.** We call a PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$

i) *platform-optimal* if it maximizes the ex-ante expected probability of success $\mathbb{P}(R_T \geq G)$ among all PBE,

ii) *donor-optimal* if it maximizes the ex-ante expected donor payoff $\mathbb{E}\big[\mathbb{1}(R_T \geq G)\cdot(W - D_T)\big]$ among all PBE,

iii) *buyer-optimal* if it maximizes the ex-ante expected buyer payoff

$$\frac{1}{\frac{T}{\Delta}} \sum_{t\in\mathbb{T}^\Delta} \mathbb{E}\big[\mathbb{1}(R_T \geq G)\cdot \tilde{b}^\Delta(\mathbf{h}_t^{B,\Delta})\cdot(v-p) + (1-\tilde{b}^\Delta(\mathbf{h}_t^{B,\Delta}))\cdot v_0\big]$$

among all PBE.

Note that we use lower-case $w$ to denote a given realized wealth type of a donor and use upper-case $W$ to denote the random variable.

**Payoff-relevant state and Markov equilibria.** All players' payoffs only depend on the aggregate number of purchases and the aggregate donation amount. Therefore, it is useful to define the payoff-relevant state to be

$$\mathbf{x} := (N, D, u) \in \mathbb{X}^{\Delta} := \mathbb{N} \times [0, \overline{W}] \times \mathbb{U}^{\Delta},$$

being equal to $(N_{t-\Delta}, D_{t-\Delta}, T-t)$ for a buyer and $(N_t, D_{t-\Delta}, T-t)$ for a donor, in period $t$.

Donor strategies, buyer strategies, and buyer beliefs are said to be *Markovian* if they only depend on the state (on and off-equilibrium path). They are represented by $D^{\Delta} : \mathbb{X}^{\Delta} \times W \to \mathbb{R}$, $b^{\Delta} : \mathbb{X}^{\Delta} \to [0,1]$ and $(F(\cdot;\mathbf{x}))_{\mathbf{x}}$, respectively. We call PBEs in Markovian strategies and beliefs *Markov equilibria,* described by a Markovian assessment $(b^{\Delta}, D_{+}^{\Delta}, F^{\Delta})$.

# 4 Dynamics of Reward-based Contribution Games

In this section, we characterize two Markov equilibria with a common structure given any arbitrary set of campaign parameters $\mathbf{y}$. We show that one is platform- and donor-optimal and the other buyer-optimal. We are explicit when we vary $\Delta$ and take limits. All proofs are in the Appendix.

## 4.1 Preliminaries

We introduce two additional pieces of notation. Let $\underline{M}(D) = \lceil \frac{G-D}{p} \rceil$ denote the number of buyers needed for success given total donations $D$, if no further donations are made. Further, given an assessment $(b^{\Delta}, D_{+}^{\Delta}, F^{\Delta})$, for any state $\mathbf{x} = (N, D, u)$, let $\pi^{\Delta}(\mathbf{x})$ denote the induced probability of reaching the goal from the perspective of the $N+1$st buyer if she buys in state $\mathbf{x}$. Since buyers can only pledge upon arrival, a buyer in state $\mathbf{x}$ is willing to purchase if and only if the expected utility of purchasing is greater than the utility of the outside option, that is

$$\pi^{\Delta}(\mathbf{x}) \cdot (v-p) \geq v_0. \quad \text{(Buyer-IC)}$$

The function $\pi^{\Delta}(\cdot)$ is uniquely pinned down by $(b^{\Delta}, D_{+}^{\Delta}, F^{\Delta})$ and the two sources of uncertainty in this game. First, buyers are uncertain about the wealth $w$ of the donor and they update their beliefs based on the observed state $\mathbf{x}$. Second, agents face uncertainty about future arrival of buyers.

**Definition 2.** For a given assessment $(b^{\Delta}, D_{+}^{\Delta}, F^{\Delta})$,

i) we call a campaign *alive* in state $\mathbf{x}$ if $\pi^{\Delta}(\mathbf{x}) \geq \frac{v_0}{v-p}$, and

ii) we call a campaign *dead* in state $\mathbf{x}$ if $\pi^{\Delta}(\mathbf{x}) < \frac{v_0}{v-p}$.

### 4.2 Pooling-Threshold Strategies and Equilibrium Structure

Analysis of the game is challenging both due to the large number of PBE typical for dynamic signaling games, and because of the large state space. We address these difficulties by first showing that platform-, donor-, and buyer-optimal outcomes can be attained by Markov equilibria in which the donor plays a simple pooling-threshold strategy that keeps aggregate donations above a state-dependent threshold as long as donor wealth $w$ is not depleted.

**Definition 3.** We call a Markovian donor strategy $D_{+}^{\Delta}$ a *pooling-threshold (PT) strategy* if for any $N$ and $u > 0$ there is a *wealth threshold* $D_{*}^{\Delta}(N, u) \geq 0$ such that

$$D_{+}^{\Delta}(\mathbf{x}; w) = \max\{D, D_{*}^{\Delta}(N, u)\} \qquad \text{for all } w \geq D_{*}^{\Delta}(N, u)$$

and $D_{+}^{\Delta}(\mathbf{x}; w) = w$ otherwise.[33]

At the deadline, the donor will always contribute any amount up to his wealth if necessary to meet the goal. Donations made before the deadline serve the purpose of signaling donor wealth to buyers. With a pooling-threshold strategy, the donor ensures, if possible, that the next buyer will purchase given $N$ and $u$. While this is a very natural strategy, one can also imagine donors of different wealth types incentivizing buying in different periods, since the incentive to "save donations for later" is different for donors of different wealth types. We show that donors cannot actually benefit from saving donations early on to incentivize purchases in later periods.

The equilibrium structure with pooling-threshold strategies is more tractable. In particular, any PBE with pooling-threshold strategies can be supported by buyer beliefs given by

$$F^{\Delta}(w; \mathbf{x}) = \begin{cases} \frac{F_0(w)-F_0(D)}{1-F_0(D)} \cdot 1(w \geq D) & \text{if } D \geq D_{*}^{\Delta}(N, u+\Delta) \\ 1(w \geq D) & \text{otherwise} \end{cases} . \qquad \text{(PT-belief)}$$

That is, as soon as the donations fall short of $D_{*}^{\Delta}(N, u)$, buyers believe that the donor has exhausted

[33]Note that at $u = 0$ *any* equilibrium strategy has this pooling-threshold structure, because at the deadline if the goal has not been reached, all donor types who can afford it will donate exactly the balance required to meet the goal.

his wealth and $w = D$.[34] The equilibria we construct further share the feature that buyers buy if and only if aggregate donations exceed the wealth threshold of the donor in the preceding period. That is,

$$b^{\Delta}(N, D, u) = 1 \iff D \geq D_*^{\Delta}(N, u + \Delta).$$

Furthermore, once a campaign reaches a state in which it is dead, it can never be alive again. Accordingly, there are cut-off times (CT) such that given wealth $w$ and $j \in \{1, \ldots, \underline{M}(w)\}$ additional buyers needed for success, the campaign dies unless the next buyer arrives before the cut-off time $u = \xi_j^{\Delta}(w)$. Formally,

$$\text{For } j \leq \underline{M}(w), \qquad \xi_j^{\Delta}(w) := \min\left\{ u \in \mathbb{U}^{\Delta} : \pi^{\Delta}(\underline{M}(w) - j, w, u) \geq \frac{v_0}{v - p} \right\}. \tag{CT}$$

For all $j < 1$ and $w \geq 0$, let $\xi_j^{\Delta}(w) := 0$.[35] In words, given state $\mathbf{x}$ with $N = \underline{M}(w) - j$, a donor with wealth $w$ is not able to satisfy (Buyer-IC) if and only if $u < \xi_j(w)$. At this point, the donor "runs out of funds" because he would need to donate more than $w$ to keep the campaign alive. Note that there is asymmetric information about $\xi_j^{\Delta}(w)$. The donor knows $w$ but buyers only know that $\xi_j^{\Delta}(w)$ is reached when donations fall short of the threshold.

The equilibrium construction is not straightforward because the payoff-relevant state contains donations. In the absence of a donor, one can construct an equilibrium by induction in the number of additional buyers needed to reach the goal, $M_0 - N$. With a donor, the induction proceeds instead in the number of additional buyers needed to reach the goal if no additional donations are made, $j = \underline{M}(D) - N$. Within each induction step in $j$, we perform an induction in time and construct the equilibrium objects $D_*^{\Delta}(N, u)$, $\pi^{\Delta}(N, D, u)$ and $\xi_j^{\Delta}(w)$.

We establish all results for fixed $\mathbf{y}$ in discrete time. For cleaner exposition and to obtain uniqueness, we present the equilibrium expressions in the limit as $\Delta \to 0$. For the limiting expressions, we simply drop the $\Delta$-superscript. We show in the Online Appendix that all point-wise/uniform limits

[34] Many other buyer belief systems can sustain a PBE in which the donor plays pooling-threshold strategies. Technically, the beliefs chosen here violate the "cannot signal what you do not know" condition off equilibrium path as introduced in Fudenberg and Tirole (1991), in the sense that early buyer purchases can affect the beliefs of later buyers independently of the donor's actions. We could recover the "cannot signal what you do not know" condition without altering anything qualitatively, by imposing that for any off-path history $\mathbf{h}_t^{B,\Delta}$ such that there exists a $s \leq t$ with $D_s < D_*^{\Delta}(N, T - s)$, we have $F(w; \mathbf{h}_t^{B,\Delta}) = 1(w \geq \min\{D_s : D_s < D_*^{\Delta}(N, T - s), s \leq t\})$. Instead of allowing such non-Markovian off-path beliefs, we choose the Markovian on- and off-path beliefs given in (PT-belief) for their clean structure.

[35] For $N > \underline{M}(w)$, $j$ can even become negative and in that case, as well, $\xi_j^{\Delta}(w) := 0$.

indeed exist. Thus, we can define the limiting wealth threshold and donor strategy

$$D_*(N,u) \equiv \lim_{\Delta\to 0} D_*^\Delta(N, \lceil \tfrac{u}{\Delta} \rceil \Delta), \quad D_+(\mathbf{x};w) \equiv \lim_{\Delta\to 0} D_+^\Delta(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta; w), \tag{L-D}$$

the limiting buyer strategy and beliefs

$$b(\mathbf{x}) := \lim_{\Delta\to 0} b^\Delta\big(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta\big), \quad F(w;\mathbf{x}) := \lim_{\Delta\to 0} F^\Delta\Big(w; \big(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta\big)\Big), \tag{L-B}$$

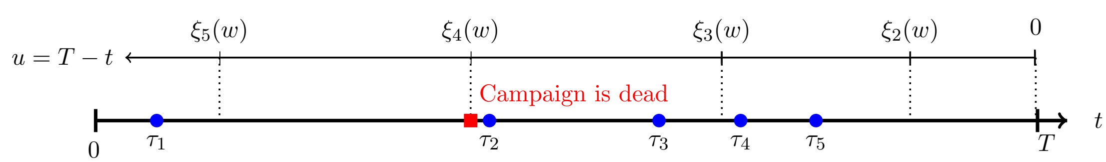
and the limiting cut-off times and probabilities of success

$$\xi_j(w) \equiv \lim_{\Delta\to 0} \xi_j^\Delta(w), \quad \pi(\mathbf{x}) \equiv \lim_{\Delta\to 0} \pi^\Delta(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta) \text{ uniform (in } D), \tag{L-P}$$

where $\lceil \frac{u}{\Delta} \rceil \Delta$ is the smallest multiple of $\Delta$ that is larger than $u$. In the limiting game, we denote the arrival process of buyers by $(\tilde{N}_s)_{s\leq t}$ and the arrival time of the $n$−th buyer by $\tau_n \equiv \inf\{t \geq 0 | \tilde{N}_t \geq n\}$.

We provide a pictorial representation of an equilibrium outcome as $\Delta \to 0$ in Figure 6. The figure shows a realization of buyer arrivals and cut-off times $\xi_j(w)$ when the realized donor wealth $w$ is such that $\underline{M}(w) = 5$ buyers are needed for success. Recall that in a Markov equilibrium with pooling-threshold strategies, the $N$-th buyer must arrive at least $\xi_{\underline{M}(w)-N+1}(w)$ time before the deadline in order to be willing buy. After that point in time, the expected utility from pledging drops below the utility from the outside option $v_0$ and the campaign dies as the donor has run out of funds. In Figure 6, the second buyer does not arrive in time, so the campaign dies because all buyers after the first buyer do not buy.

Figure 6: Sample equilibrium path given donor wealth $w$ such that $\underline{M}(w) = 5$ and deadline $T$.




The first line shows the threshold times $\xi_j(w)$ by which the $\overline{M}(w) - j + 1$-th buyer must arrive in order for the campaign to stay alive. The second line depicts realizations of buyer arrivals (by blue dots) at $\tau_1, \ldots \tau_5$ for a campaign with deadline $T$. The second arrival $\tau_2$ does not occur "in time," i.e., before time $T - \xi_4(w)$. The donor runs out of funds and the campaign dies at $T - \xi_4(w)$.

#### 4.2.1 Platform-Optimal Equilibrium

First we construct platform-optimal equilibria. The platform does not directly care about donations being made during the campaign because it knows that a donor who can afford it will ensure success of the campaign at the deadline. The platform would like buyers to have "maximally optimistic" beliefs about $W$ so that buyers are maximally incentivized to buy. In this subsection, we show that the platform-optimal PBE outcome is attained by a Markov equilibrium in pooling-threshold strategies such that the wealth threshold in any state, denoted by $\underline{D}^{\Delta}(N,u)$, is as low as possible to incentivize purchase. This threshold makes buyers just indifferent between buying and not buying. Furthermore, we show that as $\Delta \to 0$, any sequence of platform-optimal Markov equilibria in pooling-threshold strategies converges to a unique limit.

**Proposition 1** (**Platform-Optimal Equilibrium**). *Given any $\Delta > 0$, there exist platform-optimal PBE that are Markov equilibria and in which the donor plays a pooling-threshold strategy. Any sequence of these Markov equilibria $\{(b^{\Delta}, D_{+}^{\Delta}, F^{\Delta})\}_{\Delta}$ converges as $\Delta \to 0$ to a unique limit $(b, D_{+}, F)$ with the following properties.*

*i) The donor's limiting pooling-threshold strategy has wealth threshold $D_{*}(N,u) := \underline{D}(N,u)$ defined as follows: $\underline{D}(N,u) = 0$ if $\pi(N,0,u) > \frac{v_0}{v-p}$; $\underline{D}(N,u) = \overline{W}$ if $\pi(N,\overline{W},u) < \frac{v_0}{v-p}$; Otherwise, the next buyer is made indifferent:*

$$\pi(N,\underline{D}(N,u),u) = \frac{v_0}{v-p}.$$

*ii) The limiting probability of success $\pi(\mathbf{x})$ satisfies the following conditions.*

*(a) If $u = 0$ and the campaign has yet to reach success,*

$$\pi(N,D,0) = \frac{1-F_0(\max\{G-(N+1)p, D\})}{1-F_0(D)}.$$

*(b) If $N \geq \underline{M}(D)-1$, the campaign has already reached success. Hence, $\pi(\mathbf{x}) = 1$.*

*(c) For $u>0$, $N<\underline{M}(D)-1$, and $D\geq\underline{D}(N,u)$, $\pi(N,D,u)$ is equal to*

$$\mathbb{E}^{F_0}\left[\int_0^{\max\{u-\xi_{\underline{M}(W)-(N+1)}(W),0\}}\lambda e^{-\lambda s}\pi(\max\{D,\underline{D}(N+1,u-s)\},\ N+1,\ u-s)ds\,\middle|\,W\geq D\right],$$

*where, for any realized donor wealth $w$, the cutoff time $\xi_j(w)$ satisfies*

$$\pi(\underline{M}(w)-j,w,\xi_j(w))=\frac{v_0}{v-p}\text{ for }2\leq j\leq\underline{M}(w)-1, \tag{L-$\xi$}$$

*and $\xi_j(w)=0$ for $j<2$.*

*(d) If $u>0$, $N<\underline{M}(D)-1$, and $D<\underline{D}(N,u)$, then the campaign is dead, $\pi(\mathbf{x})<\frac{v_0}{v-p}$.*

*iii) Limiting buyer beliefs about the donor wealth on and off equilibrium path are given by*

$$F(w;\mathbf{x})=\begin{cases}\frac{F_0(w)-F_0(D)}{1-F_0(D)}\cdot\mathbb{1}(w\geq D) & \text{if } D\geq\underline{D}(N,u)\\ \mathbb{1}(w\geq D) & \text{otherwise}\end{cases}.$$

*iv) A buyer buys, $b(\mathbf{x})=1$, if and only if $D\geq\underline{D}(N,u)$.*

The proof is in the Appendix, and proceeds in two steps. First, we construct a Markov equilibrium with a donor pooling-threshold strategy for a fixed $\Delta$ that satisfies analogous discrete-time properties described in Proposition 1. Second, we establish that this PBE is platform-optimal and show that any platform optimal PBE must converge to the same limit as $\Delta\to 0$ .

The construction of the Markov equilibrium with pooling-threshold strategies is explained in Section 4.2. A key expression for the construction is the recursive definition of the probability of success in Proposition 1−(ii)(b): A buyer in state $(N,D,u)$ knows that after her, $\underline{M}(w)-(N+1)$ more buyers are needed for a successful campaign if the donor wealth realization is $w$. Thus, the $(N+2)$-th buyer must arrive at least $\xi_{\underline{M}(w)-(N+1)}$ before the deadline. This gives this buyer an interval of length $u-\xi_{\underline{M}(w)-(N+1)}$ to arrive. If an arrival occurs at time $s$ from the deadline, the donation level must be given by $\max\{D,\underline{D}(N+1,u-s)\}$ by the equilibrium donor strategy. Hence, the probability of success is given by $\pi(\max\{D,\underline{D}(N+1,u-s)\},\ N+1,\ u-s)$.

Intuitively, to maximize probability of success, the platform wants buyers to buy as often as possible, but it does not care about the amounts donated as long as they incentivize buying. Led by this intuition, we consider "reduced histories" which ignore the precise donation amounts but simply record whether or not a donation incentivizes the potential next buyer to buy. We recast the platform's problem as one in which it maximizes the ex-ante probability of success by directly choosing probabilities of reaching each reduced history. This relaxed platform problem—that ignores donor incentives—has a nice recursive structure and can be solved by induction in time. We finally show that the PBE constructed in the first step indeed achieves the value of this relaxed problem and that the limit outcome of all Markov equilibria with pooling-threshold strategies of the donor converges to a unique limit as $\Delta \to 0$.

#### 4.2.2 Donor-Optimal Equilibrium

The donor—like the platform—cares about success, but also wants to minimize aggregate donations. If, at the end of the campaign, the donor could pull out funds that were donated in excess of the goal, then the donor's and platform's incentives would be perfectly aligned. In this case, the equilibrium of Proposition 1 would also have been donor-optimal and Proposition 2 below would be immediate.[36] When this is not possible—as on Kickstarter—one may conjecture that the donor "over-donates" in the platform-optimal equilibrium and can do better by trading off lower donations with a lower probability of success. We show that this is not possible, and as a result, the equilibrium of Proposition 1 is indeed also donor-optimal.

**Proposition 2 (Donor-Optimal Equilibrium).** *Given any $\Delta > 0$, there exists a donor-optimal PBE that is a Markov equilibrium in which the donor plays a pooling-threshold strategy. Any sequence of such equilibria as $\Delta \to 0$ converges to the limit $(b, D_+, F)$ as described in Proposition 1.*

The key idea in the proof of Proposition 2 is to show that if there is a donor-optimal equilibrium, then there must be a donor-optimal equilibrium in which after any arbitrary history two properties hold. First, all donors who do not incentivize the next buyer to buy do not donate. Second, donors who incentivize the next buyer to buy "pool" and donate the same amount. That is, at any history $\mathbf{h}_t^{D,\Delta}$, the donor donates either nothing or $\max\{D_{t-\Delta}, D_*(\mathbf{h}_t^{D,\Delta})\}$ for some $D_*(\mathbf{h}_t^{D,\Delta})$. This allows us to restrict attention to a reduced class of assessments. The donor's problem then reduces to one in which he chooses probabilities over the same reduced histories as in the proof

[36] We briefly discuss this mechanism in Section 7.3.

of Proposition 1, and donation amounts $D_*(\mathbf{h}_t^{D,\Delta})$. As a result, the donor is essentially solving the platform's problem, while also minimizing the donation amount.

### 4.2.3 Buyer-Optimal Equilibrium

Buyers can optimally trade off the value of the outside option and the probability of success if they knew the donor wealth $w$. We show that the buyers can indeed do this in a Markov equilibrium with pooling-threshold donor strategies by coordinating to buy only if donations exceed a threshold that is so high that buyers would buy even if no more donations were made. To define this threshold, we consider a benchmark setting in which $w$ is known to be zero, that is, $F_0(w) = 1(w \geq 0)$ and consider the corresponding equilibrium constructed in Proposition 1. In particular, we can define the time thresholds $\xi_j^\Delta(w)$ for $w = 0$, given by Equation (CT).

**Definition 4.** For $w = 0$ we denote the time thresholds $\xi_j^\Delta(0)$ from Proposition 1 by $\overline{\xi}_j^\Delta$. We call $\overline{\xi}_j^\Delta$ the *no-donation thresholds*. Furthermore, $\overline{\xi}_j := \lim_{\Delta \to 0} \overline{\xi}_j^\Delta$.

This allows us to define $\overline{D}^\Delta(N, u + \Delta) := \max\{G - (j-1)p - Np, 0\}$ for $u \in \left(\overline{\xi}_{j-1}^\Delta, \overline{\xi}_j^\Delta\right]$ and in the limit $\Delta \to 0$:

$$\overline{D}(N, u) := \max\{G - (j-1)p - Np, 0\} \text{ for } u \in (\overline{\xi}_{j-1}, \overline{\xi}_j]. \tag{$\overline{D}$}$$

The wealth threshold is defined so that buyers would buy even if they believed no additional donations will occur. Buyer beliefs in this equilibrium are therefore "maximally pessimistic." Indeed, in any PBE, the donor would never donate a positive amount if the total revenue already exceeds $G - (j-1)p$ at $u \in \left(\overline{\xi}_{j-1}^\Delta, \overline{\xi}_j^\Delta\right]$. Moreover, unlike in Proposition 1, here, when a buyer buys, she has a strict incentive to do so.

**Proposition 3** (**Buyer-Optimal Equilibrium**). *Given any $\Delta > 0$, there exists a unique Markov equilibrium in which the donor plays a pooling-threshold (PT) strategy, that is buyer-optimal. As $\Delta \to 0$, any sequence of such equilibria $\{(b^\Delta, D_+^\Delta, F^\Delta)\}_\Delta$ converges to a limit $(b, D_+, F)$ with the following properties.*

*i) The donor's limiting pooling-threshold strategy's wealth threshold $D_*(N, u) := \overline{D}(N, u)$ given by ($\overline{D}$).*

*ii) The limiting probability of success $\pi(\mathbf{x})$ satisfies the following conditions.*

*(a) If $u = 0$ and the campaign has yet to reach success,*

$$\pi(N, D, 0) = \frac{1 - F_0(\max\{G - (N+1)p, D\})}{1 - F_0(D)}.$$

*(b) If $N \geq \underline{M}(D) - 1$, the campaign has already reached success. Hence, $\pi(\mathbf{x}) = 1$.*

*(c) For $N < \underline{M}(D) - 1$, $u > 0$, and $D \geq \overline{D}(N, u)$, $\pi(N, D, u)$ is equal to*

$$\mathbb{E}^{F_0}\left[\int_0^{\max\{u - \overline{\xi}_{\underline{M}(W)-(N+1)}, 0\}} \lambda e^{-\lambda s} \pi(\max\{D, \overline{D}(N+1, u-s)\}, N+1, u-s) ds \,\middle|\, W \geq D\right].$$

*(d) If $u > 0$, $N < \underline{M}(D) - 1$, and $D < \overline{D}(N, u)$, then the campaign is dead, $\pi(\mathbf{x}) < \frac{v_0}{v-p}$.*

*iii) Limiting buyer beliefs on and off equilibrium path about the donor wealth are given by*

$$F(w; \mathbf{x}) = \begin{cases} \frac{F_0(w) - F_0(D)}{1 - F_0(D)} \cdot \mathbb{1}(w \geq D) & \text{if } D \geq \overline{D}(N, u) \\ \mathbb{1}(w \geq D) & \text{otherwise} \end{cases}.$$

*iv) A buyer buys, $b(\mathbf{x}) = 1$, if and only if $D \geq \overline{D}(N, u)$.*

Importantly, the equilibria characterized in Propositions 1 and 3 have a common structure. This allows us to derive a number of testable predictions, while remaining agnostic about the specific equilibrium being played.

# 5 Testable Implications and Empirical Evidence

In this section, we derive some testable implications of the unique limiting outcomes characterized in Proposition 1 and 3 to then examine their consistency with our data. For each prediction, we first state the proposition and then provide empirical evidence. All proofs are in the Online Appendix.

## 5.1 Comparative Statics

We start with deriving some comparative statics of the probability of success of a campaign.

**Proposition 4** (**Comparative Statics**). *The limiting equilibrium outcomes described in Propositions 1 and 3 satisfy the following properties.*[37]

*i) The ex-ante probability of success of a campaign is strictly increasing in $T$ and strictly decreasing in $G$.*

*ii) In any period $t$, the probability of success is weakly increasing in revenue $R_t$. Given revenue level R, the probability of success is strictly decreasing in time elapsed.*

**Empirical Evidence (Proposition 4).**

i) The ideal experiment to measure the impact of $G$ and $T$ on success probability would be to exogenously vary $G$ and $T$ individually for many identical campaigns.[38] Although this is not feasible, we are able to measure "repeated campaigns" using our data.[39]

We map each campaign to its creator and then select creators who launched multiple campaigns in the same category. Using string matching techniques (Levenshtein distance), we identify repeated campaigns. These are instances in which the entrepreneur's first campaign failed, but the entrepreneur relaunched the campaign. We do not condition on success for the relaunched campaign. We use string matching techniques because the title of the campaign may change over launches. We then compare differences in goal and length for these repeated campaigns. Although this conditions on the initial outcome (failure), it allows us to investigate how campaigns were adjusted across relaunches. Among successful campaigns, we find that entrepreneurs decrease the goal amount (by $23,000, or 61%; significant). Among unsuccessful repeated campaigns, we find the goal amount increases by $1,000 on average. We do not find strong evidence that length is adjusted. The median change for successful campaigns is zero days added, and the mean change is -1.5 days and insignificant.[40] We find 42% of repeated campaigns succeed using the Levenshtein distance threshold at 99%.

[37]The statements hold as long as the probability of success is not zero.

[38]Table 1 confirms that lower goals are associated with greater success, but also shows that unsuccessful projects tend to have longer time horizons than successful ones—a finding seemingly inconsistent with the theory. Clearly, campaign length, goal, and other campaigns features are endogenous, however, they are treated as primitives in our model.

[39]We supplement our collected data with the Kickstarter data posted on Kaggle. This allows us to identify creators without needing to scrape the creator pages.

[40]This finding is robust to the threshold of the string matching technique as well as the string matching technique itself. Here, we use the partial ratio statistic calculated using the package `fuzzywuzzy` and set a threshold to 99%. The pairwise test statistics are 0.855 and $-3.077$ with p-values equal to 0.398 and 0.004, respectively for length and goal. We also find insignificant effects on length change for repeated campaigns that fail a second time.

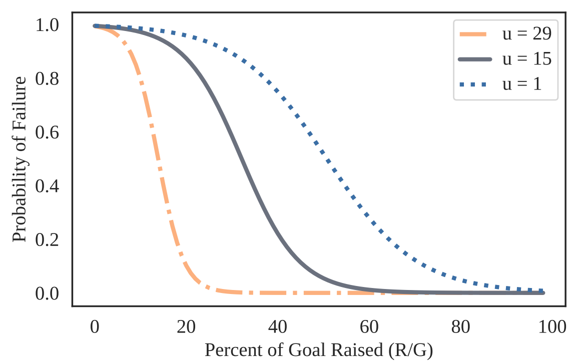
ii) In order to test the predictions of how probability of success varies with revenues raised and time remaining, we first construct a reduced-form model for the probability of success given the state of a campaign using logistic regression with regularization (LASSO). The model is

$$\mathbb{1}[\text{reaches success}]_{j,t} = x_{j,t}\beta + u_{j,t} \quad \text{such that} \quad \left|\beta\right| < \lambda,$$

where the outcome variable is an indicator function of the campaign outcome (success/failure). The hyperparameter $\lambda$ controls the amount of regularization. Included in $x_{j,t}$ are indicators of campaign $j$ and the state of the campaign $(R/G)$ at each twelve-hour interval indexed by $t$.

We split the training and testing data so that the entire time series of a campaign is assigned to either training or testing. This split-train-test assignment implies that our results are not dependent on campaign fixed effects. Our model produces an out-of-sample prediction accuracy of 91%.[41]

Figure 7: Probability of Failure given $u$ and Revenue $R_t$ as a Fraction of Goal $G$




Notes: Predictions based on a LASSO model of eventual success or failure, with campaign indicators and $R_t/G$ as predictors. The three curves plotted correspond to one, fifteen, and twenty-nine days before the deadline.

Figure 7 shows the predicted failure probabilities as a function of the percentage of the goal raised at different points in time. It shows that for 30-day campaigns, the probability of failure is decreasing in $R/G$ for the three periods plotted ($u = 1, 15, 29$). Moreover, the S-curves are increasingly bowed out as $u$ decreases. For instance, with 30% of the goal raised, the probability of failure is close to zero if $u = 29$, but is 60% if $u = 15$. Hence, the probability of failure varies considerably with time remaining for given $R/G$ (it is decreasing in $R/G$).

[41] This number would be clearly higher if we adjusted the fitting of the model to allow for campaign fixed effects. Also, we do not include interactions of campaign states with other campaign features, including campaign category.

## 5.2 Properties of Successful Campaigns

Next, we derive some testable properties of successful campaigns. Since, we derive predictions conditional on success, it is useful to formally define the event of campaign success and the time of success. Recall that $(\tilde{N}_s)_{s\leq t}$ and $\tau_n \equiv \inf\{t \geq 0 | \tilde{N}_t \geq n\}$ denote, respectively, the arrival process of buyers and the arrival time of the $n-$th buyer. For our defined equilibria, the event of a campaign becoming successful exactly at $t$ can be written as

$$\mathscr{S}_t = \{\tau_{\underline{M}(W)} = t \text{ and } \tau_j \leq \min\{T - \xi_{\underline{M}(W)-(j-1)}(W), t\} \; \forall j \leq \underline{M}(W)\},$$

where $\xi_{\underline{M}(W)-(j-1)}(W)$ corresponds to the cutoff time from Propositions 1 and must be substituted by $\bar{\xi}_{\underline{M}(W)-(j-1)}$ for the equilibrium in Proposition 3. Let $\mathscr{S} = \bigcup_{t\in[0,T]} \mathscr{S}_t$. The success time of a successful campaign is given by $\tau \equiv \inf\{t \geq 0 \mid \tilde{N}_t p + D_t \geq G\}$.

**Proposition 5** (**Successful Campaigns**). *The limiting equilibrium outcomes described in Propositions 1 and 3 satisfy the following properties.*

*i) Campaigns that succeed at the deadline raise exactly the goal and would fail without a donation, i.e., conditional on $\mathscr{S}_T$, $R_T = G$ and $D_T > D_{T-\Delta}$ almost surely.*

*ii) If a campaign succeeds prior to the deadline, then it becomes successful due to the arrival of a buyer, i.e., conditional on $\mathscr{S}$, if $\tau < T$, then $D_t < G - N_t p$ for all $t < \tau$.*

*iii) Donations drop to zero after success, i.e., conditional on $\mathscr{S}$, $D_t = D_\tau$ for all $t \geq \tau$.*

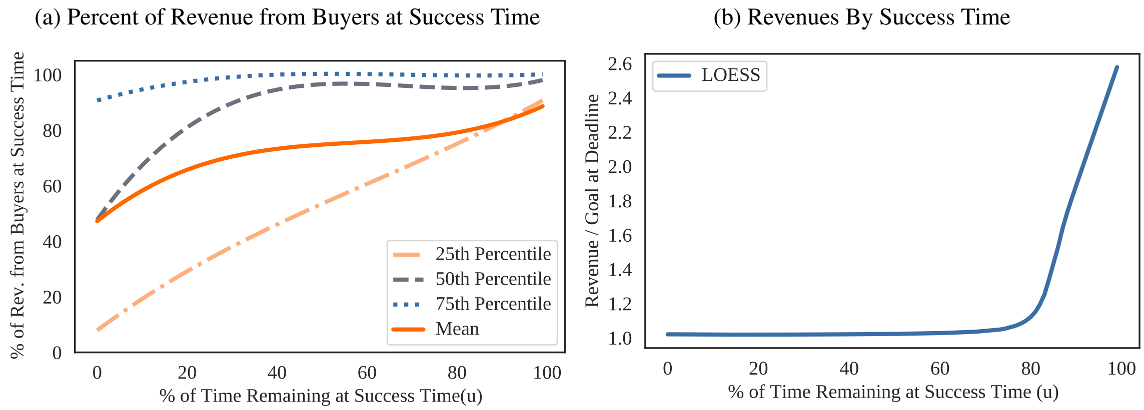
*iv) Total revenue is "essentially decreasing" in the time of success, i.e., for any $t < t'$, $\mathbb{E}[R_T|\mathscr{S}_{t'}] + \lambda(t'-t)p - p < \mathbb{E}[R_T|\mathscr{S}_t]$. For the buyer-optimal equilibrium it is strictly decreasing in the time of success. That is, for any $t < t'$, $\mathbb{E}[R_T|\mathscr{S}_{t'}] + \lambda(t'-t)p < \mathbb{E}[R_T|\mathscr{S}_t]$.*

*v) The percentage of revenue from donations is smaller for successful campaigns than for unsuccessful ones. Formally, given a a donor wealth realization $w$, $\frac{D_t}{R_t}$ is smaller for an arrival process such that $\mathscr{S}$ is realized than for an arrival process such that $\mathscr{S}$ is not realized.*

**Empirical Evidence (Proposition 5).**

i) Consistent with our prediction, we find that campaigns that succeed in the last twelve hours before the deadline raise on average 1.04 of the goal. We also investigate if these campaigns would have failed absent donations, by subtracting any last-minute donations and checking if $R_T > G$. We find that 73% of campaigns would have failed.[42]

Figure 8: Revenue Analysis of Successful Campaigns




Notes: (a) Fitted values of (25th, 50th, 75th percentile) quantile regressions of percent of revenue from buyers in the period in which a campaign succeeds on success time. The covariates are a third-order polynomial of the fraction of time remaining at success. The x-axis shows the percent time remaining at the success time and the y-axis denotes the predicted value. (b) LOESS estimates of final revenue over the goal amount ($R_T/G$), as a function of time remaining ($u$) at the success time.

ii) To verify that buyers cause campaigns to succeed before the deadline, we estimate quantile regressions, where the outcome is the percentage of revenue coming from purchases in the period in which a campaign succeeds. The conditional quantile function at the $\phi$ percentile is equal to $Q_\phi(y_i) = P_i^T \beta(\phi)$, where we specify $P_i$ to be an orthogonal polynomial function (3rd order) in the percentage of time remaining. This specification allows us to investigate the relative importance of purchases versus donations at the time of success across the distribution of successful projects (within and over time).

