The Economic Motives for Foot-binding*

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Job Market Paper[‡]

Abstract

What are the origins of gender-biased social norms? As a painful custom that persisted in historical China, foot-binding reshaped the feet of girls during early childhood. This paper presents a unified theory to explain the key stylized facts about foot-binding, and investigates its historical dynamics driven by a gender-asymmetric mobility system in historical China (the Civil Examination System). The exam system marked the transition from heredity aristocracy to meritocracy, which generated a more heterogeneous composition of men compared to women, triggering intensive competition among women in the marriage market. Embodying both aesthetic and moral values, foot-binding was gradually adopted by women as a social ladder, first by the upper class and later by the lower class. However, since foot-binding impedes non-sedentary labor but not sedentary labor, its adoption in the lower class exhibited distinctive regional variation: it was highly prevalent in regions where women specialized in household handicraft, and was less popular in regions where women specialized in intensive farming, e.g. rice cultivation. Empirically, we analyze data from county-level Republican archives on foot-binding to test the model predictions, and find evidence supporting the key theoretical predictions.

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"Foot-binding as a ladder of success for women thus mirrored the fate of the civil service exam, a similar vehicle for men...Foot-binding was useful for social climbing, not mountain climbing."

- Dorothy Ko, Cinderella's Sisters: A Revisionist History of Footbinding, Chapter 6, 2005

1 Introduction

Gender-biased social norms reflect the evolution of gender inequality. Carrying significant disutility for women, gender-biased social norms have existed widely across the globe and in different historical periods, with profound impacts on women's economic, social, physical, and psychological well-being. For instance, as a cruel procedure that violates a woman's rights to health and security, female genital mutilation (FGM) has a long history and still persists in Africa, the Middle East and Asia (WHO, 2012). In other historical contexts, we also observe corsets in Victorian Europe and the U.S. and foot-binding in historical China. These practices share striking similarities: all involve devastating body modifications, all are practiced at young ages for girls, all have significant marriage market implications, and all carry significant individual and social costs. Moreover, if we extend our horizon beyond cruel body modifications, we observe other gender-biased norms that harm women's welfare, for instance high dowries in post-modernization India, which often result in dowry death, female infanticide, and related violence (e.g. Bloch and Rao 2002, Anderson 2003, Bhalotra et al. 2016). Given that some such norms were successful eliminated (corsets and foot-binding) while others persist (FGM and high dowries in India), one may wonder about the origins of gender-biased social norms, as well as the institutional and economic factors that determine their spread, persistence, and disappearance. This paper sheds light on these questions by examining foot-binding, a representative gender-biased social norm in historical China that had been successfully eradicated in the early 20^{th} century.

As a painful practice in historical China, foot-binding targeted girls whose feet were reshaped systematically during their early childhood. Originating from a female dancer in imperial palace during the Five Dynasties (907-960), foot-binding persisted for nearly a millennium in historical China. Initially appreciated as a major icon of feminine beauty, from the early 20^{th} century onwards, it increasingly was viewed as a brutal cultural practice reflecting the oppression of women by a masculine-dominated society. Considerable efforts were made by the government and social activists to eradicate the practice (e.g. Yang, 2012), and scholars from multiple disciplines have provided perspectives to explain its popularity. Many

explanations focus on a specific aspect of foot-binding, but none can explain the timing of its emergence and decline, as well as regional and class differences in its prevalence, within a consistent framework.

This paper presents a unified economic theory of foot-binding that can explain variation in its practice over time, among classes, and across regions. Foot-binding is modeled as a premarital investment made by girls' parents for marriage market competition, where there is a key trade-off between higher expected marrying-up benefits with foot-binding and disutility from the pain of foot-binding and the labor opportunity cost in non-sedentary work (e.g., farmland work). A key feature of the theory is that it explicitly models how changes in the gender-specific social mobility system affect matching in the marriage market and thus the marrying-up benefits of foot-binding for upper class and lower class women.

To explain the dynamics of foot-binding in response to changes in marriage market competition, we consider a gender-asymmetric shock to the social mobility system in historical China – the Civil Examination System (in Chinese, the *Keju*, 607-1905). The exam system triggered a transition from heredity aristocracy to meritocracy. By taking and passing the exams at different levels, talented males could climb the social ladder while those who failed the exams would move downwards. We model how the change in the exam system generated a more heterogeneous distribution in the quality of men than of women, and triggered intensive marriage market competition among women which increased their premarital investments. Foot-binding, which embodies both Confucianism moral codes and men's aesthetic appreciation of women, was adopted to distinguish themselves in the marriage market and served as a social ladder for them to climb up.

In addition to the marriage market value of foot-binding, we also consider the labor opportunity cost of foot-binding by examining different types of women's labor in diverging agricultural regimes. Given that foot-binding deforms women's feet, it sharply limits physical mobility thus precludes them from engaging in intensive non-sedentary activities (e.g. rice farming), while having much less of an effect on sedentary activities such as household handicraft production. Therefore, among lower class women who played an active income-earning role, foot-binding prevalence exhibited regional variations driven by different agricultural regimes. In particular, foot-binding of lower class women was highly prevalent in regions where women specialized in sedentary labor (e.g. handicrafts in Northern China), and less popular in regions requiring labor-intensive farmland work (e.g. rice cultivation in the Pearl River Delta).

To test the model predictions, we draw upon a wide range of quantitative and qualitative sources, including cross-sectional data from Republican archives on county-level foot-binding prevalence, as well as historical, anthropological and archaeological evidence. In particular, we use cross-sectional variation in the exam quota of entry-level degree holders as a proxy for the proportion of men succeeding in the exams (Kun et al., 1899) to capture the marriage market benefits from foot-binding. Empirically, we find that more exam quotas predict a higher incidence of foot-binding at the county level, with a one standard deviation increase in exam quota leading to a 6 percentage point increase in the probability of foot-binding. As additional evidence, we use archaeological findings to show that foot-binding shoes of upper class women are more likely to have occurred in regions with more exam quotas. In addition, local folk ballads reveal a systematic association between foot-binding and marrying up benefits.

To empirically test the impact of labor opportunity cost on foot-binding, we combine the cross-sectional data on county-level foot-binding prevalence with the agricultural suitability index developed by GAEZ (Global Agro-Ecological Zones, FAO) to test whether labor-intensive crop cultivation can be protective for women in terms of foot-binding. We find that a greater suitability for rice (a major labor-intensive crop) relative to wheat predicts less foot-binding prevalence, and this finding is robust across multiple model specifications, sub-samples, and inclusion of a rich set of county-level socioeconomic, demographic and geographic variables. Specifically, a one standard deviation increase in relative rice suitability lowers the probability of foot-binding by 9 percentage points. As an additional check to rule out potential confounding factors, we use a qualitative example of spatial discontinuity in Jiangsu Province to support our causal interpretation of this relationship.

Lastly, we present three extensions of the model. The first adopts a three-layer hierarchy of exam results rather than a binary structure, reflecting high, medium and low rankings in the entry-level exam. Second, we take foot-binding as a continuous practice to explain variation in the size of bound feet. (i.e. large versus small). Finally, we introduce multiple tools for marriage market competition, allowing foot-binding and dowry payments to coexist. Results from our extensions are qualitatively similar to our baseline model, yielding richer implications for explaining differences in foot-binding practice.

Our study makes contributions to several different literatures. First, it increases understanding of the origins of gender inequality, by showing that a gender-asymmetric mobility system can generate gender-biased cultural customs that carry high disutility for women. Previous scholarly work has explained the origins of gender norms from agriculture activities (e.g.

Boserup, 1970; Qian, 2008; Alesina, Giuliano and Nunn, 2013; Xue 2017), marriage market competition (e.g. Grossjean and Khattar, 2016), and education and technology (e.g. Goldin 1990, 2006; Goldin and Katz 2002). Our paper provides a case similar to the association between the caste system and high dowries in post-modernization India (Anderson, 2003), and therefore serves an additional case to explain the historical roots of gender norms (see Giuliano, 2017 for a summary in this field).

Second, our research contributes to the literature on the economics of marriage and premarital investments (e.g. Becker, 1973, 1974; Peters and Siow 2002; Iyigun and Walsh, 2007; Chiappori et al. 2009; Bhaskar and Hopkins 2016). We add to this literature by showing that non-monetary utility transfers (foot-binding) can be one type of premarital investment. The perspective is similar to that of Mariani (2010) who takes women's chastity as a special type of premarital investment. By zooming in foot-binding as a case study, we examine the impact of a specific shock in mobility system on the evolution of foot-binding, providing both theoretical and empirical analysis.

Third, our work contributes to the growing literature in cultural economics, including Carvalho's (2013) explanation of the New Veiling movement among Muslim women; Chesnokova and Vaithianathan (2010), Bellemare et. al. (2015) and Poyker (2016) on the motives and persistence of FGM; and studies by Rao (1993), Edlund (2000) and Anderson (2003, 2007) on dowries in India. In this regard, our paper facilitates understanding of how destructive gender-biased cultural practices can arise, considering similar cultural practices such as corsets in the Victorian period in the US and Europe¹. While foot-binding and corsets have disappeared, understanding the evolution of foot-binding is helpful to policy makers seeking to eradicate harmful customs such as FGM and high dowries.

Last but not least, this paper contributes to the literature on foot-binding by providing a unified economic theory of the practice, and a quantitative assessment using a new historical data set. It builds upon previous studies of foot-binding have come from different disciplines such as history, anthropology (e.g. Blake, 1994; Gates, 2001; Bossen et al., 2011; Brown et al., 2012; Yang, 2012; Bossen and Gates, 2017), sociology (e.g. Mackie, 1996), and literature (e.g. Wang, 2002; Ko, 2005), all of which provide useful insights and evidence². In the economic

¹The Victorian period also was a peak period for gender-asymmetric social mobility (e.g. Galor and Moav, 2006; Lindert, 2000).

²Previous explanations of foot-binding emphasized its aesthetic, erotic, social and cultural motives. Perhaps the most widely acknowledged interpretation for foot-binding is the aesthetic and erotic motive (e.g. Yao, 1936; Levy, 1960; Ko, 2005; Gates, 2008). A second explanation of foot-binding is the biological motive, that it was restrictive in order to guarantee women's purity and enhance women's loyalty to their husbands

literature, the earliest analysis of foot-binding is from Veblen (1899), who considered foot-binding to be a parallel phenomenon to Victorian corsets in Europe and America, where in both cases the investment was a symbol of social status. Another discussion of foot-binding is by Cheung (1972), who argues that it reduces the cost of enforcing property rights over wives. However, both of these studies are limited in that neither systematically reviewed the stylized facts of foot-binding and cannot explain foot-binding's temporal and regional variations.

The rest of this paper is organized as follows. Section 2 provides a brief review of foot-binding history and documents key stylized facts, as well as a historical description of the Civil Examination System. Section 3 presents the model, and Section 4 discusses the data and empirical results. Three extensions of our theory are discussed in Section 5, and Section 6 concludes.

2 Background

2.1 Foot-binding and The Stylized Facts

The custom of foot-binding targeted girls whose feet were reshaped systematically during their early childhood, usually from age 5 to 12. The process is often initiated and practiced by mothers or grandmothers and can last for years, during which the bones and muscles were gradually modified towards a certain shape. A pair of bound feet carries lifetime painfulness, and is often accompanied by other types of health issues (e.g. infections). There were even cases when the girl became handicapped and unable to walk without the assistance of others for the rest of her life. However, given the significant disutility and welfare loss induced by foot-binding, the appreciation of foot-binding by men and the society as a whole stayed mysterious³.

⁽Daly, 1978; Mackie, 1996). The prevalence of foot-binding also has been viewed as a consequence of the revival of Confucianism to control women (Chen, 1928), or as a way to consolidates the patriarchal kinship system (Greenhalgh, 1977). In addition, the ethnicity origin of foot-binding argues that the Han Chinese women wanted to distinguish themselves from invading northern barbarians with such dressing code (Ebrey, 1990). Recent anthropological work has revealed an association with women's household handicraft (Ebery, 1990; Mann, 1997; Gates, 2001; Bossen et al.,2011; Bossen and Gates, 2017). In particular, Bossen and Gates (2017) argue that foot-binding was adopted to boost the economic contribution of girls in household production, especially for handicrafts which call for sedentary labor.

³The obsession for foot-binding could be depicted from numerous articles, poems, and prose from the Six Dynasties (420-589) to the late Qing (1636-1911). Bound feet were often the smaller the better, and

In terms of the earliest evidence of foot-binding, the greatest consensus is that it emerged during the Five Dynasties (907-960) (e.g., Ko, 2005). Yao Niang, who was a dancer of the palace, wrapped her feet tightly to support them and enhance their daintiness. In its early phase, foot-binding was often practiced by dancing girls and prostitutes for the purpose of entertaining men. Starting in the Song dynasty (960-1279), foot-binding gradually gained popularity among upper class women⁴. Later, the practice diffused from the upper class to the lower class during the Ming and Qing dynasties. While the Qing government tried to prohibit foot-binding in 1636, 1638 and 1664, these prohibitions were largely ignored. In the late Qing, with a surging influence of foreign influence and domestic reforms, intellectuals and social activists raised awareness about the detrimental effects of foot-binding. In the 1930s, the Republican government officially prohibited this practice by implementing nationwide anti-foot-binding campaigns. By the middle of the 20th century, foot-binding finally had faded into the past.

To establish the stylized facts of foot-binding, we investigate a wide range of historical sources and present a detailed summary in Appendix 9.1. Briefly, four stylized facts merit emphasis: (1) Time variation: while scattered instances of foot-binding can be found in earlier historical periods, it did not gain popularity until the post-Song period; (2) Class variation: foot-binding was first adopted by the upper class and later by the lower class, and its prevalence within the upper class was greater that within the lower class; (3) Regional variation: among lower class women, the prevalence of foot-binding varies by region. As illustrated by Figure 1, foot-binding was most prevalent in Northern China, while it was much less prevalent in Southern China, especially in the Pearl River Delta⁵; (4) Size variation: the higher the social status, the more tightly bound the feet⁶. In addition, within the upper class, the size of feet grew smaller over time. In Song dynasty, the shape of bounded feet were thin and straight,

the highest class of bound feet was labeled as the "Three-Inch Golden Lotus". A throughout checklist for beautiful bound feet includes the following seven items: thin, tiny, sharp, arched, scented, soft and straight (Yao, 1936). Readers could also refer to Zhang (2015) for a summary of poems and prose on the appreciation over foot-binding.

⁴Archaeological evidence show that bureaucrats' wives and daughters have all practiced foot-binding (Ebrey,1990).

⁵This pattern has been well established by scholars (e.g. Qian, 1969; Hu Pu-An; Ko, 2005; Davin, 1976; Turner, 1997). Another piece of evidence illustrating the intensity of foot-binding in Northern China was the *Bound Feet Beauty Contest* (in Chinese, *Sai Zu Hui*). This unique beauty contest was often organized in either early spring or Autumn (Yao, 1991), when women proud of their lotus feet came out to show off their feet in public. At the same time, crowds of men came to appreciate the lotus feet one by one to decide the winner (see Appendix Figure 3).

⁶The smallest bounded feet were often found in Beijing, Hebei, Shandong, Shanxi, and Shaanxi, and the largest ones were often found in Suzhou, Hangzhou, Jiaxing, and Songjiang (Qian, 1969).

which was referred to as knife-slim feet. However, in the Qing dynasty, the shape of bound feet became highly curved, which were often called the Three-inch $Lotus^7$.

>>Figure 1. Regional Variation of Foot-binding Prevalence<<

2.2 Social Stratification and The Civil Examination

In this paper, we focus on a gender-biased institutional shock: the Civil/Imperial Examination System (the *Keju* exams)⁸, which was a meritocratic elite recruitment system adopted by Chinese emperors triggering changes in social mobility. The system was established during the Sui (581-618) and the Tang (618-907) dynasties, consolidated and expanded during the Song (960-1276) and fully institutionalized during the Ming (1368-1644) and Qing (1644-1911). During the post-Song period, the most important central and local officials and bureaucrats were selected through this system.

Several features of the exam system are worth highlighting. First, only men were eligible to participate in the examination⁹. Second, almost men of all class were eligible to take the exams regardless of family background, social status or age¹⁰. Third, the number of officials selected and the criteria of the exams changed over time. In particular, during the Sui (581-618) and the Tang (618-907), only a very limited proportion of officials were selected by the exams, while after the Song (960-1279) there was a significant increase in recruitment size, and the procedures became much more meritocratic¹¹ (Chafee,1995). Forth, the exam

⁷One may also observe variation in terms of ethnicity. While most Han Chinese women intensively practiced foot-binding, the ethnically minority women living in Canton, Fujian and Yunnan do not bind their feet at all. However, following the Han Chinese, some Manchurian women also imitate the foot-binding practice. It was also recorded that some Hui people in Gansu province also did foot-binding. So overall, there's no uniform pattern of foot-binding adoption by ethnic minorities. For the consistency of analysis, in the following we will focus on the Han Chinese women and their foot-binding practice.

⁸For a general description of the examination system, see Magone (2015). On the *Keju* system and changing social mobility as a consequence, see Ho (1962), Hartwell (1982), Elman (2000, 2013), Kung and Jiang (2015) and recently Shiue(2017). For the impact of the abolition of *Keju*, see Bai and Jia (2016). For the cultural persistence of the exams, see Chen, Kung and Ma (2017).

⁹Except for the exams hosted by the Taiping regime during their rebellion (1851-1864) that permitted women to take the exams, which is geographically and temporally very limited.

 $^{^{10}}$ One exception is men with pariah status, defined as people who are excluded from the four major catagories of Chinese society: the *shi* (gentry scholars), the *nong* (peasant farmers), the *gong* (artisans and craftsmen), and the *shang* (merchants and traders). The proportion of pariah status is very low in the whole population.

¹¹The key improvements to the exam system made by the Song include: (1) the local examiners were relocated once a year, in order to prevent collusion between candidates and examiners; (2) during the examination period, examiners had no contact with the candidates (*Suo Yuan*,); (3) the names and birth place

system structure was hierarchical. Specifically, during the post-Song period, there were three levels of exams including: (i) the Licensing exam (Tongshi) at the county level, those who passed the exam were entitled as Literati (Shengyuan or Xiucai), and the number of degree holders at this level is subject to quotas assigned by the central government; (ii) the Qualifying exam (Xiangshi) at the provincial level, where those who passed were entitled as Recommended man (Juren); (iii) the Academy exam (Huishi), where those earned the title Presented Scholar (Jinshi)¹². Finally, with respect to content, the exams mainly focused on canonical texts from the Five Classics (Wujing) and the Four Books (Sishu)¹³. With the above features, the exam system served as a recruitment system that was highly inclusive and meritocratic.

In terms of regional variation in elite recruitment, the exam was regulated by a quota system during the Ming and Qing dynasties. Just as the structure of the exams was hierarchical, the quota system was also hierarchical at the level of *Literati*, *Recommended Man* and *Presented Scholar*. The quota allocation at the entry level of the exam was determined by the following factors: population, tax obligations, and historical talent distribution (Chang, 1955, Shang, 2004, Liang and Zhang, 2013). While there were modifications to the quota allocation driven by sociopolitical shocks, the overall quota allocation pattern across regions was stable during the Ming and Qing period.