Figure 8-(a) shows the estimated quantile functions over time. We find that, among campaigns that succeed with about 30% or more time remaining, in the period of success, the median campaign obtains nearly 90% of its revenue from buyers. For campaigns that succeed closer to the deadline, the median drops to 60% of revenue in the period of success coming from buyers. The 75th percentile remains above 90% for almost the entire time horizon. For the 25th percentile, the

[42] Our model predicts that 100% would have failed. However, recall that our data collection occurs once every twelve hours.

fraction of revenue coming from purchases decreases almost linearly over time. In other words, our model predictions closely match over 50% of successful projects. The bottom 25% of projects fit the model poorly, which also lowers the conditional mean (plotted in solid orange).

iii) Figure 4 shows that average donations drop after a campaign succeeds. We find that 77.1% of campaigns see lower donations, or donations stay at zero, after success. We also test for significant donations after success. Note that some spurious donations will be imputed because donations are inferred after estimating shipping costs. To test for significant donations, we estimate the following model,

$$\text{donation revenue}_{j,t} = \alpha_j \mathbb{1}\left[\text{success}_j = 0\right] + \beta_j \mathbb{1}\left[\text{success}_j = 1\right] + u_{j,t},$$

where $j$ is a campaign and $t$ is a twelve hour time period. We include campaign-specific parameters to account for the heterogeneity in campaigns on Kickstarter. Our hypothesis is that $\beta_j = 0$ and we reject the null for a reasonably small percentage of projects.[43] We reject the null (that donations are zero) for 37.1% of successful campaigns at the 5% level and 18.2% at the 1% level.[44]

iv) There exists a strong relationship between revenues raised and the time of success. Table 3 shows that, on average, early-finishing campaigns raise 7.4 times their goal, middle-finishing campaigns raise 1.5 times the goal, and late-finishing campaigns raise almost exactly the goal. In general, the magnitude of $R_T/G$ is strongly decreasing in the success time $\tau$. This is shown in Figure 8-(b), which plots the local regression (LOESS) of $R_T/G$ as a function of the success time. The fact that many middle-finishing campaigns tend to raise just over the goal amount may be surprising. We suspect this is largely due to Kickstarter promoting projects that are yet to succeed. We consider this scenario indirectly in a model extension, where we adjust the arrival rates over time.

v) Our data confirm that unsuccessful projects see a larger percentage of revenues coming from donations. Table 1 shows that the percentage of revenues from donations is 32.8% for unsuccessful campaigns and 28.0% for successful ones.

[43] To estimate the model, we exploit the sparsity of the covariates and implement a block-bootstrap procedure, resampling within project, to compute standard errors. We exclude the period in which a campaign reaches success from the models, which lowers the pre-success mean.

[44] In practice, some donor activity may be due to measurement error, but it may also be the case that some donors care about revenue raised. This is a feature that we abstract away from in the model.

## 5.3 Properties of Unsuccessful campaigns

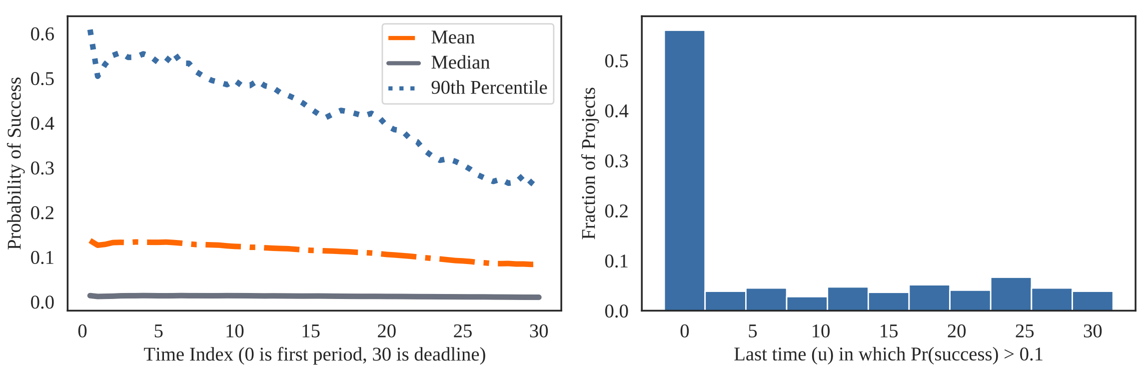
Next, we derive testable properties of unsuccessful campaigns. In our model, unsuccessful campaigns die at some point before the deadline if buyers do not arrive early enough, and the donor runs out of wealth. Formally, the time of death of a campaign can be written as

$$\begin{aligned} \iota &:= \inf\left\{ t \geq 0 \,\middle|\, \pi(\tilde{N}_t, D_t, T-u) < \tfrac{v_0}{v-p} \right\} \\ &= \min_j \left\{ \tau_j \geq 0 \,\middle|\, \tau_j > \xi_{\underline{M}(W)-(j-1)}(W) \right\}. \end{aligned} \tag{$\iota$}$$

We use the logistic regression with LASSO regularization from Section 5.1 to explore properties of unsuccessful campaigns and to infer the realized $\iota$ for each failed campaign.[45]

Figure 9: Logistic Regression: Probability of Success for Campaigns that Eventually Failed

(a) Campaign Success Probability over Time (b) Last Period Alive for each Failed Campaign




Notes: (a) The probability of success for failed campaigns over time. Plotted is the mean project, the median campaign, and the 90th percentile of projects. The results suggest that more than half of projects have lower probability of success at the start. (b) A histogram of the last time a campaign had a probability of success greater than 10%. This shows that campaigns fail throughout time, with a large mass at the end. Patterns are similar for 5% as well as 1%. Campaigns are rounded to three-day bins.

**Proposition 6** (**Unsuccessful Campaigns**). *The limiting equilibrium outcomes described in Propositions 1 and 3 satisfy the following properties.*

*i) Consider a campaign so that initial donations are required to keep the campaign alive, i.e.,* $D_*(0,T) > 0$.

[45] Alternatively, we could have derived $\iota$ by looking at the last contribution—which should be a donation. However, Kickstarter promotes projects near the deadline driving some last-minute contributions. This approach therefore makes campaign death seem to commonly occur at the deadline even though our reduced-form model predicts that death occurred much earlier.

- *In the limiting equilibrium outcome described in Proposition 1, the distribution of* $\iota$ *is continuous on* $(0,T)$ *and has mass points on* $t=0$ *and* $t=T$.
- *In the limiting equilibrium outcome described in Proposition 3, the distribution of* $\iota$ *has mass points on* $t=0$ *and* $t=T$, *and on* $\bar{\xi}_j,\ j<M_0$.[46]

*ii) Conditional on failing, campaigns with larger donor wealth realizations* $w$ *die later. Formally, if* $w>w'$ *and the campaign is unsuccessful for both wealth realizations, then the time of death* $\iota$ *is larger for* $w$ *than for* $w'$.

**Empirical Evidence (Proposition 6).**

i) The data closely matches this prediction. We use the LASSO model to predict the last time period in which an unsuccessful campaign had a probability of success greater than 10%. In Figure 9(a), we plot the predicted probability of success for projects that end up failing. More than half of the campaigns fail early on, and the probability of success decreases over time for campaigns that start out with a positive probability of success. Figure 9(b) shows a histogram of the last time in which unsuccessful campaigns had a probability of success greater than 10%.[47] In our model, the timing of death depends on the realization of arrivals and the realization of $W$. Hence, realizations can cause death to occur at any point in time. Note that Figure 9(b) does not include a large mass of campaigns that fail at $u=T$. This mass is large due to the large percentage of unsuccessful campaigns that raise little revenue (66% of failed campaigns). Therefore, the distribution of death times is bimodal at $t\in\{0,T\}$ with failures occurring throughout time between the two extremes.

ii) To verify the prediction, we simply correlate cumulative donations for failed projects with the time of death estimated using the LASSO model. We find the correlation is .13, meaning projects with larger donations die closer to the deadline.[48]

## 5.4 Donation Dynamics

Finally, we derive some properties of donation dynamics.

[46] If parameters $\mathbf{y}$ (excluding $\Delta$) are drawn randomly from a continuous distribution, such that the marginal distribution of the arrival rate $\lambda$ is continuous with support on $(0,\infty)$, then for the limiting equilibrium outcome described in Proposition 1, the distribution of $\iota$ is continuous on $(0,T)$ and has mass points on $t=0$ and $t=T$.

[47] This result is to robust to tighter thresholds, including at the 1%. level

[48] This relationship is also significant, $t=9.310$.

**Proposition 7 (Donation Dynamics).** *The limiting equilibrium outcomes described in Propositions 1 and 3 satisfy the following properties.*

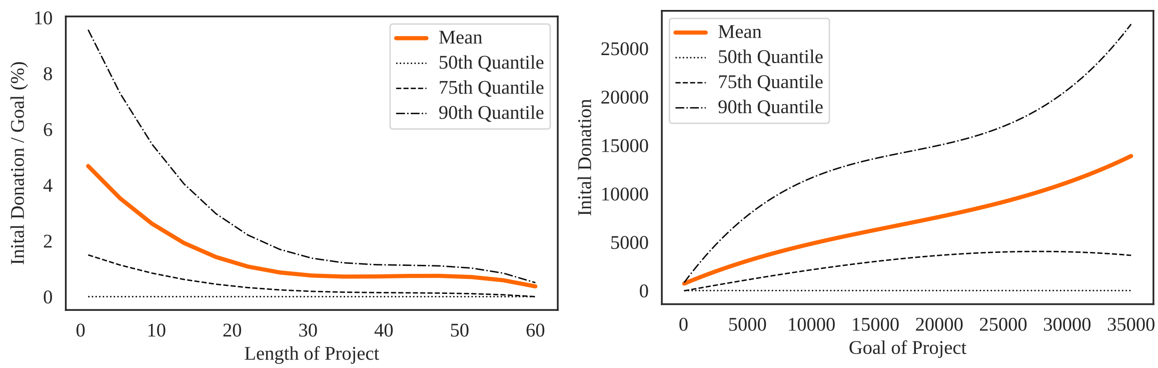
*i) The initial donation level $D_*(0,T)$ is strictly decreasing in $T$, and increasing in $G$.*

*ii) Donations drop to zero for a positive amount of time after a buyer pledges. Formally, if $b(N-1,D,u)=1$, $D_+(N,D,u';w)=D$ for $u' \in (\xi_{\underline{M}(D)-N}(D),u)$ where $\xi_{\underline{M}(D)-N}(D) < u$. If $\xi_{\underline{M}(D)-N}(D) > 0$, then $D_+(N,D,u';w) > D$ for $u' < \xi_{\underline{M}(D)-N}(D)$.*

**Empirical Evidence (Proposition 7).**

Figure 10: Initial Donation as a Function of Length and Goal

(a) Initial Donation as a Function of Length of a Project (b) Initial Donation as a Function of Goal Amount



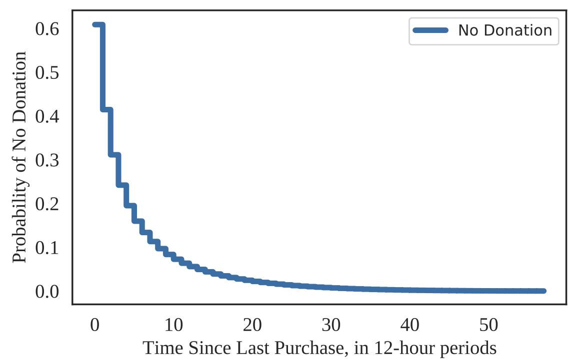

(a) Quantile regression of the initial donation level over the goal, as a function of the length of projects. As project length increases, the initial donation (over goal) is decreasing. The conditional mean is also plotted. (b) Quantile regression of the initial donation as a function of the goal amount. The initial donation is increasing in goal amount. The conditional mean is also plotted.

i) We estimate quantile regressions to investigate the distribution of the initial donation as a function of campaign length and goal. In the conditional quantile function, we include a polynomial of degree three in the covariate of interest. In Figure 10-(a), we show results of the quantile regression predictions of the initial donation divided by the goal as a function of project length. In Figure 10-(b), we show results of the quantile regression of the initial donation as a function of the goal amount. The plots show that the proportion of the campaign goal met by an initial donation decreases with the length of the campaign and increases with the goal amount. The initial donation $D_*(0,T)$ ensures that the campaign does not die right away.[49]

[49]This spike at the start of the campaign is reminiscent of the role played by "seed money" at the start of charitable fund-raising campaigns. See for instance, Andreoni (1998) for a discussion.

ii) Figure 11 shows the results of a hazard rate model of the occurrence of a donation as a function of the number of periods since the last buyer purchased. Plotted is the probability of no donation. Consistent with the prediction, the probability of no donation straight after a purchase is high. The probability of no donation is decreasing as time elapses since the last purchase. We find that the probability of no donation is especially low after five days of no buyer activity.

Figure 11: Donation Hazard Function




Notes: Hazard rate model of donations (indicator) on the number of periods since the last purchase.

# 6 Model Extensions

We consider a few extensions of the baseline model. We first describe how Propositions 1-3 are robust to a changing outside option and changing arrival rates. We then discuss a stylized example of social learning about product quality and show how social learning affects the donor's effectiveness.

## 6.1 Changing Outside Option

Although we focus on a time-invariant outside option, a natural alternative is to interpret $v_0$ as arising due to discounting: The buyer incurs an opportunity cost of pledging money for the campaign duration. Formally, this means that the outside option $v_0(u)$ is exponentially increasing in $u$. For example, Kickstarter allows for a credit card authorization for purchases, so that the opportunity cost is lower closer to the deadline. The equilibrium constructions of Propositions 1 and 3 are valid—that is, we can simply replace $v_0$ by $v_0(u)$—even for a time dependent outside option $v_0(u)$, as long as the outside option does not drop too fast with time so that in the proposed equilibrium early buyers refuse to buy, but buyers closer to the deadline buy:

$$\pi^{\Delta}(\mathbf{x})(v-p) < v_0(u) \Rightarrow \pi^{\Delta}(N, D, u-\Delta)(v-p) < v_0(u-\Delta).$$

A sufficient condition for the above to hold is

$$\frac{v_0(u) - v_0(u-\Delta)}{\Delta} < v_0(u-\Delta)\lambda(1-\Delta\lambda)^{\frac{u}{\Delta}}. \tag{1}$$

With exponential discounting, we have $v_0(u) = p(1-e^{-ru}) + \bar{v}_0$ for some $r > 0$, $\bar{v}_0 > 0$, and as $\Delta \to 0$, Equation 1 converges to $\frac{pre^{-ru}}{p(1-e^{-ru})+\bar{v}_0} < \lambda e^{-\lambda u}$. This is satisfied for sufficiently large and sufficiently small $r$.

## 6.2 Changing Arrival Rate

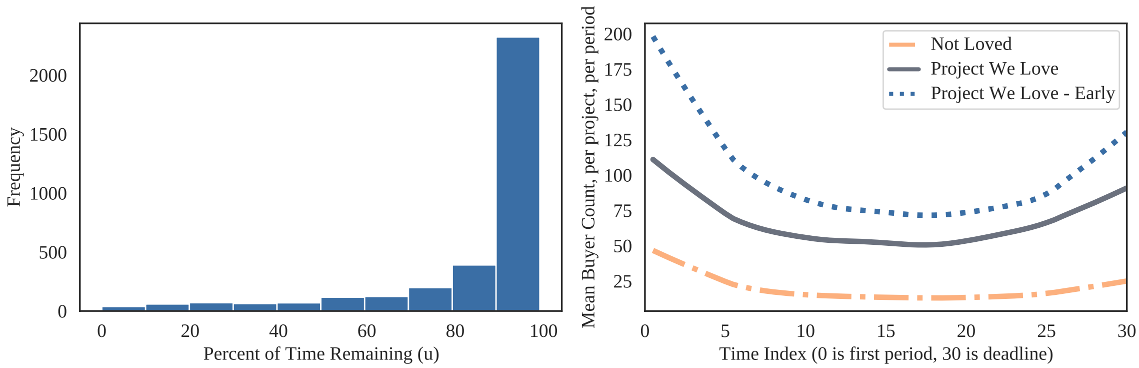
We assume a constant arrival rate of buyers $\lambda$. However, we may expect that arrival rates change over time. Indeed, arrival rates may be affected by advertising campaigns from the creator or promotions from Kickstarter. In Figure 2 we show Kickstarter promotions through "A Project We Love." This is a label attached by Kickstarter to select campaigns. In the left panel, we show when the labels are applied. In the right panel, we show purchasing rates for campaigns that receive the label early, receive the label at some point before the deadline, and never receive the label. Campaigns that receive the label early on have the highest purchases. They also have a significant spike of purchases early on, which helps drive the initial spike in purchases observed in Figure 2.

In addition, Kickstarter advertises campaigns close to the goal in the "Nearly funded" tab on their website, which may drive some of the late-purchasing behavior observed in the data.

All results from Propositions 1- 3 can be generalized to a setting in which $\lambda$ depends on exogenous variables such as time $t$, or on the realized buyer arrivals $\tilde{N}_t$, simply by replacing $\lambda$ by a corresponding function.[50] We can also accommodate a change in the arrival rate after success since purchases after success are irrelevant to both buyers and the donor. Note, however, that the dynamics would change if $\lambda$ depended on the purchases $N_t$.

[50] If the buyer arrival rate changes, there can be times when purchases increase or decrease on average, but these are anticipated by the donor and buyers. The donor's strategy remains unchanged: he donates just enough to incentivize the next buyer to buy.

Figure 12: Projects We Love, Timing and Revenues




Projects We Love is a designation assigned to campaigns by Kickstarter staff. These campaigns may be featured on the site homepage as well as advertised in emails. The left panel (a) presents a histogram of when the designation is applied, as a function of time remaining in the campaign. The right panel (b) presents average buyer revenue for three scenarios: (1) campaigns that never receive the designation, (2) campaigns that receive the designation after 10% of time has elapsed and (3) campaigns that receive the designation within the first 10% of time.

### 6.3 Social Learning

A widely-mentioned benefit of crowdfunding is that it enables potential buyers to learn about product quality from the behavior of other buyers. In this section, we incorporate social learning about product quality to study how it interacts with the dynamic incentives of the donor and buyers. Here, we focus on constructing an equilibrium in pooling-threshold strategies, analogous to the platform- and donor-optimal PBE of Proposition 1. We show that similar trade-offs emerge, but now the donor is “less effective" in solving the coordination problem of buyers.

Let $q \in \{0, 1\}$ denote the unknown quality of the product. All players (the donor and buyers) share a prior that $q = 1$ with probability $\mu_0 \in (0, 1)$ . We view $q$ as the inherent quality of the product or an unknown common value component of demand. Buyers value a product of quality $q$ at $v(q) = v \cdot q$. So, if a buyer pledges, she gets payoff $vq - p$ if the campaign is successful and zero otherwise. If she does not pledge, she receives the outside option $v_0 > 0$. As before, the donor values a successful campaign at $W$ and gets payoff $W - D_T$ if the campaign is successful, and zero otherwise.

On arrival, every buyer privately observes a signal $s \in \{0, 1\}$ before she makes her decision. For simplicity, we consider a “bad news" signal process: A buyer who receives a bad signal $s = 0$ knows with certainty that quality is low ($q = 0$), i.e. $\Pr(s = 1|q = 1) = 1$ and $\Pr(s = 1|q = 0) = \alpha \in (0, 1)$. Let $\mu(\mathbf{h}_t^{\Delta,B})$ denote the public belief about quality after a buyer history $\mathbf{h}_t^{\Delta,B}$ prior to observing the private signal. Social learning can occur because buyers only buy after a good signal.

We construct a Markov equilibrium where the public belief is simply a function of $\mathbf{x} \in \mathbb{X}$ and can be written as $\mu(\mathbf{x})$. Finally, let $\pi_q^{\Delta}(\mathbf{x})$ denote the probability of success from the perspective of buyer $N+1$ conditional on quality $q \in \{0,1\}$. A buyer who arrives at state $\mathbf{x}$ with private signal $s = 1$ buys if and only if the benefit from pledging $B^{\Delta}(\mathbf{x})$ is higher than the opportunity cost, i.e.,

$$\begin{aligned} B^{\Delta}(\mathbf{x};\mu(\mathbf{x})) \quad &:= \mathbb{E}\left[(vq-p)\cdot \mathbb{1}(N_T p + D_T \geq G)|\mathbf{x}, s\right] \\ &= \pi_1^{\Delta}(\mathbf{x})\cdot \tfrac{\mu(\mathbf{x})}{\mu(\mathbf{x})+(1-\mu(\mathbf{x}))\alpha}\cdot(v-p) - \pi_0^{\Delta}(\mathbf{x})\cdot\left(1-\tfrac{\mu(\mathbf{x})}{\mu(\mathbf{x})(1-\mu(\mathbf{x}))\alpha}\right)\cdot p \geq v_0. \end{aligned} \tag{BIC-learning}$$

Given $\pi_q^{\Delta}(\mathbf{x})$, let $\bar{\mu}(\mathbf{x})$ be the solution to Equation BIC-learning if the inequality holds with an equality, which is the smallest $\mu(\mathbf{x})$ so that Equation BIC-learning is satisfied. It is also useful to define:

$$\mu^+(N,u) := \frac{\mu_0(\Delta\lambda)^N(1-\Delta\lambda)^{\frac{T-u}{\Delta}-N}}{\mu_0(\Delta\lambda)^N(1-\Delta\lambda)^{\frac{T-u}{\Delta}-N} + (1-\mu_0)(\Delta\lambda\alpha)^N(1-\Delta\lambda\alpha)^{\frac{T-u}{\Delta}-N}}$$

to be the posterior belief if $N$ positive signals ($s=1$) and $\frac{T-u}{\Delta}-N$ negative signals ($s=0$) are observed. We can also define a donation threshold $\underline{D}^{\Delta,\mu}(N,u)$ analogously to Proposition 1. Let $\underline{D}^{\Delta,\mu}(N,u)$ be such that it makes the next buyer indifferent, $B^{\Delta}(N,\underline{D}^{\Delta,\mu}(N,u),u;\mu^+(N,u)) = v_0$, whenever an interior solution exists. If $B^{\Delta}(N,0,u;\mu^+(N,u)) > v_0$, let $\underline{D}^{\Delta,\mu}(N,u) = 0$ and if $B^{\Delta}(N,\overline{W},u;\mu^+(N,u)) < v_0$, let $\underline{D}^{\Delta,\mu}(N,u) = \overline{W}$. The following proposition characterizes an equilibrium analogous to that in Proposition 1.

**Proposition 8.** *For any $\Delta > 0$, there exists a Markov equilibrium $(b^{\Delta}, D_+^{\Delta}, F^{\Delta})$ such that:*

*i) The public belief about $q = 1$ is given by $\mu(\mathbf{x}) = \mu^+(N,u)$ as long as $\mu^+(N,u) \geq \bar{\mu}(\mathbf{x})$. Otherwise, $\mu(N,D,u) = \max_{u'>u}\{\mu^+(N,u'-\Delta)\,\big|\,\mu^+(N,u') \geq \bar{\mu}(N,D,u')\}$.*

*ii) The equilibrium has the same structure as in Proposition 1 so that*

- *The donor plays a pooling-threshold strategy with wealth threshold $D_*(N,u) := \underline{D}^{\Delta,\mu}(N,u)$; Buyer beliefs about $W$ are as in Equation PT-belief with wealth threshold $\underline{D}^{\Delta,\mu}(N,u)$; A buyer buys if and only if $D \geq \underline{D}^{\Delta,\mu}(N,u+\Delta)$.*
- *For $u > 0$, $N < \underline{M}(D)-1$ and $D \geq \underline{D}^{\Delta,\mu}(N,u)$, the probability of success $\pi_q^{\Delta}(\mathbf{x})$ has the*

*analogous recursive structure to Proposition 1 with cutoff time* $\xi_j^{\Delta,\mu}(w;q)$ *satisfying*

$$\xi_j^{\Delta,\mu}(w;q) = \min\left\{u \in \mathbb{U}^{\Delta} \,\middle|\, B^{\Delta}(\underline{M}(w) - j, w, u; \mu^{+}(\underline{M}(w) - j, u)) \geq v_0\right\}$$

*and* $\xi_j^{\Delta,\mu}(w) = 0$ *for* $j < 2$.

The proof is similar to that of Proposition 1 and is in the online appendix along with the complete formal statement of the proposition. The characterization shows that a campaign can now die not only if the donor runs out of funds but also if public belief $\mu(\mathbf{x})$ drops below the threshold level $\bar{\mu}(\mathbf{x})$. If there are so few purchases that buyers believe success is unlikely and the product is likely of low quality, a donor can increase the probability of success by chipping in money. But that also lowers the expected quality of the product conditional on success, and so can make donations ineffective. To see the intuition, it is useful to write Equation BIC-learning as

$$\mathbb{P}(N_T p + D_T \geq G)|\mathbf{x})\left(\frac{\mathbb{E}\left[(vq - p)\cdot \mathbf{1}(N_T p + D_T \geq G)|\mathbf{x}\right]}{\mathbb{P}(N_T p + D_T \geq G)|\mathbf{x})} - p\right) \geq v_0.$$

Now, a buyer cares about quality *conditional on success* $\frac{\mathbb{E}\left[(vq-p)\cdot \mathbf{1}(N_T p + D_T \geq G)|\mathbf{x}\right]}{\mathbb{P}(N_T p + D_T \geq G)|\mathbf{x})}$, and not just $v$. This valuation is decreasing in $D$, because as the fraction of contributions coming from donations increases, success becomes a weaker signal of quality, and less learning occurs. In fact, the probability of success is now non-monotonic in $G$ and $T$. A low goal $G$ or a long deadline $T$ can reduce the probability of success.

# 7 Features of Platform Design

In this section, we provide a limited exploration of some platform design questions.

## 7.1 Restricting Timing of Donations

Consider two natural counterfactual mechanisms: one in which the donor can donate only once at the start of the campaign ($u = T$) and another in which he can donate only once at the end ( $u = 0$). Both these mechanisms perform worse than the Kickstarter mechanism from each party's perspective.

**Proposition 9.** *Assume that the ex-ante probability of success is strictly positive. Then, the ex-ante probability of success, the ex-ante expected donor utility and the ex-ante expected buyer utility in any equilibrium of the counterfactual mechanisms are strictly lower than in the PBE constructed in Propositions 1 and 3.*

The proof is in the Online Appendix. A first conjecture might have been that allowing donations only at the deadline may be better for the donor, because when continuous donations are allowed, buyers can learn about low realizations of donor wealth. However, since the donor signals his wealth by donations only if buyers would not buy otherwise, he reveals his low wealth only if arrivals of buyers were so low that the campaign would have died in the counterfactual anyway.

## 7.2 “No Information" Environment

In our setting, equilibrium play conditions on $D$ and $N$. This raises the question of whether the dynamically arriving information improves the probability of success.[51] Consider a “no-information” benchmark: Suppose that buyers and the donor got no interim information, but only observed the campaign parameters and calendar time. We show in the Online Appendix that this does not necessarily imply a lower probability of success for a campaign, compared to our baseline. If donations are not crucial for success, then providing more information to buyers can hinder coordination. The intuition is that, without information about $N$, a success requires simply that enough buyers arrive by the deadline, while success in our setting requires also that they arrive early enough.

## 7.3 Entrepreneur as the Donor

Kickstarter does not allow the entrepreneur to donate. The probability of success and buyers' outcomes remain unchanged if at the end of a successful campaign, the donor was given back donations that are in excess of the goal. This observation implies that the outcomes would remain unchanged if the entrepreneur herself was the donor.[52]

[51] We thank Heski Bar-Isaac and Alessandro Bonatti for raising this question.

[52] Crosetto and Regner (2018) report that on the German reward-based crowdfunding platform Start Next, self pledges account for about 10% of all initial pledges and 9% of all pledges that secure funding.

# 8 Conclusion

We provide an empirically consistent model for reward-based crowdfunding as a finite-horizon dynamic contribution game for a private good. This paper is a first step in understanding crowdfunding and other contribution games, and leads to new research questions like optimal mechanism design, optimal price discrimination and the effect of platform competition, in the presence of both benevolent and self-interested players.

# References


**Abrams, Eliot**, "Securities Crowdfunding: More than Family, Friends, and Fools?," *working paper*, 2017.

**Admati, Anat R and Motty Perry**, "Joint projects without commitment," *The Review of Economic Studies*, 1991, *58* (2), 259–276.

**Agrawal, Ajay, Christian Catalini, and Avi Goldfarb**, "Some simple economics of crowdfunding," *Innovation Policy and the Economy*, 2014, *14* (1), 63–97.

**_ , _ , and _** , "Crowdfunding: Geography, social networks, and the timing of investment decisions," *Journal of Economics & Management Strategy*, 2015, *24* (2), 253–274.

**Alaei, Saeed, Azarakhsh Malekian, and Mohamed Mostagir**, "A dynamic model of crowdfunding," *working paper*, 2016.

**Andreoni, James**, "Toward a theory of charitable fund-raising," *Journal of Political Economy*, 1998, *106* (6), 1186–1213.

**Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan**, "Dynamic global games of regime change: Learning, multiplicity, and the timing of attacks," *Econometrica*, 2007, *75* (3), 711–756.

**Babich, Volodymyr, Simone Marinesi, and Gerry Tsoukalas**, "Does crowdfunding benefit entrepreneurs and venture capital investors?," *Manufacturing & Service Operations Management*, 2020.

**Bagnoli, Mark and Barton L. Lipman**, "Provision of Public Goods: Fully Implementing the Core through Private Contributions," *The Review of Economic Studies*, 1989, *56* (4), 583–601.

**Belavina, Elena, Simone Marinesi, and Gerry Tsoukalas**, "Rethinking crowdfunding platform design: mechanisms to deter misconduct and improve efficiency," *Management Science*, 2020.

**Belleflamme, Paul, Thomas Lambert, and Armin Schwienbacher**, "Crowdfunding: Tapping the right crowd," *Journal of business venturing*, 2014, *29* (5), 585–609.

**Bonatti, Alessandro and Johannes Hörner**, "Collaborating," *American Economic Review*, 2011, *101* (2), 632–63.

**Campbell, Arthur, Florian Ederer, and Johannes Spinnewijn**, "Delay and deadlines: Freeriding and information revelation in partnerships," *American Economic Journal: Microeconomics*, 2014, *6* (2), 163–204.

**Chakraborty, Soudipta and Robert Swinney**, "Designing Rewards-Based Crowdfunding Campaigns for Strategic (but Distracted) Contributors," *Available at SSRN 3240094*, 2019.

**_ and _** , "Signaling to the crowd: Private quality information and rewards-based crowdfunding," *Manufacturing & Service Operations Management*, 2020.

**Chang, Jen-Wen**, "The economics of crowdfunding," *American Economic Journal: Microeconomics*, 2020, *12* (2), 257–80.

**Chemla, Gilles and Katrin Tinn**, "Learning through crowdfunding," *Management Science*, 2020, *66* (5), 1783–1801.

**Crosetto, Paolo and Tobias Regner**, "It's never too late: Funding dynamics and self pledges in reward-based crowdfunding," *Research Policy*, 2018, *47* (8), 1463–1477.

**Cvitanić, Jakša and George Georgiadis**, "Achieving efficiency in dynamic contribution games," *American Economic Journal: Microeconomics*, 2016, *8* (4), 309–42.

**Dai, Hengchen and Dennis J Zhang**, "Prosocial goal pursuit in crowdfunding: Evidence from kickstarter," *Journal of Marketing Research*, 2019, *56* (3), 498–517.

**de Rijt, Arnout Van, Soong Moon Kang, Michael Restivo, and Akshay Patil**, "Field experiments of success-breeds-success dynamics," *Proceedings of the National Academy of Sciences*, 2014, p. 201316836.

**Ellman, Matthew and Sjaak Hurkens**, "Fraud tolerance in optimal crowdfunding," *Economics Letters*, 2019, *181*, 11–16.