In addition to the exam system as the major elite recruitment channel during the post-Song dynasties, there also existed alternative ways to become government officials or obtain exam degrees, including purchasing exam degrees by making donation to the government (especially in times of disasters or wars), or inheriting their senior relative's position. However, those cases were relatively rare, and very often they were assigned with less important positions in the bureaucratic system. Moreover, the career prospects of these informal degree holders were not as good as those with formal degrees through passing the examination, so they were with lower social status in general¹⁴.

information of candidates on the exam papers were obscured, to create an anonymous grading process (Hu Ming); (4) the answers of candidates were transcribed, making it impossible to identify specific candidates by their handwriting (Teng Lu); (5) if candidates were the sons or close relatives of examiners, an alternate examiner would be assigned (Bie-Tou Shi).

¹²The final national level exam is the Palace exam (*Dianshi*), which is hosted in front of the emperor. This level of exam aims at ranking the Presented scholars (Jinshi) into three levels.

¹³However, there still existed variation in the exams in terms of the way these classics were tested and the criteria. During the Sui and the Tang, the gracefulness of the articles written by candidates was the key. After the Song, the major content for the exams had been focused on memorizing the classics of Confucianism. In the Qing dynasty, the exam content and format were highly standardized.

¹⁴For an in-depth discussion comparing these two types of degree holders (those with formal and informal

Social Stratification. The exam system generated a social hierarchy and deeply affected social mobility in historical China, serving as a major social ladder for men to climb up. Figure 2 provides an illustration of this social ladder based upon Chang (1955), mapping each level of exam degree holders to their corresponding positions in the social hierarchy. Under the exam system, degrees and achievements through the exam played a key role in determining the social status of men.

>>Figure 2. The Civil Exam System and the Social Ladder<<

The upper class consisted of degree holders and officials had the greatest prestige, influence, and wealth in the society. About two percent of the population belonged to this stratum (Chang, 1955). In general, government officials and higher level degree holders (i.e. Presented Scholars and Recommended Men) belonged to the upper gentry class, and the lower level degree holders (mainly consisted of *Literati*, sometimes *Tribute Students*, those who were of the Imperial Academy) were the lower gentry class. The gentry class played a dominant role in local administration and enjoyed many aspects of privileges. They were distinguished from commoners in style of dress and costumes, were protected against insults from commoners and interference from officials, and were exempted from coercive labor service, and also excused from paying the poll tax (Hsu, 1970). The income differences of a commoner's family and a gentry family is also profound. In Appendix Table A2, we present the relative income of gentry versus commoners based on Chang (1962). The ratio of average annual income of gentry to the average annual income of a male labor varies from 1.5 to 36¹⁵. In particular, for three most popular occupations of gentry – gentry services, secretaries to officials, and teaching – the ratios range from 20 to 50, revealing large income disparities across classes. We do similar calculation within gentry class, and show that gentry at different rankings had a clear hierarchical structure as well.

Social Mobility. There has been rich studies of to what extent the exam system has affected social mobility¹⁶. Most studies reveal a profound association between the exam system and degrees), see Chang (1955).

¹⁵By examining the aggregate income profile of the 19th century elite, Chang (1962, Table 26) illustrated that the major income sources of the upper class came from officeholding in the imperial bureaucracy, landownership, mercantile activities, gentry services, teaching, services as secretaries to officials, and other gentry services (i.e. practice of traditional medicine and scholarship awardees). In terms of income composition, the upper gentry class enjoyed higher proportions of their income from landownership, officeholding in the imperial bureaucracy, mercantile activities, while the key income resource of lower gentry class were teaching, secretaries services and other gentry services.

¹⁶For reference in social history, see Ho (1962), Hartwell (1982), Hymes (1986) and Elman (1991, 2000). For recent studies in the field of economic history, see Kung and Jiang (2015) and Shiue (2017).

the social mobility, especially by comparing the pre-Song and post-Song dynasties. For post-Song dynasties, scholars used a wide variety of sources to illustrate within-dynasty variation of mobility introduced by the exam system¹⁷. To get a sense of how the exam system changed social mobility across dynasties, we examine data from the China Biographical Database Project (CBDB)¹⁸ to calculate a surname fractionalization index among Chinese figures and celebrities (mostly officials and scholars) across dynasties. By using the surnames of these people as a measure of social mobility, we hypothesize that a concentrated distribution of surnames indicates restricted mobility, while a high level of fractionalization of surnames captures greater mobility. We focus on the vast majority of the population that are of the Han ethnicity, which are identified as those having single character surnames (as a proxy), and calculate a surname fractionalization index following Alesina et. al (2003). As illustrated by Figure 3, we see a jump in the fractionalization index during the Song¹⁹. For dynasties of the post-Song period, the surname fractionalization index stays relatively stable.

>>Figure 3. Men's Mobility Trend: Evidence from CBDB<<

3 The Theory

In this section, we develop a unified theory of foot-binding which models foot-binding as a premarital investment by the bride's family to compete for better marriage opportunities when the quality dispersion of grooms is larger than that of brides. We first present the setup of the model, and then consider how a gender-biased social mobility institution triggered the evolution of foot-binding in both the upper and lower classes.

Preferences: Foot-binding, Beauty and Women's Virtue. Men's preference for foot-binding was a complex combination of appreciation of women's beauty and feminine virtue. With respect to the aesthetic value of foot-binding, there exists a long-standing admiration of women's small feet and elegant gait, traceable in Chinese poems, prose and other forms

¹⁷See appendix 9.2 for further discussion.

¹⁸The CBDB is an online relational database with biographical information about approximately 360,000 individuals as of April 2015, primarily from the 7th through 19th centuries. It is a relational database of biographical information for China before the early twentieth century. This version: 2015-03-18.

¹⁹Here's an illustrative example: during the Tang, there were 369 chancellors in total, distributed among 98 families; while the Song's 134 chancellors were distributed among 126 families. Among 1935 scholar-officials, those from non-bureaucratic families constituted around 55.12%. The cases with multiple generations all being chancellors during Tang were very common, i.e. "Many families with more than ten chancellors", but very rare during the Song (only three cases).

of literature. Walking slowly and daintily, even timidly, was appreciated from an aesthetic perspective, while walking fast and boldly was considered vulgar²⁰. By reshaping the feet with huge pain and physical restriction, foot-binding helps women approach this aesthetic goal. The specific obsession over foot-binding and tiny feet was depicted from the Six Dynasties (420-589) to the late Qing (1636-1911)²¹.

In addition to aesthetic value, foot-binding was also considered as carrying a "vector of status", as Mann (1997) puts it. It was a symbol of elegance and good breeding (e.g. Mann, 1997, Ko, 2005), and a mark of status and virtue (Bossen and Gates, 2017). Since foot-binding is a painful process for women to undertake, well-shaped and tiny bound feet also help to reveal their endurance, obedience and submissiveness, which conform with the moral codes of women suggested by Confucianism²². Taken together, as a package of beauty and women's feminine virtue, foot-binding captured the key elements of men's moral and aesthetic appreciation of women.

Setup. We look at a society with two classes: the upper class and the lower class, denoted by $\tau \in \{u, l\}$. The upper class occupies share μ of the population, with the remainder in the lower class $(1 - \mu)$. As the upper class occupies a small proportion of the population, we assume $\mu < \frac{1}{2}$. We normalize both the potential bride's and groom's population to 1, and assume the sex ratio to be 1 and identical across classes²³. We index individual women by i and individual men by j, respectively.

Marriage. Traditional Chinese marriages are arranged by brides' and grooms' families, and are facilitated by matchmakers²⁴. Thus, we take the decision makers to be the bride

²⁰In addition to its aesthetic values, foot-binding is also considered to enhance women's erotic value due to men's foot fetishism (Howard Levy, 1966).

²¹Bound feet were often the smaller the better, and the highest class of bound feet was labeled as the "*Three-Inch Golden Lotus*". A throughout checklist for beautiful bound feet includes the following seven items: thin, tiny, sharp, arched, scented, soft and straight (Yao, 1936). Readers could also refer to Zhang(2015) for a summary of poems and prose on the appreciation over foot-binding.

²²However, Neo-Confucianism itself did not propose foot-binding, that none of its classics had advocated foot-binding. Cheng, Yi (1033-1107), one of the leading philosophers of Neo-Confucianism in the Song, was against foot-binding practice and his female family members till the Yuan dynasty(1271-1368) did not conduct foot-binding.

²³Though the *de facto* sex ratio of male to female could be greater than one (especially considering the existence of historical female infanticide), assumptions about sex ratios do not drive our theoretical results. Our main results go through as long as there is greater quality heterogeneity of males than that of females, after the exam system is established.

²⁴Matchmakers have a long history in historical China (Chen, 1936), the earliest evidence can be dated back to the Zhou dynasty (1046-256 BCE). Matchmakers played a key role in the marriage market matching regardless of social class and geographic regions, and they are often local senior women with good connections with families and established reputation. There are two types of matchmakers: (1) official matchmakers, who

and groom's parents. We also assume that matching between brides and grooms in the marriage market is one-to-one²⁵. Next, we define the quality attributes of brides and grooms, respectively. In particular, we posit two quality attributes – family socioeconomic status θ and ability γ , where $\theta, \gamma \in \{h, l\}$, and h > l > 0. We assume that the ability distribution is independent of class and gender, and the proportion of individuals with high ability h revealed by the exams is p. We further assume that $p < \mu$, reflecting the fact that the examination is highly selective and elite recruitment through the examination system is not perfect²⁶.

When parents seek marriages for their daughters, the quality of a man q_i is the weighted average of his family status θ_j and ability γ_j , where the relative weight for ability of the groom is α^G , so that $q_i = \alpha^G \gamma_i + (1 - \alpha^G)\theta_i$. The quality of a women q_i , on the other hand, is determined by her family status, ability as well as her labor value affected by footbinding, thus $q_i = \alpha^B \gamma_i + (1 - \alpha^B)\theta_i + \beta_{\tau_i}(\mathbb{I}_{FB})l$, where α^B is the relative weight on ability for brides, l is women's labor value, and $\beta_{\tau_i}(\mathbb{I}_{FB})$ captures the return to women's labor as a function of foot-binding as well as the husband's social class τ_i . In particular, we assume that $\beta_u = 0$, or that women who marry upper class men do not play a laborious role in household production. However, for lower class families, since foot-binding impedes labor, we assume that $\beta_w|_{\mathbb{I}_{FB}=0}=\beta_0>\beta_w|_{\mathbb{I}_{FB}=1}=\beta_1>0$, or that women with natural feet have greater labor value than those with bound feet. In the equation for women's quality, we also assume women's meritocracy parameter $\alpha^B = 0$, since the return to women's intellectual ability is almost zero for most of the historical period we consider (women cannot be officials or be involved in commerce). Combining the two sides, for bride i marrying groom j, the matching value of their marriage is $v(q_i, q_i)$. Since α^B is zero in the following analysis, we suppress the notation of α^G to be α , so the matching value function can be simplified to be $v(q_i, q_j) = v(\theta_i + \beta_{\tau_i}(\mathbb{I}_{FB}) l, \ \alpha \gamma_j + (1 - \alpha)\theta_j).$

help facilitating matching in the marriage market, taking records of local marriage market matching, and dealing with law suits regarding women and marriage; (2) private matchmakers, with a focus on marriage market matching only and helping to proceed marital procedures.

²⁵This is motivated by China's historical marriage patterns, which are monogamous with multiple concubines. This paper focuses on the market for wives, not that of concubines. The marriage markets for wives and concubines are separated and very different. To marry a wife, the groom's family needs to go through formal marital procedures (Three Letters and Six Etiquettes, San Shu Liu Li). A wife has well established economic and legal status and cannot be easily divorced. For concubines, the marital procedures are often informal, and they are often bought from brothel and other places. Concubines have low economic and legal status compared with wives and can be easily dismissed or traded.

²⁶This assumption of talent composition does not affect our baseline analysis, yet only changes the marriage market competition pattern when there's a high degree of meritocracy (when $\alpha > \frac{1}{2}$), which case is presented in Appendix 9.2.

Taking the benefits and costs of foot-binding for both grooms and brides into account, the total marriage benefit for each side in the market is the following:

Bride:
$$V_B\left(q_i, q_j, \mathbb{I}_{FB,i}\right) = v\left(\theta_i + \beta_{\tau_j}\left(\mathbb{I}_{FB,i}\right)l, \ \alpha\gamma_j + (1-\alpha)\theta_j\right) - \mathbb{I}_{FB,i}C$$
; and

Groom:
$$V_G(q_i, q_j, \mathbb{I}_{FB,i}) = v\left(\theta_i + \beta_{\tau_j}\left(\mathbb{I}_{FB,i}\right)l, \ \alpha\gamma_j + (1-\alpha)\theta_j\right) + \mathbb{I}_{FB,i}B.$$

Foot-binding creates disutility for brides (denoted by C) and affects their labor value; on the other hand, it serves as a lump-sum utility transfer to grooms (denoted by B).

Disutility of Foot-binding. The bride's family has the option to bind their daughter's feet for better marriage opportunities²⁷. Foot-binding generates a uniform benefit B > 0 to the grooms, and induces disutility C > 0 to the bride. We assume $B \le C$ to reflect the extremely painful and unnatural process of foot-binding, and it is socially costly. Often mothers and grandmothers bound the feet of their daughters/granddaughters, so the process itself is not economically costly and no medical specialists are needed²⁸. Thus, both rich and poor households could afford the cost of the "surgery" of foot-binding²⁹.

Equilibrium. In our model, an equilibrium specifies a matching of the whole population and foot-binding choices of every woman, where three conditions are satisfied: (1) feasibility: all candidates in the marriage pool are paired; (2) optimal foot-binding: no bride will be better off in expected marital payoffs by altering her foot-binding choice; and (3) no blocking pairs: no one in a pair has an incentive to find a better partner who also prefers the new pair. We only consider pure strategy equilibrium, because foot-binding is a childhood commitment and cannot be altered upon marriage. Formally, we have the following definition and assumptions:

 $^{^{27}}$ There are, of course, other options to seek for marriage opportunities, such as dowry. Here we assume away dowry to highlight the role of foot-binding and its evolution over the years. An extension of how dowry fits in the model is provided in Section 5.

²⁸Essential tools for foot-binding includes long and sliced cloth, scissors, sewing kit and a couple of shoes with decreasing size.

²⁹One may also take the girl's family labor into consideration in this framework. In particular, as girls could start work from around age 10 until getting married at age 16, their labor value is considerable. Incorporating this into our model, we consider that foot-binding not only induces disutility to girls, but also brings some utility flow by increasing girl's labor, denoted as T. Here T is the difference in the girl's labor value with and without foot-binding. Bossen and Gates (2017) claimed T to be positive, since handicraft work is very boring and it's hard to have children sitting all day working. In that case, the bride's marriage benefit function would be: $V_B(q_i, q_j, \mathbb{I}_{FB,i}) = v\left(\theta_i + \beta_{\tau_j}\left(\mathbb{I}_{FB,i}\right)l, (1-\alpha)\theta_j + \alpha\gamma_j\right) - \mathbb{I}_{FB,i}(C-T)$. Thus a positive T could offset the disutility of foot-binding to some extent, making the adoption of foot-binding in lower-class families more likely.

Definition. An equilibrium in the game is a matching $M := \{(i,j)\}_{i \in I, j \in J}$ and a set of foot-binding choices $\{\mathbb{I}_{FB,i}\}_{i \in I}$ such that:

- 1. $\forall i \in I, \ \exists j \in J \ s.t. \ (i,j) \in M$; Symmetrically, $\forall j' \in J, \ \exists i' \in I \ s.t. \ (i',j') \in M$.
- 2. Given $\left\{\mathbb{I}_{FB,k}\right\}_{k\neq i}$, $\forall i\in I$, $\mathbb{E}_{j\in J}\left[V_{B}\left(q_{i},q_{j},\mathbb{I}_{FB,i}\right)\right]\geq\mathbb{E}_{j'\in J}\left[V_{B}\left(q_{i},q_{j'},1-\mathbb{I}_{FB,i}\right)\right]$.
- 3. Given $\{\mathbb{I}_{FB,i}\}_{i\in I}$, there does not exist $(i,j),(i',j')\in M$, such that $V_B\left(q_{i'},q_j,\mathbb{I}_{FB,i'}\right)\geq V_B\left(q_{i'},q_{j'},\mathbb{I}_{FB,i'}\right), V_G\left(q_{i'},q_j,\mathbb{I}_{FB,i'}\right)\geq V_G\left(q_i,q_j,\mathbb{I}_{FB,i}\right)$

To make the analysis more tractable, we make the following assumptions on the matching value function v and foot-binding disutility C:

Assumption 1. (Reservation utility). Getting married always dominates staying single. The marriage benefits to brides and grooms are both larger than their reservation utilities, $V_B > V_B$ and $V_G > V_G$, where $V_B > 0$ and $V_G > 0$.

Assumption 2. (Complementarity). v is continuous, convex and differentiable. In particular, $v_1, v_2, v_{11}, v_{22}, v_{12}, v_{21} > 0$. That is, v is increasing in both arguments and v exhibits complementarity, so that for $q_1 \neq q_2$, $v(q_1, q_1) + v(q_2, q_2) > v(q_1, q_2) + v(q_2, q_1)$. Viewing men and women symmetrically, we also have $v(q_1, q_2) = v(q_2, q_1)$.

Assumption 3. (Marrying up is beneficial). For any $\beta \in \{\beta_0, \beta_1\}$, linear combinations of h and l are greater than women's quality, $xh + (1-x)l > l + \beta l$, $\forall x \in (\underline{x}, 1]$, where $\underline{x} < \frac{1}{2}$.

That is, judging from the matching value, the gap between h and l is sufficiently large that a lower class woman's quality is always dominated by the gain of marrying into an upper class groom's family.

Timing. The timing of the game is as follows: (1) at t=1, the bride's family decides whether to bind the feet of their daughter (age 5 to 12). (2) at t=2, the groom's quality is partially realized and matching in the marriage market takes place. The reason for a partial quality realization is based upon the timing of actual marriages. Namely, the marriage age for men is roughly 16-25 (Guo, 2000), an age range during which a man may have just started his career in academics, business or the bureaucratic system, but not yet achieved a well-established position. For instance, taking the lowest level exam (the Licensing exam) and obtaining the degree usually happens between the ages of 16 and 25 (Chang, 1955). So while men's abilities can be revealed by their performance on the exam at an early age, there still exists uncertainly over men's ability at the time of marriage. Therefore, the quality revealing at time t=2 may be partial. With these assumptions, in the next section we analyze the dynamics of foot-binding in different institutional environments.

3.1 Marriage Market with Inherited Status

We start with the baseline case of inherited status, where the relative importance of ability α equals zero and the only determinant of individual quality in the marriage market is family socioeconomic status for both men and women. This case corresponds to the Wei-Jin, Southern and Northern Dynasties (220-589), when commoners had little chance of becoming elite, the elite group was almost closed, and hereditary clans were well-established³⁰. In this case, the marriage market competition is illustrated by Figure 4(a), where both men and women perfectly inherit their family status, without mobility in either class. The equilibrium in this case is characterized by the following proposition.

>>Figure 4. Matching Illustration<<

Proposition 1. In a segregated society with inherited status, there exists an equilibrium where matching is positive assortative in family socioeconomic status and there is no footbinding.

Proof. See Appendix 9.3.

In this case, the quality of both brides and grooms within each class is homogeneous. The matching pattern is endogamy, with upper class brides matched with upper class grooms, and lower class brides matched with lower class grooms. This matching is stable since no one has an incentive to deviate or stay single. Moreover, there is no incentive for brides to make costly premarital investments, because the complementarity of the marriage output function guarantees that the benefit of a lower class bride to marry up is always less than the loss to an upper class groom of marrying down, which means the lower class brides cannot compensate the upper class groom enough to marry her. This reflects the marriage market outcomes well during this period, wherein powerful clans enjoyed hereditary political privilege and were interconnected through marriages. Thus, marriages were highly assortative in terms of family socioeconomic status, marriage across elites and commoners was very rare, and there was no costly beauty investment (foot-binding) by the bridal families (Zhang, 2003).

³⁰During this era, the elite recruitment system was mainly the Nine Ranks-Rectifier System (Miyazaki,1977), in which men were ranked by local rectifiers (*Zhongzheng*, in Chinese) for promotion towards officials, mainly based on their family socioeconomic status and moral conduct. Rectifiers belong to one type of government officials, who are in most cases central officials working in local prefectures.

3.2 Restricted Examination System

In this section, we focus on the Civil Examination System, which is a gender-biased shock in social mobility that significantly transformed the marriage market. Specifically, we model the introduction of the system as a shock to the quality distribution of grooms. In the pre-exam era, the groom's value was fully determined by his family's socioeconomic status, while during the post-exam era, a groom's quality became more heterogeneous as a result of an increased return to ability (that is, $\alpha > 0$). In the post-exam period, grooms could be divided into four quality categories, as illustrated in Figure 4(b): (i) the champions—the upper class grooms with high ability; (ii) the fallen elites—the upper class grooms with low ability; (iii) the new elites—the lower class grooms with high ability and (iv) the new lower class—the lower class grooms with low ability. Fitting into historical contexts, we consider two cases when the exam system has low meritocracy (i.e., the pre-Song period) and high meritocracy (i.e., the post-Song period).

As reviewed in Section 2.2, although the the exam system was a shock to men's mobility, its institutional features significantly changed over time, characterized by the pre-Song and post-Song period. During the pre-Song period, i.e. the Sui (581-618) and Tang (618-907), the size of recruitment via exams was very limited and the degree of meritocracy of the exams was low³¹. The upper class status remained primarily hereditary. We summarize the equilibrium for this scenario in Proposition 2.

Proposition 2. (Restricted Meritocracy). When meritocracy plays a minor role, i.e. there exists a cutoff value $\alpha_1 < \frac{1}{2}$, no one practice foot-binding in either class when $\alpha \le \alpha_1$. The matching equilibrium features positive assortative matching in family socioeconomic status.

Proof. See Appendix 9.3.

Proposition 2 reveals the case when family status is a dominant factor in men's quality, and the benefit of marrying up is still too small for women of both classes compared to the disutility of foot-binding. The proof of this proposition consists of two parts, where the first

³¹At the beginning of the Sui, the emperor decided to recruit three persons from each prefecture each year, judged by the quality of their written articles. By the end of the Sui, the exam system was reshaped to recruit talent based on formal examinations. During the Tang, the exams were better organized with the most popular category of exams being Presented Scholars. During this era, the exam system produced a total of 6617 Presented Scholars. However, less than 10% of officials were from the exam system. Moreover, due to a highly unequal distribution of educational resources, many successful exam-takers were still from elite families.

part describes the nature of the competition and the second part solves for the cut-off value of α . First, due to complementarity between the groom's and the bride's quality, there will be no cross-class marriage, i.e. a lower class woman marrying a champion or a fallen elite, since given a fixed value of α , the lower class bride's marrying up benefit cannot compensate for the loss to the upper class men from marrying down. Therefore, the only competition is within-class. Further, since α determines the quality gap of heterogeneous types of men within class, a larger α results in higher marrying up gain within class. Thus fixing the value of α , complementarity also means that the marrying up gain for upper class women is always higher than that for lower class women, so that the former always have a greater incentive to compete for better marriages. Therefore, given the fixed cost of foot-binding C, there exists a cut-off value α_1 , so that at this point upper class women are indifferent to do foot-binding, where α_1 balances the expected marrying up benefits and the disutility cost, formally $(1-p)[v(h,h)-v(h,(1-\alpha_1)h+\alpha_1l)]=C$.

3.3 Meritocratic Examination System

As described in Section 2, the Song dynasty (960-1127) introduced major changes to the exam system, with respect to the scope of recruitment, procedural fairness and content (Chafee, 1995)³². This meritocratic exam system was fully institutionalized during the Ming (1368-1644) and the Qing (1644-1911) dynasties. The reformed exam system exhibited a much greater degree of meritocracy, together with a revolution in printing technology that lowered book prices (Ebrey, 1990) and an expansion of private schools (e.g. Chen and Kung, 2016). All of these changes contributed to a significant increase in social mobility during the post-Song period, or a higher α in our model. Consequently, the quality of men became increasingly dispersed, which triggered competition among brides in the marriage market. Potential brides sought ways to differentiate themselves from one another, which led to the adoption of foot-binding.

The next step is to understand how large was the shock of the post-Song exam system to the marriage market. As the exam system is hierarchical, we start from the Licensing exams at the entry level, which is usually taken at a young age. According to Chang (1955), among the

 $^{^{32}}$ In terms of recruitment scale, during the three centuries of the Song the system produced almost 100,000 Jinshi, around 15 times the number recruited during the Tang dynasty. In contrast, during the Tang, the number of selected Jinshi each year was around 20-30, and sometimes less than 10. With a high level of meritocracy in elite recruitment, more than 50% of officials during the Song were selected by the exam system.

Literati (Xiucai), 63% were of marriageable ages, i.e. 16-25. So the Literati (degree holders at the county level) were the most relevant group of potential grooms³³. Our question is: among all men of marriageable ages, how large is the proportion of *Literati* degree holders? Based on Chang (1955)'s numbers, we estimate this proportion by making assumptions about population structure. Take Hunan as an example. The county quota for *Literati* degrees in 1812 was 2225 per exam, with exams held twice every three years. Given that its population in 1842 was 20 million, if we take a lower bound assumption of the sex ratio to be 110 men per 100 women and an upper bound to be 130, and assume that the proportion of males aged 16-25 account for 20% of the total male population, we estimate the number of newlyproduced *Literati* as a share of men of marriageable ages, ranging from 0.098% to 0.106%. While this may not seem very large at first glance, we should remember that this calculation is for the flow of marriageable Literati, not the stock of the Literati among the marriageable population. In addition, one could take the entrance exams multiple times and compete in the marriage market for multiple years. Therefore, being "talented" at young ages perceived by the bridal families is in fact more complex. In Table A3, we perform this calculation for all provinces in the Qing dynasty³⁴.

As an illustration of how men's performance in the exams were viewed by bridal families, we can find also evidence in local ballads (Zhang, 2015). For instance, In *Zhangde* county (Henan), a ballad goes: "If you have lotus feet, you could marry a Literati (Xiucai), eating bread with meat; If you have large feet, you would marry a blind man, eating bran with chili". In historical narratives, such phrases were often used to illustrate how mothers persuaded themselves and their daughters to bind their feet. Based upon the collection by Zhang (2015), we present Figure A1 as a summary of the regional distribution of foot-binding related ballads.

Following Proposition 2, we analyze optimal foot-binding under a meritocratic exam system. Proposition 3 formally characterizes the prevalence of foot-binding in the upper class and its diffusion to the lower class. However, since increasing α would naturally flip the ranking of men if $\alpha \geq \frac{1}{2}$, we first consider the foot-binding equilibrium without flipped ranking. To facilitate such an analysis without changing the key economic forces of our model, we introduce an additional assumption:

³³Some very gifted young men even became degree holders of higher level exams (e.g. the *Recommended men* or *Presented scholars*) at a young age.

³⁴In addition to the proportion of *Literati* degree holders among all males of marriageable ages, we also calculate the proportion of gentry (defined as men with degrees higher than the *Literati*) among male population. See Table A3 for more details.

Assumption 4. (Bounded Disutility). Foot-binding disutility has an upper bound, defined by $\overline{C} = (1-p)[v(l, \frac{h+l}{2})) - v(l+\beta_0 l, l)]$, so that women from both classes will take up foot-binding when $\alpha \leq \frac{1}{2}$.

Specifically, Assumption 4 bounds the disutility of foot-binding. The role of this assumption is to ensure both upper and lower class women will take up foot-binding when there is no flipped ranking. Without this assumption (e.g. when $C > \overline{C}$), we will see in Appendix 9.2 that lower class women will never take up foot-binding, which does not fit the historical facts.

Proposition 3. (Prevalence of Foot-binding). Foot-binding is first adopted by upper class women and later by lower class women. In particular, there exists a cutoff α_2 , where $\alpha_1 < \alpha_2 \leq \frac{1}{2}$, when lower class women starts foot-binding. The proportion of foot-binding women of upper class is $r_u = max\{\frac{p}{C}[v(h,h)-v(h,(1-\alpha)h+\alpha l)],\ 1\}$ when $\alpha \in \left[\alpha_1,\frac{1}{2}\right]$, and the proportion of foot-binding women of lower class is $r_w = max\{\frac{p[v(l,(1-\alpha)l+\alpha h)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)},\ 1\}$ when $\alpha \in \left[\alpha_2,\frac{1}{2}\right]$. Further, $0 < r_w \leq r_u \leq 1$.

Proof. See Appendix 9.3.

In Proposition 2, we analyze the case when foot-binding was first adopted by the upper class, or when α is greater than cut-off α_1 . Since complementarity implies that the marrying up gain for lower class women is always less than for upper class women, lower class women take up foot-binding at a higher cut-off value α_2 . In particular, α_2 is the value of α for which the expected marrying up benefits for lower class women equals the disutility of foot-binding, formally $(1-p)[v(l,(1-\alpha_2)l+\alpha_2h)-v(l+\beta_0l,l)]=C$. Intuitively, for any bride of a certain class, her expected payoff when foot-binding, namely marry-up benefits net of foot-binding cost, should equal her expected marriage market payoff without foot-binding. Since $\alpha_1 < \alpha_2$, the upper class women practice foot-binding earlier than the lower class.

For any α in $(\alpha_2, \frac{1}{2}]$, foot-binding was used by women of both classes, and the proportion of foot-binding in the upper class is higher than that of the lower class. Two elements contribute to this result: (i) the complementarity of brides and grooms, so that lower class women always have fewer marrying-up benefits than upper class women given a fixed α ; and (ii) there exists a labor opportunity cost of foot-binding for lower class women, which is not the case for upper class women. In particular, because foot-binding impedes heavy labor and only lower class women play a laborious role in household production, lower class women consider *both* the disutility and the opportunity cost of foot-binding. In equilibrium, the

proportions of women foot-binding in the upper and lower classes are pinned down by the net expected benefit of marrying up by foot-binding equaling the disutility of foot-binding.

Historically, Proposition 3 captures the situation in the post-Song dynasties, when foot-binding first appeared among upper class women then spread to lower class women. The result in Proposition 3 characterizes an asymmetric equilibrium in a symmetric game with many players³⁵. This equilibrium can be thought of as the approximate outcome of the play of a specific symmetric mixed-strategy equilibrium, when each woman chooses foot-binding with a probability close to the fraction of foot-binding women within her class in this asymmetric pure-strategy equilibrium. With large numbers, the ex-ante probability and ex-post frequency are approximately the same (Cabral 1988). Based upon the above proposition, we present comparative statics in the following corollary:

Corollary 1. The foot-binding percentage of upper class women r_u and lower class women r_w is non-decreasing with the proportion of high-ability men p, and non-increasing with the disutility of foot-binding C, when $\alpha \in [\alpha_1, \frac{1}{2}]$ for upper class and $\alpha \in [\alpha_2, \frac{1}{2}]$ for lower class. For lower class women, the foot-binding proportion r_w is non-increasing with β_0 when $\alpha_2 \leq \alpha \leq \frac{1}{2}$.

To test Corollary 1's prediction on p for upper class women, we describe archaeological findings of shoes for bound feet. As Ebrey (1990) documented, the evidence shows that bureaucrats' wives and daughters had all practiced foot-binding during the Song. In particular, three pairs of shoes for bound feet belonging to upper class women from the Song were found in Fuzhou (in Fujian), Quzhou (in Zhejiang) and De'An (in Jiangxi). Figure A3 in appendix illustrates the location of the three sites and the regional distribution of Northern Song degree holders. All three sets of "lotus shoes" were found in regions with a large number of degree holders.

For foot-binding among lower class women, the intuition of Corollary 1 is straightforward – foot-binding is more intense where non-sedentary labor is more valuable (high β_0)³⁶. This result can help us to understand the regional variation in foot-binding practice among lower

³⁵In the sense that if we swap players, their payoffs are not affected.

 $^{^{36}}$ For lower class women, the comparative statics of foot-binding proportion r_w in terms of β_1 is less clear. With greater value of sedentary labor (greater β_1), women face lower opportunity cost of foot-binding, yet they also have lower incentive to marry up since now their own labor value is higher. So the direction of the effect of β_1 on r_w depends on the marrying up benefits compared with disutility term C, that when the marrying up benefits is greater than disutility, higher β_1 would lead to more foot-binding; yet when marrying up benefits is less than disutility, higher β_1 would lead to less foot-binding among lower class women.

class women. In Section 4, we analyze this comparative statics in detail using historical data that captures regional variation in β and p.

Historically, the magnitude of α during the Post-Song period should be less than half for two reasons. First, at the timing of marriage market matching, potential grooms are mostly taking the entry level exams at the county level, which degree can only guarantee a lowest level of gentry class, especially compared to the higher degrees obtained at older ages (e.g. Recommended Men or Presented Scholars). Therefore, a degree of Literati can hardly dominant the importance of family background at this stage, so that most marriage market matching are still taking place within social class, though the competition within class getting intensified (Chen, 1936). Second, the de facto probability of being successful in higher rankings of the exams was different among the upper and lower class men. In a landmark study of this literature, Ho (1962) demonstrates the patterns of social mobility associated with the exam system in the Ming and the Qing, showing that the proportion of degree holders from commoner's family background was below 50% for most of the time during the two dynasties³⁷. Recent quantitative findings are largely in line with Ho's perspective (e.g. Kung and Jiang, 2015, Shiue, 2017), that the exam system increased social mobility but with noticeable heterogeneity by social status, that the higher status families had advantages over lower status families in succeeding in exams. For theoretical interest and to obtain a full picture of foot-binding dynamics with high α , we examine the case when $\alpha > \frac{1}{2}$ so that the ranking of groom's quality is flipped in Appendix 9.2.

3.4 The Decline of Foot-binding

Following the logic of the above model, we characterize the decline of foot-binding as being the consequence of two forces: (1) the decreasing benefit of foot-binding in the marriage market; and (2) the increasing opportunity cost of foot-binding in the labor market.

First, after lasting for more than a thousand years, the gender-biased exam system collapsed in 1905. After the Opium Wars in the mid-19th century, Christian missionaries spread God's message in China. An important part of the missionary work was the establishment of girl's schools, mostly in coastal cities. During the Republican years, girls had greater opportunity to attend school. The increasing equality of educational opportunities promoted women's

³⁷This finding applies to both Presented Scholars (*Jinshi*) and Recommended men (*Juren*). Appendix Figure A3 and A4 plot out the original findings of Ho (1962, Table 9 and 11). In addition to Ho (1962)'s perspective, other historians argue that Ho overestimated the degree of mobility to some degree.

upward mobility, and women's quality dispersion began to catch up with that of men. In this case, the payoff of foot-binding as a costly beauty investment decreased. In our model, this process can be characterized as α^G and α^B becoming more equal over time.

Another economic force driving Chinese women out of foot-binding was the modern industrialization process. Starting in the late Qing, industrialization imposed transport infrastructure, and more integrated markets, which deeply altered the market structure for textile production (Bossen and Gates, 2017)³⁸. First, the opportunity cost of foot-binding increased, because women now had to leave their homes to work in distant factories, increasing the value of β_0 ³⁹. In addition, factory textile products gradually replaced handicrafts decreasing β_1 . Overall, the social and economic changes led to a return to the initial situation, in that men's and women's quality distributions became more equal over time. Under these circumstances, foot-binding is no longer a desired tool to compete in the marriage market, as its benefits shrank while its cost increased significantly.

3.5 Summary

To summarize the above theoretical results, we present Figure 5 which shows how foot-binding prevalence changes with the degree of meritocracy (α) in the exams based upon Propositions 1 to 3, as well as Proposition 7 in Appendix 9.2.

When α is less than α_1 , there's no incentive for foot-binding since the benefit fails to cover the fixed cost. When α reaches a cutoff (α_1) , upper class women start foot-binding, and the proportion of foot-binding women among the upper-class jumps to a certain level. When α reaches a cutoff $(\alpha_2 \leq \frac{1}{2})$, lower class women start to take up foot-binding, and the proportion of foot-binding of upper class women is higher than that of lower class women.

³⁸Bossen and Gates (2017) conducted intensive interviews of 1800 rural elderly women in 27 villages (9 provinces), recording their foot-binding history (if any), personal life history, economic activities, and village characteristics. Combining these with data on local geographical, transportation, industrialization and economic development from a wide variety of sources, they provide evidence that the demise of foot-binding is closely related to the development of modern firms and the more integrated market that gradually replaced traditional household handicraft production.

³⁹Hill Gates interviewed women in three counties in Fujian province, and found distinguishable patterns of foot-binding varying by time and locality. She writes, "As machine-woven cloth began to flood the market in the 1930s, even married women sought other sources of income when these were available. Most alternatives required more mobility, demanded more outdoor labor, and allowed more comfort than the pre-industrial labor regime. By the early 1930s, very few Tong'an girls were being bound".

When $\alpha > \frac{1}{2}$ and there is flip in the ranking of new elites and fallen elites, women from both classes practice foot-binding, but with the two differences: (1) there is a discontinuity in the proportion of foot-binding upper class women, since now the average quality of men that upper class women are competing for decreases; and (2) there exists a kink in the proportion of foot-binding among lower class women, since now marrying the fallen elites is less attractive for lower class women than before.

One may also wonder what proportion of women practice foot-binding as a function of disutility C, for a fixed value of α . In Figure 6, we plot the proportion of foot-binding women in both upper and lower classes as a function of C, given $\alpha \in [\alpha_2, \frac{1}{2}]$ when both classes adopt foot-binding.

Figure 6 illustrates several interesting points. First, when the disutility of foot-binding C is below the expected marrying up benefits net of opportunity cost for lower class women, i.e. $C \leq p\Delta V_l - \delta$, women of both classes practice foot-binding since the cost is low and is fully offset by the benefits. Here the marrying up benefits ΔV_l is $v(l, (1-\alpha)l + \alpha h) - v(l + \beta_0 l, l)$, and labor cost δ is $v(l + \beta_0 l, l) - v(l + \beta_1 l, l)$. Second, when C is in the window of $(p\Delta V_l - \delta, p\Delta V_u)$, where ΔV_u is $v(h, h) - v(h, (1-\alpha)h + \alpha l)$, fewer lower class women take up foot-binding as the disutility of foot-binding increases. Moreover, when C is larger than $p\Delta V_u$, fewer upper class women take up foot-binding as disutility increases. Finally, when C is too large (i.e. larger than ΔV_u), neither class will practice foot-binding. However, considering Assumption 4 that imposes an upper bound of C, we have a further restriction that $C \leq \bar{C} = (1-p)[v(l,\frac{h+l}{2})-v(l+\beta_0 l,l)]$, so that C will sit in the interval of $[p\Delta V_l - \delta, \Delta V_l]$ when α is approaching $\frac{1}{2}$, thus the proportion of foot-binding women of upper class will be in the interval of (p, 1].

4 Empirical Analysis

4.1 Data: Cross-sectional Variation

The historical facts presented above with regard to changes in social mobility and the marriage market provide us validation for the key theoretical predictions. To further examine our theory, in this section we analyze cross-sectional county data on foot-binding from the Republican archives to empirically assess the comparative statics presented in Corollary 1. Specifically, we focus on the regional determinants of foot-binding, using exogeneous variation in the benefits of marrying up (corresponding to p in our theory) and in the opportunity cost of labor (corresponding to β in our theory).