**_ and _** , "Optimal crowdfunding design," *Journal of Economic Theory*, 2019, *184*, 104939.

**Fershtman, Chaim and Shmuel Nitzan**, "Dynamic voluntary provision of public goods," *European Economic Review*, 1991, *35* (5), 1057–1067.

**Fudenberg, Drew and Jean Tirole**, "Perfect Bayesian equilibrium and sequential equilibrium," *journal of Economic Theory*, 1991, *53* (2), 236–260.

**Grüner, Hans Peter and Christoph Siemroth**, "Crowdfunding, Efficiency, and Inequality," *working paper*, 2017.

**Kim, Jin-Hyuk, Peter Newberry, and Calvin Qiu**, "The role of information signals in determining crowdfunding outcomes," *working paper*, 2017.

**Kuppuswamy, Venkat and Barry L Bayus**, "Crowdfunding creative ideas: The dynamics of project backers," in "The Economics of Crowdfunding," Springer, 2018, pp. 151–182.

**Lee, Samuel and Petra Persson**, "Financing from family and friends," *The Review of Financial Studies*, 2016, *29* (9), 2341–2386.

**Li, Jiasun**, "Profit Sharing: A Contracting Solution to Harness the Wisdom of the Crowd," *working paper*, 2017.

**Liu, Shannon**, "A theory of collective investment with application to venture funding," *working paper*, 2018.

**Marx, Leslie M and Steven A Matthews**, "Dynamic voluntary contribution to a public project," *The Review of Economic Studies*, 2000, *67* (2), 327–358.

**Mollick, Ethan**, "The dynamics of crowdfunding: An exploratory study," *Journal of business venturing*, 2014, *29* (1), 1–16.

**Sahm, Marco**, "Advance-purchase financing of projects with few buyers," *Review of Industrial Organization*, 2020, *57* (4), 909–933.

**Sorenson, Olav, Valentina Assenova, Guan-Cheng Li, Jason Boada, and Lee Fleming**, "Expand innovation finance via crowdfunding," *Science*, 2016, *354* (6319), 1526–1528.

**Strausz, Roland**, "A Theory of Crowdfunding: A Mechanism Design Approach with Demand Uncertainty and Moral Hazard," *American Economic Review*, June 2017, *107* (6), 1430–76.

**Varian, Hal R**, "Sequential contributions to public goods," *Journal of Public Economics*, 1994, *53* (2), 165–186.

# Appendix

## A Proofs

### A.1 Proof of Proposition 1 (Platform-Optimal Equilibrium)

We prove the result in two parts. First, in Subsection A.1.1, we construct, for any period length $\Delta$, a Markov equilibrium in which the donor plays a pooling-threshold (PT) strategy. We show that the limit of these equilibria as $\Delta \to 0$ is as specified in Proposition 1. Next, in Subsection A.1.2, we show that for any $\Delta > 0$, the constructed equilibrium is platform-optimal and that any sequence of platform-optimal Markov equilibria in which the donor plays a PT strategy converges to the limiting expressions specified in Proposition 1.

### A.1.1 Equilibrium construction:

We first provide the formal assertions for the discrete game and its limit, and a lemma that contains properties of the limiting expressions that are used in the proofs of Section 5. We prove both assertions and the Lemma concurrently using the same inductive argument.[53] We use lower-case $w$ to denote a given realized wealth type of a donor or whenever it is an argument in a function and use upper-case $W$ to denote the random variable.

**Assertion Platform Discrete (PD):** *Fix parameters $\mathbf{y}=(G,T,p,\lambda,v,v_0,\Delta)$. There exists a wealth threshold $\underline{D}^\Delta(N,u)\in[0,\overline{W}]$ for each $N\geq 0, u>0$, and an assessment $(b^\Delta,D_+^\Delta,F^\Delta)$ with induced probability of success $\pi^\Delta(\mathbf{x})$, $\mathbf{x}\in\mathbb{X}^\Delta$, such that the following four conditions hold.*

***PD-i)*** *The wealth threshold $\underline{D}^\Delta(N,u)$ for $u>0$ satisfies*

$$\begin{cases} \underline{D}^\Delta(N,u)=0 & \text{if } \pi^\Delta(N,0,u-\Delta)>\frac{v_0}{v-p}, \\ \pi^\Delta(N,\underline{D}^\Delta(N,u),u-\Delta)=\frac{v_0}{v-p} & \text{if } \pi^\Delta(N,0,u-\Delta)\leq\frac{v_0}{v-p}\leq\pi^\Delta(N,\overline{W},u-\Delta), \\ \underline{D}^\Delta(N,u)=\overline{W} & \text{if } \pi^\Delta(N,\overline{W},u-\Delta)<\frac{v_0}{v-p}. \end{cases}$$

*Further, $\underline{D}^\Delta(N,u)$ is strictly decreasing in both $N$ and $u$ as long as $\pi^\Delta(N,0,u-\Delta)<\frac{v_0}{v-p}<\pi^\Delta(N,\overline{W},u-\Delta)$.*

***PD-ii)*** *The assessment $(b^\Delta,D_+^\Delta,F^\Delta)$ is given by the following:*

*(a) Buyers buy, i.e., $b^\Delta(N,D,u)=1$, if and only if $D\geq\underline{D}^\Delta(N,u+\Delta)$;*

*(b) The donor plays a PT strategy with wealth threshold $\underline{D}^\Delta(N,u)$, i.e., for $u>0$, $D_+^\Delta(N,D,u;w)=\min\{\max\{D,\underline{D}^\Delta(N,u)\},w\}$. For $u=0$, $D_+^\Delta(N,D,0;w)=\min\{\max\{D,G-Np\},w\}$;*

*(c) Beliefs are given by Equation PT-belief with wealth threshold $D_*^\Delta(N,u)=\underline{D}^\Delta(N,u)$.*

***PD-iii)*** *The probability of success $\pi^\Delta$ is recursively given as follows:*

*(a) For $N\geq\underline{M}(D)-1$, $\pi^\Delta(N,D,u)=1$;*

[53] We relegate the formal proofs of a few statements to the Online Appendix B.1. The existence of the relevant limits is mostly a technical exercise. Further, the statements of the lemma for the last induction step ($u>0$ and $N=\underline{M}(D)-j$) are very similar to the preceding steps.

*(b)* *For* $N < \underline{M}(D)-1$*,* $\pi^\Delta(N,D,0) = \frac{1-F_0(G-(N+1)p)}{1-F_0(D)}\mathbb{1}(D \geq \underline{D}^\Delta(N,\Delta))$*, and for* $u > 0$*, if* $D \geq \underline{D}^\Delta(N, u+\Delta)$*,*

$$\pi^\Delta(N,D,u) = \mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\frac{\max\{u-\xi^\Delta_{\underline{M}(W)-(N+1)}(W),0\}}{\Delta}} (1-\Delta\lambda)^{i-1}\Delta\lambda \cdot \pi^\Delta(N+1, \max\{D, \underline{D}^\Delta(N+1, u-\Delta(i-1))\}, u-\Delta i)\Big| W \geq D\Bigg];$$

*(c)* *For* $D < \underline{D}^\Delta(N, u+\Delta)$*,*

$$\begin{aligned}\pi^\Delta(N,D,u) &= \mathbb{P}\big(\tau^u_1 \leq T - \xi^\Delta_{\underline{M}(D)-(N+1)}(D), \tau^u_2 \leq T - \xi^\Delta_{\underline{M}(D)-(N+2)}(D)\ldots, \tau^u_{\underline{M}(D)-(N+1)} \leq T\big)\\ &< \tfrac{v_0}{v-p},\end{aligned}$$

*where for any* $w$*,* $\xi^\Delta_{\underline{M}(w)-(N+1)}(w)$ *is given by Equation CT. That is, for* $j = \underline{M}(w)-(N+1)$*,*

$$\xi^\Delta_{\underline{M}(w)-(N+1)}(w) = \min\left\{u \in \mathbb{U}^\Delta : \pi^\Delta(N+1, w, u) \geq \frac{v_0}{v-p}\right\}$$

*and* $\tau^u_i$ *is the arrival time of the* $i$*-th buyer after period* $u$*. Further,* $\pi^\Delta(N,D,u)$ *is strictly increasing in* $N$*,* $D$*, and* $u$ *as long as* $G-(N+1)p > D > \underline{D}^\Delta(N, u+\Delta)$*. The thresholds* $\xi^\Delta_j(w)$ *are strictly decreasing in* $w$ *and* $\xi^\Delta_j(w) > \xi^\Delta_{j'}(w)$ *for any* $j > j'$ *if* $\xi^\Delta_j(w) > 0$*.*

***PD-iv)*** *The assessment* $(b^\Delta, D^\Delta_+, F^\Delta)$ *is a PBE.*

Note that if $(b^\Delta, D^\Delta_+, F^\Delta)$ is a PBE, then $\pi^\Delta(N,D,u) \geq \frac{v_0}{v-p}$ is equivalent to $D \geq \underline{D}^\Delta(N,u)$.

**Assertion Platform Continuous (PC):** The limits defined in Equation L-D, Equation L-B, Equation L-P exist and correspond to the expressions in Proposition 1.

**Lemma 1.** *For* $u \in (0, T]$*, the following holds:*

***L-i)*** *The probability of success* $\pi(N,D,u)$ *is strictly increasing in* $N$*,* $D$ *and* $u$*, and strictly decreasing in* $G$ *as long as* $G-(N+1)p > D \geq \underline{D}(N,u)$*;*

***L-ii)*** *The donor wealth threshold* $\underline{D}(N,u)$ *is strictly decreasing in* $N$ *and* $u$*, and strictly increasing in* $G$ *as long as* $\pi(N,0,u) < \frac{v_0}{v-p} < \pi(N, \overline{W}, u)$*;*

***L-iii)*** *The time cutoff $\xi_j(w)$ is strictly increasing in $j$ and $G$ and strictly decreasing in $w$ as long as $\xi_j(w) > 0$.*

**Proof Outline:** Assume that at any state $(N, D, u)$, buyer beliefs are given by Equation PT-belief for some wealth threshold $D_*^\Delta(N, u)$. We now construct this threshold $D_*^\Delta(N, u) = \underline{D}^\Delta(N, u)$ inductively so that the strategies of the donor and buyers specified in **PD-ii** constitute mutual best responses, and on-path beliefs are consistent with the donor's action. Recall that $\underline{M}(D) = \lceil \frac{G-D}{p} \rceil$ denotes the number of buyers needed for success given current total donation $D$ if no further donations are made.

The organization of the nested inductive argument is shown below. The proof immediately follows using the same numbering system. We use underlined text to denote portions of the proof related to PC and Lemma 1 within each step.

**A)** Time induction starts at $u = 0$ (last period): We compute donor and buyer responses given Equation PT-belief beliefs at state $(N, D, 0)$.

**B)** Then, we conduct an induction in $j = \underline{M}(D) - N$. This is done in three steps.

- **(a)** The induction start considers all $(N, D, u)$ such that $N \geq \underline{M}(D) - 2$. Within this induction start (in $j$), we proceed by backwards induction in the time remaining $u$ using **A** as the induction start in $u$.
- **(b)** We state the induction hypothesis that takes all expressions for $(N, D, u)$, such that $N = \underline{M}(D) - j'$ for $j' < j$, as given.
- **(c)** In the induction step, we then construct all expressions for state $(\underline{M}(D) - j, D, u)$ when $u > 0$. There are four steps within this section. As in step **B-(a)**, within each step, we conduct a backward induction in $u$. In **B-(c)(1)**, we study the buyer best response given an arbitrary wealth threshold $D_*^\Delta$. In **B-(c)(2)**, we show that if $D_*^\Delta = \underline{D}^\Delta$ as defined in **PD-i**, then the buyer strategy in **PD-ii**(a) is indeed a best response given beliefs as in **PD-ii**(c), and **PD-iii** defines the corresponding probability of success. In **B-(c)(3)**, we establish that the donor strategy in **PD-ii**(b) is also a best response, and in **B-(c)(4)**, we show properties of the corresponding functions required for the induction step.

In each induction step, we also establish PC, i.e., we show the existence of the limits Equation L-D, Equation L-B, and Equation L-P, and prove Lemma 1.

**Proof:**

**A) Time induction start** $u = 0$ **(last period)** First, consider the donor at state $(N, D, 0)$. The campaign succeeds only if the total donation in that period $D_+^\Delta(N, D, 0; w)$ satisfies $Np + D_+^\Delta(N, D, 0; w) \geq G$. This gives the donor a payoff of $w - D_+^\Delta(N, D, 0; w)$. Hence, if $D + Np \geq G$, then the campaign is already successful, and the donor does not contribute, i.e., $D_+^\Delta(N, D, 0; w) = D$. Otherwise, two cases arise. If the donor's value $w$ satisfies $w \geq G - Np$, then he donates the remaining amount necessary for success. If $w < G - Np$, then he is not willing to donate to reach the goal and he receives a payoff of zero.[54] Thus, $D_+^\Delta(N, D, 0; w) = \min\{\max\{D, G - Np\}, w\}$ constitutes a best response for the donor with wealth $w$.

Next, consider the buyer at state $(N, D, 0)$. The buyer beliefs are given by Equation PT-belief with donation threshold $D_*^\Delta(N, 0)$. Given the best response of the donor in state $(N+1, D, 0)$, and if $D \geq D_*^\Delta(N, \Delta)$, the probability of success from the buyer's perspective if she buys is given by

$$\pi^\Delta(N, D, 0) = \frac{1 - F_0(\max\{G - p(N+1), D\})}{1 - F_0(D)}.$$

If she buys and $D < D_*^\Delta(N, \Delta)$, then $\pi^\Delta(N, D, 0) = \mathbb{1}(D + (N+1)p \geq G)$. Thus, it is optimal for the buyer to buy if and only if $D \geq D_*^\Delta(N, \Delta)$ and $\frac{1 - F_0(\max\{G - p(N+1), D\})}{1 - F_0(D)} \geq \frac{v_0}{v - p}$, or if $D \geq G - (N+1)p$.

For the donor, we also include in the induction start period $u = \Delta$. Define the platform-optimal wealth threshold at $u = \Delta$, $\underline{D}^\Delta(N, \Delta)$, as in **PD-i**:

i) If $1 - F_0(G - p(N+1)) > \frac{v_0}{v - p}$, then $\underline{D}^\Delta(N, \Delta) := 0$;

ii) If $\overline{W} < G - p(N+1)$, then define $\underline{D}^\Delta(N, \Delta) := \overline{W}$;

iii) Otherwise, let $\underline{D}^\Delta(N, \Delta)$ be such that $\frac{1 - F_0(G - p(N+1))}{1 - F_0(\underline{D}^\Delta(N, \Delta))} = \frac{v_0}{v - p}$.

Then, if $D_*^\Delta(N, \Delta) = \underline{D}^\Delta(N, \Delta)$, the buyer's best response is indeed $b^\Delta(N, D, 0) = 1$ if and only if $D \geq \underline{D}^\Delta(N, \Delta)$.

[54] If $D + Np < G$ and $w < G - Np$ we specify that the donor donates up to his value $w$. Recall that in this situation, the campaign is not successful and the donor gets his money back. So donating up to $w$ is equivalent to not donating.

*PC and Lemma 1 for $u=0$:* The limiting donor strategy is trivially defined as $D_+(N,D,0;w) = \min\{\max\{D, G-Np\}, w\}$. If $1-F_0(G-p(N+1)) > \frac{v_0}{v-p}$, then $\underline{D}(N,0) := \lim_{\Delta\to 0} \underline{D}^\Delta(N,\Delta) = 0$. If $\overline{W} < G-p(N+1)$, then $\underline{D}(N,0) = \overline{W}$. If $\overline{W} \geq G-p(N+1)$, $\underline{D}(N,0)$ must satisfy $\frac{1-F_0(G-p(N+1))}{1-F_0(\underline{D}(N,0))} = \frac{v_0}{v-p}$ and is well defined because $F_0$ is continuous. Hence,

$$\pi(N,D,0) := \lim_{\Delta\to 0} \pi^\Delta(N,D,\Delta) = \frac{1-F_0(\max\{G-p(N+1), D\})}{1-F_0(D)} \mathbb{1}(D \geq \underline{D}(N,0)).$$

Thus, $\underline{D}\big(N,\Delta\big)$ is strictly decreasing in $N$ and strictly increasing in $G$ as long as $\pi(N,0,u) > D > \pi(N,\overline{W},u)$. Moreover, $\pi(N,D,0)$ is strictly increasing in both $N$ and $D$ for $G-(N+1)p > D > \underline{D}(N,0)$. Finally, $\pi(N,D,0)$ is strictly decreasing in $G$ as long as $G-(N+1)p > D \geq \underline{D}(N,0)$.

**B) Induction in** $j = \underline{M}(D) - N$

**(a) Induction start:** Suppose $N \geq \underline{M}(D)-1$. An arriving buyer knows that either the campaign is already successful, or will succeed if she buys. The unique best response of such a buyer is to buy ($b^\Delta(N,D,u)=1$) because $\pi^\Delta(N,D,u)=1$ for all $u>0$. Given that this buyer always buys, it is optimal for the donor to not donate, i.e., $D_+^\Delta(N,D,u;w) = D = \min\{\max\{D, \underline{D}^\Delta(N,u)\}, w\}$ where $\underline{D}^\Delta(N,u) := 0$. This is exactly the PT strategy with wealth threshold defined in **PD-i** for any $N \geq \underline{M}(D)-1$ and $u>0$. This also implies that $\xi_1^\Delta(w)=0$ for all $w$ by Equation CT. Also, trivially, $\pi^\Delta(N,D,u)$ is weakly increasing in $D$ and $N$, $\xi_1^\Delta(w)$ is weakly decreasing in $w$, and $\underline{D}^\Delta(N,u)$ is weakly increasing in $N$ and weakly decreasing in $u$.

*PC and Lemma 1 for $u>0$ and $N \geq \underline{M}(D)-1$:* It is immediate that all expressions Equation L-D, Equation L-B, Equation L-P exist and are given by:

$$
\begin{aligned}
&(I) && \underline{D}(N,u) := \lim_{\Delta\to 0} \underline{D}^\Delta(N, \lceil \tfrac{u}{\Delta} \rceil \Delta) = 0,\\
&(II) && D_+(N,D,u;w) := \lim_{\Delta\to 0} D_+^\Delta(N,D,\lceil \tfrac{u}{\Delta} \rceil \Delta; w) = D,\\
&(III) && \pi(N,D,u) := \lim_{\Delta\to 0} \pi^\Delta(N,D,\lceil \tfrac{u}{\Delta} \rceil \Delta) = 1,\\
&(IV) && b(N,D,u) := \lim_{\Delta\to 0} b^\Delta\big(N,D,\lceil \tfrac{u}{\Delta} \rceil \Delta\big) = 1,\\
&(V) && \xi_j(w) := \lim_{\Delta\to 0} \xi_j^\Delta(w) = 0,\\
&(VI) && F(w;(N,D,u)) := \lim_{\Delta\to 0} F^\Delta\Big(w; \big(N,D,\lceil \tfrac{u}{\Delta} \rceil \Delta\big)\Big) = \tfrac{F_0(w)-F_0(D)}{1-F_0(D)} \mathbb{1}(w \geq D).
\end{aligned}
$$

Hence, Lemma 1 is trivially satisfied for $u>0$ and $N \geq \underline{M}(D)-1$. Further, all expressions are continuous in $D$ and $u$ (if applicable).

**(b) Induction hypothesis:** Suppose that for every $N = \underline{M}(D) - j'$ with $j' < j$ and all $u > 0$,

i) there exists a wealth threshold $\underline{D}^{\Delta}(N, u)$ satisfying **PD-i** , and

ii) there exists an assessment $(b^{\Delta}, D_{+}^{\Delta}, F^{\Delta})$ satisfying **PD-ii** at state $(N, D, u)$,

iii) the functions $b^{\Delta}$ and $D_{+}^{\Delta}$ are mutual best responses at such states,

iv) the corresponding $\pi^{\Delta}(N, D, u)$ satisfies **PD-iii**,

v) the probability of success $\pi^{\Delta}(N, D, u)$ is strictly increasing in $N$, $D$ and $u$, as long as $G - (N+1)p > D \geq \underline{D}^{\Delta}(N, u + \Delta)$,

vi) $\xi_{j'}^{\Delta}(w)$ is strictly decreasing in $w$ and strictly increasing in $j'$ for all $j' < j$ as long as $\xi_{j'}^{\Delta}(w) > 0$,

vii) for $N' > \underline{M}(D) - j$ the threshold $\underline{D}^{\Delta}(N', u)$ is strictly decreasing in $N$ and $u$ as long as $\pi^{\Delta}(N', 0, u - \Delta) < \frac{v_0}{v-p} < \pi^{\Delta}(N', \overline{W}, u - \Delta)$.

<u>*PC and Lemma 1*</u> Assume that all expressions Equation L-D, Equation L-B, Equation L-P exist for state $(N, D, u)$ with $N = \underline{M}(D) - j'$ and $j' < j$ and $u \geq 0$, and that they satisfy Lemma 1. Further, $\pi(N, D, u)$ is continuous in $D$ and $u$ for $D \geq \underline{D}(N, u)$. Note that for $N \geq \underline{M}(D) - 1$, all comparative statics only apply weakly. We show, that weak inequalities result in strict inequalities in the induction step.

**(c) Induction step:** In this step, we use the induction hypothesis to establish PD, PC, and Lemma 1 for $N = \underline{M}(D) - j$. We proceed in four steps.

**1)** First, consider a buyer in state $(\underline{M}(D) - j, D, u)$ for an arbitrary $u > 0$. By the induction hypothesis, the best response of a donor of wealth $w$ in those states is given by

$$D_{+}^{\Delta}(\underline{M}(D) - (j-1), D, u - \Delta(i-1); w) = \min\{\max\{D, \underline{D}^{\Delta}(\underline{M}(D) - (j-1), u - \Delta(i-1))\}, w\},$$

where $\underline{D}^{\Delta}(\underline{M}(D) - (j-1), u - \Delta(i-1))$ solves

$$\pi^{\Delta}(\underline{M}(D) - (j-1), \underline{D}^{\Delta}(\underline{M}(D) - (j-1), u - \Delta(i-1)), u - \Delta i) = \frac{v_0}{v-p}$$

if $\pi^\Delta(\underline{M}(D)-(j-1), 0, u-\Delta i) \le \frac{v_0}{v-p} \le \pi^\Delta(\underline{M}(D)-(j-1), \overline{W}, u-\Delta i)$ and

$$\begin{cases} \underline{D}^\Delta(\underline{M}(D)-(j-1), u-\Delta(i-1)) = 0 & \text{if } \pi^\Delta(\underline{M}(D)-(j-1), 0, u-\Delta i) > \frac{v_0}{v-p} \\ \underline{D}^\Delta(\underline{M}(D)-(j-1), u-\Delta(i-1)) = \overline{W} & \text{if } \pi^\Delta(\underline{M}(D)-(j-1), \overline{W}, u-\Delta i) < \frac{v_0}{v-p}. \end{cases}$$

By the induction hypothesis, a donor with wealth $w$ at state $(\underline{M}(D)-(j-1), D, u')$ for any $u' > 0$ can keep the campaign alive, i.e., for $u' = u-\Delta i$, $i \ge 0$, $w \ge \underline{D}^\Delta(\underline{M}(D)-(j-1), u-\Delta i)$, if and only if $u-\Delta i \ge \xi^\Delta_{j-1+\underline{M}(w)-\underline{M}(D)}(w)$, noting that $\underline{M}(w)-\underline{M}(D)$ is the number of additional buyers that the donor can make up for using his wealth $w$. Hence, in the continuation game, we have

$$\pi^\Delta(\underline{M}(D)-(j-1), \underbrace{\max\{D, \underline{D}^\Delta(\underline{M}(D)-(j-1), u-\Delta(i-1)\}}_{=D^\Delta_+(\underline{M}(D)-(j-1), D, u-\Delta(i-1); w)}, u-\Delta i) \ge \frac{v_0}{v-p}$$

for $u-\Delta i \ge \xi^\Delta_{j-1+\underline{M}(w)-\underline{M}(D)}(w)$, and $\pi^\Delta(\underline{M}(D)-(j-1), D^\Delta_+(\underline{M}(D)-(j-1), D, u-\Delta(i-1); w), u-\Delta i) < \frac{v_0}{v-p}$ for $u-\Delta i < \xi^\Delta_{j-1+\underline{M}(w)-\underline{M}(D)}(w)$, i.e., on equilibrium path, the buyer does not buy. Hence, by Equation PT-belief, for $u \ge \xi^\Delta_{j-1+\underline{M}(\overline{W})-\underline{M}(D)}(\overline{W})$ and $D \ge D^\Delta_*(\underline{M}(D)-j, u)$, a buyer's probability of success at state $(\underline{M}(D)-j, D, u)$ is given by

$$\pi^\Delta(\underline{M}(D)-j, D, u) := \mathbb{E}^{F_0}\Bigg[ \sum_{i=1}^{\frac{\max\{u-\xi^\Delta_{j-1+\underline{M}(W)-\underline{M}(D)}(W), 0\}}{\Delta}} \underbrace{(1-\Delta\lambda)^{i-1}\Delta\lambda}_{\text{prob. of arrival at } u-\Delta i} \cdot \underbrace{\pi^\Delta(\underline{M}(D)-(j-1), \max\{D, \underline{D}^\Delta(\underline{M}(D)-(j-1), u-\Delta(i-1))\}, u-\Delta i)}_{\ge \frac{v_0}{v-p} \text{ if } u-\Delta i \ge \xi^\Delta_{j-1}(W) \text{ (by induction hypothesis)}} \Bigg| W \ge D \Bigg],$$

for $u < \xi^\Delta_{j-1+\underline{M}(\underline{W})-\underline{M}(D)}(\overline{W})$, $\pi^\Delta(\underline{M}(D)-j, D, u) = 0$, and for $D < D^\Delta_*(\underline{M}(D)-j, u)$, $u \ge \xi^\Delta_{j-1+\underline{M}(\underline{W})-\underline{M}(D)}(\overline{W})$,

$$\pi^\Delta(\underline{M}(D)-j, D, u) := \mathbb{P}(\tau^u_1 \le T-\xi^\Delta_{j-1}(D), \tau^u_2 \le T-\xi^\Delta_{j-2}(D), \dots, \tau^u_{j-1} \le T)$$

by Equation PT-belief, where $\tau^u_i$ is the arrival time of the $i$-th buyer after period $u$.

**2)** Next, we define $\underline{D}^\Delta(\underline{M}(D)-j, u)$ and we show that if $D^\Delta_*(\underline{M}(D)-j, u) = \underline{D}^\Delta(\underline{M}(D)-j, u)$, then $\pi^\Delta(\underline{M}(D)-j,D,u) \geq \frac{v_0}{v-p}$ if and only if $D \geq \underline{D}^\Delta(\underline{M}(D)-j,u)$. Finally, we check that in that case $\underline{D}^\Delta(\underline{M}(D)-j,u)$ indeed satisfies **PD-i**. It is useful to define $\hat{\pi}^\Delta(\underline{M}(D)-j,D,u)$ to be

$$\mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\frac{\max\{u-\xi^\Delta_{j-1+\underline{M}(W)-\underline{M}(D)}(W),0\}}{\Delta}}(1-\Delta\lambda)^{i-1}\Delta\lambda\cdot \\ \pi^\Delta(\underline{M}(D)-(j-1),\max\{D,\underline{D}^\Delta(\underline{M}(D)-(j-1),u-\Delta(i-1))\},u-\Delta i)\Bigg| W \geq D\Bigg],$$

on the range $D \geq \underline{D}^\Delta(\underline{M}(D)-(j-1),u)$. In particular, this implies $\hat{\pi}^\Delta(\underline{M}(D)-j,D,u) = \pi^\Delta(\underline{M}(D)-j,D,u)$ for $D \geq D^\Delta_*(\underline{M}(D)-j,u)$.

First, assume $\hat{\pi}^\Delta(\underline{M}(D)-j,0,u) \leq \frac{v_0}{v-p} \leq \hat{\pi}^\Delta(\underline{M}(D)-j,\overline{W},u)$ and note two regularity properties of $\hat{\pi}^\Delta(\underline{M}(D)-j,D,u)$:

(1) $\hat{\pi}^\Delta(\underline{M}(D)-j,D,u)$ is strictly increasing in $D$ for $D \geq D^\Delta_*(\underline{M}(D)-j,u)$ since

$$\pi^\Delta(\underline{M}(D)-(j-1),\max\{D,\underline{D}^\Delta(\underline{M}(D)-(j-1),u-\Delta(i-1))\},u-\Delta i) \geq \frac{v_0}{v-p},$$

as long as $\pi^\Delta(\underline{M}(D)-(j-1),\overline{W},u-\Delta i) \geq \frac{v_0}{v-p}$ and $\xi^\Delta_{j-1+\underline{M}(w)-\underline{M}(D)}(w)$ are weakly decreasing in $w$ by the induction hypothesis, and $\frac{1}{1-F_0(D)}$ is strictly increasing in $D$.

(2) $\hat{\pi}^\Delta(\underline{M}(D)-j,D,u)$ is continuous in $D$ because, by the induction hypothesis, $\pi^\Delta(\underline{M}(D)-(j-1),D,u-\Delta i)$ is continuous in $D$ for $D \geq \underline{D}^\Delta(\underline{M}(D)-(j-1),u-\Delta(i-1))$, and the max operator is continuous.

Hence, there is a unique solution to $\hat{\pi}^\Delta(\underline{M}(D)-j,\underline{D}^\Delta(\underline{M}(D)-j,u+\Delta),u) = \frac{v_0}{v-p}$. If $\hat{\pi}^\Delta(\underline{M}(D)-j,0,u) > \frac{v_0}{v-p}$, then let $\underline{D}^\Delta(\underline{M}(D)-j,u) := 0$, and if $\frac{v_0}{v-p} > \hat{\pi}^\Delta(\underline{M}(D)-j,\overline{W},u)$, $\underline{D}^\Delta(\underline{M}(D)-j,u) := \overline{W}$. Now, if we set $D^\Delta_*(\underline{M}(D)-j,u) = \underline{D}^\Delta(\underline{M}(D)-j,u)$, then it follows immediately that $\pi^\Delta(\underline{M}(D)-j,D,u) \geq \frac{v_0}{v-p}$ if and only if $D \geq \underline{D}^\Delta(\underline{M}(D)-j,u)$ because, if $D < \underline{D}^\Delta(\underline{M}(D)-j,u)$,

$$\mathbb{P}(\tau^u_1 \leq T-\xi^\Delta_{j-1}(D), \tau^u_2 \leq T-\xi^\Delta_{j-2}(D), \ldots, \tau^u_{j-1} \leq T) < \\ \hat{\pi}^\Delta(\underline{M}(\underline{D}^\Delta(\underline{M}(D)-j,u))-j,\underline{D}^\Delta(\underline{M}(D)-j,u),u) = \tfrac{v_0}{v-p}.$$

Note that it is immediate that $\underline{D}^\Delta(\underline{M}(D)-j,u)$ satisfies **PD-i**.

**3)** Next, consider the donor in state $(\underline{M}(D)-j,D,u)$. We just established that the next buyer buys in periods $u' < u$ if and only if the total donations satisfy $D' \geq \underline{D}^{\Delta}(\underline{M}(D') - j, u')$. It remains to be shown that it is indeed optimal for the donor to play according to a PT strategy with wealth threshold $\underline{D}^{\Delta}(\underline{M}(D) - j, u)$ in this state.