Foot-binding. Data on the prevalence of foot-binding at the county level come from surveys conducted by the Republican government in the early 1930s. In 1928, the Republican government officially prohibited foot-binding nationwide and later conducted a survey to investigate progress on its prohibition (Yang, 2003). This survey was conducted by the Ministry of the Interior, Republic of China, with records kept in the Second Historical Archives of China (SHAC) in Nanjing. Our data covers counties in four provinces, namely Shandong, Chahaer (now part of Hebei province), Hunan and Yunnan.

The main question of interest is the following: "Before the implementation of the prohibition law, to what extent did women practice foot-binding?" For instance, in Gaoyuan county of Shandong province, the answer is "seven or eight women out of ten bound their feet". In Yongshun county of Hunan province, the answer is "women always work along with men, so there was no foot-binding custom". We code the answers that clearly describe intensive foot-binding practice as "High Prevalence", taking a value of one, and code those that describe foot-binding as being rare or nonexistent as "Low Prevalence", taking a value of zero. With respect to cases with vague information on foot-binding intensity, we code those cases to be "ambiguous". Using the above coding system, we present the regional distribution of foot-binding prevalence in the counties in the archives in Figure 7.

>>Figure 7. Foot-binding Regional Prevalence: Sample Counties<<

As shown in Figure 7, the distribution of foot-binding prevalence is very similar to that described in the styled facts supported by numerous qualitative sources described in Section 2 and Appendix 9.1, that Northern China had higher prevalence of foot-binding than Southern China.

Exam Quota. To capture the variation in the size of elite recruitment (p), and therefore the benefits of marrying up related to the exam system, we use one institutional feature of the exam system that created cross-sectional variation during the Ming and Qing: the quota allocation of the exam degree holders. Specifically, the quota system was an institution that regulated and balanced elite recruitment through the exams across regions, and allocation was hierarchical at the level of *Literati*, *Recommended Man* and *Presented Scholar*. In

particular, by controlling the fundamental entry-level exam quotas, the imperial government could de facto control the size of the local gentry, specifically those with exam degrees above the Literati level (Chang, 1955). The variation in exam quota was relatively stable across the Ming and Qing⁴⁰, and the main rational for quota allocation at the county level included the following factors: population, tax obligations, and historical talent distribution⁴¹ (Chang, 1955, Shang, 2004, Liang and Zhang, 2013). In our analysis, we use the exam quota at the county level during the late Qing (Kun et al., 1899) as a proxy for the benefits for women to marry up.

Women's Labor. Following our theory, the value of both sedentary and non-sedentary labor determines the opportunity cost of foot-binding for lower class women. Non-sedentary labor, namely agricultural work on the farmland, is much more sensitive to foot-binding than sedentary labor, which is mostly household handicrafts that are less affected by foot-binding. Using the regional variation of crops and handicraft sources, we capture the value of non-sedentary labor (β_0) and the value of sedentary labor (β_1) by comparing different agricultural regimes in China.

For cereal crops cultivation, there existed two agricultural regimes in China: rice versus wheat. In particular, in Southern China (Yangtze River and Pearl River Deltas), wetland rice was often the dominant cereal crop; while in Northern China (the Yellow River Basin), wheat, millet, maize and sorghum were the main cereals. As has been documented, cultivating rice requires highly intensive skilled labor (especially with double cropping, e.g. Bray 1994), where women's labor serves as key input and they are often considered to have a comparative advantage in the field because they are more careful and have more dexterous hands, especially valuable in rice cultivation (e.g. transplant rice seedlings). Therefore, in rice regions, women often worked alongside their husbands in the fields and paddies. To illustrate features of crops, we use John Buck's 1937 land utilization survey as a summary in Table 1.

>>Table 1: Number of Days of Man-labor Required per Crop Hectare for Major Crops<

Table 1 presents number of days of man-labor required per crop hectare for major crops. The average number of labor days required for crops in the Wheat Regime was around 100,

⁴⁰The Qing government adopted the baseline allocation scheme from the Ming government, yet for some regions, the quota allocation experienced changes in respond to conflicts (e.g. the Taiping Rebellion), exam performances or contribution/donation to the imperial government. But the regional distribution was overall stable during this period.

⁴¹Bai and Jia (2016) investigates the quota allocation at the prefecture level, using the number of small rivers (given river lengths) and exam performance before the quota system as instrument variables.

while the average number of labor days required in the rice region was almost double⁴². Empirically, we collect agricultural suitability and other county data from multiple sources. For the geographic distribution of crop suitability, we used the FAO's GAEZ (Global Agro-Ecological Zones), including rice, wheat, flax and cotton⁴³. Mapping this fact to our model, the β_0 of rice-growing areas is greater than that of non-rice-growing regions. Figure A6 illustrate the regional distribution of suitability index of rice and wheat. In our analysis, we use the relative agricultural suitability index of rice to wheat to proxy non-sedentary labor-intensive agricultural activity.

Next, we consider the value of sedentary labor – women's household handicraft work. The composition of household handicraft products had great variation across regions, depending on the endowment of resources. For instance, handicraft produced in Southwestern China included the spinning and weaving of cotton, hemp, silk, straw hats, mats and shoes, plaited bamboo baskets and pot covers, twisted fiber ropes, woven saddle pads and fish nets, dyed cloth, and so on (Bossen and Gates, 2017). But in general, the dominant handicraft source fibers were cotton and flax, where cotton outputs were in particular with higher economic value than flax outputs (e.g. Xue, 2017). In the empirical analysis, we use cotton and flax suitability as proxy for sedentary labor activity to test our prediction on the opportunity cost of foot-binding (varying β_1). Figure A7 illustrate the regional distribution of suitability index of cotton and flax.

Other Variables. In addition to the above variables, we also collect other variables as controls in our analysis. For county population and sex ratio in the early 1930s, we collect such information from Yearbook of Domestic Affairs of Republican China in 1931^{44} . To capture the social influence on foot-binding in late-Qing China, we use the records of Christian missionaries in China during the period of 1918-1921 (Stauffer, 1922) as proxies, including the number of churches, vicars, and disciples per 10,000 people. In addition, considering that the newly established firms in the late 19^{th} century and early 20^{th} century could influence foot-binding practice, we use numbers of modern firms in the prefecture level as a proxy (Chang, 1987,1989). Also to reflect the impact of ethnic minority, we use the intensity of the Mongol migration in the 13^{th} century, constructed based on Wu and Cao (1997). Finally, to control for the influence of Republican government on anti-foot-binding, we calculate the great circle

 $^{^{42}}$ It should be noted that in a double cropping system of rice, early and late rice should be counted as a whole.

⁴³The suitability index of a certain crop is estimated based on a model, which has been applied considering the average climate of baseline period 1961-1990 reflecting suitability levels and distributions within grid cells.

⁴⁴The population and sex ratio data for earlier periods on county level are not available.

distance from each county to Nanjing (which was capital of the Republican government).

4.2 Empirical Strategy

In this section, we first discuss the feature of our archival sample, and then present the empirical strategy to identify the impact of marriage market and labor incentives on foot-binding. As discussed in section 4.1, our data on foot-binding is constructed from archives conducted by the Republican government covering four provinces, which could induce two potential concerns: first, some counties are missing and not covered by the archive⁴⁵; second, among the counties covered by the archive, some reported clear information on foot-binding, yet some with ambiguous information. For our analysis, sample selection would only be an issue if the selection were based on some unobserved characteristics that are correlated with determinants of foot-binding at county level. As a robustness check for the selection issue of the first type, we conduct a balance check for out-of-archive and in-archive counties by province in Table 2. As shown in Table 2, the two types of counties do not display significant differences in terms of relative difference suitability of rice to wheat, cotton and flax suitability, and distance to Nanjing for counties of Hunan province. For Shandong, Chahaer and Yunnan, there is no clear pattern in common for selection on the in-archive counties.

For the second type of selection issue, we label counties with ambiguous information on foot-binding as in the "ambiguous category", and conduct t-test for key variables. Table 3 shows that none of the variables show significant differences in mean across the two groups of counties.

In the next step, we use those counties with clear information on foot-binding as main analytical sample, and descriptive statistics of key variables on the province and county level are shown in Table 4.

 $^{^{45}}$ This issue may stem from the wars and disturbances during the early 20^{th} century, that the archives of some counties are missing, or they didn't conduct the survey in the first place. But unfortunately, we cannot distinguish such difference based on the sources we have.

As shown in Table 4, counties with higher prevalence in foot-binding on average have higher suitability for cotton and flax, more entry-level exam quotas, and less relative suitability of rice to wheat. Given those statistics are informative, we formally analyze the determinants of foot-binding using the following regression specification:

$$Footbinding_c = \alpha + \beta Exam \ Quota_c + \gamma Suitability_c + \mu X_c + \epsilon_c$$

Here foot-binding prevalence in county c is a dummy variable, taking one for high prevalence, and zero for low or no prevalence. Our key explanatory variables are the difference in the suitability of rice compared to wheat, cotton and flax, as well as the size of the county exam quota, where all are standardized to compare the magnitudes of their effects. X_c is a vector of control variables, including the number of modern firms, international influence (proxy by Christianity activity) and geographic characteristics.

Next, we consider identifying the effects of exam quota and crop suitability on foot-binding. For crop suitability, these indexes are exogenous, as they are compound indicators that capture exogenous and predetermined geographical and ecological factors. For exam quota, the major identification challenge here is whether the factors determined quota allocation at county level are orthogonal to determinants of foot-binding. As discussed in Section 2.2, the assignment of quota across regions is not random but based on factors including population, tax obligations, historical talent distribution, and political stability⁴⁶. Therefore, it is possible that the quota may be confounded with unobservables that capturing historical talent distribution or political stability, which may affect foot-binding at the same time. To understand the endogeneity issue on exam quota, we use the following strategies: (1) we progressively add county level socioeconomic variables as a richer set of control, including province and prefecture fixed effects in different specifications; (2) we test for selection on unobservables for quota allocation at the county level using Oster (2016)'s approach.

⁴⁶For instance, the exam quotas are more generously given to the remote and border provinces, e.g. Yunnan and Guizhou, for the sake of maintaining social stability of these regions. Investigating the impact of abolition of the exam system on social uprisings at different locality, Bai and Jia (2016) uses the variation in prefecture level exam quotas and finds that a higher density of quota were associated with a higher probability of revolution participation after the abolition.

4.3 Empirical Results

Table 5 shows the baseline results using the above empirical specification, where robust standard errors are reported. As in Column 1, one standard deviation's increase in the relative suitability of rice to wheat leads to 9.4 percentage points lower probability in foot-binding prevalence, and the impact of cotton suitability as a proxy of local handicraft intensity has a slightly larger magnitude. For county exam quota, one standard deviation's increase in the quota leads to 6 percentage point's higher incidence in foot-binding prevalence. Columns 2, 3, 4, 5 progressively add variables capturing Christianity activities, socioeconomic variables (size of modern firms and historical Mongolion migration intensity), geographical distance to Nanjing and province fixed effects. Most are not statistically significantly different from zero. At the same time, the impacts of rice relative suitability, cotton suitability and county exam quota are highly stable. To sum up, this set of results supports the comparative statics of foot-binding prevalence with respect to both the proportion of high ability men and the opportunity cost of foot-binding in labor.

>> Table 5: Crop Suitability, Exam Quotas and Foot-binding Prevalence <<

Next, considering the assignment of county exam quota may be endogeneous to foot-binding by being confounded with unobserved socio-economic characteristics, we use prefecture fixed effect and also use Oster (2016)'s approach to evaluate the degree of selection on unobservables compare to observables to alleviate this concern. The results are presented in Table 6.

>> Table 6: Crop Suitability, Exam Quotas and Foot-binding: Prefecture Fixed Effects <<

In Column 1 of Table 6, we use the full sample and the only explanatory variables are county exam quota and county population, where a one standard deviation's increase in the quota increases foot-binding prevalence by 7 percentage points. In Columns 2, we add prefecture fixed effects, and find the magnitude of the exam quota's effect staying the same, while the R^2 increases a lot compared with Column 1. In Column 3, we further add the suitability variables, yet find no statistical significance of their impacts. This is not surprising, however, since the variation in suitability within prefecture is small given the relatively high homogeneity in terms of geographical conditions. In Column 4, we add other control variables, and find the coefficient of quota still very stable. In lower panel of Table 6, we practice

Oster (2016)'s test for selection on unobservables. The assumption of the test is we have a proportional selection relationship of selection on observables and unobservables, so selection on observables is informative about selection on unobservables. Let the proportion of selection be δ and label our key coefficient (of the exam quota) be β , and R_{max} be the R^2 from a hypothetical regression that includes both observables and unobservables, the testing statistics are presented for Columns 2, 3 and 4. For Column 2, the δ for $\beta = 0$ and $R_{max} = 1$ is 2.5, which passes the common heuristic value of 1. The bias-adjusted β , given $\delta = 1$ and $R_{max} = 1$ is 0.07, which is similar to our original estimates. For Column 3, we have similar magnitude and direction of selection on observables and unobservables; for Column 4, the testing results are very similar. Overall, the analysis in Table 6 adds towards our analysis in Table 5 that county exam quotas play a positive role in determining foot-binding prevalence, and presents the degree to which selection in county exam quota as a concern is limited.

4.4 A Spatial Discontinuity Case

We next extend the empirical analysis by using a case of spatial discontinuity. As Figure 8 illustrates, Jiangsu Province lies at the border on the Northern and Southern China, along the Huai River. Thus, it is a province with both wheat and rice cultivation. The northern part of Jiangsu mainly cultivates wheat, while rice dominates its southern part. With respect to foot-binding practice, according to the provincial gazetteer of Jiangsu published in the early Republican years, except for urban areas, the Northern Huai River region had the highest prevalence of foot-binding, and the Yangtze River region had very little foot-binding. Women living north of the Huai River worked within the household, and women living south of the river worked in farming. The case of Jiangsu adds a piece of evidence to our understanding of female labor specialization and foot-binding prevalence.

To sum up, our major finding in Section 4 is that greater relative suitability of rice to wheat was associated with lower prevalence of foot-binding. On the other hand, a higher suitability for cotton was correlated with a higher popularity of foot-binding. These findings are robust across model specifications and different sub-samples, as well as to the inclusion of a rich set of county-level socioeconomic and geographic variables.

5 Extensions

In this section we provide three extensions of our baseline model. First, we assume a finer hierarchy of exam results rather than a binary structure, where individual intellectual capability can take three values, i.e. $\gamma \in \{High, Medium, Low\}$. Second, we allow the choice of foot-binding to be continuous rather than binary. Finally, we introduce multiple price tools for premarital investment: dowry and foot-binding. In particular, we examine the interaction of these two competition tools and how this can change the strategy of bridal families in equilibrium.

5.1 Hierarchical Structure of the Exam System

Previously, we assume a binary exam structure, i.e. $\gamma \in \{h,l\}$, corresponding to high and low ability captured by the exams. Here in the first extension, we replace the binary structure with multi-level achievement structure, i.e. $\gamma \in \{H, M, L\}$, representing three levels of ability that reflecting high, medium and low rankings in the Licensing exam. To capture a pyramid structure of talents, we assume the probability of getting a high score H is p (the same as before), that of getting a medium score M is q, and $0 . Further, we focus on the case when <math>\alpha < \frac{1}{2}$ to fix the ranking of grooms and present the analysis comparable to Section 3.3. With the new assumptions, the quality distribution of grooms are described in Table 7, with labels A-F provided for each group of grooms:

>>Table 7: Groom's Quality Distribution with Hierarchical Ranking<<

Quality Ranking	l	$(1-\alpha)l + \alpha m$	$(1-\alpha)l + \alpha h$	$(1-\alpha)h + \alpha l$	$(1-\alpha)h + \alpha m$	h
Category	F	E	D	C	В	A
Male Population	$(1-\mu)(1-p-q)$	$(1-\mu)q$	$(1-\mu) p$	$\mu \left(1 - p - q \right)$	μq	μp

Proposition 4. With hierarchical exam rankings, the quality of grooms is further spread. Women of both classes will practice foot-binding, and it is more intensive among upper class. Moreover, the prevalence of foot-binding is higher than the benchmark case for women of both classes.

Proof. See Appendix 9.3.

When $\alpha < \frac{1}{2}$, upper class women seek for marriages of grooms with ranking A and B within class, and lower class women seek for those with ranking D and E within class. Similar to Proposition 3, here the no-cross-class marriage is a result of the complementarity between brides and grooms. To pin down the proportions of foot-binding women in each class, we have reasoning process similar to that of Proposition 3. Intuitively, in equilibrium upper class brides bind their feet to the extent that the expected payoff of marrying-up net of foot-binding costs equals to non-foot-binding marriage outcomes. For lower class brides, the cost of foot-binding for lower class brides includes both disutility and the opportunity cost of labor, driving the proportion of foot-binding less than that from upper-class brides together with the complementarity. In the end, upper class women have higher prevalence in footbinding of lower class women. Comparing the prevalence of Proposition 4 and Proposition 3, we also have that a higher proportion of foot-binding women in both classes in this extension, which is directly driven by the additional marrying up benefits of the M-type talent men within class. Given that the de facto talent measures of grooms could be a richer set (considering the number of their trials in taking the exam, their ranking in the exam), the impact of the exams on marriage market is actually much larger than the impact calculated purely based on the number of degree holders, so our quantification in Section 3.3 provides a lower bound on the marriage market effects of the exams.

5.2 Continuous Choice of Foot-binding

In this section we analyze adoption of foot-binding when the choice is continuous instead of binary. As before, we assume that foot-binding is costly and there are only two ability levels revealed by the exam (H and L). The key difference is we allow the disutility of foot-binding to be a continuous function of binding intensity, c(b), with the following assumption:

Assumption 5. (Cost of Continuous Foot-binding) (1).
$$c(b) = 0$$
 for $b \in [0, \underline{b})$, $0 < c(b) < c(\overline{b})$ for $b \in [\underline{b}, \overline{b}]$, and $c(b) = c(\overline{b})$ for $b \in (\overline{b}, \infty)$; (2). $c' > 0$, $c'' < 0$ for $b \in [\underline{b}, \overline{b}]$.

The first property says that a small degree of foot-binding, i.e. $b \in [0, \underline{b})$, is not costly until it becomes visible enough. The practice of foot-binding is physically subject to a point when it is physically impossible to further bind the feet, i.e. $b = \overline{b}$. The second property says that the cost of foot-binding increases in binding intensity and is concave. To simplify our analysis here, we assume the benefits of foot-binding B(b) = c(b). Following similar logic, now the counterpart we have of Assumption 4 (bounded disutility) is the following:

Assumption 6. (Bounded Disutility with Continuous Foot-binding). The cost of foot-binding is positive but bounded from above, i.e. $0 < c(\overline{b}) \leq \overline{C}_{cont}$, so that lower class women take up foot-binding when $\alpha \leq \frac{1}{2}$. In particular, \overline{C}_{cont} is defined by $\overline{C}_{cont} = v(l, \frac{h+l}{2}) - v(l+\beta_0 l, l)$.

The return to lower class women's labor value with continuous foot-binding now is defined by the following:

$$\beta = \begin{cases} \beta_0 & if & b < \underline{\mathbf{b}} \\ \beta_1 & if & b \ge \underline{\mathbf{b}} \end{cases}$$

Therefore, following a logic similar to that in Section 3.1 and 3.2, we define following proposition for the case of continuous foot-binding:

Proposition 5. (Continuous Foot-binding). When foot-binding is continuous:

- (1) There's no foot-binding in upper class below cutoff $\tilde{\alpha}_1$, and no foot-binding in lower class below cutoff $\tilde{\alpha}_2$, where $\tilde{\alpha}_1 < \tilde{\alpha}_2$.
- (2) When $\alpha \in [\tilde{\alpha}_1, \frac{1}{2}]$, a population of μp upper class women take up foot-binding with binding intensity b_u ; when $\alpha \in [\tilde{\alpha}_2, \frac{1}{2}]$, $(1-\mu)p$ lower class women take up foot-binding with intensity b_l , where $b_u > b_l$.
- (3) When $\alpha \in (\frac{1}{2}, 1)$, a population of μp upper class women choose binding intensity b_C , $(1 \mu) p$ choose binding intensity b_{NE} , and (μp) choose no foot-binding; for lower class women, $p(1 \mu)$ choose foot-binding with intensity b_{FE} . In particular, $b_C > b_{NE}$ and $b_C > b_{FE}$.

Proof. See Appendix 9.3.

With continuous foot-binding, brides can more precisely differentiate themselves in the marriage market. In the binary model, signal jamming takes place among upper class brides competing for champions and new elites. In the case of continuous foot-binding, upper class brides separate themselves by adopting a higher intensity of foot-binding to compete for champions, and a lower intensity to compete for the new elites. Since the population of champions and new elites combined is smaller than that of upper class brides (i.e. $p < \mu$), Bertrand competition stops when foot-binding intensities completely absorb any marrying-up benefits. This result hinges on the assumption that foot-binding is able to provide large enough benefits to achieve separation, since otherwise brides can pool to maximal binding

intensity, and we are back to the binary case. Therefore, the continuous foot-binding case predicts within-class variation in foot-binding intensity. Historically, this echoes with our last stylized fact, that the higher the social status was the bride, the smaller their feet were.

5.3 Multiple Competition Tools: Dowry and Foot-binding

In this section, we allow for dowry payments in addition to foot-binding to see if the existence of alternative means of competition in the marriage market alters our earlier results. To accommodate full generality, we assume continuous foot-binding. To highlight the interactions between dowry and foot-binding, we assume away the small labor value differences under different levels of foot-binding ($b \ge \underline{b}$), that is, $\beta = 0$. The marriage values for two sides with both dowry and foot-binding are as follows:

Brides' Utility Function:
$$V_B(q_i, q_j, b_{FB,i}, d) = v(\theta_i, \alpha \gamma_j + (1 - \alpha)\theta_j) - c(b_{FB,i}) - u(d_i)$$

Grooms' Utility Function:
$$V_G(q_i, q_j, b_{FB,i}, d) = v(\theta_i, \alpha \gamma_j + (1 - \alpha)\theta_j) + b(b_{FB,i}) + u(d_i)$$

Here we do not include the budget constraint of the bride's family to highlight the substitution between dowry and foot-binding when there are interior solutions. One can easily add in budget constraints, and the qualitative results of the analysis remain unchanged.

Now the bride's family needs to decide both foot-binding intensity $b_{FB,i}$ and the size of the dowry payment d_i . Since dowry is usually offered as a direct monetary transfer, we assume u' > 0, u'' < 0 and u'(0) = 1, that is, a monetary transfer has diminishing marginal returns. Adopt Assumption 5 and 6 in the continuous foot-binding, we have the following proposition:

Proposition 6. (Foot-binding and Dowry). When both dowry and foot-binding are available, we have the following cases:

- (1) There's no foot-binding nor dowry payment in either class when $\alpha = 0$. After a cutoff of $\tilde{\alpha}_1$ for upper class and $\tilde{\alpha}_2$ for lower class, bride's family use dowry $d_{0,u}^*$ and $d_{0,l}^*$ respectively, where $\tilde{\alpha}_1 < \tilde{\alpha}_2$; for given $\alpha \in [\tilde{\alpha}_1, \tilde{\alpha}_2]$, $d_{0,u}^* > d_{0,l}^*$.
- (2) When $\alpha \in [\tilde{\alpha}_1, \frac{1}{2}]$, a population of μp upper class women choose binding intensity b_u^* and dowry size d_u^* ; when $\alpha \in [\tilde{\alpha}_2, \frac{1}{2}]$, $(1 \mu)p$ lower class women choose binding intensity b_l^* and dowry size d_l^* ;

(4) When $\alpha \in (\frac{1}{2}, 1)$, a population of μp upper class women choose binding intensity b_C^* , dowry size d_C^* ; $p(1-\mu)$ choose binding intensity b_{NE}^* , dowry size d_{NE}^* ; and $(\mu - p)$ choose no foot-binding and pay no dowry. For lower class women, $p(1-\mu)$ choose binding intensity b_{FE}^* , dowry size d_{FE}^* . In particular, $b_C^* > b_{NE}^*$, $b_C^* > b_{FE}^*$, $d_C^* > d_{NE}^*$, and $d_C^* > d_{FE}^*$.

Proof. See Appendix 9.3.

The above proposition carries several new insights. First, as foot-binding investment requires a fixed cost, it will be adopted later than dowry payments. In this sense, dowry payments are more flexible than foot-binding. Second, when both tools play a role in marriage competition, both foot-binding intensity and dowry payments will increase with the degree of competition. In fact, this is exactly what we have observed since the Song dynasty, that more and more historical accounts include parent's complains about high dowries required to marry their daughters for both class (e.g. Guo, 2000). Third, being qualitatively consistent with our previous results, upper class women still have greater foot-binding intensity and larger dowries compared to lower class women.

6 Conclusion and Discussion

Economic incentives can deeply influence cultural practices. This paper explains the economic motives of foot-binding in historical China by developing a model of the marriage market that also considers labor opportunity cost and treats foot-binding as a premarital investment. We show that foot-binding is not only a cultural practice that carries aesthetic, erotic and social motives, but also has a deep economic rationale. The key trade-off considered in the foot-binding decision is the following: (1) foot-binding can improve marriage market outcomes for women by making themselves more attractive; and (2) foot-binding has a significant disutility and can have a large labor opportunity cost depending on specific agricultural regimes and household handicraft patterns. We analyze several cases which provide insights into the evolution of foot-binding. In a stratified society where marriage is completely assortative in terms of family status and there is no incentive for foot-binding. The introduction of a gender-biased examination system significantly increased men's mobility and quality dispersion. Historically, this led to the emergence of foot-binding among upper class women. When meritocracy increases further, as happened historically in China, marrying-up benefits continue to increase, and foot-binding diffuses from upper class women to lower class women, exactly the sequence observed in China.

For lower class women, the opportunity cost of foot-binding also plays a significant role in decision-making. In the model, we consider two agricultural regimes. Where women's farmland work dominants (e.g. rice-growing regions), foot-binding is a much less attractive investment for women. In areas with less agricultural labor demand in farming (e.g. the wheat-growing regions) or where women can specialize in household handicraft production which can be done with sedentary labor, foot-binding is more popular. Thus, different agricultural regimes and gender labor specialization translate into differential adoption of foot-binding among women. Based on the predictions of our model, our empirical analysis using county-level from the Republican archives shows that higher suitability of rice relative to wheat and higher suitability for household handicraft are associated with less/more foot-binding among lower class women respectively, and the county exam quota predicts a higher incidence of foot-binding. The results are robust to using different samples and control variables. The empirical findings directly test and support our theoretical predictions, that the prevalence of foot-binding among lower class women increases with the marriage market prospects and decreases with the labor opportunity cost of foot-binding.

Overall, our paper sheds light upon cultural practices from an economic perspective, showing that an equal mobility institution without gender bias, as well as women's active participation in economic activities can be protective against cultural customs that carry high disutility for women. Future work may focus on the dynamic interactions between foot-binding, dowries and sex ratios, and further examine similar cultural practices in our context, i.e. the corsets. We believe the analysis of foot-binding introduced here will yield broader insights for future studies in women's development.

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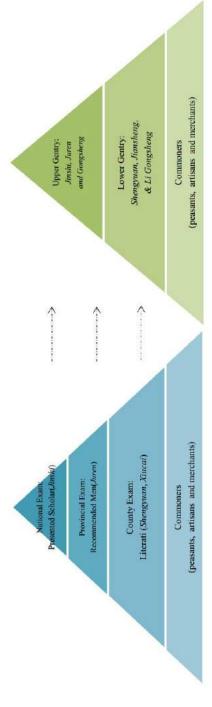
8 Maps, Figures and Tables

Foot-binding Prevalence
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Figure 1: Regional Variation of Foot-binding Prevalence

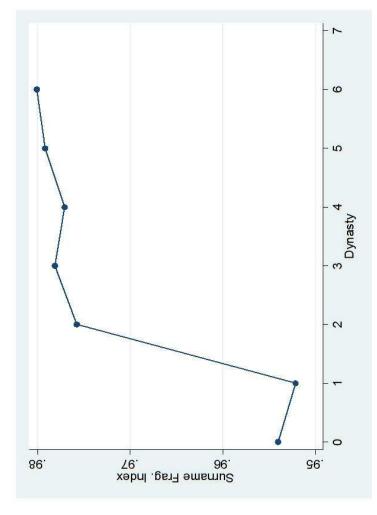
Note: The foot-binding prevalence variable on province level is based on historical qualitative evidence, including Hu(1936); Qian(1969); Xu(1984); Xu(1984); Yian(1976); Yi

Figure 2: The Civil Exam System and the Social Ladder



Scholars (Jinshi), Recommended Men (Juren), and Literati (Shengyuan). Tribute Students (Gongsheng) refer to those who are accepted into the Imperial Academy (Guozijian), and Jiansheng refers to those who purchased the Literati degree without taking actual exams at the entry Note: Illustration based on Chang (1955). Description of degree holders of the exam system is presented in Section 2, including Presented level.

Figure 3: Men's Mobility Trend: Evidence from CBDB



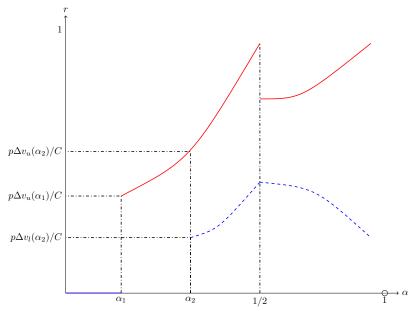
(version: 2015-03-18). We calculate the surname fractionalization index with a sample of male celebrities with single character surnames (a proxy of Han ethnicity). The surname fractionalization index is constructed following Alesina et. al (2003): $Frac_d = 1 - \sum_{i=1}^{N} S_i^2$, where S_i is Dynasty(1271-1368), 4=Ming Dynasty(1368-1644), 5=Qing Dynasty(1636-1911) and 6=Post-Qing Dynasty(1911-1949). Data source: CBDB Note: In the above graph, 0=Qin-Han Dynasty(BC221-220), 1=Sui-Tang Dynasty(581-907), 2=Song Dynasty(960-1127), 3=Yuan the share of surname group i in dynasty d.

Figure 4. Matching Illustration

Pre-Exam Case	kam C	ase	Post-Ex	Post-Exam: α ≤0.5	ı.	Post-	Post-Exam: $\alpha > 0.5$.5
0-0	Men	Women		Men	Women		Men	Women
	h	h	The Champions [μp]		h 	The Champions [μp]	, h	
Upper Class [μ]		: : : : : : : : : : : : : : : : : : : :	The Fallen Elites	(1-a)h+al	111	The New Elites $[(1 - \mu)p]$	(1-a)l+ah 	
	u		[rd - r)#]		 h	i i	$(I-\alpha)h+\alpha I$	 h
	1	$l+\beta_0 l$	The New Elites [(1- µ)p]	(1-a)I+ah	$l(+\beta_0 l)$	The Fallen Elites $[\mu(1-p)]$		l(+β ₀ l)
Lower Class	1 1	::	The New	 1	1 1	The New	1	: :
2	ï	 1+β ₀ 1	Lower Class $[(1-\mu)(1-p)]$: :~	 !(+\beta_0!)	Lower-Class $[(1-\mu)(1-p)]$	1	 l(+β₀l)
	(a)			(p)			(c)	

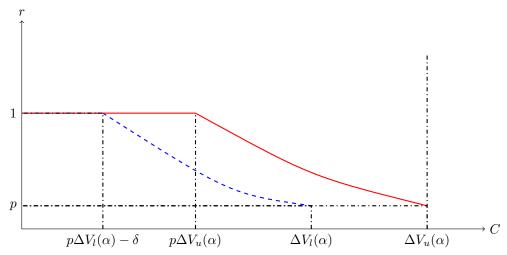
Note: We use brackets for lower class women's quality since their quality depends on their husbands' wealth status. In the case of marrying a wealthy enough husband (with quality measure higher than h), the wife do not have to work in the household. Panel (a) is corresponding to Proposition 1 (the pre-Exam case), Panel (b) refers to Proposition 2 (Restricted Meritocracy) and 3 (Prevalence of Foot-binding), and Panel (c) characterizes Proposition 7 (Flipping Ranking).

Figure 5. Foot-binding Prevalence and Exam Meritocracy α



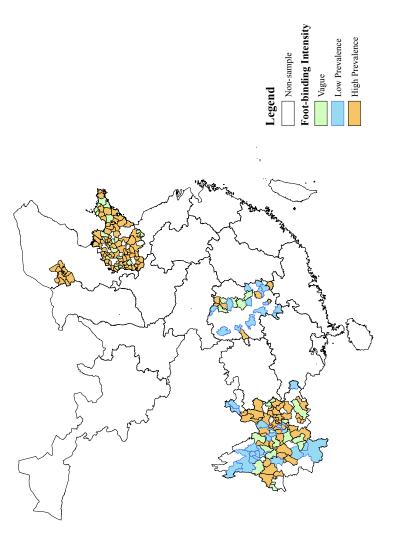
Note: This figure is drawn based on Proposition 1, 2, 3 and 7 (in appendix). The horizontal axis is α , or the relative weight of talent for men determined by the mobility institutions (i.e. the exam system). The vertical axis is the proportion of foot-binding within class. Here $\Delta V_u = v(h,h) - v(h,(1-\alpha)h + \alpha l)$, and $\Delta V_l = v(l,(1-\alpha)l + \alpha h) - v(l+\beta_0 l,l)$. Proposition 1 characterizes the case when $\alpha=0$, Proposition 2 refers to the case when $\alpha\in(0,\alpha_1]$, Proposition 3 captures the case when $\alpha\in(\alpha_1,\frac{1}{2}]$ for upper class women and $\alpha\in(\alpha_2,\frac{1}{2}]$ for lower class women, and Proposition 7 describes the case when $\alpha\in(\frac{1}{2},1)$. Here the solid line in red denotes upper class women, while the dashed line in blue denotes lower class women.

Figure 6. Foot-binding Prevalence and Disutility



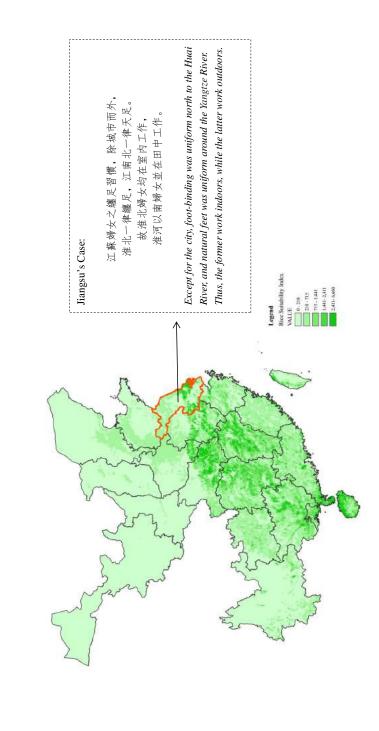
Note: This figure of foot-binding prevalence is drawn given the α discussed in Proposition 3. The red solid line denotes upper class women, while the blue dashed line denotes lower class women. Here $\Delta V_u(\alpha) = v(h,h) - v(h,(1-\alpha)h + \alpha l)$, which is the marrying up gain for upper class women. The marrying up gain for lower class women is $\Delta V_l(\alpha) = v(l,(1-\alpha)h + \alpha l) - v(l+\beta_0l,l)$. The labor opportunity cost of foot-binding δ is $[v(l+\beta 0l,l) - v(l+\beta 1l,l)]$.

Figure 7: Sample Counties by Prevalence of Foot-binding



Note: The data in this map comes from Republican archives stored in the Second Historical Archives of China. Four provinces are covered by the archives: Shandong, Chahaer (now part of Hebei), Hunan and Yunnan.

Figure 8: The Case of Jiangsu Province



Note: The data in this map comes from geographic distribution of crop suitability from the FAOs GAEZ (Global Agro-Ecological Zones). The two rivers denotes Huai River and Yangtzi River from north to south respectively. The source of this qualitative case is provincial gazetteer of Jiangsu province published in the Republican years.

Table 1: Number of days of man-labor required per crop hectare for major crops

		Majo	or Crops in	Rice Reg	gion	
Average	Rice 186.4	Rice, early 120.7	Rice, late 104.9	R	tice, glut 201.6	
		Major	Crops in	Wheat Re	gion	
Average	Wheat 95.4	Spring Wheat 114.6	Maize 108.7	Kaoliang 101.9	Millet 103.9	Millet, proso 127.8

Source: Buck, J. L. (1937). Land Utilization in China: A Study of 16,786 Farms in 168 Localities, and 38,256 Farm Familites in Twenty-two Provinces in China, 1929-1933—Statistics (Vol. 2). Commercial Press, Limited, Agents in the United States, The University of Chicago Press.

Table 2: Out-of-Archive and In-Archive Counties

	Out-o	of-Archive Counties	In-Ar	chive Counties	T-test
	N1	Mean1	N2	Mean2	p-value
Shandong					
Relative Suitability	17	-0.86	90	-0.979	0.62
Cotton Suitability	17	1.056	90	0.938	0.15
Flax Suitability	17	0.933	90	0.968	0.85
County Exam Quota	17	13.471	87	15.356	0.03
Log Dist. To Nanjing	17	6.464	90	6.482	0.58
Hunan					
Relative Suitability	45	2.412	26	2.601	0.43
Cotton Suitability	45	-0.447	26	-0.415	0.32
Flax Suitability	45	-0.333	26	-0.417	0.2
County Exam Quota	42	14.476	25	15.16	0.56
Log Dist. To Nanjing	45	6.875	26	6.848	0.4
Chahaer					
Relative Suitability	142	-0.583	10	-0.697	0.4
Cotton Suitability	142	0.429	10	-1.74	0
Flax Suitability	142	0.38	10	0.451	0.66
County Exam Quota	126	16.571	10	15.8	0.56
Log Dist. To Nanjing	142	6.824	10	7.029	0
Yunnan					
Relative Suitability	29	0.957	80	-0.143	0
Cotton Suitability	29	-0.856	80	-1.127	0.07
Flax Suitability	29	-1.241	80	-1.245	0.98
County Exam Quota	7	11.143	69	13.217	0.4
Log Dist. To Nanjing	29	7.64	80	7.581	0

Note: This table provides a balance check for out-of-archive and in-archive counties by the four provinces covered by the Republican archives. The key variables for comparison include standardized relative suitability (Rice-Wheat), cotton and flax suitability, county exam quota. Column 5 provides p-value for the t-test between two groups. All variables are standardized except for log distance to Nanjing (capital of the Republican government).