To prove this result, we first show that $\underline{D}^{\Delta}(\underline{M}(D) - j, u)$ is strictly decreasing in $u$. It suffices to show that $\hat{\pi}^{\Delta}(\underline{M}(D) - j, D, u - \Delta)$ is strictly increasing in $D$ and $u$ for $u > \xi^{\Delta}_{j-1}(w)$. We have already shown in part 2) that $\hat{\pi}(\underline{M}(D) - j, D, u - \Delta)$ is strictly increasing in $D$. For $u > \xi^{\Delta}_{j-1}(w)$,

$$\pi^{\Delta}(N+1, \max\{D, \underline{D}^{\Delta}(\underline{M}(D) - (j-1), u - (i+1)\Delta)\}, u - i\Delta) = \frac{v_0}{v-p},$$

if $D < \underline{D}^{\Delta}(\underline{M}(D) - (j-1), u - (i+1)\Delta)$, and otherwise

$$\pi^{\Delta}(N+1, \max\{D, \underline{D}^{\Delta}(\underline{M}(D) - (j-1), u - (i+1)\Delta)\}, u - i\Delta) \geq \frac{v_0}{v-p}.$$

and $\underline{D}^{\Delta}(\underline{M}(D) - (j-1), u - (i+1)\Delta)$ is weakly decreasing in $u$ by induction hypothesis (and the induction start). Then, given state $(D, \underline{M}(D) - j, u)$, if exactly $\underline{M}(D) - (j-1)$ buyers have pledged, then $\xi^{\Delta}_{j-1}(D)$ is the cutoff time such that $D$ suffices to keep the campaign alive until then, but after that cut-off time, a total donation of more than $D$ is needed to keep the campaign alive, i.e.,

$$\begin{cases} D < \underline{D}(\underline{M}(D) - (j-1), u) & \text{for } u < \xi^{\Delta}_{j-1}(D) \\ D \geq \underline{D}(\underline{M}(D) - (j-1), u) & \text{for } u \geq \xi^{\Delta}_{j-1}(D). \end{cases}$$

Then, because $\pi^{\Delta}(\underline{M}(D) - (j-1), D, u - \Delta i) \geq \pi^{\Delta}(\underline{M}(D) - (j-1), D, u - \Delta(i+1))$ and $\pi^{\Delta}(\underline{M}(D) - (j-1), D, \xi^{\Delta}_{j-1}(D)) \geq \frac{v_0}{v-p}$ by the induction hypothesis, we have for $u > \xi^{\Delta}_{j-1}(D)$

$$\begin{aligned}
&\hat{\pi}^{\Delta}(\underline{M}(D) - j, D, u) - \hat{\pi}^{\Delta}(\underline{M}(D) - j, D, u - \Delta) = \\
&\mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\frac{\max\{u - \Delta - \xi^{\Delta}_{j-1}(D), 0\}}{\Delta}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \\
&\qquad \left(\pi^{\Delta}(\underline{M}(D) - (j-1), D, u - \Delta i) - \pi^{\Delta}(\underline{M}(D) - (j-1), D, u - \Delta(i+1))\right) \Bigg| W \geq D\Bigg] \\
&\quad + \mathbb{E}^{F_0}\left[(1 - \Delta\lambda)^{\frac{u - \xi^{\Delta}_{j-1}(D)}{\Delta} - 1} \Delta\lambda \left(\pi^{\Delta}(\underline{M}(D) - (j-1), D, \xi^{\Delta}_{j-1}(D)) - \frac{v_0}{v-p}\right) \middle| W \geq D\right] \\
&\quad + \mathbb{E}^{F_0}\left[(1 - \Delta\lambda)^{\frac{u - \xi^{\Delta}_{j-1+\underline{M}(W) - \underline{M}(D)}(W)}{\Delta} - 1} \Delta\lambda \cdot \frac{v_0}{v-p} \middle| W \geq D\right] \qquad > 0.
\end{aligned}$$

Hence, $\underline{D}^{\Delta}(\underline{M}(D)-j,u)$ is strictly decreasing in $u$. This implies that the donor cannot make a buyer purchase in later periods with a lower donation amount. If he can afford it, it is optimal for the donor to attract a buyer right away, in the next period. Therefore, the donor's best response is to play according to the PT strategy specified in **PD-ii**(b). Note that this also implies that the buyer beliefs from the previous step 1) with $D^{\Delta}_{*}(\underline{M}(D)-j,u)=\underline{D}^{\Delta}(\underline{M}(D)-j,u)$ can be derived from Bayes' rule given the donor strategy.

**4)** Finally, we show the following properties of $\xi^{\Delta}_{j}(W)$ and $\underline{D}^{\Delta}(\underline{M}(D)-j,u)$:

(1) $\xi^{\Delta}_{j}(w)$ is strictly decreasing in $w$ as long as $\xi^{\Delta}_{j}(w)>0$ and $\xi^{\Delta}_{j}(w)>\xi^{\Delta}_{j-1}(w)$ as long as $\xi^{\Delta}_{j}(w)>0$;

(2) The threshold $\underline{D}^{\Delta}(\underline{M}(D)-j,u)$ is strictly decreasing in $u$ and $\underline{D}^{\Delta}(\underline{M}(D)-j,u)>\underline{D}^{\Delta}(\underline{M}(D)-(j-1),u)$ if $\pi^{\Delta}(\underline{M}(D)-j,D,u-\Delta)<\frac{v_0}{v-p}<\pi^{\Delta}(\underline{M}(D)-(j-1),D,u-\Delta)$.

To establish the first claim, recall that Equation CT is equivalent to $\xi^{\Delta}_{j}(W)=\min\left\{u\in\mathbb{U}^{\Delta}:\pi^{\Delta}(\underline{M}(W)-j,W,u)\geq\frac{v_0}{v-p}\right\}$. We know that $\hat{\pi}^{\Delta}(\underline{M}(D)-j,D,u-\Delta)$ is strictly increasing in $D$ and $u$ by 2) and 3). Therefore, as long an increase in $w$ (locally) leaves $\underline{M}(w)$ unchanged, an increase in $w$ strictly decreases $\xi^{\Delta}_{j}(w)$. So, consider the case in which increasing $w$ strictly decreases $\underline{M}(w)$. This happens when $\underline{M}(w)=\frac{G-w}{p}$. In this case, it suffices to show that $\pi^{\Delta}(N,D,u)$ is strictly increasing in $N$, i.e., $\pi^{\Delta}(\underline{M}(D)-(j-1),D,u)>\pi^{\Delta}(\underline{M}(D)-j,D,u)$ as long as $\xi^{\Delta}_{j}(W)>0$. By the induction hypothesis we know that $\xi_{j'}(W)\geq\xi_{j''}(W)$ for $j'>j''$ and $\underline{D}^{\Delta}(\underline{M}(D)-(j-1),u)\geq\underline{D}^{\Delta}(\underline{M}(D)-(j-2),u)$ for all $u>0$ and as before we will use

$$\begin{cases} D<\underline{D}(\underline{M}(D)-(j-2),u) & \text{for } u<\xi^{\Delta}_{j-2}(D)\\ \underline{D}(\underline{M}(D)-(j-2),u)\leq D<\underline{D}(\underline{M}(D)-(j-1),u) & \text{for } \xi^{\Delta}_{j-2}(D)\leq u<\xi^{\Delta}_{j-1}(D)\\ D\geq\underline{D}(\underline{M}(D)-(j-1),u) & \text{for } u\geq\xi^{\Delta}_{j-1}(D)\end{cases}$$

to write an expression for $\pi^{\Delta}(\underline{M}(D)-(j-1),D,u)-\pi^{\Delta}(\underline{M}(D)-j,D,u)$ for $u>\xi^{\Delta}_{j-1}(D)$:

$$\hat{\pi}^{\Delta}(\underline{M}(D)-(j-1),D,u)-\hat{\pi}^{\Delta}(\underline{M}(D)-j,D,u)$$
$$\geq\mathbb{E}^{F_0}\Bigg[\sum_{i=\frac{u-\xi^{\Delta}_{j-1}(D)}{\Delta}+1}^{\frac{\max\{u-\xi^{\Delta}_{j-2}(D),\max\{u-\xi^{\Delta}_{j-1+\underline{M}(W)-\underline{M}(D)}(D),0\}\}}{\Delta}}(1-\Delta\lambda)^{i-1}\cdot\Delta\lambda\cdot\underbrace{\left(\pi^{\Delta}(\underline{M}(D)-(j-2),D,u-\Delta i)-\frac{v_0}{v-p}\right)}_{\geq 0}\Big|W\geq D\Bigg]$$

$$
\begin{aligned}
&+\mathbb{E}^{F_0}\Bigg[\sum_{i=\frac{u-\xi^{\Delta}_{j-2}(D)}{\Delta}}^{\frac{\max\{u-\xi^{\Delta}_{j-1+\underline{M}(W)-\underline{M}(D)}(W),0\}}{\Delta}} (1-\Delta\lambda)^{i-1}\cdot\Delta\lambda\cdot \\
&\qquad \underbrace{\left(\pi^{\Delta}(\underline{M}(D)-(j-2),D,u-\Delta i)-\pi^{\Delta}(\underline{M}(D)-(j-1),D,u-\Delta i)\right)}_{\geq 0}\Big|W\geq D\Bigg] \\
&+\mathbb{E}^{F_0}\Bigg[\sum_{i=\frac{\max\{u-\xi^{\Delta}_{j-1+\underline{M}(W)-\underline{M}(D)}(W)+\Delta,0\}}{\Delta}}^{\frac{\max\{u-\xi^{\Delta}_{j-2+\underline{M}(W)-\underline{M}(D)}(W),0\}}{\Delta}} (1-\Delta\lambda)^{i-1}\cdot\Delta\lambda\cdot\underbrace{\pi^{\Delta}(\underline{M}(D)-(j-2),D,u-\Delta i)}_{>0}\Big|W\geq D\Bigg] \\
&>0,
\end{aligned}
$$

where the inequalities follow from the induction hypothesis. Therefore, $\pi^{\Delta}(\underline{M}(D)-(j-1),D,u)>\pi^{\Delta}(\underline{M}(D)-j,D,u)$ for $u>\xi^{\Delta}_{j-1}(D)$, which in turn implies that increasing $w$ strictly decreases $\xi^{\Delta}_{j}(w)$.

Finally, we show that $\xi^{\Delta}_{j}(W)>\xi^{\Delta}_{j-1}(W)$. This follows immediately from the fact that $\hat{\pi}^{\Delta}(N,D,u)$ is strictly increasing $N$ and $u$.

To establish the second claim, note that the indifference condition that defines $\underline{D}^{\Delta}(\underline{M}(D)-(j-1),u)$ and $\underline{D}^{\Delta}(\underline{M}(D)-j,u)$ and that $\hat{\pi}(N',D,u)$ is strictly increasing in $D$ for $N'\geq\underline{M}(D)-j$ and $\hat{\pi}(\underline{M}(D)-(j-1),D,u)>\hat{\pi}(\underline{M}(D)-j,D,u)$ for all $D$.

This establishes PD. Finally, it remains to complete the proofs of PC and Lemma 1, i.e., to show existence of the relevant limits and the statements from Lemma 1 for the induction step of $u>0$ and $N=\underline{M}(D)-j$. The arguments are very similar to the preceding steps, and the existence of limits is just a technical exercise. Therefore, we relegate the proofs to the Online Appendix B.1.

### A.1.2 Platform-optimality of constructed equilibrium

**Proof Outline:** Next, we show that the equilibrium constructed above is platform-optimal and that any platform-optimal sequence of Markov equilibria in pooling-threshold (PT) strategies must converge to the same limit as specified in Proposition 1. The proof proceeds in four steps. In Step **Step 1**, we formulate a relaxed problem for the platform. In **Step 2**, we solve the relaxed problem. In **Step 3** we show that the outcome of the solution is attained by the equilibrium constructed above. In **Step 4** we show that as $\Delta\to 0$, any sequence of platform-optimal Markov equilibria with PT strategies converges to the limit as specified in Proposition 1.

The key idea of the proof stems from the observation that the donor, if needed (and feasible), will always donate enough to reach the goal at the deadline. Hence, the platform does not care about the exact amount the donor donates during the campaign before the deadline as long as buyers keep buying. To find platform-optimal PBE outcomes, we consider reduced histories that ignore donation amounts and only keep track of whether a donation incentivizes the next potential buyer to purchase or not. This idea allows us to recast the platform's problem as one in which, rather than choosing over the set of PBEs, the platform chooses probabilities of reaching these reduced histories.

**Proof:**

**Step 1: The platform's problem**

Consider a particular assessment $(\tilde{D}_+^{\Delta}, \tilde{b}^{\Delta}, \tilde{F}^{\Delta})$. Given this assessment, any buyer history $\mathbf{h}_t^{B,\Delta} = \prod_{s\in\mathbb{T}^{\Delta}, s\le t} (N_{s-\Delta}, D_{s-\Delta})$ corresponds to a reduced buyer history

$$\tilde{\mathbf{h}}_t^B := \prod_{s\in\mathbb{T}^{\Delta}, s\le t} \big(N_{s-\Delta}, b_{s-\Delta}\big) := \prod_{s\in\mathbb{T}^{\Delta}, s\le t} \left(N_{s-\Delta}, \tilde{b}^{\Delta}\Big(\prod_{s'\in\mathbb{T}^{\Delta}, s'\le s} (N_{s'-\Delta}, D_{s'-\Delta})\Big)\right),$$

so that instead of the donation $D_{s-\Delta}$, the history records the probability with which a buyer arriving in period $s$ buys on observing total donation amount $D_{s-\Delta}$, denoted by $b_{s-\Delta} \in [0,1]$. Let $\mathscr{R}_{\tilde{b}^{\Delta}}$ be the mapping so that $\mathscr{R}_{\tilde{b}^{\Delta}}(\mathbf{h}_t^{B,\Delta}) = \tilde{\mathbf{h}}_t^B$ as defined above. In a platform-optimal equilibrium, the buyer should always buy when she is indifferent between buying and not buying, so henceforth we assume $b_{s-\Delta} \in \{0,1\}$. Let the set of such reduced buyer histories in period $t$ be $\tilde{\mathscr{H}}_t^B$. Further, let us denote the corresponding set of reduced donor histories in period $t$ by

$$\tilde{\mathscr{H}}_t^D := \left\{ \tilde{\mathbf{h}}_t^D = \left(\tilde{\mathbf{h}}_t^B, N_t\right) \,\middle|\, \tilde{\mathbf{h}}_t^B \in \tilde{\mathscr{H}}_t^B,\ N_t \in \{N_{t-\Delta}, N_{t-\Delta}+1\} \right\}.$$

The assessment and the arrival and wealth distributions define a probability measure $\mathbb{P}$ on the space of outcomes $\prod_{t\in\mathbb{T}^{\Delta}} (N_t, D_t)$ and hence on $\tilde{\mathscr{H}}_t^B$ and $\tilde{\mathscr{H}}_t^D$. Given this probability space, we define the following probabilities:

i) $\kappa(\tilde{\mathbf{h}}_t^B; w)$ denotes the probability that $\tilde{\mathbf{h}}_t^B \in \tilde{\mathscr{H}}_t^B$ is reached if the donor has wealth $w$;

ii) $\mathbb{P}(\tilde{\mathbf{h}}_t^D; w)$ denotes the probability that $\tilde{\mathbf{h}}_t^D \in \tilde{\mathscr{H}}_t^D$ is reached if the donor has wealth $w$.

Note that this implies that for each $w \in [0,\overline{W}]$ and $t \in \mathbb{T}^{\Delta}$, we have $\sum_{\tilde{\mathbf{h}}_t^B \in \tilde{\mathscr{H}}_t^B} \kappa(\tilde{\mathbf{h}}_t^B; w) = \sum_{\tilde{\mathbf{h}}_t^D \in \tilde{\mathscr{H}}_t^D} \mathbb{P}(\tilde{\mathbf{h}}_t^D; w) = 1$. Further, note that $\mathbb{P}(\tilde{\mathbf{h}}_t^D; w) := \kappa(\tilde{\mathbf{h}}_t^D, 1; w) + \kappa(\tilde{\mathbf{h}}_t^D, 0; w)$.

In the relaxed maximization problem, the platform's choice variable is the vector of probabilities of reaching the reduced buyer histories at which they buy. The set of such reduced buyer histories in period $t$ is given by

$$\tilde{\mathscr{H}}_t^1 := \left\{ \tilde{\mathbf{h}}_t^B = \left(\tilde{\mathbf{h}}_{t-\Delta}^D, 1\right) \middle| \tilde{\mathbf{h}}_{t-\Delta}^D \in \tilde{\mathscr{H}}_{t-\Delta}^D \right\} \subset \tilde{\mathscr{H}}_t^B.$$

It must be that

$$\kappa(\tilde{\mathbf{h}}_t^D, 1; w) \leq \mathbb{P}(\tilde{\mathbf{h}}_t^D; w) \text{ for all } \tilde{\mathbf{h}}_t^D \in \tilde{\mathscr{H}}_t^D. \tag{$\mathbb{P}$}$$

Further, the following inter-temporal link between histories must hold,

$$\left.\begin{array}{lcl} \mathbb{P}(\tilde{\mathbf{h}}_t^D, 1, N_t + 1; w) & = & \Delta\lambda\, \kappa(\tilde{\mathbf{h}}_t^D, 1; w) \\ \mathbb{P}(\tilde{\mathbf{h}}_t^D, 1, N_t; w) & = & (1-\Delta\lambda)\, \kappa(\tilde{\mathbf{h}}_t^D, 1; w) \\ \mathbb{P}(\tilde{\mathbf{h}}_t^D, 0, N_t; w) & = & \mathbb{P}(\tilde{\mathbf{h}}_t^D; w) - \kappa(\tilde{\mathbf{h}}_t^D, 1; w) \end{array}\right\} \text{ for all } \tilde{\mathbf{h}}_t^D \in \tilde{\mathscr{H}}_t^D. \tag{$\mathbb{P}-t$}$$

Hence, $(\kappa(\tilde{\mathbf{h}}_t^B; w))_{\tilde{\mathbf{h}}_t^B \in \tilde{\mathscr{H}}_t^1}$ and $\mathbb{P}(0; w) = 1$ uniquely define $(\mathbb{P}(\tilde{\mathbf{h}}_t^D; w))_{\tilde{\mathbf{h}}_t^D \in \tilde{\mathscr{H}}_t^D}$ and $(\kappa(\tilde{\mathbf{h}}_t^D, 0; w))_{\tilde{\mathbf{h}}_t^D \in \tilde{\mathscr{H}}_t^B}$. Our strategy of solving the platform's problem will be to find the maximal probability of success of a campaign conditional on reaching an arbitrary donor history $\tilde{\mathbf{h}}_{t-\Delta}^D$. So, given an arbitrary donor history $\tilde{\mathbf{h}}_{t-\Delta}^D$, it is useful to define for $t' \geq t$, the set of $t'$-period reduced buyer histories that are consistent with $\tilde{\mathbf{h}}_{t-\Delta}^D$, i.e.,

$$\tilde{\mathscr{H}}_{t'}^1(\tilde{\mathbf{h}}_{t-\Delta}^D) := \{\tilde{\mathbf{h}}_{t'}^B \in \tilde{\mathscr{H}}_{t'}^1 : \text{the first entries of } \tilde{\mathbf{h}}_{t'}^B \text{ are } \tilde{\mathbf{h}}_{t-\Delta}^D\},$$

and the vector of corresponding probabilities of reaching these reduced buyer histories is

$$\boldsymbol{\kappa}_t(\tilde{\mathbf{h}}_{t-\Delta}^D; w) := \{\kappa(\tilde{\mathbf{h}}_{t'}^B; w)\}_{\tilde{\mathbf{h}}_{t'}^B \in \tilde{\mathscr{H}}_{t'}^1(\tilde{\mathbf{h}}_{t-\Delta}^D), t' \geq t}.$$

We write $\boldsymbol{\kappa}_t$ to denote this in order to shorten notation when there is no ambiguity. This vector will be the choice variable when we maximize the probability of success conditional on reaching a donor history $\tilde{\mathbf{h}}_{t-\Delta}^D$.

Given this notation, we can write the probability of success for donor type $w$ after a history $\tilde{\mathbf{h}}^D_{t-\Delta}$ recursively as a function of $\boldsymbol{\kappa}_t = \boldsymbol{\kappa}_t(\tilde{\mathbf{h}}^D_{t-\Delta}; w)$, $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)$, and $N_{t-\Delta}$. For $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w) > 0$, we have

$$\begin{aligned}
&\Pi_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N_{t-\Delta}; w) = \\
&\quad \underbrace{\Delta\lambda}_{\text{arrival}} \underbrace{\frac{\kappa(\tilde{\mathbf{h}}^D_{t-\Delta}, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)}}_{\text{buyer buys}} \Pi_t(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}, 1, N_{t-\Delta}+1; w), N_{t-\Delta}+1; w) \\
&\quad + \underbrace{(1-\Delta\lambda)}_{\text{no arrival}} \frac{\kappa(\tilde{\mathbf{h}}^D_{t-\Delta}, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)} \Pi_t(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}, 1, N_{t-\Delta}; w), N_{t-\Delta}; w) \qquad \text{(W-}\Pi\text{)} \\
&\quad + \underbrace{\left(1 - \frac{\kappa(\tilde{\mathbf{h}}^D_{t-\Delta}, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)}\right)}_{\text{buyer does not buy}} \Pi_t(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}, 0, N_{t-\Delta}; w), N_{t-\Delta}; w),
\end{aligned}$$

and for $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w) = 0$, we set $\Pi_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N_{t-\Delta}; w) = 0$ without loss. We will solve the platform's problem backwards, by induction in time. So, the initial condition is the probability of success for donor type $w$ at a terminal ($T$-period) history is given by

$$\Pi_T(\boldsymbol{\kappa}_{T+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_T; w), N_T; w) = \mathbb{1}(G - N_T p \leq w), \qquad \text{(W-}\Pi_T\text{)}$$

where $\boldsymbol{\kappa}_{T+\Delta} = \emptyset$.

Finally, note that the assessment $(D^\Delta_+, b^\Delta, F^\Delta)$ is a PBE only if the incentive compatibility constraint for buyers is satisfied after any history $\tilde{\mathbf{h}}^B_t \in \mathscr{H}^1_t(\tilde{\mathbf{h}}^D_{t-\Delta})$, i.e.,

$$\frac{\displaystyle\int \kappa(\tilde{\mathbf{h}}^B_t; W)\, \Pi_t(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^B_t, N_{t-\Delta}+1; W), N_{t-\Delta}+1; W)\, dF_0(W)}{\displaystyle\int \kappa(\tilde{\mathbf{h}}^B_t; W)\, dF_0(W)} \geq \frac{v_0}{v-p}. \qquad \text{(Buyer IC)}$$

We are now equipped to state the relaxed maximization problem of the platform formally as

$$\max_{(\boldsymbol{\kappa}_\Delta(0;w))_w} \mathbb{E}^{F_0}[\Pi_0(\boldsymbol{\kappa}_\Delta(0; W), \mathbb{P}(0; W), 0; W)],$$

where $\mathbb{P}(0; w) = 1$, and the maximization problem is subject to Equation $\mathbb{P}$, Equation $\mathbb{P} - t$, Equation W-Π, Equation W-$\Pi_T$, and Equation Buyer IC for all $\tilde{\mathbf{h}}^B_t \in \mathscr{H}^1_t, t \in \mathbb{T}^\Delta, N \in \mathbb{N}, w \in [0, \overline{W}]$.

This is a relaxed problem because the vectors $(\boldsymbol{\kappa}_\Delta(0;w))_w$ that satisfy the above constraints do not necessarily correspond to a PBE. Further, we are ignoring donor incentives by considering reduced histories.

Finally, note that for a Markov equilibrium with pooling-threshold strategies, it must be that there exists a $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)\geq 0$ such that for all $\tilde{\mathbf{h}}_t^D\in\mathscr{H}_t^D$,

$$\kappa(\tilde{\mathbf{h}}_t^D,1;w)=\begin{cases}\mathbb{P}(\tilde{\mathbf{h}}_t^D;w) & \text{for } w\geq\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)\\ 0 & \text{otherwise,}\end{cases} \tag{PT-$\kappa$}$$

**Step 2: Solution to the relaxed problem**

In this step, we show the following two statements:

i) $(\boldsymbol{\kappa}_\Delta(0;w))_w$ as in Equation PT-$\kappa$ can be a solution to the relaxed problem only if

$$\begin{aligned}&\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)\\&\in\left[\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t),\ \max\left\{G-\left(N_t+\tfrac{T-t}{\Delta}\right)p,\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)\right\}\right],\end{aligned} \tag{$\tilde{D}^*$-Region}$$

where

$$\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t):=\min\left\{w\in[0,\overline{W}]\;\middle|\;\frac{\int_w^{\overline{W}}\mathbb{P}(\tilde{\mathbf{h}}_t^D;W)\,\Pi_{t-\Delta}(\boldsymbol{\kappa}_t^*,\mathbb{P}(\tilde{\mathbf{h}}_t^D,1,N_t+1;w),N_t+1;W)\,dF_0(W)}{\int_w^{\overline{W}}\mathbb{P}(\tilde{\mathbf{h}}_t^D;W)\,dF_0(W)}\geq\frac{v_0}{v-p}\right\}. \tag{$\underline{W}$}$$

We set $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)=\overline{W}$ if the set on the right-hand side is empty.

ii) $(\boldsymbol{\kappa}_\Delta(0;w))_w$ as in Equation PT-$\kappa$ with $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)=\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)$ is always a solution.

First note that it follows immediately that if $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)<\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}_t^D;w))_w,N_t)$, Equation PT-$\kappa$ cannot be a solution because it would violate Equation Buyer IC.

We proceed by backwards induction in time. We repeatedly use the following decomposition for any period $t$,

$$\mathbb{E}^{F_0}[\Pi_0(\boldsymbol{\kappa}_\Delta, \mathbb{P}(0;W), 0; W)] = \sum_{\tilde{\mathbf{h}}^D_{t-\Delta} \in \tilde{\mathscr{H}}^D_{t-\Delta}} \mathbb{E}^{F_0}[\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; W) \cdot \Pi_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; W), N_{t-\Delta}; W)]. \tag{$\Pi_0$-Decomp}$$

**(a) Induction start:** Since we need an intertemporal statement for the induction hypothesis, we consider both the last and second to last period.

**1)** Consider the last period ($t = T$). We characterize the optimal probabilities $(\boldsymbol{\kappa}^*_T(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_{\tilde{\mathbf{h}}^D_{T-\Delta} \in \tilde{\mathscr{H}}^D_{T-\Delta}, w \in [0,\overline{W}]}$ given $\left(\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)\right)_{\tilde{\mathbf{h}}^D_{T-\Delta} \in \tilde{\mathscr{H}}^D_{T-\Delta}, w \in [0,\overline{W}]}$ and $N_{T-\Delta}$. Applying Equation $\Pi_0$-Decomp at $T$, we know that $(\boldsymbol{\kappa}^*_T(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_{\tilde{\mathbf{h}}^D_{T-\Delta} \in \tilde{\mathscr{H}}^D_{T-\Delta}, w \in [0,\overline{W}]}$ must maximize

$$\sum_{\tilde{\mathbf{h}}^D_{T-\Delta} \in \tilde{\mathscr{H}}^D_{T-\Delta}} \mathbb{E}^{F_0}[\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; W) \cdot \Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; W), N_{T-\Delta}; W)],$$

subject to Equation $\mathbb{P}$, Equation $\mathbb{P}-t$, Equation W-$\Pi$, Equation W-$\Pi_T$ and the buyer IC constraint rewritten as

$$\frac{\int \kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w)\, \mathbb{1}(G-(N_{T-\Delta}+1)p \le W)\, dF_0(W)}{\int \kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; W)\, dF_0(W)} \ge \frac{v_0}{v-p}.$$

By Equation W-$\Pi$ and Equation W-$\Pi_T$, we can write the probability of success in period $T-\Delta$ as

$$\begin{aligned}\Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N_{T-\Delta}; W) = {} & \Delta\lambda \cdot \tfrac{\kappa(\tilde{\mathbf{h}}^{\mathbf{D}}_{\mathbf{T-\Delta}}, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)} \cdot \mathbb{1}(G-(N_{T-\Delta}+1)p \le w) \\ & + \left((1-\Delta\lambda) \cdot \tfrac{\kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)} + \left(1 - \tfrac{\kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)}\right)\right) \cdot \mathbb{1}(G - N_{T-\Delta}p \le w).\end{aligned}$$

Thus, given $\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)$,$\kappa^*(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)$ almost surely for $w \ge G-(N_{T-\Delta}+1)p$ because increasing $\kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w)$ for a positive mass of $w \ge G-(N_{T-\Delta}+1)p$ relaxes the buyer IC constraint in period $T-\Delta$ and strictly increases the objective function. For $w < G-(N_{T-\Delta}+1)p$, the probability of success is zero, so the maximum probability of success is attained by Equation PT-$\kappa$ if and only if Equation $\tilde{D}^*$-Region is satisfied.

Finally, the maximal probability of success is

$$\Pi_{T-\Delta}(\boldsymbol{\kappa}^*_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N_{T-\Delta}; w) = \Delta\lambda\, \mathbb{1}(w \ge G-(N_{T-\Delta}+1)p) + (1-\Delta\lambda)\, \mathbb{1}(w \ge G - N_{T-\Delta}p). \tag{$\Pi_{T-\Delta}$}$$

**2)** Next, consider the second to last period ($t = T-\Delta$). We characterize the optimal $(\boldsymbol{\kappa}^*_{T-\Delta}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_{\tilde{\mathbf{h}}^D_{T-2\Delta}\in\tilde{\mathscr{H}}^D_{T-2\Delta}, w\in[0,\overline{W}]}$ given $\left(\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w)\right)_{\tilde{\mathbf{h}}^D_{T-2\Delta}\in\tilde{\mathscr{H}}^D_{T-2\Delta}, w\in[0,\overline{W}]}$ and $N_{T-2\Delta}$. Applying Equation $\Pi_0$-Decomp at $T-\Delta$, we know that $\boldsymbol{\kappa}^*_{T-\Delta}$ must maximize

$$\sum_{\tilde{\mathbf{h}}^D_{T-2\Delta}\in\tilde{\mathscr{H}}^D_{T-2\Delta}} \mathbb{E}^{F_0}[\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; W)\cdot \Pi_{T-2\Delta}(\boldsymbol{\kappa}_{T-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; W), N_{T-2\Delta}; W)]$$

subject to Equation $\mathbb{P}$, Equation $\mathbb{P}-t$, Equation W-$\Pi$, Equation W-$\Pi_T$ and the buyer IC constraint for a buyer in period $T-\Delta$

$$\frac{\displaystyle\int \kappa(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W)\, \Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}+1; W), N_{T-2\Delta}+1; W)\, dF_0(W)}{\displaystyle\int \kappa(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W)\, dF_0(W)} \geq \frac{v_0}{v-p}.$$

Given any $(\kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w))_{\tilde{\mathbf{h}}^D_{T-2\Delta}, w}$, we have constructed the platform-optimal $(\boldsymbol{\kappa}^*_T(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_{\tilde{\mathbf{h}}^D_{T-\Delta}, w}$ in the previous step and hence for any $\tilde{\mathbf{h}}^D_{T-2\Delta}$, the platform is choosing $(\kappa(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w))_w$ to maximize

$$\begin{aligned}
\mathbb{E}^{F_0}\big[\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; W)\, &\Pi_{T-2\Delta}((\kappa(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W), \boldsymbol{\kappa}^*_T), \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w), N_{T-2\Delta}; W)\big] = \\
&\mathbb{E}^{F_0}\big[\underbrace{\Delta\lambda\, \kappa(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W)}_{=\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}+1; W)} \Pi_{T-\Delta}(\boldsymbol{\kappa}^*_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}+1; W), N_{T-2\Delta}+1; W) \\
&+\underbrace{(1-\Delta\lambda)\, \kappa(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W)}_{=\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}; W)} \Pi_{T-\Delta}(\boldsymbol{\kappa}^*_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}; W), N_{T-2\Delta}; W) \\
&+\underbrace{(\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; W) - \kappa(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W))}_{=\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; W)} \Pi_{T-\Delta}(\boldsymbol{\kappa}^*_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; W), N_{T-2\Delta}; W)\big],
\end{aligned}$$

by Equation W-$\Pi$.

We first show that Equation PT-$\kappa$ attains the maximal outcome for $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$. We argue by contradiction. Suppose not. Then, there exists $(\kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w))_w$ that attains a strictly higher probability of success and attains the maximum. Therefore, there must be a positive mass of donor types greater than $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$ for which $\kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W) < \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w)$.

It is now useful to define the wealth level

$$W' := G - (\underline{M}(\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})) + 1)p.$$

Since we assumed that $(\kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w))_w$ attains a higher probability of success than Equation PT-$\kappa$ with threshold $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$, there must also be a positive mass of donor types $w < W'$ so that $\kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w) > 0$. In particular, there must exist sets $A \subset [\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta}), \overline{W}]$ and $B \subset [0, W']$ so that $\int_A \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w) - \kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W) dF(W) = \int_B \kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W) dF(W) > 0$. Now, let $\kappa''(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w) = \kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w)$ for all $w \notin A \cup B$ and $\kappa''(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w)$ for $w \in A$ and $\kappa''(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w) = 0$ for $w \in B$. Then, by Equation $\Pi_{T-\Delta}$, we know that

$$\min_{w \in A} \Pi_{T-\Delta}(\boldsymbol{\kappa}^*_T, \Delta\lambda\kappa''(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w), N_{T-2\Delta} + 1; w) > \max_{w \in B} \Pi_{T-\Delta}(\boldsymbol{\kappa}^*_T, \Delta\lambda\kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w), N_{T-2\Delta} + 1; w)$$

because $\min_{w \in A} w \geq G - N_{T-2\Delta} p > p + \max_{w \in B} w$. Thus, $\kappa''$ strictly relaxes the buyer IC constraint while strictly increasing the objective function relative to $\kappa'$. Hence, $(\kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; w))_w$ cannot be a solution to the problem. Thus, Equation PT-$\kappa$ with $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$ is a solution to the maximization problem that is a continuous maximization over a compact set.

We next argue that $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; w))_w, N_{T-2\Delta}) \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$, i.e., $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; w), N_{T-2\Delta})_w) = \overline{W}$. This statement says that if the donor does not incentivize buying in period $T - 2\Delta$, he does not incentivize buying in period $T - \Delta$. Assume not. Then $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; w))_w, N_{T-2\Delta}) < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$. Now, using the definition of $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; w))_w, N_{T-2\Delta})$, and the fact that $w < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$ at history $(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta})$, we can write

$$\frac{\displaystyle\int_{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; w))_w, N_{T-2\Delta})}^{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})} \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; W)\, \mathbb{1}(G - (N_{T-2\Delta} + 1)p \leq W)\, dF_0(W)}{\displaystyle\int_{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; w))_w, N_{T-2\Delta})}^{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})} \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; W)\, dF_0(W)} \geq \frac{v_0}{v - p}.$$

Now, note that $\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w)$ for $w \leq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$ and $\Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta} + 1; w), N_{T-2\Delta} + 1; w) \geq \mathbb{1}(G - (N_{T-2\Delta} + 1)p \leq w)$. Hence, by replacing $\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; W)$ by $\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; W)$ and $\mathbb{1}(G-(N_{T-2\Delta}+1)p \le W)$ by $\Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}+1; W), N_{T-2\Delta}+1; W)$ and increasing the integral bounds we must have

$$\frac{\displaystyle\int_{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta},0,N_{T-2\Delta};w))_w, N_{T-2\Delta})}^{\overline{W}} \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; W)\, \Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}+1; W), N_{T-2\Delta}+1; W)\, dF_0(W)}{\displaystyle\int_{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta},0,N_{T-2\Delta};w))_w, N_{T-2\Delta})}^{\overline{W}} \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; W)\, dF_0(W)} \ge \frac{v_0}{v-p},$$

which is a contradiction to the definition of $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$. We can conclude $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{T-2\Delta}; w))_w, N_{T-2\Delta}) = \overline{W}$.

This allows us to write the probability of success in period $T-2\Delta$ as

$$\begin{aligned}
&\Pi_{T-2\Delta}(\boldsymbol{\kappa}^*_{T-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w), N_{T-2\Delta}; w) = \mathbb{1}(w \ge \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta}))\cdot \\
&\quad \big[(\Delta\lambda)^2\, \mathbb{1}(w \ge \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}+1; w))_w, N_{T-2\Delta}+1))\, \mathbb{1}(G-(N_{T-2\Delta}+2)p \le w) + \\
&\quad (\Delta\lambda)(1-\Delta\lambda)\, \mathbb{1}(w \ge \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}+1; w))_w, N_{T-2\Delta}+1))\, \mathbb{1}(G-(N_{T-2\Delta}+1)p \le w) + \\
&\quad (1-\Delta\lambda)\Delta\lambda\, \mathbb{1}(w \ge \mathbb{1}(w \ge \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}; w))_w, N_{T-2\Delta}))\, \mathbb{1}(G-(N_{T-2\Delta}+1)p \le w) + \\
&\quad (1-\Delta\lambda)^2\, \mathbb{1}(w \ge \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1, N_{T-2\Delta}; w))_w, N_{T-2\Delta}))\, \mathbb{1}(G - N_{T-2\Delta}p \le w)\big].
\end{aligned}$$

Finally, we show that this value of the objective function can be attained by $\kappa_{T-\Delta}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w)$ as in Equation PT-$\kappa$ only if

$$\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta}) \le \max\{G - \big(N_{T-2\Delta}+2\big)p, \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})\}.$$

First, assume $G-\big(N_{T-2\Delta}+2\big)p > \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$. For $w < G-\big(N_{T-2\Delta}+2\big)p$, it must be that $\Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N_{T-2\Delta}+1; w) = 0$. For $w \ge G-\big(N_{T-2\Delta}+2\big)p$, we have $\Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N_{T-2\Delta}+1; w) > 0$. Thus, $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta}) > G-\big(N_{T-2\Delta}+2\big)p$ would strictly decrease the objective function.

Next, assume $G-\big(N_{T-2\Delta}+2\big)p \le \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$. For $w \ge G-\big(N_{T-2\Delta}+2\big)p$, we have again $\Pi_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N_{T-2\Delta}+1; w) > 0$. Hence, $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta}) >$

$\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w))_w, N_{T-2\Delta})$ would strictly decrease the objective function.

In the induction step, we will also use the fact that given $(\mathbb{P}(\tilde{\mathbf{h}}^D_{T-3\Delta}; w))_w$, if there exists a $(\kappa(\tilde{\mathbf{h}}^D_{T-3\Delta}, 1; w))_w$, $X \in [0, \overline{W}]$, and $(p_w)_{w<X}$ such that for all $w < X$, $\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w) = p_w$ and $\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w) = 0$ for all $w \geq X$, then

$$\Pi_{T-2\Delta}(\boldsymbol{\kappa}^*_{T-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; w), N; w) \geq \Pi_{T-\Delta}(\boldsymbol{\kappa}^*_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N; w).$$

This follows directly from the expressions for the probabilities $\Pi_{T-2\Delta}$ and $\Pi_{T-\Delta}$ derived in **1)** and **2)**.

**(b) Induction hypothesis:** First, recall that $\boldsymbol{\kappa}^*_t$ is recursively defined by Equation PT-$\kappa$ with $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_t; w))_w, N_t) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_t; w))_w, N_t)$ given by Equation $\underline{W}$. Fix a $t$. Then, given $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)$ for $\tilde{\mathbf{h}}^D_{t-\Delta} \in \mathscr{H}^D_{t-\Delta}$, let us assume that the probability of success

$$\sum_{\tilde{\mathbf{h}}^D_{t-\Delta} \in \mathscr{H}^D_{t-\Delta}} \mathbb{E}^{F_0}[\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; W) \cdot \Pi_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; W), N_{t-\Delta}; W)],$$

is maximized by $\boldsymbol{\kappa}^*_t$, subject to Equation $\mathbb{P} - t$, Equation $\mathbb{P}$, Equation W-$\Pi$, Equation W-$\Pi_T$, and Equation Buyer IC. Furthermore, assume that any solution to the problem of the form Equation PT-$\kappa$ satisfies Equation $\tilde{D}^*$-Region.

Next, for $t' \leq t$, given $\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w)$ and $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w))_w, N_t)$ defined by Equation $\underline{W}$, we assume

$$\begin{aligned}\Pi_{t'-\Delta}(\boldsymbol{\kappa}^*_{t'}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w), N_{t'-\Delta}; w) = \mathbb{1}(w \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w))_w, N_{t'-\Delta}) \cdot \\ \big[\Delta\lambda\, \Pi_{t'}(\boldsymbol{\kappa}^*_{t'+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}, 1, N_{t'-\Delta}+1; w), N_{t'-\Delta}+1; w) + \\ (1-\Delta\lambda)\, \Pi_{t'}(\boldsymbol{\kappa}^*_{t'+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}, 1, N_{t'-\Delta}; w), N_{t'-\Delta}; w)\big],\end{aligned}$$

with the initial condition Equation $\Pi_{T-\Delta}$. Thus, the objective function is given by a linear combination of indicator functions of the form $\mathbb{1}(w \geq X)$ for some $X \in [0, \overline{W}]$ with positive weight on $\mathbb{1}(w \geq G - N_{t'-\Delta} p)$.

Finally, assume that given $(\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w$, if there exists a $(\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; w))_w$, $X \in [0, \overline{W}]$ and $(p_w)_{w<X}$ such that for all $w < X$, $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_t; w) = p_w$ and $\mathbf{P}(\tilde{\mathbf{h}}^D_t; w) = 0$ for all $w \geq X$, then

$$\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N; w) \geq \Pi_t(\boldsymbol{\kappa}^*_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_t; w), N; w).$$

**(c) Induction step:** Let us assume the induction hypothesis is satisfied and consider period $t-\Delta$. Fix arbitrary $(\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_{\tilde{\mathbf{h}}^D_{t-2\Delta}\in\tilde{\mathscr{H}}^D_{t-2\Delta},w\in[0,\overline{W}]}$. For brevity, we denote a vector of such probabilities simply by $(\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_{\tilde{\mathbf{h}}^D_{t-2\Delta},w}$. Then, $(\boldsymbol{\kappa}^*_{t-\Delta}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_{\tilde{\mathbf{h}}^D_{t-2\Delta},w}$ must maximize

$$\sum_{\tilde{\mathbf{h}}^D_{t-2\Delta}\in\tilde{\mathscr{H}}^D_{t-2\Delta}} \mathbb{E}^{F_0}[\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};W)\cdot\Pi_{t-2\Delta}(\boldsymbol{\kappa}_{t-\Delta},\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w),N_{t-2\Delta};W)]$$

subject to Equation $\mathbb{P}$, Equation $\mathbb{P}-t$, Equation W-$\Pi$, Equation W-$\Pi_T$, and Equation Buyer IC for all $t'\geq t-2\Delta$. Given any $(\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w))_{\tilde{\mathbf{h}}^D_{t-2\Delta},w}$, we know the platform-optimal probabilities $(\boldsymbol{\kappa}^*_t(\tilde{\mathbf{h}}^D_{t-\Delta};w))_{\tilde{\mathbf{h}}^D_{t-\Delta},w}$ and hence for any $\tilde{\mathbf{h}}^D_{t-2\Delta}$, the platform is choosing $(\kappa^*(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w))_w$ to maximize

$$\begin{aligned}
&\mathbb{E}^{F_0}\big[\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};W)\,\Pi_{t-2\Delta}((\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W),\boldsymbol{\kappa}^*_t),\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w),N_{t-2\Delta};W)\big]=\\
&\quad\mathbb{E}^{F_0}\big[\underbrace{\Delta\lambda\,\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W)}_{=\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},1,N_{t-2\Delta}+1;W)}\,\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t,\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},1,N_{t-2\Delta}+1;W),N_{t-2\Delta}+1;W)\\
&\quad+\underbrace{(1-\Delta\lambda)\,\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W)}_{=\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},1,N_{t-2\Delta};W)}\,\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t,\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},1,N_{t-2\Delta};W),N_{t-2\Delta};W)\\
&\quad+\underbrace{(\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};W)-\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W))}_{=\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};W)}\,\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t,\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};W),N_{t-2\Delta};W)\big]
\end{aligned}$$

subject to Equation $\mathbb{P}-t$, Equation $\mathbb{P}$, Equation W-$\Pi$, Equation W-$\Pi_T$, and

$$\frac{\displaystyle\int \kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W)\,\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t,\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},1,N_{t-2\Delta}+1;w),N_{t-2\Delta}+1;W)]\,dF_0(W)}{\displaystyle\int \kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W)\,dF_0(W)}\geq\frac{v_0}{v-p}.$$

We first show that Equation PT-$\kappa$ attains the maximal outcome for $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w,N_{t-2\Delta})=\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w,N_{t-2\Delta})$. To this end, it is useful to define

$$W'=G-(\underline{M}(\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w,N_{t-2\Delta}))+1)p.$$

We argue by contradiction. Assume that Equation PT-$\kappa$ does not attain the maximal outcome for $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w,N_{t-2\Delta})=\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w,N_{t-2\Delta})$ and there exists $(\kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w))_w$ that attains a strictly higher probability of success and attains the maximum. Then, there must be a positive mass of donor types greater than $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$ for which $\kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W) < \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w)$. Since we assumed that $(\kappa'(\tilde{\mathbf{h}}^D_{T-2\Delta},1;w))_w$ attains a higher probability of success than Equation PT-$\kappa$ with threshold $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$, there must also be a positive mass of donor types $w < W'$ so that $\kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W) > 0$. In particular, there must exist sets $A \subset [\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta}), \overline{W}]$ and $B \subset [0, W']$ so that $\int_A \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w) - \kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W)dF(W) = \int_B \kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W)dF(W) > 0$. Now let $\kappa''(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w) = \kappa'(\tilde{\mathbf{h}}^D_{t\,t-2\Delta},1;w)$ for all $w \notin A\cup B$ and $\kappa''(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{t\,t-2\Delta};w)$ for $w \in A$ and $\kappa''(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w) = 0$ for $w \in B$. By the expression for the probability of success in the induction hypothesis, we know that

$$\min_{w\in A} \Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \Delta\lambda\kappa''(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w), N_{t-2\Delta}+1;w) > \max_{w\in B} \Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \Delta\lambda\kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w), N_{t-2\Delta}+1;w)$$

because $\min_{w\in A} w \geq G - N_{t-2\Delta}p > p + \max_{w\in B} w$. Thus, $\kappa''$ strictly relaxes the buyer IC constraint while strictly increasing the objective function relative to $\kappa'$ Hence, $(\kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w))_w$ cannot be a solution to the problem. Thus, Equation PT-$\kappa$ with $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$ is a solution to the maximization problem that is a continuous problem over a compact set.

We next argue that $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w))_w, N_{t-2\Delta}) = \overline{W}$. Assume not. Then $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w))_w, N_{t-2\Delta}) < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$ and by the definition of $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w))_w, N_{t-2\Delta})$, we have

$$\frac{\displaystyle\int_{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w))_w,N_{t-2\Delta})}^{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w,N_{t-2\Delta})} \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};W)\,\Pi_t(\boldsymbol{\kappa}^*_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta},1,N_{t-2\Delta}+1;W), N_{t-2\Delta}+1;W)\,dF_0(W)}{\displaystyle\int_{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w))_w,N_{t-2\Delta})}^{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w,N_{t-2\Delta})} \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};W)\,dF_0(W)}$$
$$\geq \frac{v_0}{v-p}.$$

Now, note that $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w)$ for $w < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$ and hence for $w \in [\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w))_w, N_{t-2\Delta}), \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta}))$, we have

$$\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}, 1, N_{t-2\Delta}+1; w) = \Delta\lambda\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w) = \Delta\lambda\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w).$$

Further, $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w) = 0$ for $w \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta})$, i.e.,

$$\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}, 1, N_{t-2\Delta}+1; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w) = 0.$$

Let us now set $(\kappa(\mathbf{h}^D_{t-2\Delta}, 1; w))_w$ to be given by Equation PT-$\kappa$ with $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w))_w, N_{t-2\Delta})$ by the induction hypothesis. Then, for $w \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w)$, we have

$$\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \underbrace{\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}+1; w)}_{=\Delta\lambda\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)}, N_{t-2\Delta}+1; w) \geq \Pi_t(\boldsymbol{\kappa}^*_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}, 1, N_{t-2\Delta}+1; w), N_{t-2\Delta}+1; w).$$

Hence, by replacing $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; W)$ by $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; W)$, $\Pi_t(\boldsymbol{\kappa}^*_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}, 1, N_{t-2\Delta}+1; W), N_{t-2\Delta}+1; W)$ by $\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}+1; W), N_{t-2\Delta}+1; W)$, and increasing the integration bounds, we must have

$$\frac{\int_{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w))_w, N_{t-2\Delta})}^{\overline{W}} \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; W)\, \Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}+1; W), N_{t-2\Delta}+1; W)\, dF_0(W)}{\int_{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w))_w, N_{t-2\Delta})}^{\overline{W}} \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; W)\, dF_0(W)} \geq \frac{v_0}{v-p},$$

which is a contradiction to the definition of $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta})$. This concludes the proof of $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w))_w, N_{t-2\Delta}) = \overline{W}$. Hence, $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta}) <$

$\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 0, N_{t-2\Delta}; w))_w, N_{t-2\Delta})$ and we can write

$$\begin{aligned}\Pi_{t-2\Delta}(\boldsymbol{\kappa}^*_{t-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w), N_{t-2\Delta}; w) = \quad & \mathbb{1}(w \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta}))\cdot \\ & (\Delta\lambda\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}+1; w), N_{t-2\Delta}+1; w) \\ & +(1-\Delta\lambda)\Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}; w), N_{t-2\Delta}; w)).\end{aligned}$$

This implies that given $(\mathbb{P}(\tilde{\mathbf{h}}^D_{t-3\Delta}; w))_w$, if there exists a $(\kappa(\tilde{\mathbf{h}}^D_{t-3\Delta}, 1; w))_w$, $X \in [0, \overline{W}]$, and $(p_w)_{w<X}$ such that for all $w < X$, $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w) = p_w$ and $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w) = 0$ for all $w \geq X$, then

$$\Pi_{t-2\Delta}(\boldsymbol{\kappa}^*_{t-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w), N; w) \geq \Pi_{t-\Delta}(\boldsymbol{\kappa}^*_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N; w).$$

Finally, we show that this value of the objective function can be attained by $\kappa_{t-\Delta}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)$ as in Equation PT-$\kappa$ only if

$$\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta}) \leq \max\{G - \big(N_{t-2\Delta} + \frac{T-t}{\Delta}\big)p, \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta})\}.$$

First, assume $G - \big(N_{t-2\Delta} + \frac{T-t}{\Delta}\big)p > \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta})$. For $w < G - \big(N_{t-2\Delta} + \frac{T-t}{\Delta}\big)p$, it must be that $\Pi_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N_{t-2\Delta}+1; w) = 0$. For $w \geq G - \big(N_{t-2\Delta} + \frac{T-t}{\Delta}\big)p$, we have $\Pi_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N_{t-2\Delta}+1; w) > 0$. Thus, $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta}) > G - \big(N_{t-2\Delta} + \frac{T-t}{\Delta}\big)p$ would strictly decrease the objective function.

Next, assume $G - \big(N_{t-2\Delta} + \frac{T-t}{\Delta}\big)p \leq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta})$. For $w \geq G - \big(N_{t-2\Delta} + \frac{T-t}{\Delta}\big)p$, we have again $\Pi_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N_{t-2\Delta}+1; w) > 0$. Hence, $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta}) > \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w, N_{t-2\Delta})$ would strictly decrease the objective function.

Thus, for any solution of the platform's problem in period $t-\Delta$ of the form Equation PT-$\kappa$, Equation $\tilde{D}^*$-Region must be satisfied. This concludes the induction step.

**Step 3: Implementation by equilibrium**

Finally, we show that the optimal solution is achieved by the PBE constructed in Proposition 1. Consider the PBE assessment $(D^\Delta_+, b^\Delta, (F^\Delta(\cdot|\mathbf{x}))_\mathbf{x})$ from the proof of Proposition 1. This assessment induces a probability measure $\mathbb{P}$ on outcomes and also induces corresponding systems of probabilities $\kappa(\tilde{\mathbf{h}}^D_t, 1; w)$ and $\mathbb{P}(\tilde{\mathbf{h}}^D_t, ; w)$ over reduced histories, as defined in the **Step 1** above.

Consider any on-path buyer history in the last period $\mathbf{h}_T^{B,\Delta} = \prod_{s\in\mathbb{T}^\Delta, s\leq T}(N_{s-\Delta}, D_{s-\Delta})$. The PBE specifies that buyers purchase if and only if the probability of success is at least $\frac{v_0}{v-p}$. In addition, in the preceding period, unless success is already guaranteed, donors with $W \geq \underline{D}^\Delta(N_{T-\Delta}, \Delta)$ donate $\max\{D_{T-2\Delta}, \underline{D}^\Delta(N_{T-\Delta}, \Delta)\}$. This makes the next buyer just indifferent between buying and not buying if such a donation amount exists and $\underline{D}^\Delta(N_{T-\Delta}, \Delta) = \overline{W}$ otherwise. Therefore, for any on-path history $\mathbf{h}_{T-\Delta}^{D,\Delta} = \left(\prod_{s\in\mathbb{T}^\Delta, s\leq T-\Delta}(N_{T-\Delta}, D_{s-\Delta}), N_{T-\Delta}\right)$, the induced probabilities over reduced histories satisfy

$$\kappa(\tilde{\mathbf{h}}_{T-\Delta}^D, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}_{T-\Delta}^D; w) \text{ if and only if } w \geq \underline{D}^\Delta(N_{T-\Delta}, \Delta).$$

Now, notice that since $\underline{D}^\Delta(N_{T-\Delta}, \Delta)$ is calculated using the indifference condition for buyers, $\pi^\Delta(N, D, u)$ is increasing in $D$, and $F^\Delta$ is a truncation given by Equation PT-belief, this $\underline{D}^\Delta(N_{T-\Delta}, \Delta)$ is exactly $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}_{T-\Delta}^D; w))_w, N_{T-\Delta})$ defined in Equation $\underline{W}$ in the solution to the relaxed problem. Analogous arguments apply to any history $\mathbf{h}_t^{D,\Delta} = \left(\prod_{s\in\mathbb{T}^\Delta, s\leq t}(N_{s-\Delta}, D_{s-\Delta}), N_t\right)$. Therefore, the PBE assessment from the proof of Proposition 1 induces exactly $(\boldsymbol{\kappa}_\Delta^*(0; w))_w$ and it achieves the optimum in the relaxed problem above. Thus, it must be platform-optimal in the full class of PBEs.

**Step 4: Uniqueness of limits**

We have shown in **Step 2** that solutions to the reduced problem that correspond to Markov equilibria are as in Equation PT-$\kappa$ only if they satisfy Equation $\tilde{D}^*$-Region. Now, for a given $t$ if $\Delta$ is sufficiently small, then $G - \left(N + \frac{T-t}{\Delta}\right)p < 0$, so any sequence of outcomes converges point-wise to the equilibrium outcome attained by the Markov equilibrium constructed in **Step 1**.

## A.2 Proof of Proposition 2

**Proof Outline:** Given any assessment, we will use the same class of reduced histories and systems of probabilities $\kappa(\tilde{\mathbf{h}}_t^B; w)$ and $\mathbb{P}(\tilde{\mathbf{h}}_t^D, N_t; w)$, as defined in the proof of Proposition 1. As for any platform-optimal equilibrium, in any donor-optimal equilibrium, the buyer should always buy when she is indifferent between buying and not buying, so we can again assume that $b_s \in \{0, 1\}$ for all histories. The induced probability measure $\mathbb{P}$ allows us to define $(\boldsymbol{\kappa}_\Delta(0; w))_w$ which determines the outcome of the game, except for the donation amount.

The proof proceeds in four steps. **Step 1** establishes that in order to find donor-optimal equilibria, it is without loss to restrict attention to a smaller class of assessments. In **Step 2**, we formulate the relaxed donor problem, just as we did in the proof of Proposition 1. In **Step 3**, we solve the donor's problem and show that the platform-optimal solution also corresponds to a solution of the donor's problem. We also prove that all solutions that are Markov equilibria with PT strategies must converge to the same limit as $\Delta \to 0$. Finally, in **Step 4**, we verify that the donor strategy constructed in **Step 3** of the proof of Proposition 1 is consistent with the donor-optimal solution.

**Proof:**

**Step 1: Limiting the class of assessments**

To find a donor-optimal equilibrium, we first show (in Lemmata 2 and 3 below) that it is without loss to restrict attention to assessments that satisfy two properties. First, at histories at which buyers are induced to buy, all donor types that donate positive amounts make the same aggregate donation. Second, if a donor does not incentivize buying, he donates nothing. Within the class of assessments satisying these two properties the mapping from reduced histories to donations becomes unique, a fact we use when we formulate the donor's maximization problem. The proofs of Lemmata 2 and 3 are in the Online Appendix in Section B.2.

**Lemma 2.** *For any donor-optimal PBE* $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$*, there exists a donor-optimal PBE* $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ *such that*

*i) both assessments generate the same probability measures* $(\boldsymbol{\kappa}_\Delta(0;w))_w$*,*

*ii) the donor and buyer strategy is such that for each* $\mathbf{h}_t^{D,\Delta}$*, there exists a* $D_*(\mathbf{h}_t^{D,\Delta}) \in \mathbb{R}$ *such that*

$$\begin{aligned} \hat{D}_+^\Delta(\mathbf{h}_t^{D,\Delta};w) &= \begin{cases} \tilde{D}_+(\mathbf{h}_t^{D,\Delta};w) & \text{if } \tilde{b}^\Delta(\mathbf{h}_t^{D,\Delta}, \tilde{D}_+(\mathbf{h}_t^{D,\Delta};w)) = 0 \\ D_*(\mathbf{h}_t^{D,\Delta}) & \text{if } \tilde{b}^\Delta(\mathbf{h}_t^{D,\Delta}, \tilde{D}_+(\mathbf{h}_t^{D,\Delta};w)) = 1 \end{cases}, \text{ and} \\ \hat{b}^\Delta(\mathbf{h}_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) &= \begin{cases} 1 & \text{if } D_{t-\Delta} = D_*(\mathbf{h}_{t-\Delta}^{D,\Delta}) \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{2}$$

Hence, to find a donor-optimal equilibrium, it suffices to restrict attention to assessments $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ such that for any $\mathbf{h}_t^{D,\Delta}$, there exists a $D_*(\mathbf{h}_t^{D,\Delta}) \in \mathbb{R}$ with

$$\tilde{D}_+^\Delta(\mathbf{h}_t^{D,\Delta};w) = D_*(\mathbf{h}_t^{D,\Delta}), \text{ whenever } \hat{b}^\Delta(\mathbf{h}_t^{D,\Delta}, \tilde{D}_+^\Delta(\mathbf{h}_t^{D,\Delta};w)) = 1 \tag{3}$$

and $\tilde{b}^{\Delta}$ as is defined in Equation 2. Indeed, the platform-optimal equilibrium constructed in Proposition 1 is in this class.

**Lemma 3.** *For any donor-optimal PBE $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$ for which the donor strategy satisfies Equation 3 and buyer strategy Equation 2, there exists a donor-optimal PBE $(\hat{b}^{\Delta}, \hat{D}_{+}^{\Delta}, \hat{F}^{\Delta})$ so that*

*i) both assessments generate the same probability measures $(\boldsymbol{\kappa}_{\Delta}(0; w))_{w}$,*

*ii) the donor strategy is such that for each $\mathbf{h}_t^{D,\Delta}$,*

$$\hat{D}_{+}^{\Delta}(h_t^{D,\Delta}; w) = \begin{cases} D_{t-\Delta} & \text{if } \tilde{b}^{\Delta}(h_t^{D,\Delta}, \tilde{D}_{+}(\mathbf{h}_t^{D,\Delta}; w)) = 0 \\ \tilde{D}_{+}(\mathbf{h}_t^{D,\Delta}; w) & \text{if } \tilde{b}^{\Delta}(h_t^{D,\Delta}, \tilde{D}_{+}(\mathbf{h}_t^{D,\Delta}; w)) = 1 \end{cases}, \tag{4}$$

*and $\hat{b}^{\Delta} = \tilde{b}^{\Delta}$.*

Hence, in the following, we restrict attention to assessments $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$ that satisfy Equation 4. Indeed, the platform-optimal equilibrium from Proposition 1 satisfies Equation 4.

**Step 2: Relaxed donor problem.**

Consider an arbitrary assessment $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$ that satisfies Equation 4. Recall that, analogously to Proposition 1, we can define reduced histories, systems of probabilities $\kappa(\tilde{\mathbf{h}}_t^B; w)$, $\mathbb{P}(\tilde{\mathbf{h}}_t^D., N_t; w)$, and the mapping $\mathscr{R}_{\tilde{b}^{\Delta}}$ that maps general histories to the corresponding reduced history. Let $\mathscr{D}(\mathscr{R}_{\tilde{b}^{\Delta}}(\mathbf{h}_t^{D,\Delta})) := D_*(\mathbf{h}_t^{D,\Delta})$. Then, the donor's payoff given assessment $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$ can be written as

$$\begin{aligned} &\mathbb{D}_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D; w), N_{t-\Delta}, D_{t-2\Delta}; w) = \\ &\Delta\lambda \tfrac{\kappa(\tilde{\mathbf{h}}_{t-\Delta}^D, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D; w)} \mathbb{D}_t\big(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D, 1, N_{t-\Delta}+1; w), N_{t-\Delta}+1, \max\{D_{t-2\Delta}, \mathscr{D}(\tilde{\mathbf{h}}_{t-\Delta}^D)\}; w\big) \\ &+(1-\Delta\lambda) \tfrac{\kappa(\tilde{\mathbf{h}}_{t-\Delta}^D, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D; w)} \mathbb{D}_t\big(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}_{\mathbf{t-\Delta}}^{\mathbf{D}}, 1, N_{t-\Delta}; w), N_{t-\Delta}, \max\{D_{t-2\Delta}, \mathscr{D}(\tilde{\mathbf{h}}_{t-\Delta}^D)\}; w\big) \\ &+\left(1 - \tfrac{\kappa(\tilde{\mathbf{h}}_{t-\Delta}^D, 1; w)}{\mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D; w)}\right) \mathbb{D}_t(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D, 0, N_{t-\Delta}; w), N_{t-\Delta}, D_{t-2\Delta}; w), \end{aligned} \tag{W-$\mathbb{D}$}$$

with the initial condition

$$\mathbb{D}_T\big(\boldsymbol{\kappa}_{T+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}_T^D; w), N_T, D_{T-\Delta}; w\big) = \mathbb{1}(G - N_T p \le w) \cdot (w - \max\{D_{T-\Delta}, G - N_T p\}).$$

Then, we can write the donor's problem as

$$\max_{(\boldsymbol{\kappa}_\Delta(0;w))_w,\, (\mathscr{D}(\tilde{\mathbf{h}}_t^D))_{\tilde{\mathbf{h}}_t^D\in\tilde{\mathscr{H}}_t^D,\, t\in\mathbb{T}^\Delta}} \mathbb{E}^{F_0}[\mathbb{D}_0(\boldsymbol{\kappa}_\Delta, \mathbb{P}(0;w), 0, 0; W)]$$

subject to Equation $\mathbb{P}$, Equation $\mathbb{P}-t$, Equation Buyer IC, and the donor IC constraint defined as

$$\left.\begin{array}{l} \text{if} \\ \quad \mathbb{D}_t(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D, 0, N_{t-\Delta}; w), N_{t-\Delta}, D_{t-2\Delta}; w) < \\ \quad \Delta\lambda\, \mathbb{D}_t\big(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D, 1, N_{t-\Delta}+1; w), N_{t-\Delta}+1, \max\{D_{t-2\Delta}, \mathscr{D}(\tilde{\mathbf{h}}_{t-\Delta}^D)\}; w\big) \\ \quad +(1-\Delta\lambda)\, \mathbb{D}_t(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D, 1, N_{t-\Delta}; w), N_{t-\Delta}, \max\{D_{t-2\Delta}, \mathscr{D}(\tilde{\mathbf{h}}_{t-\Delta}^D)\}; w), \\ \text{then} \\ \quad \kappa(\tilde{\mathbf{h}}_{t-\Delta}^D, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D; w) \qquad \forall\, \tilde{\mathbf{h}}_t^D \in \tilde{\mathscr{H}}_t^D, t\in\mathbb{T}^\Delta, N\in\mathbb{N}, w\in[0,\overline{W}]. \end{array}\right\} \quad \text{(Donor IC)}$$

This donor problem is a relaxed problem because we have not shown that each $\boldsymbol{\kappa}$ satisfying the above constraints indeed corresponds to a PBE. Furthermore, we consider a lower bound on donations and we only impose a necessary condition for donor incentive compatibility.

We denote a solution to the above problem by $\boldsymbol{\kappa}_\Delta^{**}(0;w))_w$ and $(\mathscr{D}^{**}(\tilde{\mathbf{h}}_t^D, 1))_{\tilde{\mathbf{h}}_t^D\in\tilde{\mathscr{H}}_t^D,\, t\in\mathbb{T}^\Delta}$. Recall that the solution that we presented to the platform's relaxed problem was denoted $\boldsymbol{\kappa}_\Delta^{*}(0;w))_w$.

**Step 3: Solution to the relaxed problem**

Next, we show the following two statements:

i) $(\boldsymbol{\kappa}_\Delta(0;w))_w$ as in Equation PT-$\kappa$ can be a solution to the relaxed problem only if Equation $\tilde{D}^*$-Region is satisfied;

ii) $(\boldsymbol{\kappa}_\Delta(0;w))_w$ as in Equation PT-$\kappa$ with $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}_t^D; w))_w, N_t) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}_t^D; w))_w, N_t)$ is always a solution.

Given these two statements it follows immediately that in the limit as $\Delta \to 0$, the outcome is unique by the proof of Proposition 1. We solve the relaxed maximization problem by backwards induction in time and use an analogous decomposition to Equation $\Pi_0$-Decomp. For any period $t$, we can write

$$\mathbb{E}^{F_0}[\mathbb{D}_0(\boldsymbol{\kappa}_\Delta, \mathbb{P}(0;w), 0, 0; W)] = \sum_{\tilde{\mathbf{h}}_{t-\Delta}^D\in\tilde{\mathscr{H}}_{t-\Delta}^D} \mathbb{E}^{F_0}[\mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D; W)\cdot \mathbb{D}_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}_{t-\Delta}^D; w), N_{t-\Delta}, D_{t-2\Delta}; W)].$$

**(a) Induction start:** For donor optimality, we do not need an intertemporal statement for the induction hypothesis. Therefore, we consider only the last period ($t = T$). We characterize the optimal $(\boldsymbol{\kappa}^{**}_T(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_{\tilde{\mathbf{h}}^D_{T-\Delta}\in\tilde{\mathscr{H}}^D_{T-\Delta}, w\in[0,\overline{W}]}$ and $\mathscr{D}^{**}(\tilde{\mathbf{h}}^D_{T-\Delta})$ given $\left(\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)\right)_{\tilde{\mathbf{h}}^D_{T-\Delta}\in\tilde{\mathscr{H}}^D_{T-\Delta}, w\in[0,\overline{W}]}$, $N_{T-\Delta}$, and $D_{T-2\Delta}$. Using the analogous decomposition to Equation $\Pi_0$-Decomp above at $T$, we know that $\boldsymbol{\kappa}^{**}_T$ must maximize

$$\sum_{\tilde{\mathbf{h}}^D_{T-\Delta}\in\tilde{\mathscr{H}}^D_{T-\Delta}} \mathbb{E}^{F_0}[\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; W)\cdot \mathbb{D}_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N_{T-\Delta}, D_{T-2\Delta}; W)]$$

subject to Equation $\mathbb{P}$, Equation $\mathbb{P}-t$, Equation W-$\mathbb{D}$, Equation Buyer IC given by

$$\frac{\int \kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; W)\,\mathbb{1}(G-(N_{T-\Delta}+1)p \le W)\, dF_0(W)}{\int \kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; W)\, dF_0(W)} \ge \frac{v_0}{v-p},$$

and the Equation Donor IC constraint

$$\left.\begin{aligned}
&\text{if}\\
&\quad \mathbb{1}(G-N_{T-\Delta}p \le w)\cdot\big(w-\max\{D_{T-2\Delta}, G-N_{T-\Delta}p\}\big) <\\
&\quad \Delta\lambda\, \mathbb{1}(G-(N_{T-\Delta}+1)p \le w)\cdot\big(w-\max\{D_{T-2\Delta}, \mathscr{D}(\tilde{\mathbf{h}}^D_{T-\Delta}), G-(N_{T-\Delta}+1)p\}\big)\\
&\quad +(1-\Delta\lambda)\,\mathbb{1}(G-N_{T-\Delta}p \le w)\cdot\big(w-\max\{D_{T-2\Delta}, \mathscr{D}(\tilde{\mathbf{h}}^D_{t-\Delta}), G-N_{T-\Delta}p\}\big),\\
&\text{then}\\
&\quad \kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w) \qquad \forall\, \tilde{\mathbf{h}}^D_{T-\Delta}\in\tilde{\mathscr{H}}^D_{T-\Delta}, N\in\mathbb{N}, w\in[0,\overline{W}].
\end{aligned}\right\}$$

First, using Equation W-$\mathbb{D}$, we can write the donor's payoff at $T-\Delta$ as

$$\begin{aligned}
&\mathbb{D}_{T-\Delta}(\boldsymbol{\kappa}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N_{T-\Delta}, D_{T-2\Delta}; w) =\\
&\quad \Delta\lambda\, \tfrac{\kappa(\tilde{\mathbf{h}}^D_{T-\Delta},1;w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta};w)}\, \mathbb{1}(G-(N_{T-\Delta}+1)p \le w)\cdot\big(w-\max\{D_{T-2\Delta}, \mathscr{D}(\tilde{\mathbf{h}}^D_{T-\Delta}), G-(N_{T-\Delta}+1)p\}\big)\\
&\quad +(1-\Delta\lambda)\, \tfrac{\kappa(\tilde{\mathbf{h}}^D_{T-\Delta},1;w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta};w)}\, \mathbb{1}(G-N_{T-\Delta}p \le w)\cdot\big(w-\max\{D_{T-2\Delta}, \mathscr{D}(\tilde{\mathbf{h}}^D_{T-\Delta}), G-N_{T-\Delta}p\}\big)\\
&\quad +\left(1-\tfrac{\kappa(\tilde{\mathbf{h}}^D_{T-\Delta},1;w)}{\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta};w)}\right)\mathbb{1}(G-N_{T-\Delta}p \le w)\cdot\big(w-\max\{D_{T-2\Delta}, G-N_{T-\Delta}p\}\big).
\end{aligned}$$

For $w < G-(N_{T-\Delta}+1)p$ the donor payoff is zero. For $w \ge G-(N_{T-\Delta}+1)p$, if $\mathscr{D}(\tilde{\mathbf{h}}^D_{T-\Delta}) \le \max\{D_{T-2\Delta}, G-(N_{T-\Delta}+1)p)\}$, the donor payoff is strictly increasing in $\kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w)$. Further, increasing $\kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w)$ relaxes Equation Buyer IC. Any donation $\mathscr{D}(\tilde{\mathbf{h}}^D_{T-\Delta})$ in excess of $\max\{D_{T-2\Delta}, G-(N_{T-\Delta}+1)p)\}$ decreases this objective function. So, ignoring Equation Donor

IC, it is optimal to set $\mathscr{D}(\tilde{\mathbf{h}}^D_{T-\Delta}) \le \max\{D_{T-2\Delta}, G-(N_{T-\Delta}+1)p)\}$ and $\kappa^{**}(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)$ for $w \ge G-(N_{T-\Delta}+1)p$. Now note that if $\mathscr{D}(\tilde{\mathbf{h}}^D_{T-\Delta}) \le G - N_{T-\Delta}p$, then for $w \ge G-(N_{T-\Delta}+1)p$, Equation Donor IC implies $\kappa(\tilde{\mathbf{h}}^D_{T-\Delta}, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w)$.