Table 3: Ambiguous-Info. Counties and Clear-Info. Counties

	Ambiguou	Ambiguous-Info. Counties Clear-Info. Counties	Clear-In	fo. Counties	Γ -test
	N_1	Mean1	N_2	Mean2	p-value
Relative Suitability (Rice-Wheat)	38	0.048	168	-0.242	0.31
Cotton Suitability	38	-0.014	168	-0.199	0.36
Flax Suitability	38	-0.083	168	-0.093	0.96
County Exam Quota	35	-0.309	156	-0.071	0.22
Log Dist. To Nanjing	38	6.975	168	6.983	0.93

covered by the Republican archives. Column 5 provides p-value for the t-test between two groups. Key variables include standardized relative suitability index (Rice-Wheat) and county exam quota, cotton and flax suitability index. All variables Note: This table provides a balance check for ambiguous-info. counties and clear-info. counties, among all counties are standardized except for log distance to Nanjing.

Table 4: Descriptive Statistics

	Low 1	Low Prevalance	High]	High Prevalance	T-test	
Crop Suitability	N_1	Mean	$^{ m N}_2$	Mean	P-value	Sources
Agricultural suitability variables						
Relative Suitability (Rice-Wheat)	38	1.134	130	-0.645	0	A
Cotton Suitability	38	-0.921	130	0.013	0	A
Flax Suitability	38	-0.969	130	0.163	0	A
$Socio-economic\ variables$						
County Exam Quota	33	-0.437	123	0.027	0.02	В
County Population (1000 ppl.)	38	-0.188	130	0.001	0.28	Ö
County Sex Ratio	38	-0.094	130	0.107	0.31	Ŋ
Number of Christian Churches	36	-0.27	127	0.043	0.00	О
Number of Vicars	36	-0.242	127	-0.036	0.14	О
Number of Disciples (per 10,000 people)	36	-0.159	127	0.029	0.34	О
Number of Modern Firms	38	-0.254	130	-0.179	0.46	田
Mongol Historical Migration Intensity	38	-0.777	130	-0.146	0	ᇿ
Geographical Characteristics						
Log Distance to Nanjing (km)	38	7.327	130	6.882	0	IJ

Note: This table provides descriptive statistics of key variables for counties covered by the Republican archives. Column 5 provides p-value for the t-test between two groups. Key variables include standardized relative suitability index (Rice-Wheat) and county exam quota, cotton and flax suitability index. All variables are standardized except for log distance to Nanjing.

Table 5: Crop Suitability, Exam Quotas and Foot-binding Prevalence

			Sample=	All Counties	
	Dependent	Variable: 1	Foot-binding	g in Rural Ar	ea (1=High Prevalence)
	(1)	(2)	(3)	(4)	(5)
Relative Suitability (Rice-Wheat)	-0.094***	-0.095***	-0.095***	-0.089***	-0.088**
	(0.024)	(0.025)	(0.025)	(0.025)	(0.039)
Cotton Suitability	0.109***	0.105***	0.105**	0.129**	0.177**
	(0.027)	(0.028)	(0.045)	(0.051)	(0.078)
Flax Suitability	0.053	$0.055^{'}$	$0.055^{'}$	0.066	0.014
-	(0.038)	(0.040)	(0.051)	(0.051)	(0.054)
County Exam Quota	0.060*	0.063*	0.063^{*}	0.061^{\dagger}	0.058^{\dagger}
•	(0.034)	(0.036)	(0.038)	(0.038)	(0.038)
County Population	-0.077***	-0.060*	-0.060*	-0.057^{\dagger}	-0.022
-	(0.023)	(0.033)	(0.034)	(0.035)	(0.042)
Number of Christian Churches	,	0.026*	0.026*	0.017	0.022^{\dagger}
		(0.014)	(0.014)	(0.014)	(0.014)
Number of Vicars		-0.047	-0.047	-0.045	-0.060
		(0.045)	(0.046)	(0.049)	(0.046)
Number of Disciples		-0.010	-0.010	-0.010	-0.010
-		(0.036)	(0.036)	(0.036)	(0.034)
Log Distance to Nanjing (km)		,	0.001	0.102	-0.049
• • • • • • • • • • • • • • • • • • • •			(0.166)	(0.191)	(0.336)
Number of Modern Firms			,	0.002	0.011
				(0.028)	(0.026)
Mongol Historical Migration Intensity				0.058^{*}	-0.008
				(0.032)	(0.036)
Constant	0.782***	0.777***	0.771	0.088	$1.057^{'}$
	(0.028)	(0.028)	(1.147)	(1.316)	(2.238)
Prov.FE	N	N	N	N	Y
Observations	153	153	153	153	153
R-squared	0.348	0.351	0.351	0.362	0.396

Note: *** p<0.01, ** p<0.05, * p<0.1, \dagger p<0.15. Parentheses include robust standard errors. Dependent variable is a dummy variable indicating high prevalence of foot-binding, and key explainatory variables include relative suitability index and county exam quota, where both are standardized. A set of control variables includes standardized county population, cotton and flax suitability index, number of modern firms, Mongol historical migration intensity, Christianity activity variables, as well as log distance to Nanjing.

Table 6: Exam Quotas and Foot-binding Prevalence: Prefecture Fixed Effects

	Dependen	t Variable: Foo	t-binding Prevale:	Dependent Variable: Foot-binding Prevalence (1=High 0=Low)
		Sam	Sample= All Counties	
	(1)	(2)	(3)	(4)
County Exam Quota	0.070*	0.070**	0.084**	0.087**
	(0.041)	(0.033)	(0.033)	(0.034)
County Population	0.009	-0.044	*00.00	-0.037
	(0.049)	(0.029)	(0.028)	(0.040)
Relative Suitability (Rice-Wheat)			0.069	0.062
			(0.045)	(0.047)
Cotton Suitability			0.014	0.024
			(0.123)	(0.131)
Flax Suitability			0.066	0.068
			(0.040)	(0.044)
Constant	0.795***	1.158***	1.221***	-1.435
	(0.032)	(0.075)	(0.147)	(2.249)
Prefecture FE.	No	Yes	Yes	Yes
Other Control Variables	$N_{\rm o}$	$N_{\rm o}$	m No	Yes
Observations	153	153	153	153
R-squared	0.035	0.720	0.730	0.732
Test Unobservable Selection		Column 1&2	Column 1&3	Column 1&4
δ for $\beta = 0$ given $R_{max} = 1$		2.528	3.312	2.861
Bias-Adjusted β , given $\delta=1$ and $R_{max}=1$		0.07	0.102	0.117

Note: *** p<0.01, ** p<0.05, * p<0.1. Robust standard errors in parentheses. Dependent variable is a dummy variable indicating high prevalence of foot-binding within county. Both county exam quota and county population are standardized. Prefecture fixed effects are controlled in 2 to 4 columns. The test on selection on unobservables is based on Oster (2016). A set of control variables includes standardized county population, cotton and flax suitability index, number of modern firms, Mongol historical migration intensity, Christianity activity variables, as well as log distance to Nanjing.

9 Appendix

9.1 Foot-binding in Historical Accounts

As we have discussed above, foot-binding was an unpresentable and cryptic topic to discuss in public domain, and even taboo in official genres. Therefore, there is a lack of systematic record of foot-binding by the imperial government during the Song, Ming and Qing. In this section, we will summarize and compare different sources recorded and compiled by a variety of authorship on foot-binding, trying to piece out the picture of foot-binding and cross-check the evidence one from another.

Overall, sources on foot-binding have significant variation, ranging from semi-official records, privately compiled history, personal history, literature, and folklore, to archaeological evidence. Appendix Table A1 provides a summary, presenting the types and contents, author-ship/compilers, time range, accessibility and potential bias. It could be clearly seen that those sources are very different by nature, thus relying merely on one type of the sources may result in bias in understanding foot-binding. As listed as 1 in Appendix Table A1, the first type of sources on foot-binding we examine is official records by the Republican government. In the early 20th century, the Republican government prohibited foot-binding and surveyed to investigate the process of prohibition from 1931 to 1934 (Yang, 2003). In this process, the local county government conducted survey on previous foot-binding prevalence and the progress of anti-footbinding campaign, then sent reports to the Republican government, which were later compiled by the Ministry of The Interior. This is the primary source used by this paper for empirical analysis.

The second type of the sources on foot-binding is from gazetteers (on province, prefecture, and county level), which is semi-official compared with governmental records. The authors of gazetteers are mostly local gentries, scholars, and officials, and the compiling were largely based on local files, records, and personal observations. Foot-binding in gazetteers was barely mentioned from the Song to mid-Qing dynasty, and mostly mentioned during the late Qing and Republican Years. For gazetteers published during the late Qing, most of the text on foot-binding appears in the chapter of "local customs". During the Republican years, the gazetteers almost uniformly mentioned foot-binding as a bad custom along with description on anti-footbinding campaigns. Taken together, only around 230 localities (mostly counties) have foot-binding texts in gazetteers, and most of the content is anti-foot-binding, not about the pre-existing culture of foot-binding. Therefore, while gazetteers could be a good source

for supplementary purpose, they will be most useful for understanding the anti-footbinding process.

The third type of our sources on foot-binding are jottings (in Chinese, Biji), literature and privately compiled history. The authors of such type are mostly scholars, historians, philologists, and sometimes even amateurs of foot-binding, e.g., Qian, Yong(1759-1844), Xu, Ke(1869-1928), Hu, Pu-an (1878-1947), Yao, Ling-hsi(1899-1963). Their accounts cover the widest aspect of foot-binding history, and are irreplaceable in the value of first-hand observation on foot-binding. Their accounts of foot-binding are often a key source for contemporary scholars, including Dorothy Ko(2005) and Susan Mann(1997).

The fourth type of the sources is anthropology, personal history, and interviews (e.g. see a rich source summarized by Turner, 1997, and Gates, 2015; Bossen and Gates, 2017 for contemporary interviews with foot-binding women). The authors of this type of sources include foreign visitors/scholars, Christian missionaries, travelers, and foot-binding women themselves. The key merit of this type of source is its first-hand observation and experiences. For instance, Bossen and Gates (2017) show their interviews with 1800 rural senior women in 27 villages (9 provinces), consulting their foot-binding history (if any), personal life history, economic activities, village characteristics, and so on. This source is again irreplaceable in helping us to understand foot-binding on the grassroots.

The fifth source we look at is the folkloristic evidence and oral literature, i.e., ballads (e.g. see Zhang, 2015). The authors of the ballads are the folk. Compiled by domestic or foreign observers, the folk ballads described foot-binding in various ways, often bringing remarkable insight into this phenomenon. However, the drawback of this source is also clear--we do not know its time span and what kind of ballads survived and why. Thus the folk ballads will be taken as complementary sources when we examine our theory and empirical predictions.

The final source we discuss in this paper is the archaeological evidence (bound feet shoes/lotus shoes) (see Ebrey 1992 for example). The clear advantage of this source is that it is the only source that can help us to date back to the early dynasties (e.g., the Song, 960-1279). By examining the owner of the shoes, her family status, as well as the location of shoes, this type of source will be important for us to understand foot-binding at its earlier phase.

9.2 Flipped Rankings under High Meritocracy

Section 3 of our paper has analyzed cases when $\alpha < \frac{1}{2}$. To get a full picture of foot-binding dynamics with high α , we further examine the case when $\alpha > \frac{1}{2}$ so that the ranking of groom's quality is flipped. Using the group labels in Figure 4(b), now the marriage market ranking from the highest to lowest is: the champions, the new elites, the fallen elites, and the new lower class. An illustration of the equilibrium matching for this case can be found in Figure 4(c). Proposition 7 characterizes the foot-binding practice when the exam system is highly meritocratic and the society has high mobility.

Proposition 7. When men's competence dominates family status $(\frac{1}{2} < \alpha < 1)$, foot-binding is adopted by women of both classes, and is more prevalent in the upper class. The proportion of foot-binding women in the upper class is $r_u = max\{\frac{p}{C}\{\mu[v(h,h) - v(h,(1-\alpha)h + \alpha l)] + (1-\mu)[v(h,(1-\alpha)l+\alpha h) - v(h,(1-\alpha)h + \alpha l)]\}$, 1} and in the lower class is $r_w = max\{\frac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)}$, 1}. Further, $0 < r_w \le r_u \le 1$.

In this case, upper class brides compete for two types of men – the champions and new elites. The remaining upper class brides then compete with lower class brides for the fallen elites. In equilibrium, the expected net payoff from foot-binding in the marriage market is equal to the disutility of foot-binding, which condition pins down the proportion of foot-binding women in the upper and lower classes. With similar logic to that of Proposition 3, the proportion of upper class women who bind their feet is greater than that of lower class women. It should be noted that, when the upper class women compete for two types of men with only one tool for competition (the binary foot-binding option), the marriage benefits between marrying champions or new elites cannot be further absorbed by the competition. In the extensions of our model in Section 5, we will discuss foot-binding as a continuous competition tool, and also consider the availability of multiple competition tools (i.e. dowry payments) for further investigation.

Corollary 2. The foot-binding percentage of upper class women and of lower class women is non-decreasing in the proportion of high ability men p, and non-increasing with the disutility of foot-binding C when $\frac{1}{2} < \alpha < 1$. For lower class women, the foot-binding percentage is non-increasing in β_0 .

Corollary 2 establishes comparative static predictions for the case of $\frac{1}{2} < \alpha < 1$. It delivers similar predictions to Corollary 1. Thus, the key comparative statics are robust whether α is larger than one half or not.

9.3 Proofs of Propositions and Corollaries

Proof of Proposition 1

Proof. In the pre-exam period, we start from the matching where upper class brides are matched with upper class grooms, lower class brides are matched with lower class groom, and there is no foot-binding. We check such matching is in equilibrium.

The only players who have incentive to seek for better marriage opportunities are the lower class brides, because the upper class brides are already paired with the best grooms possible. If one of them chooses to foot-binding, then the condition that an upper class groom is willing to switch for her is:

$$v(l,h) + B > v(h,h) \Rightarrow B > v(h,h) - v(l,h)$$

That is, the foot-binding benefits outweigh the costs of marry-down. And the condition that the lower class bride is willing to choose foot-binding is:

$$v(l,h) - C > v(l + \beta_0 l, l) \Rightarrow C < v(l,h) - v(l + \beta_0 l, l) < v(l,h) - v(l,l)$$

However, since $B \leq C$, if the two conditions hold, we have v(h,h) - v(l,h) < v(l,h) - v(l,l), which contradicts with v being complementary. Thus lower class bride has no incentive to choose foot-binding. The matching is in equilibrium.

However, there might be multiple equilibria in the game. To require the equilibrium to be unique, we need an additional assumptions on the value function that $v(x, x) \ge v(x + a, x - a)$ for all $0 < a \le x$.

Among all assortative matching outcome the non-foot-binding matching is obviously unique because without competition there is no need to bind feet. If there exists another non-assortative matching, then there exists at least a pair (h, l) and a pair (l, h). To start with, the lower class bride in (l, h) must have bound her feet, because otherwise the groom should choose an upper class bride.

If the h bride deviates to pair up with the h groom, then the payoff difference is $v(h, h) - v(h + \beta l, l)$. By Assumption 3, when x = 1, $h - l > \beta l$, $v(h, h) - v(h + \beta l, l) \ge v(h + \beta l, h - \beta l) - v(h + \beta l, l) > v(h + \beta l, l) - v(h + \beta l, l) = 0$ therefore $v(h, h) - v(h + \beta l, l) > 0$, regardless of foot-binding choices. So the h bride has incentive to deviate.

Next we check whether the h groom prefers the h bride to the original l bride. There are two cases: (1) the h bride has bound feet. Then the h groom prefers the h bride immediately, and the original matching is not stable; (2) the h bride has natural feet. Then the h groom compares a non-foot-binding upper class bride and a foot-binding lower class bride: if the groom prefers the foot-binding lower class bride, it means $v(h,h) \leq v(l,h) + B$, in which case for the lower class bride $C \geq v(h,h) - v(l,h) > v(l,h) - v(l,l)$. Therefore the lower class bride prefers not to bind the feet and marry a lower class groom in the first place. Again, the original matching is not in equilibrium. Thus uniqueness is proven under the additional assumption.

Proof of Proposition 2

Proof. The proof is constructed as follows. First, we discuss any possible competition between or within class. Second, we determine the conditions of no foot-binding in upper class and lower class respectively. Lastly we compare the conditions to prove the proposition.

First, due to complimentarity (Assumption 2), the lower class bride never seeks for cross-class marriage. Specifically, for a lower class bride matched with, for example, a champion through foot-binding, the marrying-up benefit of the bride's side should be able to compensate the marrying-down loss of the champion, i.e.: $v(l,h) - v(l+\beta_0 l, l) > v(h,h) - v(l,h)$. However, due to complimentarity, this will not be feasible. Thus cross-class marriage is not feasible, and there only exists competition within class.

Second, we examine within class competition among women, that upper brides are matched with champions and fallen elites, lower class brides are matched with new elites and new lower class.

Lemma 1. There exists α_1 such that no foot-binding is adopted in intra-class marriage in upper class when $\alpha < \alpha_1$.

For the upper bride to stay in class without foot-binding, we require the cost of foot-binding is too large compared with the benefit of marrying up, that is, $v(h,h) - C \leq pv(h,h) + (1-p)v(h,(1-\alpha)h + \alpha l)$, then $C \geq (1-p)[v(h,h) - v(h,(1-\alpha)h + \alpha l)]$. Denote the cutoff value of α as α_1 .

Lemma 2. There exists α_2 such that no foot-binding is adopted in intra-class marriage in lower class when $\alpha_1 < \alpha_2 \leq \frac{1}{2}$.

For the lower class bride to stay in class without foot-binding, we require when everyone else choose no foot-binding, a lower class bride has no incentive to do so either: $v(l, (1-\alpha)l + \alpha h)$

 $C \leq pv\left(l, (1-\alpha)l + \alpha h\right) + (1-p)v\left(l + \beta_0 l, l\right)$, then $C \geq (1-p)\left[v\left(l, (1-\alpha)l + \alpha h\right) - v\left(l + \beta_0 l, l\right)\right]$. Denote the cutoff value as α_2 .

Since v is convex, we have $v(h,h) - v(h,(1-\alpha)h + \alpha l) > v(l,(1-\alpha)l + \alpha h) - v(l,l) > v(l,(1-\alpha)l + \alpha h) - v(l + \beta_0 l,l)$, where the first equality is guaranteed by mean value theorem. Therefore $\alpha_1 < \alpha_2 < \frac{1}{2}$. Here $\alpha_2 < \frac{1}{2}$ is directly guaranteed by Assumption 4, where $C < \bar{C} = (1-p) \left[v\left(l,\frac{l+h}{2}\right) - v\left(l + \beta_0 l,l\right) \right]$.

Proof of Proposition 3

Proof. Follow the proof of Proposition 2, we know that when $\alpha \leq \alpha_1$ there is no foot-binding in upper class, and when $\alpha_1 < \alpha \leq \alpha_2$, there is foot-binding in upper class but not in the lower class. When $\alpha_2 \leq \alpha \leq \frac{1}{2}$ there are foot-binding in both class. Here it remains to show the foot-binding percentage in upper class and lower class, denoted as r_u and r_l respectively.

For upper class, when $\alpha \in [\alpha_1, \frac{1}{2}]$, in equilibrium the percentage of foot-binding women should make the marriage benefit with and without foot-binding indifferent, if the solution is interior. That is:

$$\frac{p}{r_u}v\left(h,h\right) + \left(1 - \frac{p}{r_u}\right)v\left(h,(1-\alpha)h + \alpha l\right) - C = v\left(h,(1-\alpha)h + \alpha l\right)$$

Thus $r_u = \frac{p}{C} [v(h, h) - v(h, (1 - \alpha)h + \alpha l)]$. Of course there is a natural upper bound of r_u , where everyone binds their feet. So, $r_u = max\{\frac{p}{C} [v(h, h) - v(h, (1 - \alpha)h + \alpha l)], 1\}$.