Thus, we have established that $\boldsymbol{\kappa}^{**}_T$ is as defined in Equation PT-$\kappa$ with $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_w, N_t) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_w, N_{T-\Delta})$, and

$$\mathscr{D}^{**}(\tilde{\mathbf{h}}^D_{T-\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_w, N_{T-\Delta}) < G-(N_{T-\Delta}+1)p$$

is a solution to the donor optimization problem in period $t = T$. Further, $\boldsymbol{\kappa}^{**}_T$ as defined in Equation PT-$\kappa$ can be a solution only if Equation $\tilde{D}^*$-Region is satisfied by an analogous argument to Proposition 1.[55]

The maximal payoff of the donor can therefore be written as

$$\begin{aligned}\mathbb{D}_{T-\Delta}(\boldsymbol{\kappa}^{**}_T, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w), N_{T-\Delta}, D_{T-2\Delta}; w) = \mathbb{1}\big(w \ge G-(N_{T-\Delta}+1)p\big)\cdot \\ \big(w - \Delta\lambda \max\{D_{T-2\Delta}, G-(N_{T-\Delta}+1)p\} - \mathbb{1}\big(w \ge G - N_{T-\Delta}p)(1-\Delta\lambda)\max\{D_{T-2\Delta}, G-N_{T-\Delta}p\}\big).\end{aligned}$$

**(b) Induction hypothesis:** Fix a $t$. Then, given $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)$ for $\tilde{\mathbf{h}}^D_{t-\Delta} \in \tilde{\mathscr{H}}^D_{t-\Delta}$, let us assume that

$$\sum_{\tilde{\mathbf{h}}^D_{t-\Delta} \in \tilde{\mathscr{H}}^D_{t-\Delta}} \mathbb{E}^{F_0}[\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; W) \cdot \mathbb{D}_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N_{t-\Delta}, D_{t-2\Delta}; W)]$$

subject to Equation $\mathbb{P}-t$, Equation $\mathbb{P}$, Equation Buyer IC and Equation Donor IC is maximized by the platform-optimal $\boldsymbol{\kappa}^*_t$ and $\mathscr{D}(\tilde{\mathbf{h}}^D_{t-\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-\Delta})$. Hence, for $t' \ge t$,

$$\begin{aligned}\mathbb{D}_{t'-\Delta}(\boldsymbol{\kappa}^{**}_{t'}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w), N_{t'-2\Delta}, D_{t'-3\Delta}; w) = \mathbb{1}\big(w \ge \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w))_w, N_{t'-\Delta})\big)\cdot \\ \big(\Delta\lambda\, \mathbb{D}_{t'}(\boldsymbol{\kappa}^{**}_{t'+\Delta}, \Delta\lambda \mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w), N_{t'-\Delta}+1, \max\{D_{t'-3\Delta}, \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w))_w, N_{t'-\Delta}+1)\}; w) \\ +(1-\Delta\lambda)\, \mathbb{D}_{t'}(\boldsymbol{\kappa}^{**}_{t'+\Delta}, (1-\Delta\lambda)\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w), N_{t'-\Delta}, \max\{D_{t'-3\Delta}, \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w))_w, N_{t'-\Delta})\}; w)\big).\end{aligned}$$

with the initial condition given by the induction start.

Finally, assume that the following *Donor Monotonicity Condition* is satisfied: Given a history $\tilde{\mathbf{h}}^D_{t'-\Delta}$, consider two sets of probabilities $(\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w))_w$ and $(\mathbb{P}'(\tilde{\mathbf{h}}^D_{t'-\Delta}; w))_w$ with $\int \mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; W) dF_0(W) = \int \mathbb{P}'(\tilde{\mathbf{h}}^D_{t'-\Delta}; W) dF_0(W)$. Then, for any two sets $A, B$ such that

[55] $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_w, N_{T-\Delta}) \ge \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{T-\Delta}; w))_w, N_{T-\Delta})$ can be a solution because for $w < G-(N_{T-\Delta}+1)p$ the donor payoff is zero regardless of the donation amount.

i) all $w \in A$ satisfy $w > w'$ for all $w' \in B$,

ii) $\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w) = \mathbb{P}'(\tilde{\mathbf{h}}^D_{t'-\Delta}; w)$ for all $w \notin A \cup B$,

iii) for $w \in A$, $\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w) > \mathbb{P}'(\tilde{\mathbf{h}}^D_{t'-\Delta}; w)$, and for $w \in B$, $\mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w) < \mathbb{P}'(\tilde{\mathbf{h}}^D_{t'-\Delta}; w)$,

it must be that

$$\min_{w\in A} \mathbb{D}_{t'-\Delta}(\boldsymbol{\kappa}^{**}_{t'}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t'-\Delta}; w), N_{t'-2\Delta}, D_{t'-3\Delta}; w) > \max_{w\in B} \mathbb{D}_{t'-\Delta}(\boldsymbol{\kappa}^{**}_{t'}, \mathbb{P}'(\tilde{\mathbf{h}}^D_{t'-\Delta}; w), N'_{t'-2\Delta}, D_{t'-3\Delta}; w).$$

**(c) Induction step:** Let us assume the induction hypothesis is satisfied and consider period $t-\Delta$. Fix arbitrary $(\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_{\tilde{\mathbf{h}}^D_{t-2\Delta}, w}$. For any $\tilde{\mathbf{h}}^D_{t-2\Delta}$, the donor is choosing $(\kappa^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; w))_w$ to maximize

$$\begin{aligned}
&\mathbb{E}^{F_0}\big[\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; W)\, \mathbb{D}_{t-2\Delta}((\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; W), \boldsymbol{\kappa}^{**}_t), \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w), N_{t-2\Delta}, D_{t-3\Delta}; W)\big] = \\
&\mathbb{E}^{F_0}\big[\Delta\lambda\, \kappa(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; W)\, \mathbb{D}_{t-\Delta}(\boldsymbol{\kappa}^{**}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}+1; W), N_{t-2\Delta}+1, \max\{D_{t-3\Delta}, \mathscr{D}(\tilde{\mathbf{h}}^D_{t-2\Delta})\}; W) \\
&+(1-\Delta\lambda)\, \kappa(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; W)\, \mathbb{D}_{t-\Delta}(\boldsymbol{\kappa}^{**}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}; W), N_{t-2\Delta}, \max\{D_{t-3\Delta}, \mathscr{D}(\tilde{\mathbf{h}}^D_{t-2\Delta})\}; W) \\
&+(\mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}; W) - \kappa(\tilde{\mathbf{h}}^D_{T-2\Delta}, 1; W))\, \mathbb{D}_{t-\Delta}(\boldsymbol{\kappa}^{**}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{T-2\Delta}, 0, N_{t-2\Delta}; W), N_{t-2\Delta}, D_{t-3\Delta}; W)\big]
\end{aligned}$$

subject to Equation $\mathbb{P}-t$, Equation $\mathbb{P}$, Equation Buyer IC and Equation Donor IC.

If we ignore Equation Donor IC, for a fixed $\mathscr{D}(\tilde{\mathbf{h}}^D_{t-2\Delta})$, we can use an analogous argument as for platform optimality in Proposition 1 to show that the donor payoff is maximized by setting $\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)$ for all $w \geq W'' := \max\{\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)_w, N_{t-2\Delta}), \mathscr{D}(\tilde{\mathbf{h}}^D_{t-2\Delta})\}$. We proceed by contradiction. Assume not and let

$$W' = G - (\underline{M}(W'') + 1)p\ (< W'').$$

Then, any candidate solution $(\kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w$ must be such that there exist sets $A \subset [W'', \overline{W}]$ and $B \subset [0, W']$ so that $\int_A \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w) - \kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; W) dF(W) = \int_B \kappa'(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; W) dF(W) > 0$. Then, the donor has a deviation such that donor payoffs can be strictly increased without violating Equation Buyer IC by Condition Donor Monotonicity. Further, because donor payoff is decreasing in $\mathscr{D}(\tilde{\mathbf{h}}^D_{t-2\Delta})$, if the donor-optimal donation is $\mathscr{D}^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta}) \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)_w$, it must be that $\mathscr{D}^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)_w$ and $\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1; w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)$ for all $w \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)_w$. If

$\mathscr{D}^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta}) < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w)_w, N_{t-2\Delta})$, it cannot be that $\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w)$ for all $w \geq W''$ as if that was the case, then for all donor types $w \in (\mathscr{D}^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta})]$, we must have $\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w) = \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w)$ by Equation Donor IC (because $\mathbb{D}_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w), N_{t-2\Delta}, D_{t-3\Delta};w) = 0$ by the induction hypothesis), which in turn would violate Equation Buyer IC.

Thus, there must be a positive mass of $w > \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w)_w, N_{t-2\Delta})$ such that $\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w) < \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w)$. Let $W_0 > W''$ be the infimum of such donor types. Then, it must be that

$$\frac{\int_{W_0}^{\overline{W}} (\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};W)) - \kappa^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W))\,\Pi_t(\boldsymbol{\kappa}_{t+\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^B_{t-2\Delta}, N_{t-\Delta}+1;W), N_{t-\Delta}+1;W)\,dF_0(W)}{\int_{W_0}^{\overline{W}} (\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};W)) - \kappa^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta},1;W))\,dF_0(W)} < \frac{v_0}{v-p}$$

because there must also be a mass of donor types $w < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$ such that $\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w) > 0$ . This in turn implies that $W_0 < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w))_w, N_{t-2\Delta})$. But then, it must be that for all $w \in (W_0, \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w))_w, N_{t-2\Delta}))$,

$$\mathbb{D}_{t-\Delta}(\boldsymbol{\kappa}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta},0,N_{t-2\Delta};w), N_{t-2\Delta}, D_{t-3\Delta};w) = 0,$$

so the left-hand side of condition Equation Donor IC is satisfied. This is a contradiction to how we defined $W_0$. Thus, we can conclude that it cannot be optimal that $\mathscr{D}^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta}) < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$. Hence, Equation PT-$\kappa$ with $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_t;w))_w, N_t)$ and $\mathscr{D}^{**}(\tilde{\mathbf{h}}^D_{t-2\Delta}) \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$ must be a solution to the optimization problem.

Next, we show that any other solution of the form Equation PT-$\kappa$ must satisfy Equation $\tilde{D}^*$-Region. We only need to consider the case $G - \left(N+2+\frac{T-t}{\Delta}\right)p > \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$. For $w \in [\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta}), G - \left(N+2+\frac{T-t}{\Delta}\right)p]$, the probability of success and $\mathbb{D}_{t-2\Delta}((\kappa(\tilde{\mathbf{h}}^D_{t-2\Delta},1;w), \kappa^{**}_{\mathbf{t}}), \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w), N_{t-2\Delta}, D_{t-3\Delta};w))$ are zero, but for $w > G - \left(N+2+\frac{T-t}{\Delta}\right)p$, both are positive. Thus, if $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta}) > G - \left(N+2+\frac{T-t}{\Delta}\right)p$, then Equation Buyer IC is slack, the donor payoff is strictly smaller, and the Equation Donor IC remains satisfied relative to $\tilde{D}^*((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta}) = \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta};w))_w, N_{t-2\Delta})$. Hence, Equation $\tilde{D}^*$-Region must be satisfied. This implies that

$$\mathbb{D}_{t-2\Delta}(\boldsymbol{\kappa}^{**}_{t-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w), N_{t-2\Delta}, D_{t-3\Delta}; w) = \mathbb{1}(w \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^{\mathbf{D}}_{\mathbf{t-2\Delta}}; w))_w))$$

$$\Delta\lambda\, \mathbb{D}_{t-\Delta}\Big(\boldsymbol{\kappa}^{**}_{t-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}+1; w), N_{t-2\Delta}+1, \max\{D_{t-3\Delta}, \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w)\}; w\Big)$$

$$+(1-\Delta\lambda)\, \mathbb{D}_{t-\Delta}\Big(\boldsymbol{\kappa}^{**}_{t-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}, 1, N_{t-2\Delta}; w), N_{t-2\Delta}, \max\{D_{t-3\Delta}, \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w)\}; w\Big).$$

It is increasing in $N_{t-2\Delta}$, weakly decreasing in $D_{t-3\Delta}$, and weakly increasing in $w$. For $D_{t-3\Delta} \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w))_w)$, $\mathbb{D}_{T-2\Delta}(\boldsymbol{\kappa}^{**}_{t-\Delta}, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w), N_{t-2\Delta}, D_{t-3\Delta}; w)$ is strictly decreasing and continuous in $D_{t-3\Delta}$.

Finally, *Donor Monotonicity Condition* is satisfied. To see this, note that for $\int \mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; W)dF_0(W) = \int \mathbb{P}'\tilde{\mathbf{h}}^D_{t-2\Delta}; W)dF_0(W)$, let us consider sets $A, B$ such that

i) all $w \in A$ satisfy $w > w'$ for all $w' \in B$,

ii) $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w) = \mathbb{P}'(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)$ for all $w \notin A \cup B$,

iii) for $w \in A$, $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w) > \mathbb{P}'(\tilde{\mathbf{h}}^D_{t-2\Delta}; w)$, and for $w \in B$, $\mathbb{P}(\tilde{\mathbf{h}}^D_{t-2\Delta}; w) < \mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w)$.

Since $\underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-2\Delta}) < \underline{W}((\mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-2\Delta})$ the expressions

$$(\Delta\lambda\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)\mathbb{1}(w \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-2\Delta})))_w \qquad \text{and}$$

$$(\Delta\lambda\mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w)\mathbb{1}(w \geq \underline{W}'((\mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-2\Delta})))_w,$$

$$((1-\Delta\lambda)\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)\mathbb{1}(w \geq \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-2\Delta})))_w \qquad \text{and}$$

$$((1-\Delta\lambda)\mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w)\mathbb{1}(w \geq \underline{W}'((\mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-2\Delta})))_w$$

$$(\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w)\mathbb{1}(w < \underline{W}((\mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-2\Delta})))_w \qquad \text{and}$$

$$((1-\Delta\lambda)\mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w)\mathbb{1}(w < \underline{W}'((\mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w))_w, N_{t-2\Delta})))_w,$$

all satisfy the properties of the Donor Monotonicity Condition in period $t-\Delta$. Thus, by the induction hypothesis, it must be that

$$\mathbb{D}_{t'-\Delta}(\boldsymbol{\kappa}^{**}_t, \mathbb{P}(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N_{t-2\Delta}, D_{t-3\Delta}; w) > \mathbb{D}_{t'-\Delta}(\boldsymbol{\kappa}^{**}_{t'}, \mathbb{P}'(\tilde{\mathbf{h}}^D_{t-\Delta}; w), N'_{t-2\Delta}, D_{t-3\Delta}; w).$$

This concludes the induction step.

**Step 4: Implementation by equilibrium**

We have already shown in Proposition 1 that $(\kappa^*_\Delta(0;w))_w$ is induced by the constructed assessment and established that the wealth threshold $\underline{D}^\Delta(N,u)$ corresponds to $\underline{W}(\mathbb{P}(\tilde{\mathbf{h}}^D_t;w)_w)$ if there is a history $\mathbf{h}^{D,\Delta}_t$ with $\mathscr{R}_{b^\Delta}(\mathbf{h}^{D,\Delta}_t)=\tilde{\mathbf{h}}^D_t$ and $u=T-t$, $N_t=N$. This concludes the proof.

## A.3 Proof of Proposition 3

We prove the result in two parts. Just as we did for Proposition 1, first in Section A.3.1, we construct, for any period length $\Delta$, a Markov equilibrium in which the donor plays a pooling-threshold (PT) strategy. We show that the limit of these equilibria as $\Delta\to 0$ exists, and is as specified in Proposition 3. We also establish some properties of the limiting expressions in Lemma 4. Then, in Section A.3.2, we establish buyer-optimality of this PBE.

### A.3.1 Equilibrium construction

We construct an equilibrium in the discrete time game by proving the following assertions.

**Assertion Donor Discrete (DD):** *Fix $\mathbf{y}=(G,T,p,\lambda,v,v_0,\Delta)$. Define the wealth threshold as in the main text:*

$$\overline{D}^\Delta(N,u+\Delta):=\max\{G-(j-1)p-Np,0\}\ \textit{for}\ u\in(\overline{\xi}^\Delta_j,\overline{\xi}^\Delta_{j-1}].$$

*The following assessment $(b^\Delta,D^\Delta_+,\{F^\Delta(\cdot|\mathbf{x})\}_\mathbf{x})$ constitutes a PBE:*

*i)* $b^\Delta(N,D,u)=1$ *if and only if* $D\geq\overline{D}^\Delta(N,u+\Delta)$.

*ii)* $$D^\Delta_+(N,D,u;w)=\begin{cases}\min\{\max\{D,G-Np\},w\} & \text{if } u=0\\ \min\{\max\{D,\overline{D}^\Delta(N,u)\},w\} & \text{if } u>0.\end{cases}$$

*iii) Beliefs $F^\Delta(w;(N,D,u))$ are given by Equation PT-belief with $D^\Delta_*(N,u)=\overline{D}^\Delta(N,u)$.*

*The probability of success is recursively given as follows:*
*For $N\geq\underline{M}(D)-1$, $\pi^\Delta(N,D,u)=1$.*
*For $N<\underline{M}(D)-1$, $\pi^\Delta(N,D,0)=\frac{1-F_0(G-(N+1)p)}{1-F_0(D)}\mathbb{1}(D\geq\overline{D}^\Delta(N,\Delta))$ and if $u>0$, then for $D\geq$*

$\overline{D}^{\Delta}(N,u+\Delta)$

$$\pi^{\Delta}(N,D,u)=\mathbb{E}^{F_0}\left[\sum_{i=1}^{\frac{\max\{u-\overline{\xi}_{\underline{M}(W)-(N+1)},0\}}{\Delta}}(1-\Delta\lambda)^{i-1}\Delta\lambda\pi^{\Delta}(N+1,\,\max\{D,\overline{D}^{\Delta}(N+1,u-(i-1)\Delta)\},\,u-\Delta i)\,\middle|\,W\geq D\right]$$

*and for* $D<\overline{D}^{\Delta}(N,u+\Delta)$

$$\pi^{\Delta}(N,D,u)=\mathbb{P}\big(\tau_1^u\leq T-\overline{\xi}^{\Delta}_{\underline{M}(D)-(N+1)},\tau_2^u\leq T-\overline{\xi}^{\Delta}_{\underline{M}(D)-(N+2)},\ldots,\tau^u_{\underline{M}(D)-(N+1)}\leq T\big)<\frac{v_0}{v-p},$$

*where* $\tau_i^u$ *is the arrival time of the* $i$*-th buyer after period* $u$*. Further,* $\pi^{\Delta}(N,D,u)$ *is strictly increasing in* $N$*,* $D$ *and* $u$ *for* $G-(N+1)p>D\geq\underline{D}^{\Delta}(N,u+\Delta)$*. The thresholds* $\xi_j^{\Delta}(w)$ *are decreasing in* $w$ *and* $\xi_j^{\Delta}(w)>\xi_{j'}^{\Delta}(w)$ *for any* $j>j'$ *as long as* $\xi_j^{\Delta}(W)>0$*.*

Note that the donor is playing a PT strategy, but unlike in Proposition 1, the wealth threshold $\overline{D}^{\Delta}(N,u)$ can be defined in terms of primitives and the cutoffs $\bar{\xi}_j^{\Delta},\ j=0\ldots M_0$. Thus, we only need to check that players are best-responding in every state $\mathbf{x}$ and that the belief system is consistent with Bayes' rule on equilibrium path.

**Assertion Donor Continuous (DC):** *From Proposition 1 we know that the point-wise limits* $\bar{\xi}_j:=\lim_{\Delta\to 0}\bar{\xi}_j^{\Delta}$ *and*

$$\overline{D}(N,u):=\lim_{\Delta\to 0}\overline{D}^{\Delta}\Big(N,\Big\lceil\frac{u}{\Delta}\Big\rceil\Delta\Big)=\max\{G-(j-1)p-Np,0\}\text{ for }u\in(\bar{\xi}_j,\bar{\xi}_{j-1}]$$

*exist.*

This implies that the point-wise limits $D_+(N,D,,u;w):=\lim_{\Delta\to 0}D_+^{\Delta}\big(N,D,\lceil\frac{u}{\Delta}\rceil\Delta;w\big)$, $b(N,D,u)=\lim_{\Delta\to 0}b^{\Delta}\big(N,D,\lceil\frac{u}{\Delta}\rceil\Delta\big)$, and $F(w;(N,D,u))=\lim_{\Delta\to 0}F^{\Delta}\Big(w;\big(N,D,\lceil\frac{u}{\Delta}\rceil\Delta\big)\Big)$ exist and are equal to the expressions specified in Proposition 3. Similarly, it is straight-forward to show that all limits defined in Equation L-D, Equation L-B, Equation L-P exist and correspond to the expressions in Proposition 3. Finally, an analogous lemma to Lemma 1 holds, stated below.

**Lemma 4.** *For* $u\in(0,T]$*, the following holds:*

***L-i)*** $\pi(N,D,u)$ *is strictly increasing in* $N$*,* $D$ *and* $u$*, and strictly decreasing in* $G$ *for* $G-(N+1)p>D\geq\overline{D}(N,u)$ *;*

***L-ii)*** *$\overline{D}(N,u)$ is strictly decreasing in $N$ and weakly decreasing $u$ and strictly increasing in $G$ if $\pi(N,0,u) \leq \frac{v_0}{v-p} \leq \pi(N,\overline{W},u)$;*

***L-iii)*** *$\bar{\xi}_j(W)$ is strictly increasing in $j$ and $G$ and weakly decreasing in $W$ as long as $\bar{\xi}_j(W) > 0$.*

Condition (iii) follows directly from Lemma 1. Further, since $\bar{\xi}_j^{\Delta}$ is strictly increasing in $j$, $\overline{D}^{\Delta}(N,u)$ is weakly decreasing in $u$. It also follows from the definition of $\overline{D}^{\Delta}(N,u)$ that it is strictly decreasing in $N$ and strictly increasing in $G$. Condition (i) follows analogously to the proof in Proposition 1 given that conditions (ii) and (iii) hold true. Thus, nothing remains to be shown for DC once DD is shown.

**Proof** We conduct an analogous induction to Proposition 1:

**A) Time induction start $u = 0$ (last period)**

In any state $(N,D,0)$, $D_+^{\Delta}(N,D,0;w) = \min\{\max\{D, G-Np\}, w\}$ is a donor best-response by the same argument as in Proposition 1. For a buyer in that state, if $N \geq \underline{M}(D)-1$, either the campaign is already successful, or the buyer can complete the campaign herself. Hence $\pi^{\Delta}(N,D,0) = 1$ and $b^{\Delta}(N,D,0) = 1$. If $N < \underline{M}(D)-1$, then the buyer believes that conditional on buying, the campaign will succeed if and only if $W \geq G - p(N+1)$. By Equation PT-belief the buyer best response is to buy ($b^{\Delta}(N,D,0) = 1$) if and only if $D \geq \overline{D}^{\Delta}(N,\Delta) = G-(N+1)p$ and in that case

$$\pi^{\Delta}(N,D,0) = 1 = \frac{1-F_0(G-p(N+1))}{1-F_0(D)} \geq \frac{v_0}{v-p}.$$

For $D < G-(N+1)p$, $\pi^{\Delta}(N,D,0) = 0$.

**B) Induction in $j = \underline{M}(D) - N$ for $u > 0$**

**(a) Induction start:** For $N \geq \underline{M}(D)-1$, the campaign is either already successful, or a buyer can complete the campaign. Hence $\pi^{\Delta}(N,D,u) = 1$ and $b^{\Delta}(N,D,u) = 1$ for all $u \in \mathbb{U}^{\Delta}$, and $D \in [0,\overline{W}]$ in any equilibrium. Note that $\overline{\xi}_1^{\Delta} = 0$ and $D_+^{\Delta}(N,D,u;w) = D$.

**(b) Induction hypothesis:** Assume that we have shown that the above strategy profiles are best responses for the donor and buyers for all states $(N,D,u)$ with $N = \underline{M}(D) - j'$ with $j' < j$.

**(c) Induction step:** Consider a buyer in state $(N,D,u)$ with $N = \underline{M}(D) - j$. If $D < \overline{D}^{\Delta}(\underline{M}(D) - j, u+\Delta)$, then $u < \overline{\xi}_j^{\Delta}$, and the belief system dictates that a buyer assigns a probability of success equal to

$$\pi^{\Delta}(\underline{M}(D)-j,D,u) \quad = \quad \mathbb{P}(\tau_1^u \leq T-\overline{\xi}_{j-1}^{\Delta},\ldots,\tau_{j-2}^u \leq T-\overline{\xi}_2^{\Delta},\tau_{j-1}^u \leq T) < \frac{v_0}{v-p},$$

where $\tau_i^u$ is the arrival time of the $i$-th buyer after period $u$. The inequality follows directly from the definition of $\overline{\xi}_j^{\Delta}$ since $u < \overline{\xi}_j^{\Delta}$. Hence, $b^{\Delta}(\underline{M}(D)-j,D,u)=0$.

If $D \geq \overline{D}^{\Delta}(\underline{M}(D)-j,u+\Delta)$, then $u \geq \overline{\xi}_j^{\Delta}$; by the induction hypothesis, we have

$$\begin{aligned}
&\pi^{\Delta}(\underline{M}(D)-j,D,u)= \\
&\quad \mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\frac{\max\{u-\overline{\xi}_{j-1-(\underline{M}(D)-\underline{M}(W))}^{\Delta},0\}}{\Delta}}(1-\Delta\lambda)^{i-1}\Delta\lambda\cdot \\
&\qquad \pi^{\Delta}(\underline{M}(D)-(j-1),\max\{D,\overline{D}^{\Delta}(\underline{M}(D)-(j-1),u-\Delta(i-1))\},u-\Delta i)\Big|W \geq D\Bigg] \\
&> \mathbb{P}(\tau_1^u \leq T-\overline{\xi}_{j-1}^{\Delta},\ldots,\tau_{j-2}^u \leq T-\overline{\xi}_2^{\Delta},\tau_{j-1}^u \leq T) \geq \tfrac{v_0}{v-p},
\end{aligned}$$

where the last inequality follows because $u \geq \overline{\xi}_j^{\Delta}$ and the definition of $\overline{\xi}_j^{\Delta}$ via Proposition 1. Hence, indeed $b^{\Delta}(\underline{M}(D)-j,D,u)=1$.

Next, consider a donor in state $(N,D,u)$ with $N=\underline{M}(D)-j$. It is immediate that given the buyer's strategy, it is optimal for the donor to donate $\max\{D,\overline{D}^{\Delta}(\underline{M}(D)-j,u)\}$ because $\overline{D}^{\Delta}(N,u)$ is strictly decreasing in $N$ and weakly decreasing in $u$.

It is immediate that the belief system is consistent with donor strategies.

### A.3.2 Buyer optimality

Next, we show that the equilibrium just constructed is buyer optimal in the class of PBE. In any PBE $(\tilde{b}^{\Delta},\tilde{D}_+^{\Delta},(\tilde{F}^{\Delta})$, the buyer strategy $\tilde{b}^{\Delta}$ solves

$$\max_{\tilde{b}^{\Delta}(\mathbf{h}_t^{B,\Delta})} \mathbb{E}\big[\mathbb{1}(R_T \geq G)\cdot \tilde{b}^{\Delta}(\mathbf{h}_t^{B,\Delta})\cdot(v-p)+(1-\tilde{b}^{\Delta}(\mathbf{h}_t^{B,\Delta}))\cdot v_0\big],$$

given donor strategy $\tilde{D}_+^{\Delta}$ and induced buyer beliefs $\tilde{F}^{\Delta}$. The expectation is over arrivals of future buyers and donor wealth. A buyer-optimal PBE therefore selects $\tilde{D}_+^{\Delta}$ (which induces a $\tilde{F}^{\Delta}$) to maximize the buyers' ex-ante maximization problem, i.e.,

$$\max_{(\tilde{b}^{\Delta},\tilde{D}_{+}^{\Delta},\tilde{F}^{\Delta})} \frac{1}{\frac{T}{\Delta}} \sum_{t\in\mathbb{T}^{\Delta}} \mathbb{E}\Big[\mathbb{1}(W+pN_T \geq G)\cdot \tilde{b}^{\Delta}(\mathbf{h}_t^{B,\Delta})\cdot(v-p)+(1-\tilde{b}^{\Delta}(\mathbf{h}_t^{B,\Delta}))\cdot v_0$$

$$\text{s.t. } (\tilde{b}^{\Delta},\tilde{D}_{+}^{\Delta},\tilde{F}^{\Delta}) \text{ constitutes a PBE.}$$

We consider a relaxed problem by ignoring incentive constraints and only imposing donor rationality, i.e., that no donor type donates more than his wealth. An upper bound for the value of the buyer's objective in the above relaxed problem can be calculated by imposing that the donor always donates his entire wealth, i.e., $\tilde{D}_{+}^{\Delta}(\mathbf{h}_t^{B,\Delta};W)=W$. This results in a payoff of buyers of

$$\mathbb{E}^{F_0}\Bigg[\max_{(\tilde{b}^{\Delta}(\mathbf{h}_t^{B,\Delta}))_{\mathbf{h}_t^{B,\Delta}}} \frac{1}{\frac{T}{\Delta}} \sum_{t\in\mathbb{T}^{\Delta}} \mathbb{E}^{(\tilde{N}_t)_t}\Big[\mathbb{1}(W+pN_T \geq G)\cdot b^{\Delta}(\mathbf{h}_t^{B,\Delta})\cdot(v-p)+(1-b^{\Delta}(\mathbf{h}_t^{B,\Delta})\cdot v_0\Big]\Bigg],$$

where the inner expectation is over arrivals of future buyers. It suffices to show that this upper bound is exactly achieved in the PBE constructed above.

Consider the buyer's problem in using the payoff function above. It is easy to see that the solution is for the buyer to set $b^{\Delta}(\mathbf{h}_t^{B,\Delta})=1$ whenever $\mathbb{P}(W+pN_T \geq G) \geq \frac{v_0}{v-p}$ and 0 otherwise. Moreover, we have

$$\mathbb{P}(W+pN_T \geq G)=\mathbb{P}(\tau_1^u \leq T-\overline{\xi}^{\Delta}_{\underline{M}(W)-(N_{T-u}+1)},\ldots,\tau^u_{\underline{M}(W)-(N_{T-u}+2)} \leq T-\overline{\xi}^{\Delta}_2,\tau^u_{\underline{M}(W)-(N_{T-u}+1)} \leq T),$$

where $\tau_i^u$ is the arrival time of the $i$-th buyer after period $u$. This is because at any history $\mathbf{h}_t^{B,\Delta}$, the buyer knows that $\underline{M}(W)$ buyers are required, and no further donations are expected.