For lower class, when $\alpha \in [\alpha_2, \frac{1}{2}]$, in equilibrium the percentage of foot-binding women should make the marriage benefit with and without foot-binding indifferent. That is:

$$r_{l} = \frac{p}{r_{l}}v\left(l, (1-\alpha)l + \alpha h\right) + \left(1 - \frac{p}{r_{l}}\right)v\left(l + \beta_{1}l, l\right) - C = v\left(l + \beta_{0}l, l\right), \text{ thus}$$

$$r_{l} = \frac{p[v(l, (1-\alpha)l + \alpha h) - v(l + \beta_{1}l, l)]}{C + v(l + \beta_{0}l, l) - v(l + \beta_{1}l, l)}.$$

Again, combining the boundary case, we have:

$$r_l = max\{\frac{p[v(l,(1-\alpha)l+\alpha h)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)}, 1\}$$

Lastly, by complementarity of v, we have foot-binding is less prevalent in the lower class, i.e. $r_l \leq r_u$.

Proof of Proposition 4:

Proof. The proof is constructed as follows. First, we examine the possibility of inter-class marriages when $\alpha < \frac{1}{2}$. With complimentarity (Assumption 2), the lower class bride never seeks for cross-class marriage. In particular, a lower class bride cannot compensate the marrying-down loss of ranking A and B grooms (in Table 4) through foot-binding. Thus cross-class marriage is not feasible, and there only exists competition within class.

Second, we examine within class competition among women, that upper brides are matched with ranking A and B grooms, lower class brides are matched with D and E grooms.

Lemma 3. There exists $\alpha_1^{Ext.}$ such that no foot-binding is adopted among upper class women when $\alpha < \alpha_1^{Ext.}$.

For the upper bride to stay in class without foot-binding, we require the cost of foot-binding is too large compared with the benefit of marrying up, that is, $v(h,h)-C \leq pv(h,h)+qv(h,(1-\alpha)h+\alpha m)+(1-p-q)v(h,(1-\alpha)h+\alpha l)$, then $C \geq (1-p)\left[v(h,h)-v(h,(1-\alpha)h+\alpha l)\right]-q\left[v(h,(1-\alpha)h+\alpha m)-v(h,(1-\alpha)h+\alpha l)\right]$. Denote the cutoff value of α as α_1^{Ext} , thus $\alpha_1 < \alpha_1^{Ext}$.

Lemma 4. There exists $\alpha_2^{Ext.}$ such that no foot-binding is adopted among lower class women when $\alpha_1^{Ext.} < \alpha_2^{Ext.} \le \frac{1}{2}$.

For the lower class bride to stay in class without foot-binding, we require when everyone else choose no foot-binding, a lower class bride has no incentive to do so either:

$$v(l, (1 - \alpha)l + \alpha h) - C \le pv(l, (1 - \alpha)l + \alpha h) + qv(l, (1 - \alpha)l + \alpha m) + (1 - p - q)v(l + \beta_0 l, l),$$

then
$$C \ge (1-p) \left[v\left(l, (1-\alpha)l + \alpha h \right) - v\left(l + \beta_0 l, l \right) \right] - q \left[v\left(l, (1-\alpha)l + \alpha m \right) - v\left(l + \beta_0 l, l \right) \right].$$

Denote the cutoff value as α_2^{Ext} .

Since v is convex, $\alpha_1^{Ext.} < \alpha_2^{Ext.}$. Moreover, to make sure that lower class women adopt footbinding when α is less than half for the multiple talent case, the upper bound of disutility is $C^{Ext.} = (1-p)\left[v\left(l,\frac{l+h}{2}\right) - v\left(l+\beta_0l,l\right)\right] - q\left[v\left(l,\frac{l+m}{2}\right) - v(l+\beta_0l,l)\right]$, which is smaller than \bar{C} given in Assumption 4. Thus, we have $\alpha_1^{Ext.} < \alpha_2^{Ext.} < \frac{1}{2}$.

Upper class brides now compete on grooms with ranking A and B. The lower class brides compete on grooms with ranking D and E within class. Suppose equilibrium foot-binding percentages in both class are r_u^{Ext} and r_l^{Ext} , respectively, then apply the indifference condition and assume interior solution:

 $\frac{p}{r_u}v\left(h,h\right) + \frac{q}{r_u}v\left(h,(1-\alpha)l + \alpha m\right) + \left(1 - \frac{p+q}{r_u}\right)v\left(h,(1-\alpha)h + \alpha l\right) - C = v\left(h,(1-\alpha)h + \alpha l\right),$ so that the proportion of foot-binding women in upper class is:

$$r_u^{Ext} = \frac{p[v(h,h)-v(h,(1-\alpha)h+\alpha l)]+q[v(h,(1-\alpha)l+\alpha m)-v(h,(1-\alpha)h+\alpha l)]}{C}.$$

Add the boundary, we have:

$$r_u^{Ext} = max \{ \frac{p[v(h,h) - v(h,(1-\alpha)h + \alpha l)] + q[v(h,(1-\alpha)l + \alpha m) - v(h,(1-\alpha)h + \alpha l)]}{C}, 1 \}.$$

For lower class women, in equilibrium the percentage of foot-binding women should make the marriage benefit with and without foot-binding indifferent. That is:

$$\frac{p}{r_l} v \left(l, (1 - \alpha)l + \alpha h \right) + \frac{q}{r_l} v \left(l, (1 - \alpha)l + \alpha m \right) + \left(1 - \frac{p + q}{r_l} \right) v \left(l + \beta_1 l, l \right) - C = v \left(l + \beta_0 l, l \right), \text{ so that } r_l = \frac{p[v(l, (1 - \alpha)l + \alpha h) - v(l + \beta_1 l, l)] + q[v(l, (1 - \alpha)l + \alpha m) - v(l + \beta_1 l, l)]}{C + v(l + \beta_0 l, l) - v(l + \beta_1 l, l)}.$$

Add the boundary, we have:

$$r_l^{Ext} = \max\{\tfrac{p[v(l,(1-\alpha)l+\alpha h)-v(l+\beta_1 l,l)]+q[v(l,(1-\alpha)l+\alpha m)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)},\ 1\}.$$

By complementarity of v it is easy to check that $r_l^{Ext} \leq r_u^{Ext}$ in this case. Moreover, comparing the proportion of foot-binding women in either class in this case with our baseline model, it's easy to check: $r_u < r_u^{Ext}$ and $r_l < r_l^{Ext}$.

Proof of Proposition 5:

Proof. (1), (2) and (3): Following the logic of the proof of Proposition 1, when $\alpha = 0$ there exists no incentive to practice foot-binding. In addition, following our complementarity assumption, any inter-class marriage is not feasible, so that the competition is merely within-class.

As there exists a fixed-cost for foot-binding, namely $c(\underline{b})$, upper class women will only start foot-binding when $\alpha \in [\tilde{\alpha}_1, \frac{1}{2}]$, where $\tilde{\alpha}_1$ is defined by $c(\underline{b}) = v(h, h) - v(h, (1 - \tilde{\alpha}_1)h + \tilde{\alpha}_1 l)$, when μ of upper class women take up foot-binding, and their foot-binding intensity choice b_u perfectly absorbs the marry-up benefit: $c(b_u) = v(h, h) - v(h, (1 - \alpha)h + \alpha l)$. Similarly, when $\alpha \in [\tilde{\alpha}_2, \frac{1}{2}]$, where $\tilde{\alpha}_2$ is defined by $c(\underline{b}) = v(l, (1 - \tilde{\alpha}_2)l + \tilde{\alpha}_2 h) - v(l + \beta_0 l, l)$, μ of lower class women take up foot-binding, and their foot-binding intensity choice b_l perfectly absorbs the marry-up benefit: $c(b_l) = v(l, (1 - \alpha)l + \alpha h) - v(l + \beta_0 l, l)$.

(4) When $\alpha \in (\frac{1}{2}, 1)$, that we have a flipping ranking of new elites and fallen elites, the upper class women are now competing for both the champions and the new elites, while the lower

class women is competing for fallen elites left. Now the Bertrand competition leads to the unique outcome where all rents from marrying up are dissipated. That is,

$$c(b_C) = v(h, h) - v(h, (1 - \alpha)h + \alpha l)$$

$$c(b_{NE}) = v(h, (1-\alpha)l + \alpha h) - v(h, (1-\alpha)h + \alpha l)$$

It is immediate that both binding intensities increase with α . In this case, upper class brides are indifferent between $b = 0, b = b_C$ or $b = b_{NE}$. Therefore the share of brides who adopt the corresponding intensities should coincide with the share of champions and new elites. The same logic applies to lower class brides, where

$$c(b_{FE}) = v(l, (1 - \alpha)h + \alpha l) - v(l + \beta_0 l, l)$$

The share of lower class bride's population who bind their feet should also coincide with the rest of the fallen elites, which has size $\mu + (1 - \mu) p - \mu = (1 - \mu) p$. Due to complimentarity, we have $b_C > b_{NE}$, and $b_C > b_{FE}$. When α is slightly larger than $\frac{1}{2}$, $b_{FE} > b_{NE}$, and when α is close to 1, $b_{FE} < b_{NE}$.

Proof of Proposition 6.

Proof. (1) Similar logic of the proof of Proposition 1.

(2) As the foot-binding investment calls for a fix amount of cost $c(\underline{b})$, it will not be used until the marry-up benefits within class can compensate the minimum level of cost. Following similar analysis of Proposition 6, upper class women start foot-binding when $\alpha \in [\tilde{\alpha}_1, \frac{1}{2}]$, where $\tilde{\alpha}_1$ is defined by $c(\underline{b}) = v(h, h) - v(h, (1 - \tilde{\alpha}_1)h + \tilde{\alpha}_1 l)$, and lower class women take up foot-binding when $\alpha \in [\tilde{\alpha}_2, \frac{1}{2}]$, where $\tilde{\alpha}_2$ is defined by $c(\underline{b}) = v(l, (1 - \tilde{\alpha}_2)l + \tilde{\alpha}_2 h) - v(l + \beta_0 l, l)$. For upper class, when $\alpha \in (0, \tilde{\alpha}_1)$, bride's family only uses dowry payments. The dowry payments will perfectly absorb the marrying up benefit, so now the dowry by the upper

payments will perfectly absorb the marrying up benefit, so now the dowry by the upper class brides is defined by: $u(d_{0,u}^*) = v(h,h) - v(h,(1-\alpha)h + \alpha l)$. Similarly, when $\alpha \in (0,\tilde{\alpha}_2)$, the lower class brides' family only use dowry payments. The dowry payment level in lower class is defined by $u(d_{0,l}^*) = v(l,(1-\alpha)l + \alpha h) - v(l + \beta_0 l, l)$. By complementarity of v it is easy to check that $d_{0,u}^* > d_{0,l}^*$ given a certain level of $\alpha \in (0,\tilde{\alpha}_1)$. (3): When $\alpha \in [\tilde{\alpha}_1, \frac{1}{2}]$, both foot-binding and dowry payments will be used by upper class women; and when $\alpha \in [\tilde{\alpha}_2, \frac{1}{2}]$, both price tools will be used by lower class women. We can construct proof in following steps:

Step 1. We solve the continuous "package benefit" choice problem where

Bride:
$$V_B(q_i, q_i, h_i) = v(\theta_i, \alpha \gamma_i + (1 - \alpha)\theta_i) - h_i$$

Groom:
$$V_G(q_i, q_j, h_i) = v(\theta_i, \alpha \gamma_j + (1 - \alpha)\theta_j) + h_i$$

where bride's family chooses an effective marriage package benefit h_i . Denote the solution of the above problem h_i^* .

Step 2. We solve the substitution problem between foot-binding and dowry choice.

$$\min_{b_i,d_i} c\left(b_i\right) + u\left(d_i\right)$$

$$s.t. \ c(b_i) + u(d_i) \ge h_i^*$$

Solving Step 1 is exactly the same as in Section 3.5.2. So it only remains to show how the optimal b_i and d_i is solved. Given h_i^* , this minimization problem is solved when the following two equations hold:

$$c^{'}(b_i) = u^{'}(d_i)$$

$$c(b_i) + u(d_i) = h_i^*$$

With both $c(\cdot)$ and $u(\cdot)$ concave, it is immediate that the optimal b_i^* and d_i^* increase with h_i^* .

Therefore, when $\alpha \in [\tilde{\alpha_1}, \frac{1}{2}]$, both foot-binding and dowry payments will be used by upper class women. By Bertrand competition, the marry-up benefits will be completely absorbed by these two price tools. There will be μp upper class women choose binding intensity b_u^* , dowry size d_u^* , where b_u^* and d_u^* satisfy the following conditions:

$$c^{'}\left(b_{u}^{*}\right) = u^{'}\left(d_{u}^{*}\right)$$

$$c(b_u^*) + u(d_u^*) = v(h, h) - v(h, (1 - \alpha)h + \alpha l)$$

When $\alpha \in [\tilde{\alpha}_2, \frac{1}{2}]$, both foot-binding and dowry payments will be used by lower class women, where $(1 - \mu)p$ lower class women choose binding intensity b_l^* , dowry size d_l^* , satisfying the following conditions:

$$c^{'}(b_{l}^{*}) = u^{'}(d_{l}^{*})$$

$$c(b_l^*) + u(d_l^*) = v(l, (1 - \alpha)l + \alpha h) - v(l, l)$$

It is obvious to check that $b_u^* > b_l^*$ and $d_u^* > d_l^*$ when $\alpha \in [\tilde{\alpha}_2, \frac{1}{2}]$.

(4) When $\alpha \in (\frac{1}{2}, 1)$, both foot-binding and dowry payments will be used by both classes, and μp of upper class brides choose binding intensity b_C^* , dowry size d_C^* ; $p(1-\mu)$ of upper class brides choose binding intensity b_{NE}^* , dowry size d_{NE}^* ; and $(\mu - p)$ choose not to bind feet and pay no dowry. For the lower class brides, precisely $p(1-\mu)$ of them choose binding intensity b_{FE}^* , dowry size d_{FE}^* . In particular, $b_C^* > b_{NE}^*$, $b_C^* > b_{FE}^*$, $d_C^* > d_{NE}^*$, and $d_C^* > d_{FE}^*$. These values satisfy the following conditions:

$$\begin{split} c^{'}\left(b_{C}^{*}\right) &= u^{'}\left(d_{C}^{*}\right), \text{ and } c\left(b_{C}^{*}\right) + u\left(d_{C}^{*}\right) = v(h,h) - v(h,(1-\alpha)h + \alpha l); \\ \\ c^{'}\left(b_{NE}^{*}\right) &= u^{'}\left(d_{NE}^{*}\right), \text{ and } c\left(b_{NE}^{*}\right) + u\left(d_{NE}^{*}\right) = v(h,(1-\alpha)l + \alpha h) - v(h,(1-\alpha)h + \alpha l); \\ \\ c^{'}\left(b_{FE}^{*}\right) &= u^{'}\left(d_{FE}^{*}\right), \text{ and } c\left(b_{FE}^{*}\right) + u\left(d_{FE}^{*}\right) = v(l,(1-\alpha)l + \alpha h) - v(l,l) \; . \end{split}$$

Proof of Corollary 1

Proof. From Proposition 3, we know that $r_l = max\{\frac{p[v(l,(1-\alpha)l+\alpha h)-v(l+\beta_1l,l)]}{C+v(l+\beta_0l,l)-v(l+\beta_1l,l)}, 1\}$, so it's immediate that r_l decreases in β_0 . Next, since we have $r_l = max\{p + \frac{p[v(l,(1-\alpha)l+\alpha h)-v(l+\beta_0l,l)-C]}{C+v(l+\beta_0l,l)-v(l+\beta_1l,l)}, 1\}$, thus $\frac{\partial r_l}{\partial \beta_1}$ depends on the numerator. Consider Assumption 4 (Bounded Disutility), we have $\overline{C} = (1-p)[v(l,\frac{h+l}{2})) - v(l+\beta_0l,l)]$. Therefore, when $C \leq min\{v(l,(1-\alpha)l+\alpha h)-v(l+\beta_0l,l),\overline{C}\}$, r_l increases with β_1 ; when $C \in [v(l,(1-\alpha)l+\alpha h)-v(l+\beta_0l,l),\overline{C}]$, r_l decreases with β_1 .

Proof of Proposition 7

Proof. When the α is large enough that generating a flipping ranking between the new elites and fallen elites, that $\frac{1}{2} < \alpha < 1$, the competition will become the following:

First, due to complementarity, a lower class bride never seeks for marriage with champions and new elites. In particular, for a lower class bride matched with a champion through foot-binding, the marrying-up benefit of the bride's side should be able to compensate the marrying-down loss of the champion, i.e.: $v(l,h) - v(l+\beta_0 l,l) > v(h,h) - v(l,h)$. For a lower class bride matched with a new elite through foot-binding, the marrying-up benefits of the bride's side should be able to compensate the marrying-down loss of the new elite, i.e.: $v(l,(1-\alpha)l+\alpha h) - v(l+\beta_0 l,l) > v(h,(1-\alpha)l+\alpha h) - v(l,(1-\alpha)l+\alpha h)$. However, due to complementarity, both conditions will not be feasible. Thus cross-class marriage is not possible, and there only exists competition for lower class women competing for fallen elites.

Upper class brides now competes on both champions and new elites⁴⁷. The unmarried brides then compete with lower class brides on the fallen elites. Suppose equilibrium foot-binding percentages in both class are r_u and r_l , respectively, then apply the indifference condition and assume interior solution:

$$\frac{p}{r_u}v\left(h,h\right) + \frac{(1-\mu)p}{\mu r_u}v\left(h,(1-\alpha)l + \alpha h\right) + \left(1 - \frac{p}{\mu r_u}\right)v\left(h,(1-\alpha)h + \alpha l\right) - C = v\left(h,(1-\alpha)h + \alpha l\right),$$
 so that the proportion of foot-binding women in upper class is:

$$r_u = \frac{p}{C} \{ \mu[v(h,h) - v(h,(1-\alpha)h + \alpha l)] + (1-\mu)[v(h,(1-\alpha)l + \alpha h) - v(h,(1-\alpha)h + \alpha l)] \}.$$
 Add the boundary, we have:

$$r_{u} = max\{\frac{p}{C}\{\mu[v(h,h)-v(h,(1-\alpha)h+\alpha l)]+(1-\mu)[v(h,(1-\alpha)l+\alpha h)-v(h,(1-\alpha)h+\alpha l)]\},\ 1\}.$$

For lower class women, in equilibrium the percentage of foot-binding women should make the marriage benefit with and without foot-binding indifferent. That is:

 $^{^{47}}$ Here the champions and the new elites act as a whole group for the brides. The reason is that foot-binding in this model is binary thus competition is insufficient. One might expect the upper class brides further compete on the champions using other tools. There are two ways to address the issue: First, if we model foot-binding as a continuous choice, where brides can adopt different sizes of bound-foot, then competition towards the champions drives the brides to bind their feet smaller and smaller. This is also consistent with the historical records. Second, we can further model the dowry process shortly before marriage, where bridal families pay dowry to compete for the champions. Since brides are assumed homogeneous in intrinsic quality, Bertrand competition bids away any extra benefit, and the dowry amount would equalize the marriage value difference between marrying a new elite and marrying a champion. Again such difference is increasing in social mobility parameter α , which suggests that dowry payment increases as Keju system expands. This is also consistent with the stylized facts.

$$\frac{p}{r_l}v\left(l, (1-\alpha)h + \alpha l\right) + \left(1 - \frac{p}{r_l}\right)v\left(l + \beta_1 l, l\right) - C = v\left(l + \beta_0 l, l\right), \text{ so that}$$

$$r_l = \frac{p[v(l, (1-\alpha)h + \alpha l) - v(l + \beta_1 l, l)]}{C + v(l + \beta_0 l, l) - v(l + \beta_1 l, l)}.$$

Add the boundary, we have:

$$r_l = max\{\frac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_1l,l)]}{C+v(l+\beta_0l,l)-v(l+\beta_1l,l)}, 1\}.$$

By complementarity of v it is easy to check that $r_l \leq r_u$ in this case. Comparing the proportion of lower class women foot-binding when $\frac{1}{2} < \alpha < 1$ and $\alpha \leq \frac{1}{2}$, we also have for interior solutions, $r_l|_{\frac{1}{2} < \alpha < 1} < r_l|_{\alpha \leq \frac{1}{2}}$, since now the fallen elites are less attractive compared to the new elites for lower class women.