Consider the equilibrium constructed in Proposition 3. A buyer does not buy in state $(N,D,u)$ if and only if $D<\overline{D}^{\Delta}(N,u+\Delta)$ which is equivalent to

$$\mathbb{P}(\tau_1^u \leq T-\overline{\xi}^{\Delta}_{\underline{M}(D)-(N-1)},\ldots,\tau^u_{\underline{M}(D)-(N+2)} \leq T-\overline{\xi}^{\Delta}_2,\tau^u_{\underline{M}(D)-(N+1)} \leq T) < \frac{v_0}{v-p}.$$

On equilibrium path, a donor of type $W$ chooses $D_{+}^{\Delta}(N,D',u+\Delta;W)<\overline{D}^{\Delta}(N,u+\Delta)$ for $D'\leq D$ if and only if $D=W$. Hence the purchase decision (and hence, payoff) for the buyer in this equilibrium is exactly the same as that in the optimal value of the buyers' payoff function above. Thus, this equilibrium is buyer optimal.

# Online Appendix: Not for Publication

## B Omitted Proofs

### B.1 Proof of Assertion Platform-Continuous (PC) and Lemma 1 for last induction step $u>0$ and $N=\underline{M}(D)-j$

*Limits PC for $u>\Delta$ and $N=\underline{M}(D)-j$:*

By the induction hypothesis, all expressions Equation L-D, Equation L-B, Equation L-P exist for state $\mathbf{x}=(N',D,u)$ with $N'=\underline{M}(D)-j'$ and $j'=1,\ldots,j$, and they satisfy Lemma 1. Recall that $N=\underline{M}(D)-j$ implies $(N+j-1)p+D<G$.

L.1) *Uniform convergence (in $D$) of $\pi^{\Delta}\big(\underline{M}(D)-j,\cdot,\lceil\frac{u}{\Delta}\rceil\Delta\big)$ for $D\le G-(\underline{M}(D)-1)p$:* Recall that

$$\lim_{\Delta\to 0}\hat{\pi}^{\Delta}(\underline{M}(D)-j,D,\lceil\tfrac{u}{\Delta}\rceil\Delta)=$$
$$\lim_{\Delta\to 0}\mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\frac{\max\left\{\lceil\frac{u}{\Delta}\rceil\Delta-\xi^{\Delta}_{j-1+\underline{M}(W)-\underline{M}(D)}(W),0\right\}}{\Delta}}(1-\Delta\lambda)^{i-1}\Delta\lambda\cdot$$
$$\pi^{\Delta}\Big(\underline{M}(D)-j+1,\max\{D,\underline{D}^{\Delta}(\underline{M}(D)-j+1,\Delta(\lceil\tfrac{u}{\Delta}\rceil-(i-1)))\},\Delta(\lceil\tfrac{u}{\Delta}\rceil-i)\Big)\Bigg|W\ge D\Bigg].$$

The uniform convergence of $\pi^{\Delta}\big(\underline{M}(D)-(j-1),D,u\big)$ in $D$ (by the induction hypothesis) and the Arzelà-Ascoli Theorem imply that the family of functions $D\mapsto\pi^{\Delta}\big(\underline{M}(D)-(j-1),D,u\big)$ is equicontinuous with respect to $\Delta$. Hence, because of point-wise convergence (by the induction hypothesis) of $\underline{D}^{\Delta}(N',u)$ for $N'\ge\underline{M}(D)-j$, we can replace $\underline{D}^{\Delta}$ by $\underline{D}$. Further, we may replace $\pi^{\Delta}$ by $\pi$. Finally, because $\lim_{\Delta\to 0}\xi^{\Delta}_{j-1}(w)=\xi_{j-1}(w)$, the dominated convergence theorem allows us to conclude that

$$\lim_{\Delta\to 0}\hat{\pi}^{\Delta}(\underline{M}(D)-j,D,\lceil\tfrac{u}{\Delta}\rceil\Delta)=\hat{\pi}(D,\underline{M}(D)-j,u)=$$
$$\mathbb{E}^{F_0}\Bigg[\int_0^{\max\{u-\xi_{j-1+\underline{M}(W)-\underline{M}(D)}(W),0\}}\lambda e^{-\lambda s}\cdot$$
$$\pi(\underline{M}(D)-j+1,\max\{D,\underline{D}(\underline{M}(D)-j+1,u-s)\},u-s)ds\Bigg|W\ge D\Bigg].\qquad\text{(L-}\hat{\pi}\text{)}$$

Note that $\hat{\pi}^{\Delta}(\underline{M}(D)-j,D,\lceil\frac{u}{\Delta}\rceil\Delta)$ indeed converges uniformly in $D\le G-(\underline{M}(D)-1)p$ for fixed $u$ because $\hat{\pi}^{\Delta}(\underline{M}(D)-(j-1),D,\lceil\frac{u}{\Delta}\rceil\Delta)$ converges uniformly in $D\le G$ for fixed $u$, the sum is bounded by one, $F_0$ is (uniformly) continuous on $[0,G]$, and $F_0(G) < 1$. Similarly it follows that $\pi(\underline{M}(D)-j,D,u) = \lim_{\Delta\to 0} \pi^{\Delta}(\underline{M}(D)-j,D,\lceil \frac{u}{\Delta} \rceil \Delta)$ converges uniformly (in $D$).

L.2) *Continuity and strict monotonicity of $\hat{\pi}$ in $D$ and $u$:* First, $\hat{\pi}(\underline{M}(D)-j,D,u)$ is continuous in $D$ and $u$ because $\hat{\pi}(\underline{M}(D)-(j-1),D,u)$ is continuous in $D$ and $u$, $\underline{D}(\underline{M}(D)-(j-1),u)$ is continuous in $u$ by the induction hypothesis and because $F_0$ is continuous.

Furthermore, $\hat{\pi}(\underline{M}(D)-j,D,u)$ is strictly increasing in $D$ for $D \leq \overline{W}$ because $\hat{\pi}(\underline{M}(D)-(j-1),D,u)$ is weakly increasing in $D$ by the induction hypothesis and $\frac{1}{1-F_0(D)}$ is strictly increasing.

Now the integrand is strictly positive as long as $u > \xi_{j-1+\underline{M}(W)-\underline{M}(D)}(W)$. Hence, $\hat{\pi}(\underline{M}(D)-j,D,u)$ is strictly increasing in $u$ for $u > \xi_{j-1+\underline{M}(\overline{W})-\underline{M}(D)}(\overline{W})$ because $\hat{\pi}(\underline{M}(D)-(j-1),D,u)$ is weakly increasing in $u$ by the induction hypothesis and because $u-\xi_{j-1+\underline{M}(W)-\underline{M}(D)}(W)$ is strictly increasing in $u$. Also, $\hat{\pi}(\underline{M}(D)-j,D,u)$ is weakly increasing in $u$ for $u < \xi_{j-1+\underline{M}(\overline{W})-\underline{M}(D)}(\overline{W})$ by a similar argument.

L.3) *Point-wise convergence of $\underline{D}^{\Delta}(\underline{M}(D)-j,\lceil \frac{u}{\Delta} \rceil \Delta)$ and $D_+^{\Delta}(\underline{M}(D)-j,D,\lceil \frac{u}{\Delta} \rceil \Delta;W)$:* First, note that if for a $u>0$, $\hat{\pi}^{\Delta}\big(\underline{M}(D)-j,0,\lceil \frac{u}{\Delta} \rceil \Delta\big) \geq \frac{v_0}{v-p}$ then $\hat{\pi}(\underline{M}(D)-j,0,u) \geq \frac{v_0}{v-p}$ and hence, $\underline{D}(\underline{M}(D)-j,u) := \lim_{\Delta\to 0} \underline{D}^{\Delta}(\underline{M}(D)-j,\lceil \frac{u}{\Delta} \rceil \Delta) = 0$.

For $u>0$ such that $\hat{\pi}^{\Delta}(\underline{M}(D)-j,0,u) < \frac{v_0}{v-p}$, $\hat{\pi}(\underline{M}(D)-j,0,u) \leq \frac{v_0}{v-p}$. Then, since $\hat{\pi}(N,D,u)$ is continuous and strictly increasing in $D$, there is a unique solution $D'(\underline{M}(D)-(j-1),u)$ to

$$\hat{\pi}(\underline{M}(D)-j,D'(\underline{M}(D)-j,u),u) = \frac{v_0}{v-p}.$$

Because we established that given $N = \underline{M}(D)-j$, $\hat{\pi}^{\Delta}(\underline{M}(D)-j,D,\lceil \frac{u}{\Delta} \rceil \Delta)$ converges uniformly, we have $\underline{D}(\underline{M}(D)-j,u) := \lim_{\Delta\to 0} \underline{D}^{\Delta}(\underline{M}(D)-j,\lceil \frac{u}{\Delta} \rceil \Delta) = D'(\underline{M}(D)-(j-1),u)$. It follows immediately that for all $u>0$,

$$\begin{aligned} D_+(\underline{M}(D)-j,D,u;w) &:= \lim_{\Delta\to 0} D_+^{\Delta}\big(\underline{M}(D)-j,D,\lceil \tfrac{u}{\Delta} \rceil \Delta;w\big) \\ &= \lim_{\Delta\to 0} \min\big\{\max\big\{D,\underline{D}^{\Delta}\big(\underline{M}(D)-j,\lceil \tfrac{u}{\Delta} \rceil\big)\big\},w\big\} \\ &= \min\{\max\{D,\underline{D}(\underline{M}(D)-j,u)\},w\}. \end{aligned}$$

L.4) *Point-wise convergence of $b^{\Delta}(\underline{M}(D)-j,D,u)$:* Note that $b^{\Delta}\big(\underline{M}(D)-j,D,\lceil \frac{u}{\Delta} \rceil \Delta\big) = 1$ if

$D \geq \underline{D}^{\Delta}\Big(\underline{M}(D)-j, (\lceil \frac{u}{\Delta} \rceil \Delta+1)\Delta\Big)$ and $b^{\Delta}\Big(\underline{M}(D)-j, D, (\lceil \frac{u}{\Delta} \rceil \Delta+1)\Delta\Big)=0$ otherwise. Since $\lim_{\Delta\to 0} \underline{D}^{\Delta}\Big(\underline{M}(D)-j, (\lceil \frac{u}{\Delta} \rceil \Delta+1)\Delta\Big) = \underline{D}(\underline{M}(D)-j, u)$, $b^{\Delta}(\underline{M}(D)-j, D, u)$ converges point-wise to

$$\lim_{\Delta\to 0} b^{\Delta}(\underline{M}(D)-j, D, u) = \begin{cases} 1 & \text{if } D \geq \underline{D}(\underline{M}(D)-(j-1), u) \\ 0 & \text{if } D < \underline{D}(\underline{M}(D)-(j-1), u) \end{cases}.$$

L.5) *Point-wise convergence of $\xi_j(w)$ and Equation L-$\xi$:* If $\hat{\pi}^{\Delta}(\underline{M}(w)-j, w, 0) \geq \frac{v_0}{v-p}$, then it follows immediately that $\xi_j^{\Delta}(w)=0$. If $\hat{\pi}^{\Delta}(\underline{M}(w)-j, w, 0) < \frac{v_0}{v-p}$, it follows that $\xi_j^{\Delta}(w)>0$ and

$$\begin{cases} \hat{\pi}^{\Delta}(\underline{M}(w)-j, W, \xi_j^{\Delta}(w)) \geq \frac{v_0}{v-p} \\ \hat{\pi}^{\Delta}(\underline{M}(w)-j, W, \xi_j^{\Delta}(w)-\Delta) < \frac{v_0}{v-p}. \end{cases}$$

Furthermore, since $\hat{\pi}(\underline{M}(w)-j, w, u)$ is continuous and strictly increasing in $u$ for $u \geq \xi_{j-1}(\overline{W})$ and weakly increasing for $u < \xi_{j-1}(\overline{W})$, there is a unique solution $\xi'(w)$ to

$$\hat{\pi}(\underline{M}(w)-j, W, \xi'(w)) = \frac{v_0}{v-p}.$$

Hence, as $\Delta \to 0$, it must be that $\lim_{\Delta\to 0} \xi_j^{\Delta}(w) = \xi'(w)$.

L.6) *Point-wise convergence of $F^{\Delta}\Big(w; \big(\underline{M}(D)-j, D, \lceil \frac{u}{\Delta} \rceil \Delta\big)\Big)$:* It follows immediately from point-wise convergence of $\underline{D}^{\Delta}\big(\underline{M}(D)-j, \lceil \frac{u}{\Delta} \rceil \Delta\big)$ that

$$F(w; (\underline{M}(D)-j, D, u)) := \lim_{\Delta\to 0} F^{\Delta}\Big(w; \big(\underline{M}(D)-j, D, \lceil \tfrac{u}{\Delta} \rceil \Delta\big)\Big)$$
$$= \begin{cases} \frac{F_0(w)-F_0(D)}{1-F_0(D)} \mathbb{1}(w \geq D) & \text{if } D \geq \underline{D}(\underline{M}(D)-j, u) \\ \mathbb{1}(w \geq D) & \text{otherwise} \end{cases}.$$

*Lemma 1 for $u > \Delta$ and $N = \underline{M}(D)-j$:*

(i) *$\pi(N, D, u)$ is strictly increasing in $N$, $D$, and $u$, and strictly decreasing in $G$, as long as $G-(N+1)p > D \geq \underline{D}(N, u)$.*

First, using an analogous argument as for showing that $\hat{\pi}^{\Delta}\big(N+1, D, u\big) - \hat{\pi}^{\Delta}\big(N, D, u\big) > 0$ in Equation 4), it follows from Equation L-$\hat{\pi}$ that $\pi(N+1, D, u) - \pi(N, D, u) > 0$. Similarly, $\pi(N, D, u)$ is strictly increasing in $D$ and u according to an analogous argument to 2) and 3) in A.1.1.

It remains to be shown that $\pi(N, D, u)$ is strictly decreasing in $G$. Note that if $G-(N+1)p > D > \underline{D}(N, u)$, then $\frac{v_0}{v-p} \leq \pi(N, D, u) < 1$. Recall that by the induction hypothesis, $\pi(N+1, D, u)$ is weakly decreasing in $u$ and $\pi(N+1, \underline{D}(N+1, u), u) = \frac{v_0}{v-p}$ for any $u > 0$, so $\underline{D}(N+1, u')$ is strictly increasing in $G$ and strictly decreasing in $u$. Similarly, $\xi_j(w)$ is strictly increasing in $G$. Thus, by the recursive definition of $\pi(N, D, u)$, it is strictly decreasing in $G$.

(ii) *$\underline{D}(N, u)$ is strictly decreasing in $N$ and $u$, and strictly increasing in $G$ as long as $\pi(N, 0, u) < \frac{v_0}{v-p} < \pi(N, \overline{W}, u)$.*

Strict monotonicity of $\underline{D}(N, u)$ in $N$ and $u$ follows from the strict monotonicity properties in $N, D,$ and $u$ of $\hat{\pi}(N, D, u)$ and because $\hat{\pi}(N, \underline{D}(N, u), u) = \frac{v_0}{v-p}$ for $\pi(N, 0, u) < \frac{v_0}{v-p} < \pi(N, \overline{W}, u)$. Further, because $\pi(N, D, u)$ is strictly decreasing in $G$ and strictly increasing in $D$ at $D = \underline{D}(N, u)$, an increase in $G$ must strictly increase $\underline{D}(N, u)$.

(iii) *$\xi_j(w)$ is strictly increasing in $j$ and $G$ and decreasing in $w$ as long as $\xi_j(w) > 0$.*

Recall that $\pi(N, w, \xi_j(w)) = \frac{v_0}{v-p}$ if $\xi_j(w) > 0$. Since $\pi(N, D, u)$ is strictly increasing in $D$ and $u$, it follows immediately that $\xi_j(w)$ is strictly decreasing in $w$. Further, since $\pi(N+1, w, \xi_{j-1} w)) = \frac{v_0}{v-p}$ and $\pi(N, D, u)$ is strictly increasing in $N$, $\xi_j(w) > \xi_{j-1}(w)$. Finally, because $\pi(N, D, u)$ is strictly decreasing in $G$ at $(N, D, u) = (N, w, \xi_j(w))$, $\xi_j(w)$ is increasing in $G$.

## B.2 Proofs of Lemma 2 and Lemma 3

*Proof of Lemma 2.* Given a donor-optimal PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, define

$$D_*(\mathbf{h}_t^{D,\Delta}) := \inf\left\{\tilde{D}_+^\Delta(\mathbf{h}_t^{D,\Delta}; w) \,\middle|\, \tilde{b}^\Delta(\mathbf{h}_t^{D,\Delta}, \tilde{D}_+(\mathbf{h}_t^{D,\Delta}; w)) = 1\right\},$$

which is the smallest donation amount that incentivizes buying at a history $\mathbf{h}_t^{D,\Delta}$. Donating this amount is feasible for all donor types $w \geq D_*(\mathbf{h}_t^{D,\Delta})$. Moreover, it is consistent with play on equilibrium path. In particular, donating this amount is feasible for all types that incentivize buying after $\mathbf{h}_t^{D,\Delta}$ in $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$.

Then, define a new assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ where $\hat{b}^\Delta$ and $\hat{D}_+^\Delta$ are given by Equation 2. On equilibrium path, $\hat{F}(w; \mathbf{h}_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ is derived by Bayes' rule. Off path, if $D_{t-\Delta} > D_*(\mathbf{h}_{t-\Delta}^{D,\Delta})$, then let $\hat{F}(w;\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ be such that it is optimal for the buyer not to buy (e.g. $\hat{F}(w;\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta}) = \mathbb{1}(w=0)$), and let $\hat{F}(w;\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta}) = \tilde{F}(w;\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ otherwise.

Note that the strategies are such that $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$ and $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$ result in the same probability measures $(\boldsymbol{\kappa}_\Delta(0;w))_w$, i.e. the same purchasing outcome after any realization of arrivals and donor type. The donation amount with $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$ is by definition weakly lower after any arrival and donor type realization. Hence, if $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$ is a PBE, then it must be donor-optimal by donor-optimality of $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$. It remains to be shown that $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$ is a PBE.

First, consider donor incentives. Given a PBE $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$, a donor type $w$ with $\tilde{b}^\Delta(\mathbf{h}^{D,\Delta}_t, \tilde{D}_+(\mathbf{h}^{D,\Delta}_t; w)) = 0$ does not find it profitable to incentivize buying after a history $\mathbf{h}^{D,\Delta}_t$. Buying can be incentivized by donations of at least $D_*(\mathbf{h}^{D,\Delta}_{t-\Delta})$. Hence, also with assessment $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$, deviating to incentivize buying cannot be profitable. For a donor type $w$ with $\tilde{b}^\Delta(h^{D,\Delta}_t, \tilde{D}_+(\mathbf{h}^{D,\Delta}_t; w)) = 1$ it is optimal to donate in the PBE $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$. Given the assessment $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$, the donor can donate weakly less and still incentivize buying, but the donor has a larger set of feasible donations in any future period. Thus, no donor type has an incentive to deviate given the assessment $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$.

Next, consider buyer incentives. Buyers at a history $(\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ where $D_{t-\Delta} < D_*(\mathbf{h}^{D,\Delta}_{t-\Delta})$ have identical beliefs about donor types in both assessments, and the purchasing outcome is also identical as argued above. Hence, the probability of success is the same across assessments and a buyer with such a history must prefer not to buy given the assessment $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$ because $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$ is a PBE. Buyers at a history $(\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ where $D_{t-\Delta} = D_*(\mathbf{h}^{D,\Delta}_{t-\Delta})$ believe that they face donor types that they would face if they played a PBE $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$ and if they were at any of the histories $(\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ after which a buyer buys. Hence, buyers must prefer to buy at a history $(\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ where $D_{t-\Delta} = D_*(\mathbf{h}^{D,\Delta}_{t-\Delta})$ given the assessment $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$. A history $(\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ with $D_{t-\Delta} > D_*(\mathbf{h}^{D,\Delta}_{t-\Delta})$ is now off equilibrium path for assessment $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$, and we assumed that $\hat{F}$ is such that the buyer does not wish to buy in this case.

It follows that $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$ is a PBE.

*Proof of Lemma 3.* Given the donor-optimal PBE $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$ satisfying Equation 3, let $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$ be given by Equation 4, $\hat{b}^\Delta = \tilde{b}^\Delta$, and $\tilde{F}^\Delta(w;\mathbf{h}^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ so that it is consistent with Bayes' rule on equilibrium path and $\tilde{F}^\Delta(w;h^{D,\Delta}_{t-\Delta}, D_{t-\Delta}) = \hat{F}^\Delta(w;h^{D,\Delta}_{t-\Delta}, D_{t-\Delta})$ off path. Then, it follows immediately that the two assessments generate the same outcomes and hence the same probability measures $(\boldsymbol{\kappa}_\Delta(0;w))_w$. It remains to show that $(\hat{b}^\Delta, \hat{D}^\Delta_+, \hat{F}^\Delta)$ constitutes a PBE. The donor does not have a profitable deviation after histories after which the buyer is incentivized to buy as the donor plays exactly the same strategy as in $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$. Whenever the donor does not incentivize buying, the donor cannot have a profitable deviation since incentivizing buying is not profitable for $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$, and moreover, $\hat{D}^\Delta_+(\mathbf{h}^{D,\Delta}_t; w) = D_{t-\Delta} \leq \tilde{D}^\Delta_+(\mathbf{h}^{D,\Delta}_t; w)$ implies that every donor type $w$ has a weakly larger set of feasible donations in the future under $\hat{D}^\Delta_+$ than under $\tilde{D}^\Delta_+$ . Each buyer is also best-responding as she buys after the same histories in both assessments, and whenever she does not buy, her belief is a mixture of beliefs after histories after which she did not buy in $(\tilde{b}^\Delta, \tilde{D}^\Delta_+, \tilde{F}^\Delta)$.

### B.3 Proof of Proposition 4 (Comparative Statics)

Consider a Markov equilibrium in PT strategies as in Proposition 1 and 3.

i) **Claim:** The ex-ante probability of success is strictly increasing in $T$ and strictly decreasing in $G$.

Let us denote the ex-ante probability of success of a campaign as a function of $G$ and $T$ by $\Pi(G,T)$. Then, for the platform-optimal equilibrium in Proposition 1 we can write

$$\Pi(G,T) \quad = \quad \mathbb{E}^{F_0}\left[\int_0^{\max\{T-\xi_{\underline{M}(W)}(W),0\}} \lambda e^{-\lambda s} \max\left\{\pi(0,\,0,\,T-s), \tfrac{v_0}{v-p}\right\} ds \,\middle|\, W \geq D\right].$$

We know from Lemma 1 L-i) and L-iii) that $\pi(0,0,T-s)$ is strictly decreasing in $G$ and $\xi_{\underline{M}(W)}$ is strictly increasing in $G$. It follows immediately that $\Pi(G,T)$ is strictly decreasing in $G$. Further, $\Pi(G,T)$ is strictly increasing in $T$ because $\pi(0,0,u)$ is strictly increasing in $u$ by Lemma 1 L-i).

For the buyer-optimal equilibrium in Proposition 3, an analogous argument holds using Lemma 4 L-i) and L-iii).

ii) **Claim:** In any period $t$, the probability of success is weakly increasing in revenue $R_t$. Given revenue level $R$, the probability of success is strictly decreasing in time elapsed.

For the platform-optimal equilibrium in Proposition 1, this follows immediately from the fact that $R_t = R_{T-u} = D + Np$ in state $(N, D, u)$ and Lemma 1 (i) that shows that $\pi(N, D, u)$ is strictly increasing in $N$ and $D$.

For the buyer-optimal equilibrium in Proposition 3, an analogous argument holds using Lemma 4 L-i).

## B.4 Proof of Proposition 5 (Successful Campaigns)

Consider a Markov equilibrium in PT strategies as in Proposition 1 and 3.

i) **Claim:** Conditional on $\mathscr{S}_T$, $R_T = G$ and $D_T > D_{T-\Delta}$ almost surely.

If the campaign has not succeeded by the beginning of the last period, then $D_{T-\Delta} + N_{T-\Delta}p < G$. If a possible arrival in the last period does not suffice to reach the goal amount, i.e., $D_{T-\Delta} + (N_{T-\Delta} + 1)p < G$, then the donor will complete the campaign as long as $w \geq G - N_{T-\Delta}p$ and will donate exactly the remaining amount so that $R_T = G$. If a buyer arrives in the last period and completes the campaign, then the amount raised must satisfy $R_T < G + p$. In the limit, $\Delta \to 0$, the probability of arrival in the last period converges to zero, i.e., $R_T = G$ almost surely.

ii) **Claim:** Conditional on $\mathscr{S}$, if $\tau < T$, then $D_t < G - N_t p$ for all $t < \tau$.

In any state $\mathbf{x} = (N, D, u)$ with $u > \Delta$, the donor never donates more than $\max\{D, D_*^\Delta(N, u)\}$. Moreover, $\underline{D}^\Delta(N, u) < \overline{D}^\Delta(N, u) < G - Np$ as long as $u > \Delta$ by definition.

iii) **Claim:** Conditional on $\mathscr{S}$, $D_t = D_\tau$ for all $t \geq \tau$.

This follows immediately from the donor preferences because once a campaign is successful, the equilibrium belief $\pi(N, D, u) = 1$, which means that every subsequent buyer will buy. In this case, the donor has no reason to donate.

iv) **Claim:** Total revenue is "essentially decreasing" in the time of success, i.e., for any $t < t'$, $\mathbb{E}[R_T|\mathscr{S}_{t'}] + \lambda(t' - t)p - p < \mathbb{E}[R_T|\mathscr{S}_t]$. For the buyer-optimal equilibrium it is strictly decreasing in the time of success, or more precisely, for any $t < t'$, $\mathbb{E}[R_T|\mathscr{S}_{t'}] + \lambda(t' - t)p < \mathbb{E}[R_T|\mathscr{S}_t]$.

For $t < t'$, we can write

$$\begin{aligned}\mathbb{E}[R_T|\mathscr{S}_t] &= \mathbb{E}[R_t + \tilde{N}_{T-t}p|\mathscr{S}_t] = \mathbb{E}[R_t|\mathscr{S}_t] + \lambda(T-t)p \\ &= \mathbb{E}[R_{t'}|\mathscr{S}_{t'}] + \lambda(T-t')p + \underbrace{(\mathbb{E}[R_t|\mathscr{S}_t] - \mathbb{E}[R_{t'}|\mathscr{S}_{t'}])}_{>-p} + \lambda(t'-t)p \\ &> \mathbb{E}[R_T|\mathscr{S}_{t'}] + \lambda(t'-t)p - p.\end{aligned}$$

For the buyer-optimal equilibrium, $(\mathbb{E}[R_t|\mathscr{S}_t] - \mathbb{E}[R_{t'}|\mathscr{S}_{t'}]) = G - G = 0$, so $\mathbb{E}[R_T|\mathscr{S}_t] > \mathbb{E}[R_T|\mathscr{S}_{t'}] + \lambda(t'-t)p$ where $\lambda(t'-t)p > 0$, so $\mathbb{E}[R_T|\mathscr{S}_t] > \mathbb{E}[R_T|\mathscr{S}_{t'}]$.

v) **Claim:** Given $w$, $\frac{D_t}{R_t}$ is smaller for an arrival process such that $\mathscr{S}$ is realized than for an arrival process such that $\mathscr{S}$ is not realized.

The following argument holds for both equilibria constructed in Proposition 1 and 3. Consider a campaign with parameters **y** and fix a $w$. If $\mathscr{S}$ is realized, then the number of purchases by the end of the campaign satisfies $N_T^{\mathscr{S}} \geq \underline{M}(D_T) > \underline{M}(w)$, and total donations satisfy $D_T^{\mathscr{S}} \leq w$. If $\mathscr{S}$ is not realized, then total donations are $D_T^{\neg\mathscr{S}} = w$ and $N_T^{\neg\mathscr{S}} < \underline{M}(w)$. It follows immediately that

$$\frac{D_T^{\mathscr{S}}}{D_T^{\mathscr{S}} + N_T^{\mathscr{S}}p} < \frac{D_T^{\neg\mathscr{S}}}{D_T^{\neg\mathscr{S}} + N_T^{\neg\mathscr{S}}p}.$$

### B.5 Proof of Proposition 6 (Unsuccessful Campaigns)

Consider a Markov equilibrium in which the donor plays a PT strategy as in Proposition 1 and 3.

i) **Claim:** If $\overline{W} > D_*(0,T) > 0$, then the distribution of $\iota$ is continuous on $(0,T)$ and has mass points on $t \in \{0,T\}$.

First, consider the equilibrium outcome of Proposition 1. Then, $\mathbb{P}(\iota = 0) = F(D_*(0,T)) > 0$ and $\mathbb{P}(\iota = T) > \mathbb{P}\big(\forall i \leq N_T : \tau_i \leq T - \xi_{\underline{M}(W)-(i-1)}(W),\ D_*(N_T,T) \leq W < G - N_T p\big) > 0$. Finally, the probability of death at a given time $t \in (0,T)$ is zero and the distribution of $\iota$ admits a density as $\mathbb{P}(t_1 < \iota \leq t_2) \leq \mathbb{P}(\exists i \leq \underline{M}(W) - 1 : \tau_i > T - \xi_{\underline{M}(W)-(i-1)}(W) \in (t_1,t_2)) \to_{t_2 \to t_1} 0$ because $F$ is continuous.

For the buyer-optimal equilibrium outcome of Propositions 3, the cutoff times $\bar{\xi}_j$, $j \leq M_0$ are independent of $W$ and are the only times at which the campaign can possibly die.

ii) **Claim:** If $w > w'$ and the campaign is unsuccessful for both wealth realizations, then the time of death $\iota$ is larger for $w$ than for $w'$.

This follows immediately from the definition in Equation $\iota$ and Lemma 1(iii) that implies that $\xi_{\underline{M}(w)-(j-1)}(w)$ is strictly decreasing in $w$.

For the buyer-optimal equilibrium in Proposition 3, an analogous argument holds using Lemma 4 L-iii).

### B.6 Proof of Proposition 7 (Donation Dynamics)

*Proof.* Consider a Markov equilibrium where the donor plays a PT strategies with wealth threshold $D_*^\Delta(N,u)$ as in Proposition 1 and 3.

i) **Claim:** $D_*(0,T)$ is strictly decreasing in $T$, and increasing in $G$.

$\underline{D}(0,T)$ is strictly decreasing in $T$ and strictly increasing in $G$ by Lemma 1 (ii). $\overline{D}(0,T)$ is strictly decreasing in $T$ and strictly increasing in $G$ by Lemma 4 L-ii).

ii) **Claim:** Donations drop to zero for a positive amount of time after a buyer pledges. Formally, if $b(N-1,D,u)=1$, $D_+^\Delta(N,D,u';w)=D$ for $u' \in (\xi_{\underline{M}(D)-N}(D),u)$ where $\xi_{\underline{M}(D)-N}(D) < u$. If $\xi_{\underline{M}(D)-N}(D) > 0$, then $D_+^\Delta(N,D,u';w) > D$ for $u' < \xi_{\underline{M}(D)-N}(D)$.

On equilibrium path, for both equilibria in Proposition 1 and 3, a buyer only buys in a state $(N-1,D,u)$ if donations are above the wealth threshold $D_*(N-1,u)$. If the buyer buys, the next wealth cutoff decreases to $D_*(N,u) < D_*(N-1,u)$ by Lemma 1 L-ii) and Lemma 4 L-ii). By Lemma 1 L-ii) and 4 L-ii) $D_*(N,u)$ is weakly decreasing in $u$. In particular, if enough time passes, i.e. $u$ decreases, $D_*(N,u)$ increases beyond the current donation level $D$, namely at $u = \xi_{\underline{M}(D)-N}(D)$, and a donor with enough wealth starts donating again.

### B.7 Proof of Section 6.1 (Changing Outside Option)

We show that Equation 1 is a sufficient condition for

$$\pi^\Delta(N,D,u)(v-p) < v_0(u) \Rightarrow \pi^\Delta(N,D,u-\Delta)(v-p) < v_0(u-\Delta).$$

For $N \geq \underline{M}(D)-1$, $\pi^{\Delta}(N,D,u)=1$ and the condition is empty. Assume Equation 1 holds. For $N<\underline{M}(D)-2$, assume $\pi^{\Delta}(N,D,u)(v-p)<v_0(u)$ and $\pi^{\Delta}(N,D,u-\Delta)(v-p)\geq v_0(u-\Delta)$ . Recall, that $\pi^{\Delta}(N,D,u)$ is the probability of success if the buyer buys and further note that $\pi^{\Delta}(N+1,D,u-\Delta)(v-p)>\pi^{\Delta}(N,D,u-\Delta)(v-p)\geq v_0(u-\Delta)$ in a Markov equilibrium with PT donor strategies. If the buyer in state $(N,D,u)$ buys, the probability of success is

$$\pi^{\Delta}(N,D,u)=\Delta\lambda\pi^{\Delta}(N+1,D,u-\Delta)+(1-\Delta\lambda)\pi^{\Delta}(N,D,u-\Delta)$$

because $\pi^{\Delta}(N,D,u-\Delta)$ is the probability of success if the buyer had made the decision to buy in $u-\Delta$ instead of in $u$. The donor would not donate in $u-\Delta$ because a buyer would buy anyway given aggregate donations $D$ by assumption. By assumption, $\frac{\pi^{\Delta}(N,D,u)-\pi^{\Delta}(N,D,u-\Delta)}{\Delta}(v-p)<\frac{v_0(u)-v_0(u-\Delta)}{\Delta}$. However,

$$\begin{aligned}
\frac{v_0(u)-v_0(u-\Delta)}{\Delta} &> \frac{\pi^{\Delta}(N,D,u)-\pi^{\Delta}(N,D,u-\Delta)}{\Delta}(v-p)\\
&= \lambda\big((\pi^{\Delta}(N+1,D,u-\Delta)-\pi^{\Delta}(N,D,u-\Delta)\big)(v-p)\\
&> \lambda\pi^{\Delta}(N+1,D,u-\Delta)(1-(1-(1-\Delta\lambda)^{\frac{u}{\Delta}}))(v-p)\\
&\geq v_0(u-\Delta)\lambda(1-\Delta\lambda)^{\frac{u}{\Delta}}.
\end{aligned}$$

This contradicts Equation 1.

## B.8 Proof of Proposition 8 (Social Learning)

We prove the following formal statement which incorporates Proposition 8.

**Proposition.** *The following Markov assessment $(b^{\Delta},D_{+}^{\Delta},F^{\Delta})$ constitutes a PBE:*

*i) Given any state $\mathbf{x}$, if $\mu^{+}(N,u)\geq\bar{\mu}(\mathbf{x})$, then the public belief about $q=1$ is given by $\mu(\mathbf{x})=\mu^{+}(N,u)$. Otherwise, $\mu(N,D,u)=\max_{u'>u}\{\mu^{+}(N,u'-\Delta)\,\big|\,\mu^{+}(N,u')\geq\bar{\mu}(N,D,u')\}$.*

*ii) The donor plays a pooling-threshold strategy with wealth threshold $D_{*}(N,u):=\underline{D}^{\Delta,\mu}(N,u)$ defined as*

$$\underline{D}^{\Delta,\mu}(N,u)=0 \quad \text{if } B^{\Delta}(N,0,u-\Delta;\mu^{+}(N,u-\Delta))>v_0,$$
$$\underline{D}^{\Delta,\mu}(N,u)=\overline{W} \quad \text{if } B^{\Delta}(N,\overline{W},u-\Delta;\mu^{+}(N,u-\Delta))<v_0,$$

*otherwise,* $\underline{D}^{\Delta,\mu}(N,u)$ *makes the next buyer indifferent, i.e.,* $B^{\Delta}(N,\underline{D}^{\Delta,\mu}(N,u),u;\mu^{+}(N,u)) = v_0$.

iii) *The probability of success* $\pi_q^{\Delta}(\mathbf{x})$ *satisfies the following: For* $u = 0$, $\pi_q^{\Delta}(N,D,0) = \frac{1-F_0(\max\{G-(N+1)p,D\})}{1-F_0(D)}$. *For* $u > 0$ *and* $N \geq \underline{M}(D)-1$, $\pi_q^{\Delta}(N,D,0) = 1$. *For* $u > 0$, $N < \underline{M}(D)-1$ *and* $D \geq \underline{D}^{\Delta,\mu}(N,u)$,

$$\pi_q^{\Delta}(\mathbf{x}) = \mathbb{E}^{F_0}\Bigg[\sum_{i=0}^{\frac{\max\{u-\xi^{\Delta,\mu}_{\underline{M}(W)-(N+1)}(W;q),0\}}{\Delta}} (1-\Delta\lambda)^{i-1}\Delta\lambda \\ \pi_q^{\Delta}(N+1,\max\{D,\underline{D}^{\Delta,\mu}(N+1,u-\Delta(i-1)\},u-\Delta i)\Big|W \geq D\Bigg],$$

*where the cutoff time* $\xi_j^{\Delta,\mu}(W;q)$ *satisfies*

$$\xi_j^{\Delta,\mu}(w;q) = \min\left\{u \in \mathbb{U}^{\Delta}\Big|B^{\Delta}(\underline{M}(w)-j,w,u;\mu^{+}(\underline{M}(w)-j,u)) \geq v_0\right\}$$

*and* $\xi_j^{\Delta,\mu}(w) = 0$ *for* $j < 2$. *If* $u > 0$, $N < \underline{M}(D)-1$ *and* $D < \underline{D}^{\Delta,\mu}(N,u)$, *then* $\pi_q^{\Delta}(N,D,u) < \frac{v_0}{v-p}$.

iv) *Buyer beliefs about* $W$ *are given by Equation PT-belief with wealth threshold* $\underline{D}^{\Delta,\mu}(N,u)$.

v) *A buyer buys, i.e.,* $b^{\Delta}(N,D,u) = 1$, *if and only if* $D \geq \underline{D}^{\Delta,\mu}(N,u+\Delta)$.

*Proof. (Proposition 8)* We proceed in two steps. First, we show that given the assessment described in Proposition 8, public beliefs indeed are given as in Proposition 8. Then, we proceed with an inductive proof analogous to Proposition 1.

**Step 1: Public beliefs $\mu(\mathbf{x})$:** First, $\mu(\mathbf{x})$ as defined in the proposition is strictly increasing in $u$ and $N$, independent of $D$, and once it drops below $\bar{\mu}(\mathbf{x})$ it remains constant. This is consistent with Bayes' rule given the assessments defined in Proposition 8:

- If $\mu^{+}(N,u) \geq \bar{\mu}(\mathbf{x})$, then in all previous periods, buyers with signal $s = 1$ buy. (Recall that buyers with $s = 0$ never buy.) Given that buyers arrive with probability $\Delta\lambda$, applying Bayes' rule yields $\mu(\mathbf{x})$ as described in the proposition.

- If $\mu^{+}(N,u) < \bar{\mu}(\mathbf{x})$, then no buyer buys in in the current period or in any future period. No further learning occurs.

**Step 2:** We can conduct an analogous induction to Proposition 1 assuming that the buyer beliefs are given as specified in the proposition.

**i) Time induction start** $u = 0$ **(last period)** In a state $\mathbf{x} = (N, D, 0)$, it follows analogously to Proposition 1 that $D_+^\Delta(N, D, 0; W) = \min\{\max\{D, G - Np\}, W\}$ is a best response for the donor.

Next consider the buyer who arrives in the last period at $\mathbf{x} = (N, D, 0)$. Clearly, if she receives $s = 0$ or if $\mu^+(N, 0) < \bar{\mu}(\mathbf{x})$, then she does not buy. If she receives $s = 1$ and $\mu^+(N, u) \geq \bar{\mu}(\mathbf{x})$, it is optimal for her to buy.

To calculate $\bar{\mu}(\mathbf{x})$, we need to determine $\pi_q^\Delta(N, D, 0)$ for $q \in \{0, 1\}$. If $N + 1 \geq \underline{M}(D)$, then $\pi_q^\Delta(N, D, , 0) = 1$ for $q \in \{0, 1\}$. If $N + 1 < \underline{M}(D)$, then PT-strategies imply that $\pi_q^\Delta(N, D, 0) = \frac{1-F(G-p(N+1))}{1-F(D)} \mathbb{1}(D \geq \underline{D}^{\Delta,\mu}(N, \Delta))$, where as in Proposition 1, we define the wealth threshold $\underline{D}^{\mu,\Delta}(N, \Delta)$ by

- $\underline{D}^{\Delta,\mu}(N, \Delta) = 0$ if $N + 1 \geq M_0$ or $B^\Delta(N, \overline{W}, 0; \mu^+(N, 0)) \geq v_0$;
- $\underline{D}^{\Delta,\mu}(N, \Delta) = \overline{W}$ if $\overline{W} < G - p(N + 1)$ or $\mu^+(N, 0) < \bar{\mu}(N, \overline{W}, 0)$;
- Otherwise $\underline{D}^{\Delta,\mu}(N, \Delta)$ is the unique solution to

$$\pi_1^\Delta(N, \underline{D}^{\Delta,\mu}(N, \Delta), 0)\, \mu^+(N + 1, 0)\, (v - p) - \pi_0^\Delta(N, \underline{D}^{\Delta,\mu}(N, \Delta), 0) \left(1 - \mu^+(N + 1, 0)\right) p = v_0.$$

Let for $N = \underline{M}(D) - 1$, $\xi_1^{\Delta,\mu}(w; q) := 0$.

**ii) Induction in** $j = \underline{M}(D) - N$

**(a) Induction start:** Consider a buyer in state $\mathbf{x} = (N, D, u)$ such that $N + 1 \geq \underline{M}(D)$. Clearly, if she receives private signal $s = 0$ or if $\mu^+(N + 1, u)(v - p) - (1 - \mu^+(N + 1, u))p < \frac{v_0}{v-p}$, then she does not buy. If she receives $s = 1$, then either the campaign is already successful or the buyer can complete it by purchasing, so $\pi_1^\Delta(N, D, u) = \pi_0^\Delta(N, D, u) = 1$ for any $u > 0$. Therefore, she buys if and only if $\mu^+(N + 1, u)(v - p) - (1 - \mu^+(N + 1, u))p \geq v_0$, which is equivalent to $\mu^+(N, u) \geq \bar{\mu}(\mathbf{x})$ or $D \geq \underline{D}^{\Delta,\mu}(N, u + \Delta)$. Accordingly, the donor does not donate, i.e., $D_+^\Delta(N, D, u; w) = D$ if $\mu(N, u) \geq \bar{\mu}(N, D, u - \Delta)$. If $\mu(N, u) < \bar{\mu}(N, D, u - \Delta)$, it is a best response for the donor to donate $D_+^\Delta(N, D, u; w) = w$ as no buyer will buy in the future anyway. Thus, indeed $D_+^\Delta(\mathbf{x}; w) = \min\{\max\{D, \bar{W}\}, w\} = w$.

**(b) Induction hypothesis:** Suppose that for all $N = \underline{M}(D) - j'$ with $j' < j$ and all $u > 0$, the expressions for $\underline{D}^{\Delta,\mu}(N, u)$, $b^\Delta(\mathbf{x})$, $D_+^\Delta(\mathbf{x}; w)$, and $F^\Delta(w; \mathbf{x})$ are given by the expressions in Proposition 8. Furthermore, let $\xi_{j'}^{\Delta,\mu}(W)$ be given by

$$\xi_{j'}^{\Delta,\mu}(w) = \min\Big\{u \in \mathbb{U}^\Delta \;\Big|\; \begin{array}{l} \mu^+(M(w) - j' + 1, u) \cdot \pi_1^\Delta(\underline{M}(w) - j', w, u) \cdot (v - p) \\ -(1 - \mu^+(M(w) - j' + 1, u)) \cdot \pi_0^\Delta(\underline{M}(w) - j', w, u) \cdot p \geq v_0 \Big\}. \end{array} \tag{5}$$

Furthermore, assume that the corresponding $\pi_q^\Delta(N, D, u)$, $q \in \{0, 1\}$ satisfies the recursive structure specified in Proposition 8 for those states and is weakly increasing in $D$ and $u$, strictly increasing in $N$ for $G - (N+1)p > D \geq \underline{D}^{\Delta,\mu}(N, u + \Delta)$, and $\xi_{j'}^\Delta(w)$ is weakly decreasing in $w$ and weakly increasing in $j'$ for all $j' < j$. We also assume that $B^\Delta(N, D, u; \mu)$ is weakly increasing in $D$ for all $\mu \geq \bar{\mu}(N, D, u)$. Finally, assume that the threshold $\underline{D}^\Delta(N', u)$ is weakly decreasing in $u$ and $\underline{D}^\Delta(N - 1, u + \Delta) > \underline{D}^\Delta(N, u + \Delta)$ for all $N = M_0 - j'$, $j' < j$.

**(c) Induction step:** Consider a state $\mathbf{x} = (N, D, u)$ where $N = \underline{M}(D) - j$ and $u \geq T - \xi_{j-1}^{\Delta,\mu}(D)$. Then, by the induction hypothesis and with an analogous argument as in the proof of Proposition 1, we can write the probability of success as

$$\pi_q^\Delta(\mathbf{x}) \;=\; \mathbb{E}^{F_0}\Bigg[ \sum_{i=0}^{\frac{\max\{u - \xi_{j-1+\underline{M}(W)-\underline{M}(D)}^{\Delta,u}(W), 0\}}{\Delta}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \\ \pi_q^\Delta(N + 1, \max\{D, \underline{D}^{\Delta,\mu}(N + 1, u - \Delta(j - 1)\}, u - \Delta i) \Big| W \geq D \Bigg].$$

Analogous to Proposition 1, it follows that $\pi_q^\Delta(\mathbf{x})$ is strictly increasing in $D$ for $D \geq D_*(N, u + \Delta)$, and a buyer with $s = 1$ buys if and only if

$$B^\Delta(\mathbf{x}; \mu^+(N, u)) = \mu^+(N + 1, u)\, \pi_1^\Delta(\mathbf{x})\, (v - p) - (1 - \mu^+(N + 1, u)) \cdot \pi_0^\Delta(\mathbf{x})\, p \geq v_0$$

and $\mu^+(N+1,u) \geq \bar{\mu}(\mathbf{x})$. Then,

$$B^\Delta(\mathbf{x};\mu^+(N,u)) = \mathbb{E}^{F_0}\Bigg[\sum_{i=0}^{\frac{\max\{u-\xi^{\Delta,u}_{j-1+\underline{M}(W)-\underline{M}(D)}(W),0\}}{\Delta}} (1-\Delta\lambda)^{i-1}\cdot\Delta\lambda\cdot B^\Delta(N+1,\max\{D,\underline{D}^{\Delta,\mu}(N+1,u-\Delta(j-1)\},u-\Delta i;\mu^+(N+1,u)))\Big|W \geq D\Bigg]$$

is strictly increasing in $D$ because $B^\Delta(N+1,\max\{D,\underline{D}^{\Delta,\mu}(N+1,u-\Delta(j-1)\},u-\Delta i;\mu^+(N+1,u)))$ is weakly increasing in $D$ by the induction hypothesis, and $\frac{1}{1-F_0(D)}$ is strictly increasing in $D$. In particular, $B^\Delta(\mathbf{x};\mu)$ is strictly increasing in $D$ for any $\mu \geq \bar{\mu}(\mathbf{x})$.

Since $B^\Delta(\mathbf{x};\mu^+(N,u))$ is strictly increasing in $D$, there exists a $D \leq G-p(N+1)$ such that the buyer is just indifferent. Therefore, it is optimal to donate just enough to make the next buyer with $s=1$ indifferent between buying and not buying absent additional donations and the donor's optimal strategy is given by

$$D^\Delta_+(D,N,u) = \max\{D,\min\{\underline{D}^{\Delta,\mu}(N,u),W\}\}.$$

Analogous properties to Proposition 1 carry through, and we can define $\xi^{\Delta,\mu}_j$ as in Equation 5.

## B.9 Alternate Platform Designs

Let us consider a campaign with parameters $\mathbf{y} = (G,T,p,\lambda,v,v_0,\Delta)$ and denote the ex ante probability of success in the platform-optimal equilibrium of Proposition 1 as $\Delta \to 0$ by

$$\underline{\Pi}(G,T) := \mathbb{E}^{F_0}\left[\int_0^{\max\{u-\xi_{\underline{M}(W)}(W),0\}} \lambda e^{-\lambda s}\pi(\underline{D}(0,u-s),0,u-s)ds\Big|W \geq D\right]$$

and the ex ante probability of success in the buyer-optimal equilibrium of Proposition 3 as $\Delta \to 0$ by

$$\overline{\Pi}(G,T) := \mathbb{E}^{F_0}\left[\int_0^{\max\{u-\overline{\xi}_{\underline{M}(W)},0\}} \lambda e^{-\lambda s}\pi(\overline{D}(0,u-s),0,u-s)ds\Big|W \geq D\right].$$

### B.9.1 Restricting Timing of Donations

If the donor can only donate at the start (at $u = T$), then given any initial donation level $D_0$, the game is identical to one with no donations, but with $\underline{M}(D_0)$ buyers required instead of $\underline{M}(0)$. This game has a unique PBE for every $\Delta$. Let us denote the limiting probability of success of this game from the $(N+1)$–th buyer's perspective in period $u$ if she buys by $\pi^{\text{no}}(N, u)$ and the ex ante limiting probability of success of a campaign by

$$\Pi^{\text{no}}(G - D_0, T) := \int_0^{\max\{T-\overline{\xi}_{\underline{M}(D_0)},0\}} \lambda e^{-\lambda s} \pi^{\text{no}}(0, T - s) ds.$$

Then, the probability of success if the donor can only donate prior to the campaign is given by

$$\Pi^0(G, T) := \mathbb{E}^W \left[ \max_{D \leq W} \Pi^{\text{no}}(G - D, T) \cdot (W - D) \right].$$

Alternatively, if the donor can only donate at the deadline then there exists a unique equilibrium for any $\Delta$ that can be constructed analogously to the construction in Proposition 1 but forcing donations to be zero prior to the deadline. In particular, as $\Delta \to 0$, the probability of success from the $(N+1)$–th buyer's perspective in period $u$, denoted by $\pi^T(N, u)$ can be described recursively by

i) If $N \geq M_0 - 1$, buyer $M_0$ buys, and assigns probability 1 to the campaign being completed;

ii) If $N < M_0 - 1$, the probability of success for the $N + 1$st buyer is given by

$$\pi^{\text{T}}(N, u) = \int_0^{\max\{u-\xi^{\text{T}}_{M_0-N-1},0\}} \lambda e^{-\lambda s} \pi^{\text{T}}(N+1, u-s) ds + e^{-\lambda(u-\xi^{\text{T}}_{M_0-N-1})}(1 - F(G - Np)),$$

where for $j \leq M_0$, $\xi^{\text{T}}_j = \inf\left\{ u \in [0, T] : \pi^T(M_0 - j, u) \geq \frac{v_0}{v-p} \right\}$.

Let the ex-ante probability of success in this equilibrium be denoted by

$$\Pi^T(G, T) := \int_0^{\max\{T-\xi^T_{M_0},0\}} \lambda e^{-\lambda s} \pi^T(0, T - s) ds.$$

*Proof.* *(Proposition 9)*

i) **Donations at the start of the campaign only:**

Suppose that donations are allowed only at the start. Consider a realization of buyer arrivals and donor wealth $w$ that leads to a success. We show that this realization also results in a success in the limit of Markov equilibria of Propositions 1 and 3.

Let $D_0$ denote the optimal donation made at the start of the campaign and by assumption, the probability of success with that donation is positive. If continuous donations were allowed, the donor could replicate the strategy of donating $D_0$ at the start. Then, in any state $(N, D, u)$, the probability of success is greater if the donor can keep donating after an initial donation $D_0$. Consequently, whenever a buyer buys in the game with only donations at the beginning, the buyer also buys in the game with continuous donations and an initial donation of $D_0$. In the platform-optimal equilibrium, the probability of success must be weakly larger, and we have established that the platform-optimal outcome is also donor optimal.

Finally, there is always a realization of arrivals that leads to a success with continuous donations, but not in the counterfactual, i.e., $\Pi(G, T) > \Pi^0(G, T)$ for all $G, T > 0$. In all such events, the donor is strictly better off when continuous donations are allowed because he can induce a success with lower donations.

Next, consider a realization of buyer arrivals. There are two cases. First, consider a state in which a buyer buys in the setting in which donations are allowed only at the start. Then, she must also buy if donations are allowed throughout. For all such realizations, the probability of success is strictly higher if donations were allowed throughout, making the buyer strictly better off.

Second, consider a state in which a buyer does not buy in the setting with donations only at the start. If donations were allowed throughout, then she buys whenever $w \geq D^*(N, u)$ (the relevant wealth threshold of the Markov equilibria in Propositions 1 and 3) and receives an expected payoff of more than $v_0$. Thus, this buyer is better off in expectation upon arrival.

ii) **Donations at the deadline only:**

First, consider a realization of buyer arrivals and donor wealth $w$ that leads to a success when donations are allowed only at the end of the campaign. We show that this realization must result in a success with continuous donation.

When donations are allowed only at the end, buyers believe that $W \sim F_0$ throughout. Their beliefs cannot be updated upward. Thus, for the same realization of arrivals, under continuous donations, whenever a buyer buys if a donation is only allowed at the end of the campaign, the buyer would also buy with any positive donation. Thus, any campaign that is successful with only donations at the end is also successful with continuous donations.

Note that there are realizations of arrivals (with few early arrivals) such that the donor can induce a success with continuous donations given an equilibrium belief system (as in Propositions 1 and 3), but the campaign would die if donations are only allowed at the deadline. Thus, $\Pi(G,T) > \Pi^T(G,T)$.

Now, note that for all realizations that end in success in a setting with donations only at $T$, the donor is indifferent between this counterfactual and the model with continuous donations since he chooses the same strategy and not donate a positive amount until the deadline. Moreover, there exist realizations with $D_T < W$ such that the model with continuous donations makes the donor strictly better off by causing a success when the campaign would have died under the counterfactual.

Finally consider buyer payoffs. If a buyer bought in the setting with donations allowed only at $T$, then she must also buy if donations are allowed throughout. For all those realizations, the probability of success is higher if donations were allowed throughout, making the buyer strictly better off. If a buyer did not buy in the counterfactual, then she buys under continuous donations if and only if $w \geq D^*(N, D, u)$ the relevant wealth threshold for the donor's PT strategies. Thus, this buyer is better off in expectation upon arrival.

### B.9.2 "No Information" Environment

If buyers and the donor only observe $G$, $p$, $T$ and $u$, but never observe $D$ or $N$ over time, then the belief about the probability of success for all buyers must be the same, regardless of when they arrive. The example below shows that in this case the induced probability of success in this equilibrium can be higher or lower than that in the Kickstarter mechanism.

Denote the ex ante probability of success if buyers do not observe the progress of the campaign by $\overline{\pi}$. Then, any buyer pledges if and only if $\overline{\pi} \geq \frac{v_0}{v-p}$. Therefore, an equilibrium is described by a belief threshold $\bar{\pi}$ and a donor strategy $\tilde{W}(w)$ given donor wealth $w$, such that

$$\tilde{W}(w) = \arg\max_D \sum_{k=M(D)}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!}(w-D) \qquad \text{and} \qquad \overline{\pi} = \int \sum_{k=M(\tilde{W}(W))}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} dF(W).$$

Consider a campaign in which no buyer contributes if players do not observe any dynamic information, i.e., $\overline{\pi} = 0$. In that case, the Kickstarter mechanism trivially performs better and makes all players better off.

Next, consider a campaign for which $\overline{W} \approx 0$ but $\overline{\pi} > 0$. In that case, the Kickstarter mechanism results in a lower probability of success as not all buyers will buy and donations are not relevant.

In contrast, if the donor could observe $N$ over time but buyers could not, we would get a similar result. In this case, the donor can condition on the number of arrivals. This means that the belief of any buyer is given by $\bar{\bar{\pi}} = \int \sum_{k=M(W)}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} dF(W) > \overline{\pi}$. A buyer pledges if and only if $\bar{\bar{\pi}} > \frac{v_0}{v-p}$. In this case, the probability of success is higher than that in the Kickstarter mechanism for campaign primitives such that $\bar{\bar{\pi}} > \frac{v_0}{v-p}$. However, if $\bar{\bar{\pi}} = 0$, then revealing information as in the Kickstarter mechanism can yield a positive probability of success.

# C Data Appendix: Bounding Donations

Our definition of a donation comes from contributors entering an amount in the donation box, or from contributors paying more than the reward price. However, some rewards may be better interpreted as donations. Examples include a low priced reward that approximates a thank-you, or an expensive reward that includes the product but also includes special recognition. The bias is in only one direction: we are possibly understating the magnitude of donations on the platform. This is not a problem, per se, but we would like to investigate what role this plays in our results.

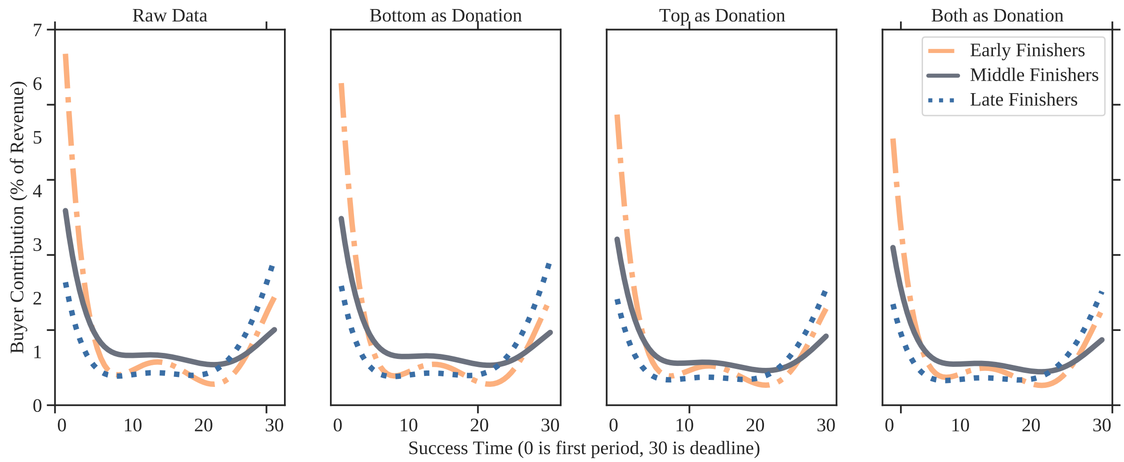
Given the number of projects and buckets per project, manually assigning a reward or part of a reward as a donation is infeasible. There are over 500,000 rewards in the data. Instead, we perform the following analyses. First, we assume the least expensive bucket represents a donation. Next, we assume the most expensive bucket represents a donation. Finally, we assume both the least and most expensive buckets constitute donations.

We reprocess the data and repeat the analyses in the previous subsections. For brevity, we only show one result and describe the others. Figure 13 and Figure 14 show a comparison of the raw data with the three robustness exercises, replicating the analysis of Figure 2—purchase and donation revenue, as percentage, over time for early, middle, and late completing campaigns. The figure shows an intuitive result: as reward purchases are assigned as donations, the amount of revenue attributed to donations increases. However, the figure also shows that qualitatively, our key finding remains—donations spike at the deadline for late-completing campaigns. There are no noticeable spikes in donations for early- or middle-finishing campaigns.

Our other empirical results are also qualitatively unaltered.

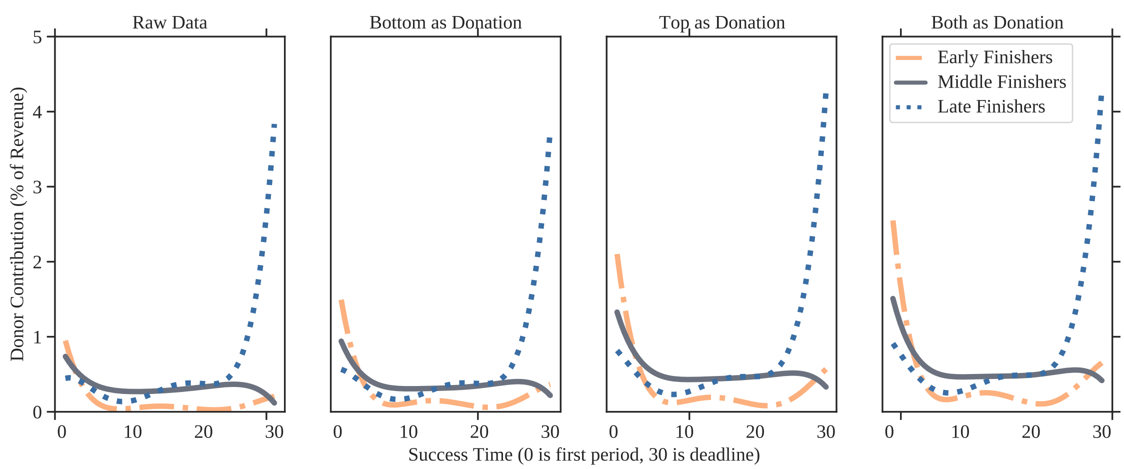
We also conduct robustness to our calculation of shipping costs. This is important because donations are determined after subtracting off shipping costs. So, if we understate shipping costs, we overstate donations. We reprocess all the data assuming all purchases are made from the country with the lowest, and then most expensive, shipping costs. Across all reward buckets, the average shipping cost in our baseline analysis is $16; this lowers to $10 under min-cost shipping and increases to $25 in max-cost shipping. Moreover, these statistics are simply the overall mean; after assigning shipping costs to observed purchases, the average shipping cost drops by nearly 50%. That is, most transactions on the platform involve rewards with lower shipping costs. Figure 15 recreates Figure 3, and shows buyer and donor purchases for US, min-cost, and max-cost shipping.

Figure 13: Robustness to Buyer Contributions over Time for Early-Middle-Late Campaigns




Note: Replication of Figure 2(b) using different definitions of donation. The left panel donates the origin version. Next, we assign the lowest priced bucket as donation. The following assigns the highest priced bucket to donations. Finally, the last panel moves both the lowest- and highest-priced buckets to donations.

Figure 14: Robustness to Donor Contributions over Time for Early-Middle-Late Campaigns

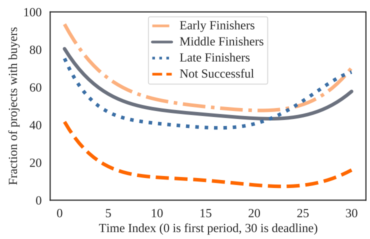



Note: Replication of Figure 2(b) using different definitions of donation. The left panel donates the origin version. Next, we assign the lowest priced bucket as donation. The following assigns the highest priced bucket to donations. Finally, the last panel moves both the lowest- and highest-priced buckets to donations.

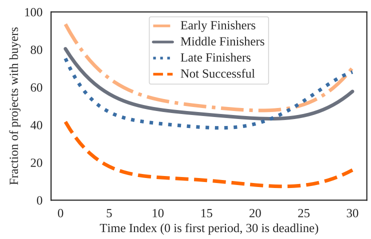
All our results are robust to the various estimates of shipping costs. Utilizing the lower and upper bounds on shipping costs provides us with lower and upper bounds on donation amounts; we find all our results hold. We bound donations at the deadline to be between 25.1% and 29.6% under min- and max-cost shipping. In our baseline results, we estimate this to be 28.0%.

Figure 15: Robustness: Percentage of Projects that Receive Purchases/Donations over Time

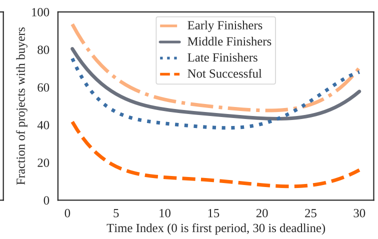
(a) Purchases - US



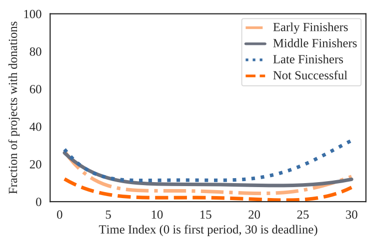

(b) Purchases - Min Cost



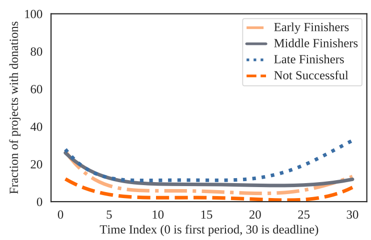

(c) Purchases - Max Cost



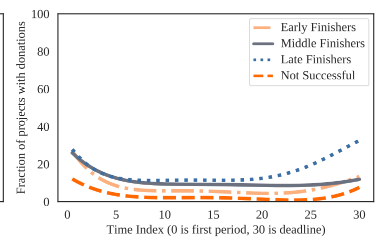

(d) Donations - US



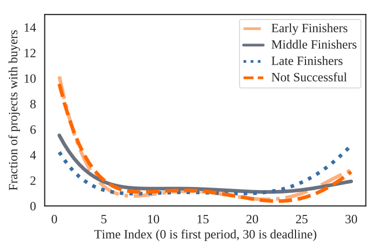

(e) Donations - Min Cost



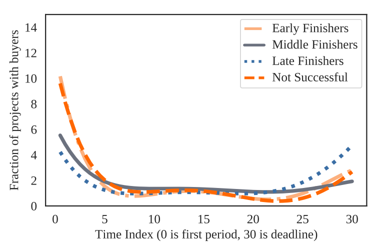

(f) Donations - Max Cost



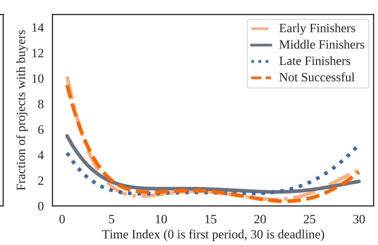

(g) Weighted Purchases - US



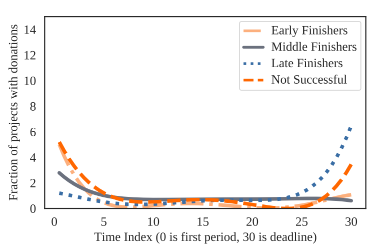

(h) Weighted Purchases - Min Cost



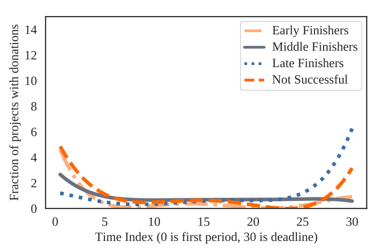

(i) Weighted Purchases - Max Cost



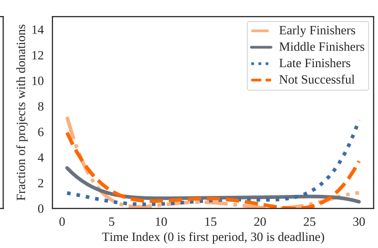

(j) Weighted Donations - US




(k) Weighted Donations - Min Cost




(l) Weighted Donations - Max Cost




These figures show the percentage of projects that have donations or purchases over time, for 30 day projects. 30 denotes the campaign deadline. Four lines are shown: early finishers, middle finishers, late finishers and unsuccessful campaigns. The weighted panels are weighted by within-project revenue.