Proof of Corollary 2

Proof. When $\frac{1}{2} < \alpha < 1$, recall from Proposition 7 that $r_l = max\{\frac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_1l,l)]}{C+v(l+\beta_0l,l)-v(l+\beta_1l,l)}, 1\}$. So it is immediate that r_l decreases in β_0 . Next, $r_l = max\{\frac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_1l,l)]}{C+v(l+\beta_0l,l)-v(l+\beta_1l,l)}, 1\} = max\{p + \frac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_0l,l)-C]}{C+v(l+\beta_0l,l)-v(l+\beta_1l,l)}, 1\}$. Consider Assumption 4 (Bounded Disutility), we have $\overline{C} = (1-p)[v(l,\frac{h+l}{2})) - v(l+\beta_0l,l)]$. Therefore, when $C \le min\{v(l,(1-\alpha)h+\alpha l)-v(l+\beta_0l,l),\overline{C}\}$, r_l increases with β_1 ; when $C \in [v(l,(1-\alpha)h+\alpha l)-v(l+\beta_0l,l),\overline{C}]$, r_l decreases with β_1 .

9.4 Appendix Tables and Figures

- Table A1: Historical Sources of Foot-binding: A Comparison
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- Table A4: Correlation Table of Variables
- Table A5: Crop Suitability, Exam Quotas and Foot-binding: Control for Sex Ratio
- Figure A1: The Distribution of Ballads on Foot-binding
- Figure A2: The Distribution of Bound Feet Beauty Contest
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- Figure A4: The Proportion of Degree Holders from Commoners' Families: Jinshi
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Table A1: Historical Sources of Foot-binding: A Comparison

Sources on Foot-binding	Examples	Authors/Compilers	Contents	Time Range	Accessbility
1. Official records by the Republican Government	Republican government archives(1931-1934)	The Republican government prohibited foot-binding in the 1930s. Original report from local county government. Compiled by the Ministry of Interior, Republic of China.	Describing foot-binding in the history, and anti-footbinding process.	Not restricted, mostly Qing-Republican.	The Second Historical Archives of China, Nanjing
 Semi-official records: gazetteers (province, prefecture, Historical gazetteers county) 	Historical gazetteers	local gentries, scholars and officials	Foot-binding as a folk practice and anti-footbinding process.	The late Qing-Republican. Around 230 localities	Around 230 localities
3. Jottings(Biji), literature, privately compiled history	Qian, Yong(1759-1844), Xu, Ke(1869-1928), Yao,Ling-hsi(1899-1963), Hu,Pu-an(1878-1947)	Scholars, historians, philologists	A wide range of foot-binding history	From the Song to the Republican.	Mostly published during the Qing and Republican.
4.A. Anthropology, personal history, and interviews (historical)	Turner(1997)	Foreign visitors/scholars, missionaries, Travellers, foot-binding women	First-hand sources		See Turner (1997) and Missionary journals.
4.B. Anthropology, personal history, and interviews (contemporary)	$ {\rm Gates}(2015), \\ {\rm Bossen}\& {\rm Gates}(2017) $	Personal history described by footbinding women themselves	First-hand sources	Rural elderly women in 27 villages, 9 provinces	e.g. Bossen&Gates(2017).
5. The folkloristic evidence and oral literature, e.g. ballads and bound-feet beauty contest.	Zhang(2015)	The folk. Compiled by domestic or foreign observers.	Describtion in various ways.	Unknown.	Zhang(2015)
6. Archaeological evidence (bound feet shoes/lotus shoes)	Ebrey (1992)	Officials' wives and daughters.	The owener of shoes, her family status, the location of shoes	The Song $(960-1279)$	Ebrey(1992)

Table A2: The Income of Gentry Class by Occupation

			Avg. Annual Income	Avg. Annual Income/Avg. Labor Income	
Category	Gentry Occupation	Average Annual Income (in taels)	Col.3/Lower Bound	Col.3/Lower Bound Col.3/Upper Bound Col.3/Category 14	Col.3/Category 14
(1)	(2)	(3)	(4)	(2)	(9)
	Rank (Central/local 1st)	180	36	18	12.00
2	Rank (Central/local 2nd)	150	30	15	10.00
က	Rank (Central/local 3rd)	130	26	13	8.67
4	Rank (Central/local 4th)	105	21	10.5	7.00
ರ	Rank (Central/local 5th)	80	16	∞	5.33
9	Rank (Central/local 6th)	09	12	9	4.00
7	Rank (Central/local 7th)	45	6	4.5	3.00
∞	Rank (local 8th)	40	∞	4	2.67
6	Rank (local 9th or none)	33.11	6.62	3.31	2.21
10	Gentry services	120	24	12	8.00
11	Secretaries to officials	250	20	25	16.67
12	Teaching	100	20	10	29.9
13	Practice of traditional medicine	200	40	20	13.33
14	Scholarship Awardees	15	3	1.5	1.00
		(0001)			

Note: Data source: Calculation based on Chang (1962). The lower bound of the male labor annual income is 5 taels and the upper bound is 10 taels.

Table A3: Proportion of Gentry and Literati by Province

				%Gentry	%Gentry among Male Pop.		$\% \mathrm{Quota}$ among Male Pop. (16-25)	25)
Province	#Quota	#Gentry	Pop.	SR=110	SR=130	SR=110	SR=130	
Fengtian	131		ı		ı	ı		
Zhili		I	36900	1	I	I	I	
Jiangsu		75831	29600	0.489%	0.453%	0.084%	0.078%	
Anhui		53713	36600	0.280%	0.260%	0.062%	0.058%	
Zhejiang		69698	30400	0.546%	0.506%	0.105%	0.097%	
Jiangxi		78382	26500	0.565%	0.523%	0.090%	0.083%	
Fujian		55152	25800	0.408%	0.378%	0.081%	0.075%	
Henan		70874	29100	0.465%	0.431%	0.099%	0.092%	
Shandong		64949	36200	0.343%	0.317%	0.089%	0.083%	
Shanxi		51366	10300	0.952%	0.882%	0.263%	0.244%	
Hubei		52027	28600	0.347%	0.322%	0.067%	0.062%	
Hunan		63372	20000	0.605%	0.561%	0.106%	0.098%	
Shanxi and Gansu		70269	29800	0.450%	0.417%	0.111%	0.102%	
Sichuan		55263	22300	0.473%	0.438%	0.108%	0.100%	
Guangdong	2453	77380	21100	0.700%	0.649%	0.111%	0.103%	
Guangxi		39638	8100	0.934%	0.866%	0.222%	0.206%	
Yunnan	2453	43165	6200	1.329%	1.232%	0.378%	0.350%	
Guizhou	1393	25448	4800	1.012%	0.938%	0.277%	0.257%	

Note: Data source: Chang (1955). To facilitate our calculation, we take a lower bound assumption of the sex ratio to be 110 men per 100 women and an upper bound to be 130, and assume that the proportion of males aged 16-25 account of 20% of the total male population.

Table A4: Correlation Table of Variables

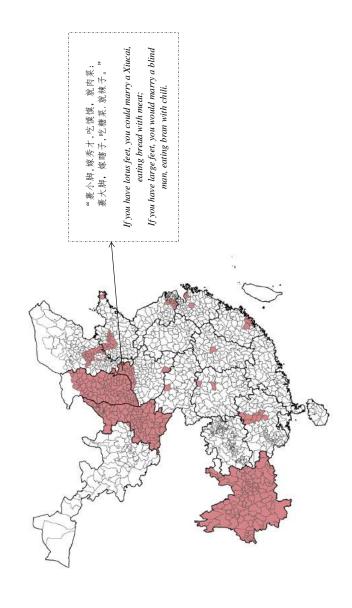
		A	B C D	C		日	E F G H	5	H
A	Relative Suitability (Rice-Wheat)								
В	Cotton Suitability	-0.05	1						
O	Flax Suitability	-0.37	0.62	1					
Ω	Cotton Suitability	-0.15	0.07	0.00					
田	County Population	-0.01	0.41	0.49	0.33	П			
伍	Log Distance to Nanjing	0.15	-0.82	-0.83	-0.22	-0.56	П		
U	Number of Modern Firms	-0.04	80.0	0.08	0.0906	0.26	-0.17	\vdash	
Η	Mongol Historical Migration Intensity	-0.23	0.16	0.32	0.23	0.22	0.22 -0.40 0.19	0.19	\vdash

Table A5: Crop Suitability, Exam Quotas and Foot-binding Prevalence: Control For Sex Ratio

		Sample=F	ull Archival	Sample (Four	Provinces)
	Dependent	Variable:	Foot-binding	g in Rural Are	a (1=High Prevalence)
	(1)	(2)	(3)	(4)	(5)
Relative Suitability (Rice-Wheat)	-0.100***	-0.100***	-0.100***	-0.093***	-0.088**
,	(0.026)	(0.027)	(0.027)	(0.027)	(0.039)
Cotton Suitability	0.114***	0.111***	0.119**	0.143**	0.175**
·	(0.029)	(0.031)	(0.052)	(0.058)	(0.078)
Flax Suitability	0.039	0.042	0.049	0.061	0.014
·	(0.043)	(0.045)	(0.051)	(0.052)	(0.055)
County Exam Quota	0.059^{*}	0.062*	0.064*	0.063 *	0.056^{\dagger}
	(0.035)	(0.036)	(0.038)	(0.038)	(0.038)
County Population	-0.077***	-0.058*	-0.057^{\dagger}	-0.055^{\dagger}	-0.021
· -	(0.023)	(0.034)	(0.035)	(0.036)	(0.042)
Number of Christian Churches	,	0.022^{\dagger}	0.023^{\dagger}	0.014	0.027^{*}
		(0.015)	(0.015)	(0.015)	(0.015)
Number of Vicars		-0.048	-0.048	-0.047	-0.060
		(0.046)	(0.046)	(0.049)	(0.046)
Number of Disciples		-0.008	-0.009	-0.009	-0.012
-		(0.035)	(0.035)	(0.035)	(0.035)
Log Distance to Nanjing (km)		,	0.040	0.139	-0.069
• • • • • • • • • • • • • • • • • • • •			(0.176)	(0.201)	(0.343)
Number of Modern Firms			,	0.006	$0.006^{'}$
				(0.028)	(0.027)
Mongol Historical Migration Intensity				0.057^{*}	-0.013
				(0.032)	(0.037)
County Sex Ratio	0.029	0.026	0.030	0.028	-0.033
	(0.040)	(0.041)	(0.045)	(0.044)	(0.050)
Constant	0.780***	0.775***	0.501	-0.170	$1.192^{'}$
	(0.029)	(0.029)	(1.215)	(1.387)	(2.281)
Prov. FE.	N	N	N	N	Y
Observations	153	153	153	153	153
R-squared	0.350	0.353	0.353	0.364	0.398

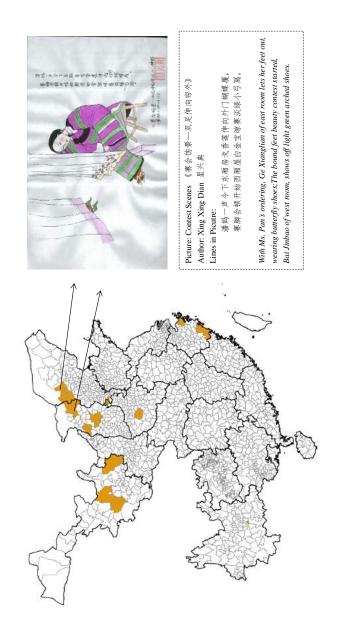
Note: *** p<0.01, ** p<0.05, * p<0.1, † p<0.15. Parentheses include robust standard errors. Dependent variable is a dummy variable indicating high prevalence of foot-binding, and key explainatory variables include relative suitability index and county Shengyuan exam quota, where both are standardized. A set of control variables includes standardized county population, cotton and flax suitability index, number of modern firms, Mongol historical migration intensity, christianity activity variables, as well as log distance to Nanjing. Sex ratio data is from Yearbook of Domestic Affairs(1931).

Figure A1: The Distribution of Ballads on Foot-binding



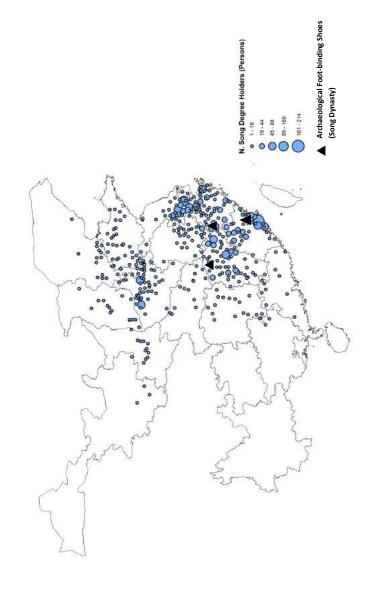
Note: The shaded counties are those with ballads on foot-binding and marriage. Source: Zhang(2015). Base map is CHGIS v4. An example is Zhangde county (Henan: "If you have lotus feet, you could marry a Literati (Xiucai), eating bread with meat; If you have large feet, you would marry a blind man, eating bran with chili."

Figure A2: The Distribution of Bound Feet Beauty Contest (Sai Zu Hui)



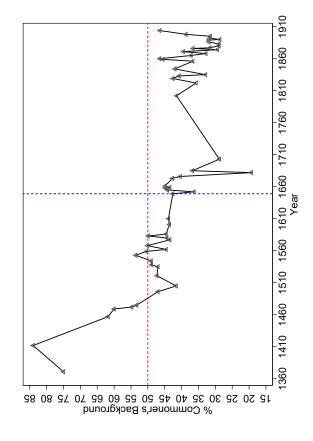
Note: The shaded counties are those with Bound Feet Beauty Contest (in Chinese, Sai Zu Hui), as recorded by Zhang(2015), Nagao(1973) and Yao(1936). Base map here is CHGIS v4. The source of picture is from the Foot-binding Forum (organized by Dr. Ke Jisheng), http://www.footbinding.com.tw/.

Figure A3. Northern Song Entry Exams(persons) and Location of Archaeological Lotus Shoes



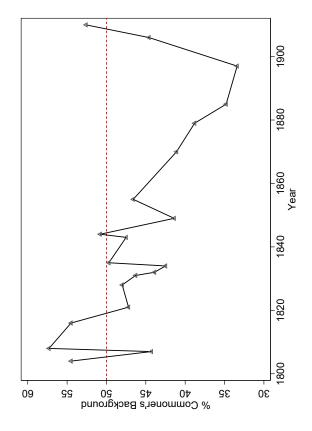
Note: Northern Song Entry Exams(persons) comes from Harvard China Map system, and the original data is from CBDB. The information of archaeological lotus shoes is from Ebrey(1992). Stars denote the locations were lotus shoes were found: Fuzhou (Fujian), Quzhou (Zhejiang) and De'An (Jiangxi).

Figure A4: The Proportion of Degree Holders from Commoners' Families: Jinshi



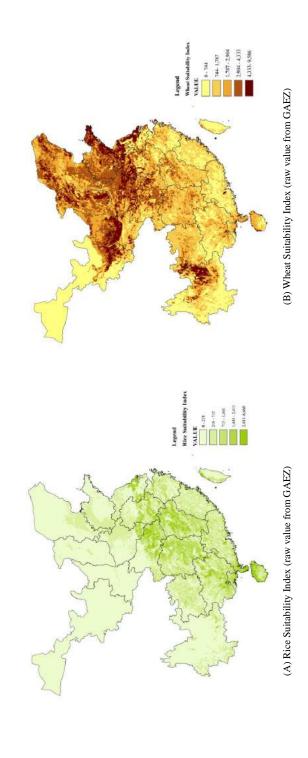
Note: Data source: Ho(1962), Table 9-The Social Backgrounds of Jinshi during the Ming and Qing. Here the Y-axis is the proportion of three generations. Type B families are those with Shengyuan degree or Jiansheng (during the Qing) among male immediate family members Jinshi from type A and B families. Type A families are those without any degree holders among male immediate family members of recent of recent three generations. Type C families are those with higher degrees than Shengyuan (i.e. Jiansheng, Juren, or Jinshi) among male immediate family members of recent three generations.

Figure A5: The Proportion of Degree Holders from Commoners' Families: Juren



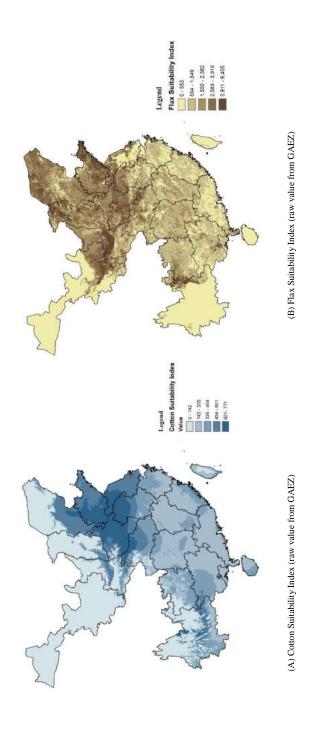
members of recent three generations. Type B families are those with Shengyuan degree or Jiansheng (during the Qing) among male immediate Note: Data source: Ho(1962), Table 11-The Social Backgrounds of Juren and Gongsheng during the late Qing. Here the Y-axis is the proportion of Juren and Gongsheng from type A and B families. Type A families are those without any degree holders among male immediate family family members of recent three generations. Type C families are those with higher degrees than Shengyuan (i.e. Jiansheng, Juren, or Jinshi) among male immediate family members of recent three generations.

Figure A6: Regional Distribution of Rice/Wheat Suitability in China



Note: The data in this map comes from geographic distribution of crop suitability from the FAOs GAEZ (Global Agro-Ecological Zones). The suitability index of a certain crop is estimated based on a model, which has been applied considering the average climate of baseline period 1961-1990 reecting suitability levels and distributions within grid cells. Panel A shows the suitability index for rice, and panel B shows that for wheat. The base map is from CHGIS v4.

Figure A7: Regional Distribution of Cotton and Flax Suitability in China



Note: The data in this map comes from geographic distribution of crop suitability from the FAOs GAEZ (Global Agro-Ecological Zones). The suitability index of a certain crop is estimated based on a model, which has been applied considering the average climate of baseline period 1961-1990 reecting suitability levels and distributions within grid cells. Panel A shows the suitability index for cotton, and panel B shows that for flax. The base map is from CHGIS v4.